CECS545-Artificial Intelligence

Exercises (Ch. 6)

Dr. Roman Yampolskiy

Exercise 6.1

- How many solutions are there for the map-coloring problem in Fig. 6.1 using three colors?

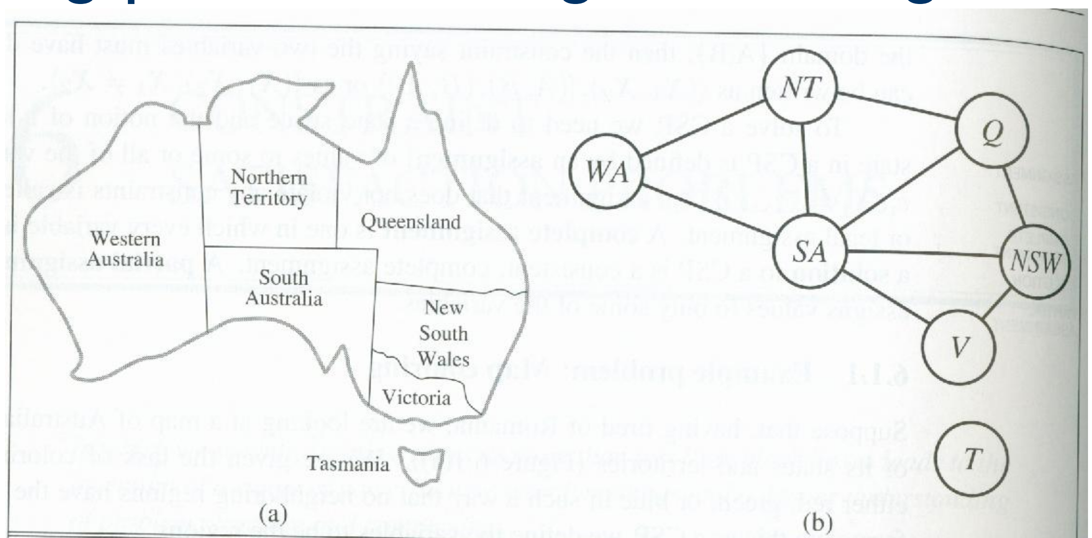

Figure 6.1 (a) The principal states and territories of Australia. Coloring this map can be viewed as a constraint satisfaction problem (CSP). The goal is to assign colors to each region so that no neighboring regions have the same color. (b) The map-coloring problem represented as a constraint graph.

Exercise 6.2

- Consider the problem of placing k knights on an n x n chessboard such that no two knights are attacking each other, where k is given and $k \leq n^2$

a. Choose a CSP formulation. In your formulation, what are the variables?

b. What are the possible values of each variable?

c. What sets of variables are constrained, and how?

d. Now consider the problem of putting *as many knights as possible* on the board without any attacks. Explain how to solve this with local search by defining appropriate ACTIONS and RESULT functions and a sensible objective function.

Exercise 6.3

6.3 Consider the problem of constructing (not solving) crossword puzzles:[5] fitting words into a rectangular grid. The grid, which is given as part of the problem, specifies which squares are blank and which are shaded. Assume that a list of words (i.e., a dictionary) is provided and that the task is to fill in the blank squares by using any subset of the list. Formulate this problem precisely in two ways:

a. As a general search problem. Choose an appropriate search algorithm and specify a heuristic function. Is it better to fill in blanks one letter at a time or one word at a time?

b. As a constraint satisfaction problem. Should the variables be words or letters?

Which formulation do you think will be better? Why?

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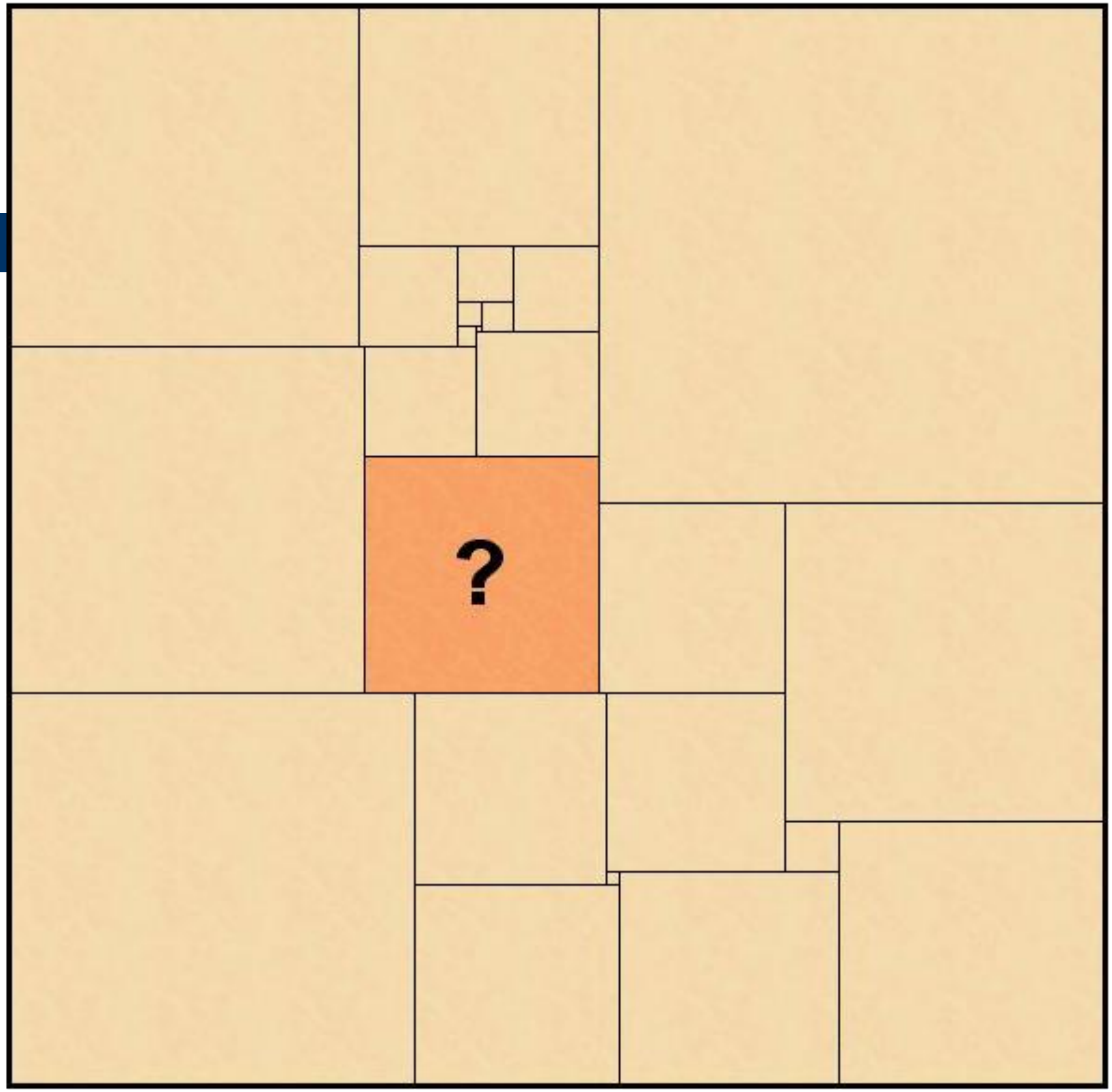
Constraint Satisfaction Problems 2

Dr. Roman Yampolskiy

The square below contains 24 smaller squares, each with a different integral size. Determine the length of the shaded square



The High IQ Exam

- From the High-IQ society entrance exam
  - Published in the Observer newspaper
  - Never been solved...
- Solved using a **constraint solver**
  - 45 minutes to **specify** as a CSP (Simon)
  - 1/100 second to solve (Sicstus Prolog)
- See the notes
  - the program, the results and the answer

Constraint Satisfaction Problems

- Set of variables $X = \{x_1, x_2, \ldots, x_n\}$
  - D**omain** for each variable (finite set of values)
  - Set of constraints
    - Restrict the values that variables can take together

- A solution to a CSP...
  - An assignment of a domain value to each variable
  - Such that no constraints are broken

Example: High-IQ problem CSP

- Variables:
  - 25 lengths (Big square made of 24 small squares)
- Values:
  - Let the tiny square be of length 1
  - Others range up to about 200 (at a guess)
- Constraints: lengths have to add up
  - e.g. along the top row
- Solution: set of lengths for the squares
- Answer: length of the 17th largest square

What we want from CSP solvers

- One solution
  - Take the first answer produced
- The ‘best’ solution
  - Based on some measure of optimality
- All the solutions
  - So we can choose one, or look at them all
- That no solutions exist
  - Existence problems (common in mathematics)

Formal Definition of a Constraint

- Informally, relationships between variables
  - e.g., $x \neq y$, $x > y$, $x + y < z$
- Formal definition:
  - Constraint $C_{xyz\ldots}$ between variables x, y, z, …
  - $C_{xyz\ldots} \subseteq D_x \times D_y \times D_z \times \ldots$ (a subset of all tuples)
- Constraints are a set of which tuples <u>**ARE**</u> allowed in a solution
- Theoretical definition, not implemented like this

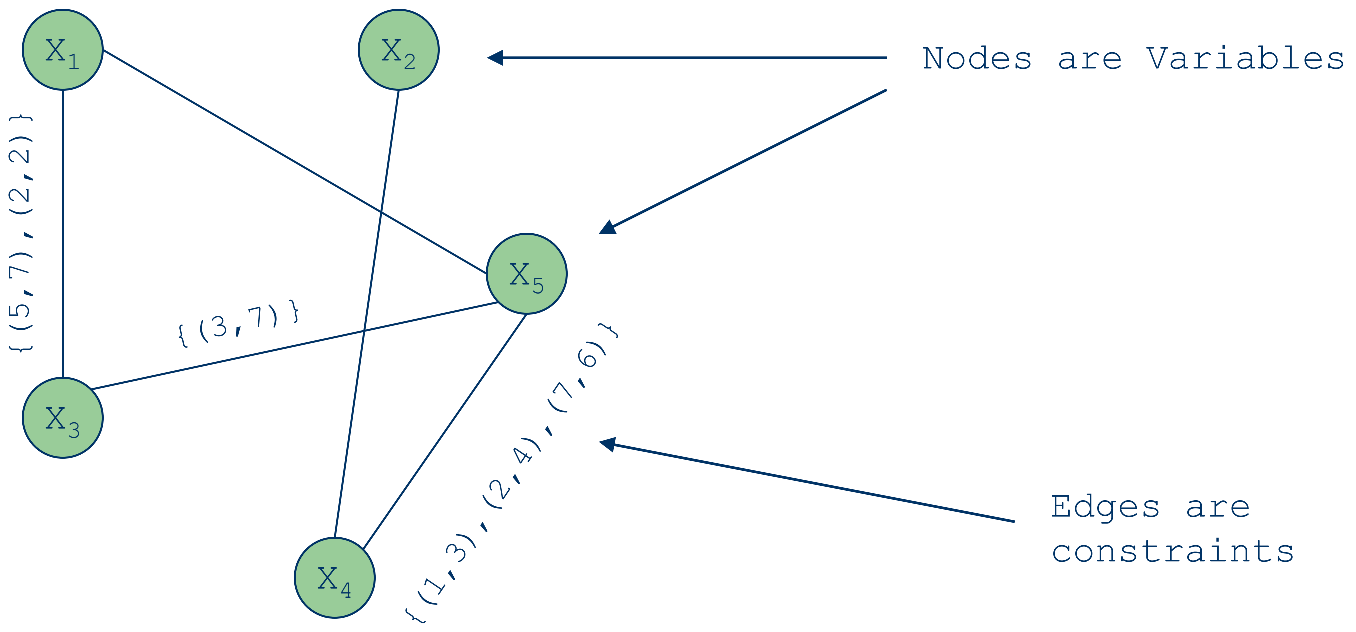
Example Constraints

- Suppose we have two variables: x and y
- x can take values {1,2,3}
- y can take values {2,3}
- Then the constraint $x=y$ is written:
  - {(2,2), (3,3)}
- The constraint $x < y$ is written:
  - {(1,2), (1,3), (2,3)}

Binary Constraints

- Unary constraints: involve only one variable
  - Preprocess: re-write domain, remove constraint
- Binary constraints: involve two variables
  - Binary CSPs: all constraints are binary
  - Much researched
    - All CSPs can be written as binary CSPs
    - Nice graphical and Matrix representations
  - Representative of CSPs in general

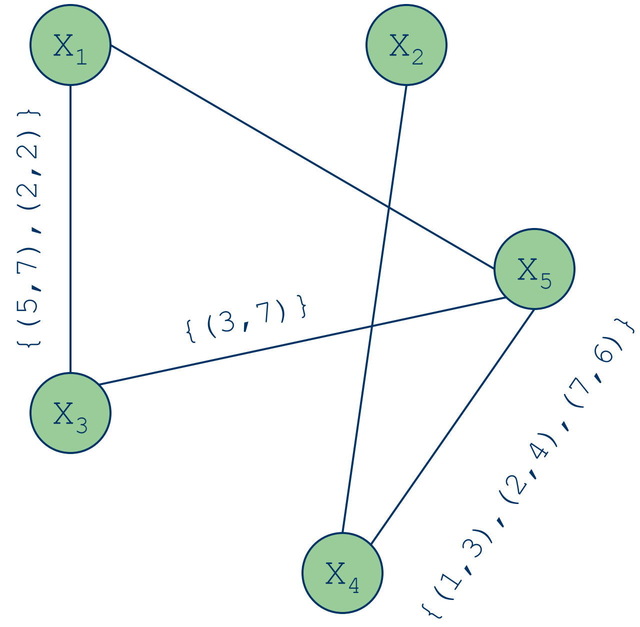
Binary Constraint Graph



Matrix Representation for Binary Constraints

C	1	2	3	4	5	6	7
1			X				
2				X			
3							
4							
5							
6							
7						X	

Matrix for the constraint between X_4 and X_5 in the graph



Random Generation of CSPs

- Generation of random binary CSPs
  - Choose a number of variables
  - Randomly generate a matrix for every pair of variables
- Used for benchmarking
  - e.g. efficiency of different CSP solving techniques

- Real world problems often have more **structure**

Rest of This Lecture

- **Preprocessing**: arc consistency
- **Search**: backtracking, forward checking
- **Heuristics**: variable & value ordering
- Applications & advanced topics in CSP

Arc Consistency

- In binary CSPs
  - Call the pair (x, y) an **arc**
  - Arcs are ordered, so (x, y) is not the same as (y, x)
  - Each arc will have a single associated constraint C_{xy}
- An arc (x,y) is consistent if
  - For all values *a* in D_x, there is a value *b* in D_y
    - Such that the assignment x = *a*, y = *b* satisfies C_{xy}
  - Does not mean (y, x) is consistent
  - Removes zero rows/columns from C_{xy}'s matrix

Making a CSP Arc Consistent

- To make an arc (x,y) consistent
  - remove values from D_x which make it inconsistent
- Use as a **preprocessing** step
  - Do this for every arc in turn
  - Before a search for a solution is undertaken
- Won't affect the solution
  - Because removed values would break a constraint
- Does not remove all inconsistency
  - Still need to search for a solution

Arc Consistency Example

- Four tasks to complete (A,B,C,D)
- Subject to the following constraints:

Task	Duration	Precedes
A	3	B,C
B	2	D
C	4	D
D	2	

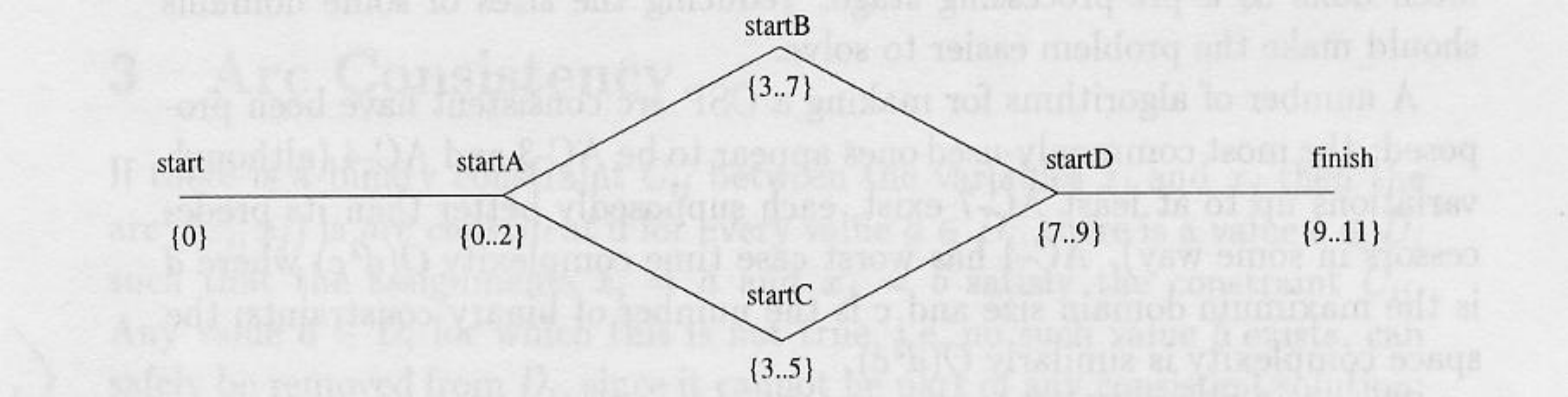
C1	start $\leq$ startA
C2	startA + 3 $\leq$ startB
C3	startA + 3 $\leq$ startC
C4	startB + 2 $\leq$ startD
C5	startC + 4 $\leq$ startD
C6	startD + 2 $\leq$ finish

- Formulate this with variables for the start times
  - And a variable for the global *start* and *finish*
    - Values for each variable are {0,1,…,11}, except *start* = 0
  - Constraints: startX + durationX $\leq$ startY

Arc Consistency Example

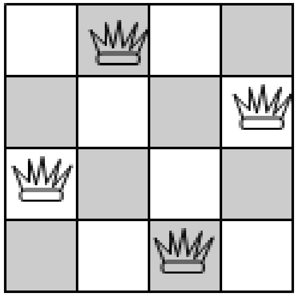
- Constraint C1 = {(0,0), (0,1), (0,2), …,(0,11)}
  - So, arc (start,startA) is arc consistent
- C2 = {(0,3),(0,4),…(0,11),(1,4),…,(8,11)}
  - These values for startA never occur: {9,10,11}
    - So we can remove them from D_{startA}
  - These values for startB never occur: {0,1,2}
    - So we can remove them from D_{startB}
- For CSPs with precedence constraints only
  - Arc consistency removes all values which can't appear in a solution (if you work backwards from last tasks to the first)
- In general, arc consistency is effective, but not enough

Arc Consistency Example



N-queens Example (N = 4)

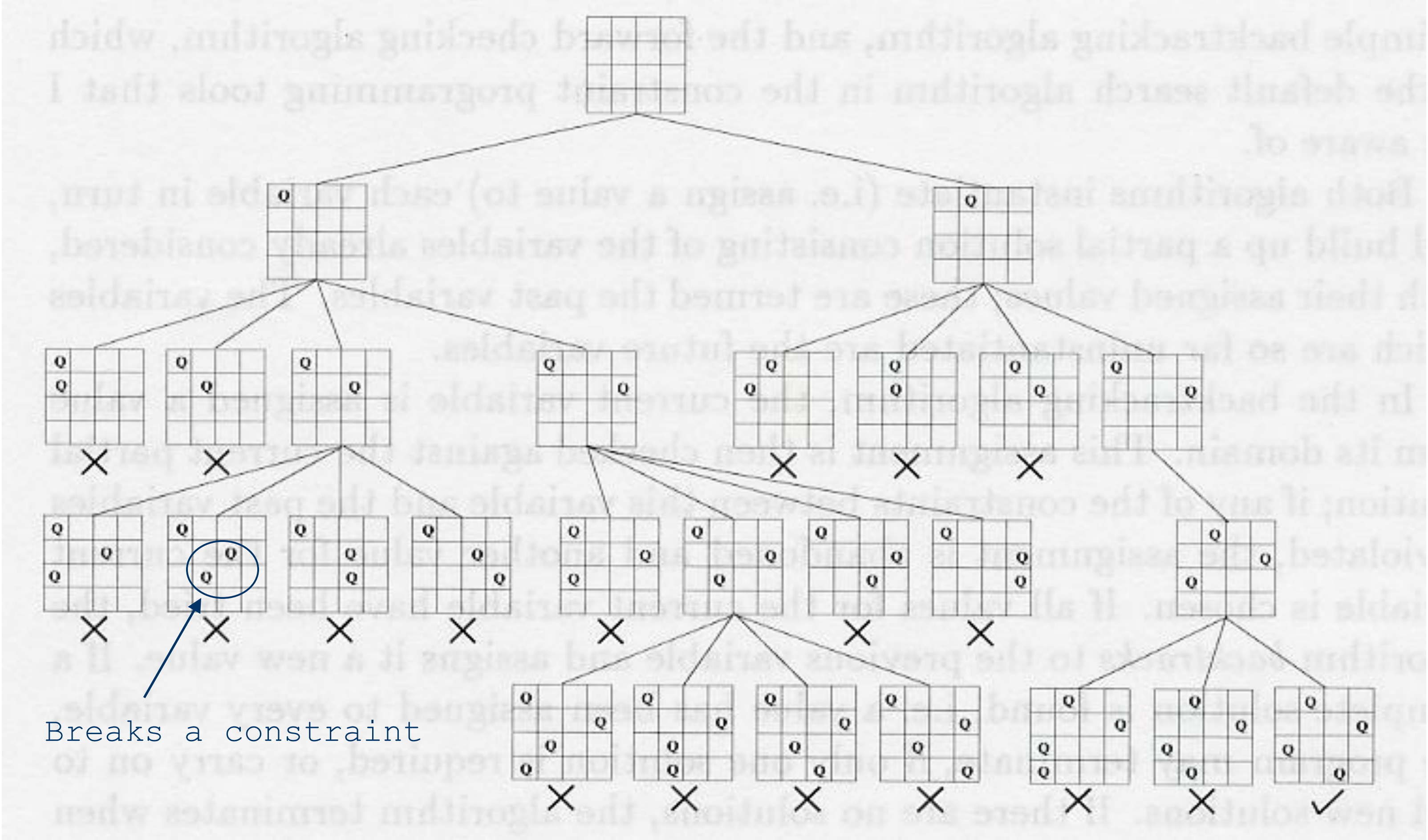
- Standard test case in CSP research
- Variables are the rows
- Values are the columns
- Constraints:
  - $C_{1,2} = \{(1,3),(1,4),(2,4),(3,1),(4,1),(4,2)\}$
  - $C_{1,3} = \{(1,2),(1,4),(2,1),(2,3),(3,2),(3,4),$
    $(4,1),(4,3)\}$
  - Etc.
  - Question: What do these constraints mean?



Backtracking Search

- Keep trying all variables in a depth first way
  - Attempt to instantiate the current variable
    - With each value from its domain
  - Move on to the next variable
    - When you have an assignment for the current variable which doesn’t break any constraints
  - Move back to the previous (past) variable
    - When you cannot find any assignment for the current variable which doesn’t break any constraints
    - i.e., **backtrack** when a deadend is reached

Backtracking in 4-queens

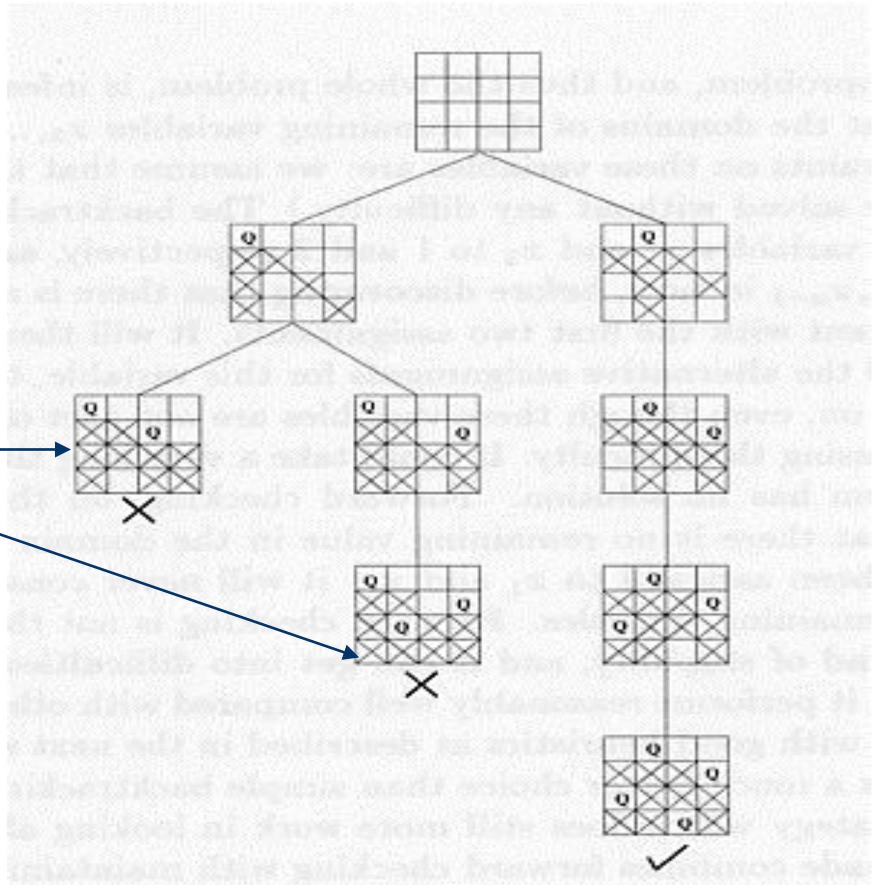


Forward Checking Search

- Same as backtracking
  - But it also looks at the future variables
    - i.e., those which haven't been assigned yet
- Whenever an assignment of value V_c to the current variable is attempted:
  - For all future variables x_f, remove (temporarily)
    - any values in D_f which, along with V_c, break a constraint
  - If x_f ends up with no variables in its domain
    - Then the current value V_c must be a "no good"
    - So, move onto next value or backtrack

Forward Checking in 4-queens

Shows that this assignment is no good



Heuristic Search Methods

- Two choices made at each stage of search
  - Which order to try to instantiate variables?
  - Which order to try values for the instantiation?
- Can do this
  - Statically (fix before the search)
  - Dynamically (choose during search)
- May incur extra cost that makes this ineffective

Fail-First Variable Ordering

- For each future variable
  - Find size of domain after forward checking pruning
  - Choose the variable with the fewest values left
- Idea:
  - We will work out quicker that this is a dead-end
  - Because we only have to try a small number of vars.
  - “Fail-first” should possibly be “dead end quickly”
- Uses information from forward checking search
  - No extra cost if we’re already using FC

Dynamic Value Ordering

- Choose value which most likely to succeed
  - If this is a dead-end we will end up trying all values anyway
- Forward check for each value
  - Choose one which reduces other domains the least
  - ‘Least constraining value’ heuristic
- Extra cost to do this
  - Expensive for random instances
  - Effective in some cases

Some Constraint Solvers

- Standalone solvers
  - ILOG (C++, popular commercial software)
  - JaCoP (Java)
  - Minion (C++)
  - ...
- Within Logic Programming languages
  - Sicstus Prolog
  - SWI Prolog
  - ECLiPSe
  - ...

Overview of Applications

- Big advantages of CSPs are:
  - They cover a big range of problem types
  - It is usually easy to come up with a quick CSP spec.
  - And there are some pretty fast solvers out there
  - Hence people use them for quick solutions

- However, for seriously difficult problems
  - Often better to write customized CSP methods
  - Operational research (OR) methods often better

Some Mathematical Applications

- Existence of algebras of certain sizes
  - QG-quasigroups by John Slaney
  - Showed that none exists for certain sizes
- Golomb rulers
  - Take a ruler and put marks on it at integer places such that *no pair* of marks have the same length between them.
    - Thus, the all-different constraint comes in
  - Question: Given a particular number of marks
    - What's the *smallest* Golomb ruler which accommodates them

Some Commercial Applications

- Sports scheduling
  - Given a set of teams
    - All who have to play each other twice (home and away)
    - And a bunch of other constraints
  - What is the best way of scheduling the fixtures
  - Lots of money in this one
- Packing problems
  - E.g., How to load up ships with cargo
    - Given space, size and time constraints

Some Advanced Topics

- Formulation of CSPs
  - It's very easy to specify CSPs (this is an attraction)
    - But some are worse than others
  - There are *many* different ways to specify a CSP
  - It's a highly skilled job to work out the best
- Automated reformulation of CSPs
  - Given a simple formulation
    - Can an agent change that formulation for the better?
    - Mostly: what choice of variables are specified
    - Also: automated discovery of additional constraints
      - Can we add in extra constraints the user has missed?

More Advanced Topics

- Symmetry detection
  - Can we spot whole branches of the search space
    - Which are exactly the same (symmetrical with) a branch we have already (or are going to) search
  - Humans are good at this
  - Can we get search strategies to do this automatically?
- Dynamic CSPs
  - Solving of problems which change while you are trying to solve them
  - E.g. a new package arrives to be fitted in

The End!