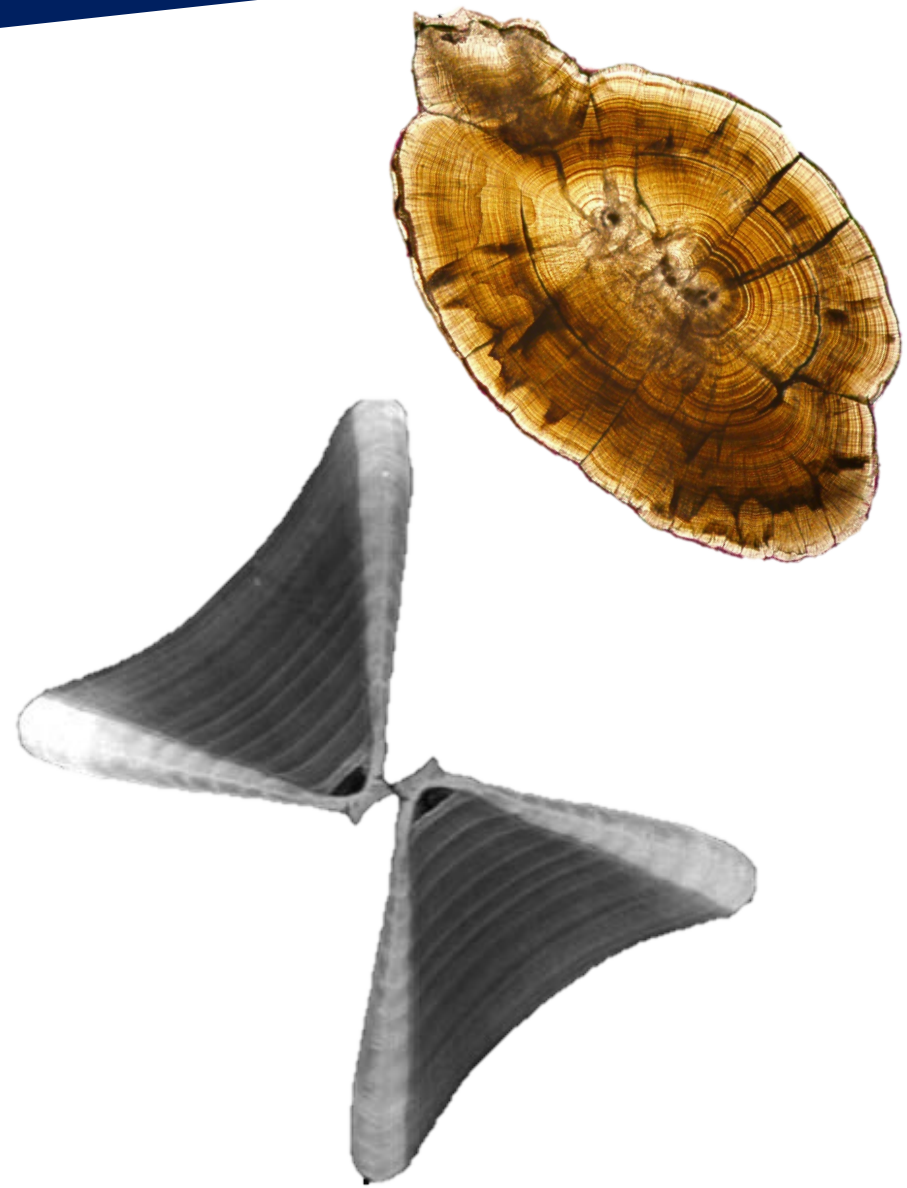
INTRODUCTION TO STATISTICAL CATCH AT AGE ASSESSMENT MODELING

FAS XXXX: Advanced Quantitative Fisheries Stock Assessment

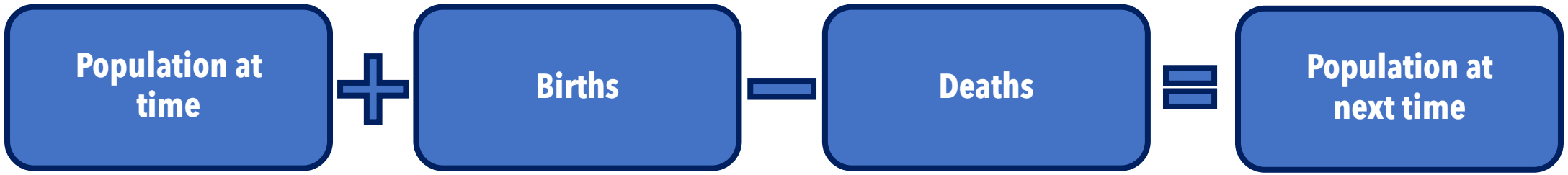
Dr. Zachary Siders

OBJECTIVES

1. Understand how adding age changes a stock assessment model's design/structure
2. Compare forward and backward projecting model structures and assumptions
3. Modify components of a stock assessment to include age
4. Learn where we will change or relax assumptions in the future sessions



POP. MODEL



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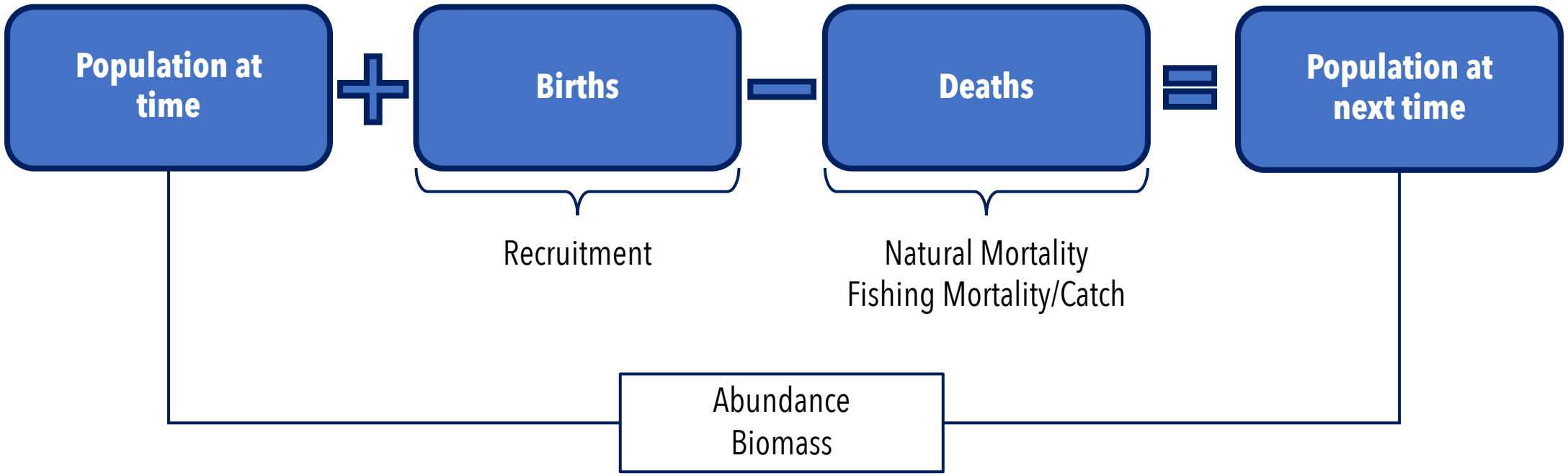
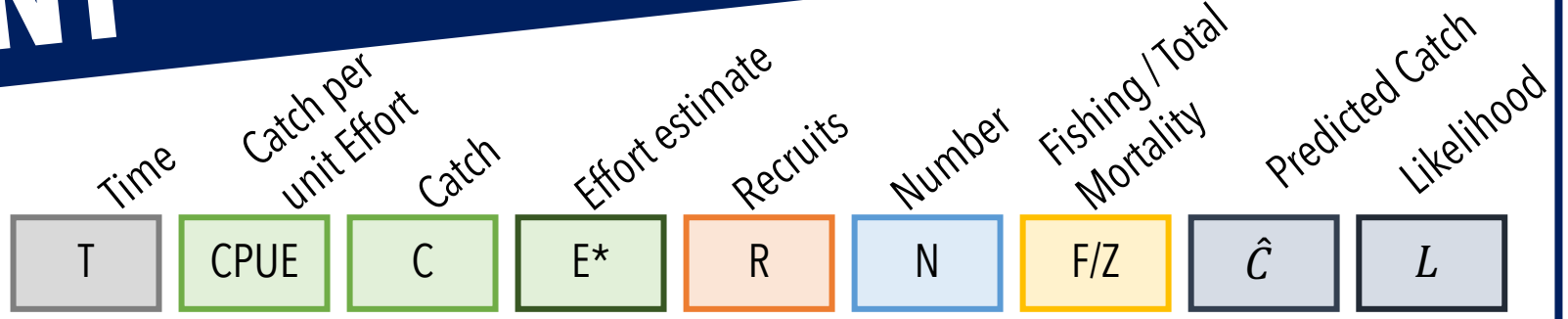

Population at time
Births
Deaths
Population at next time
Recruitment
Natural Mortality
Fishing Mortality/Catch
Abundance
Biomass

STARTING POINT



Numbers dynamic model:

Leading parameters:

- M (natural mortality)
- κ (compensation ratio)
- R_0(starting recruitment)
- F_{CUR} (Current fishing mortality)
- σ (Error rate)

Derived/Intermediate parameters:

- a, b (Beverton-Holt stock-recruitment θ)
- N_0 (starting population size)
- q (catchability)

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Time	Catch per unit Effort	Catch	Effort estimate	Recruits	Abundance	Fishing / Total Mortality	Predicted Catch	Likelihood
T	CPUE	C	E*	R	N	F/Z	$\hat{C}$	L
1								
2								
3								
4								
5								

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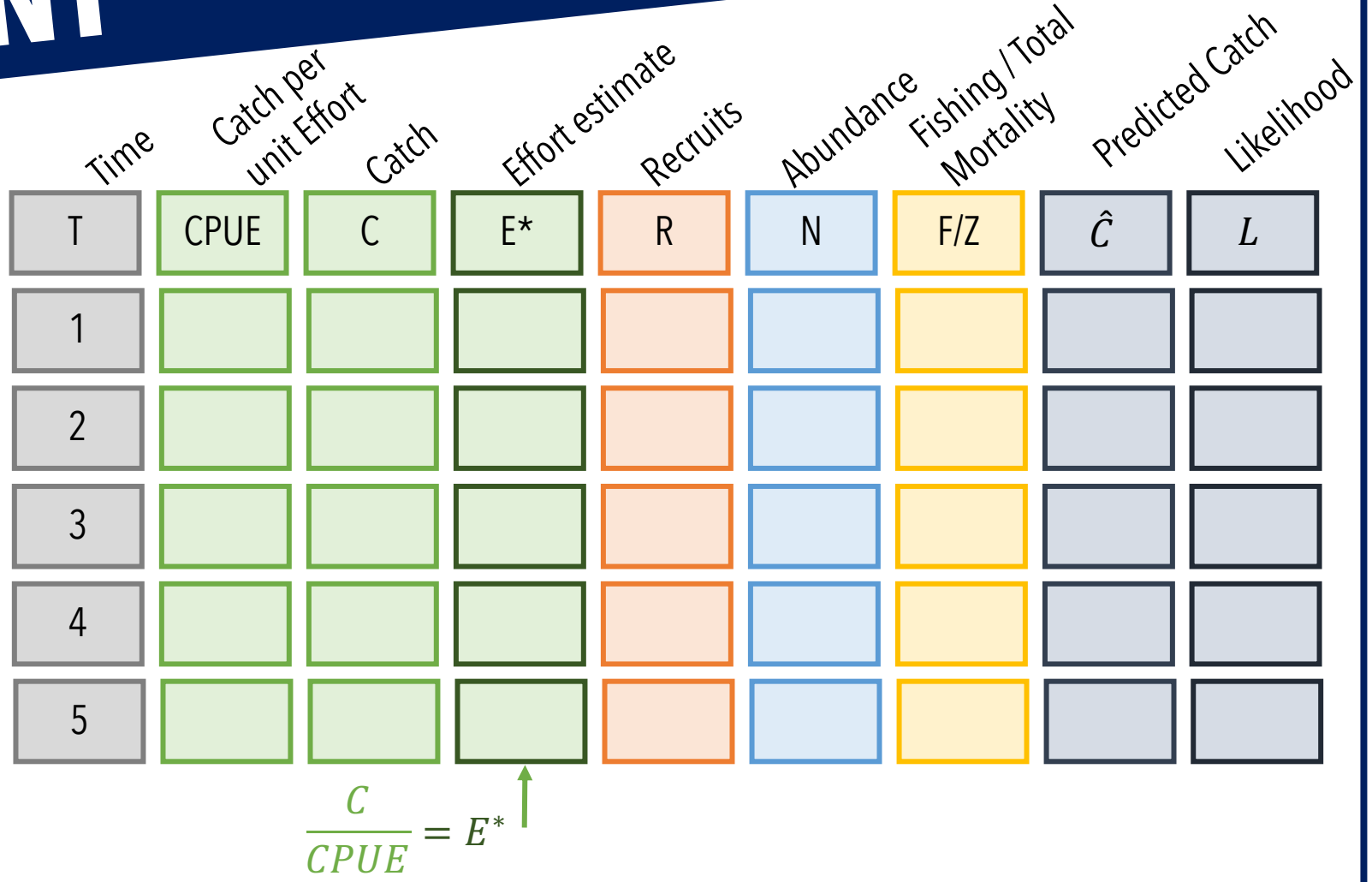

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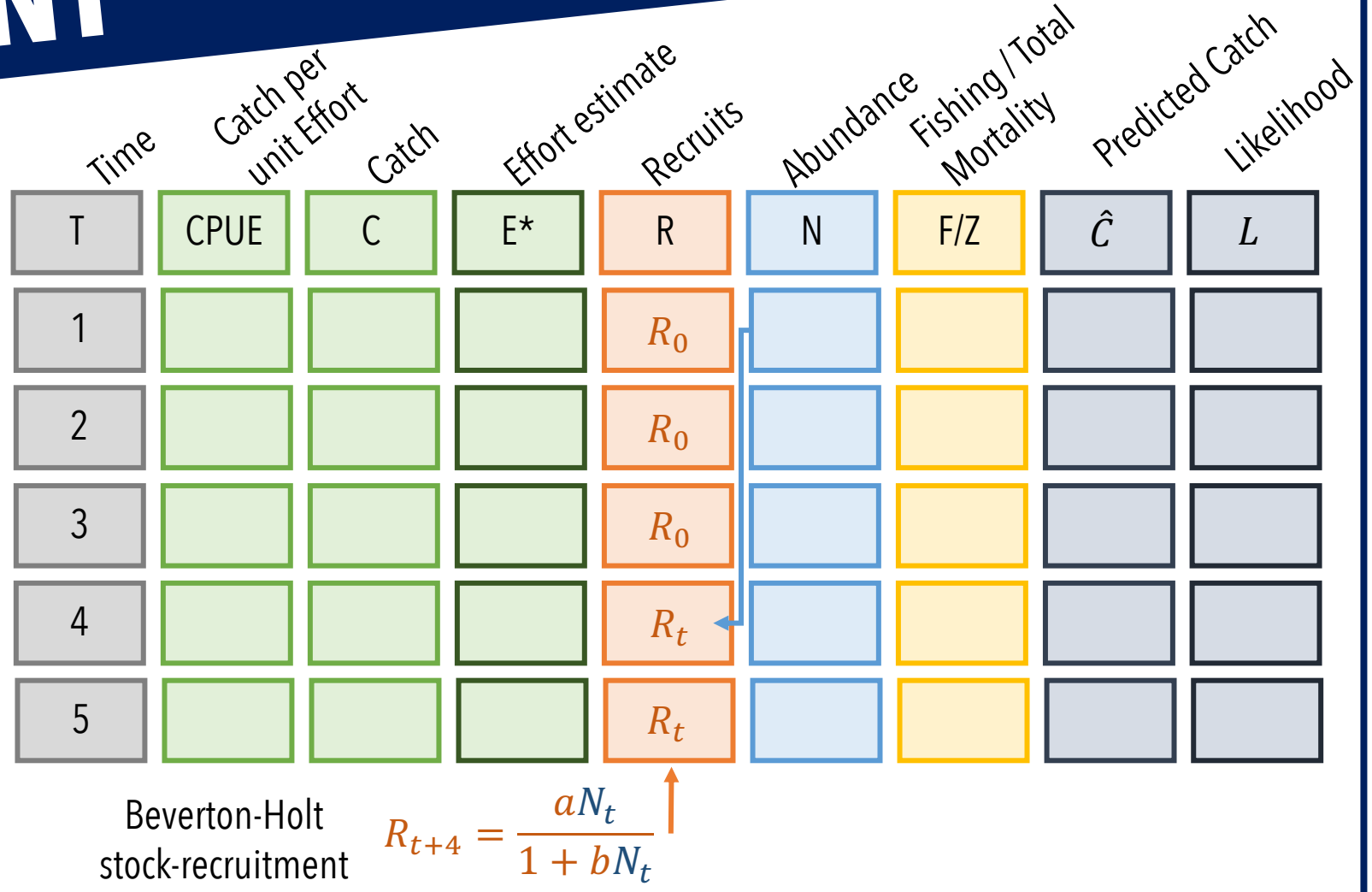

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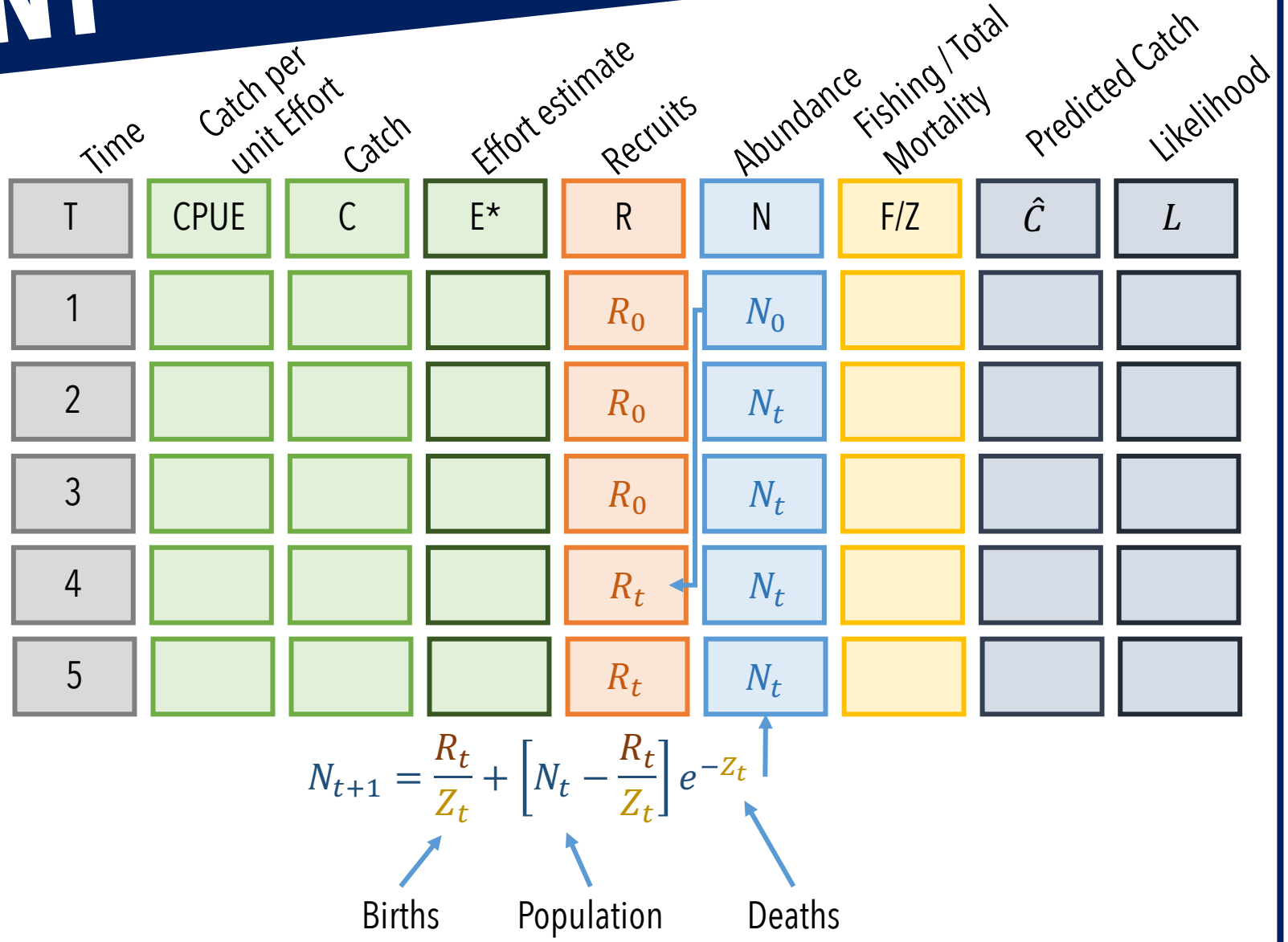

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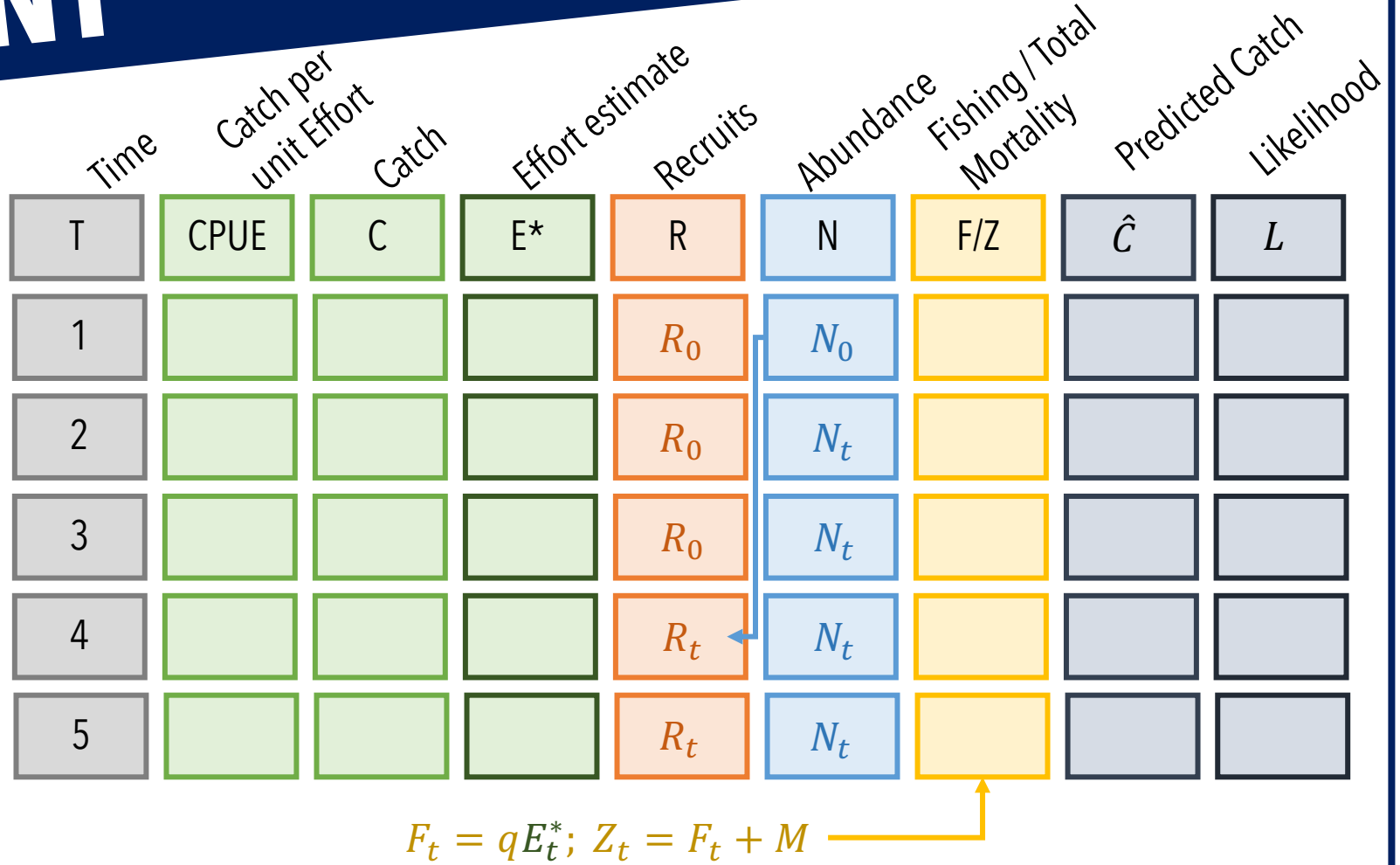

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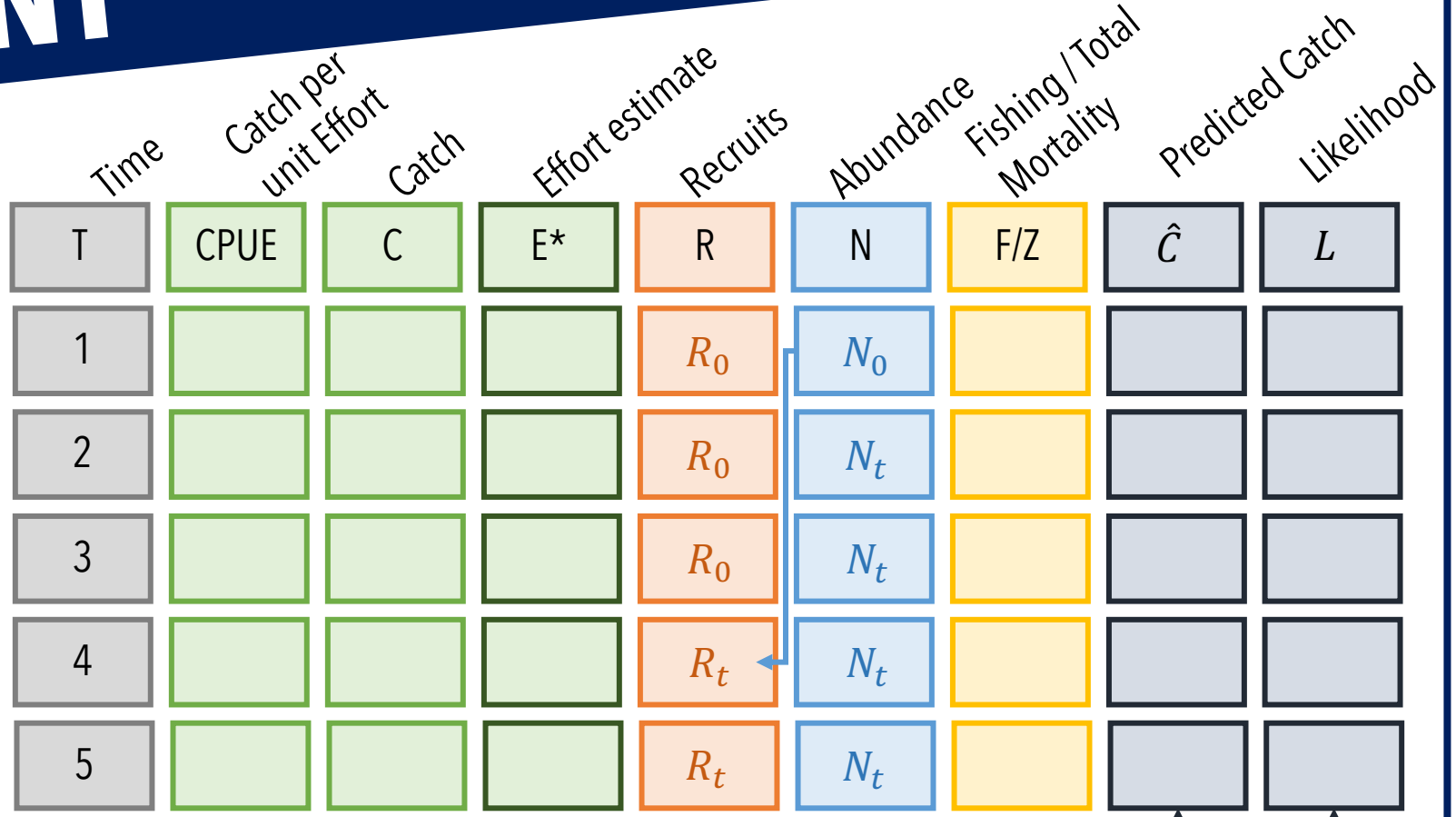

Baranov Catch Equation

$$\hat{C}_t = \left(\frac{F_t}{Z_t}\right)[R_t + N_t(1 - e^{-Z_t})]$$

$$\log\left(\frac{C_t}{\hat{C}_t}\right) \sim N(0, \sigma)$$

WHAT IS SCA?

Catch at Age Data

Time	Age-0	Age-1	Age-2	...	*A*
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
…					
T					

WHAT IS SCA?

Catch at Age Data

Time	Age-0	Age-1	Age-2	...	Age-A
1					
2					

Virtual Population Analysis

Statistical Catch at Age Analysis

WHAT IS SCA?

Catch at Age Data

Time	Age-0	Age-1	Age-2	…	Age-A
1					
2					

Virtual Population Analysis

- Assumes catch-at-age has minimal error
- Needs full coverage of catch-at-age
- Needs relative abundance index
- Backward projection
  - Uses completed cohorts & catch to project the abundance at age at *a-1, t-1*
- Estimates terminal fishing mortality (F_T) to determine current stock size
- Calculates selectivity/vulnerability over time

Statistical Catch at Age Analysis

WHAT IS SCA?

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- Calculates selectivity/vulnerability over time

Statistical Catch at Age Analysis

- Likelihood compares obs. and pred. catch
- Does not need full coverage of catch-at-age
- Needs relative abundance index
- Forward projection
  - Estimates initial abundance and fishing mortalities (and others θ's) needed to generate the catch
- Simple version assumes constant selectivity/vulnerability over time

WHAT IS SCA?

Catch at Age Data

Time	Age-0	Age-1	Age-2	...	Age-A
1					
2					

Virtual Population Analysis

Time	Age-0	Age-1	Age-2	Age-3	Age-4
1					
2					
3					
4					
5					
6					
7	Estimates F_T for uncompleted cohorts				

Statistical Catch at Age Analysis

WHAT IS SCA?

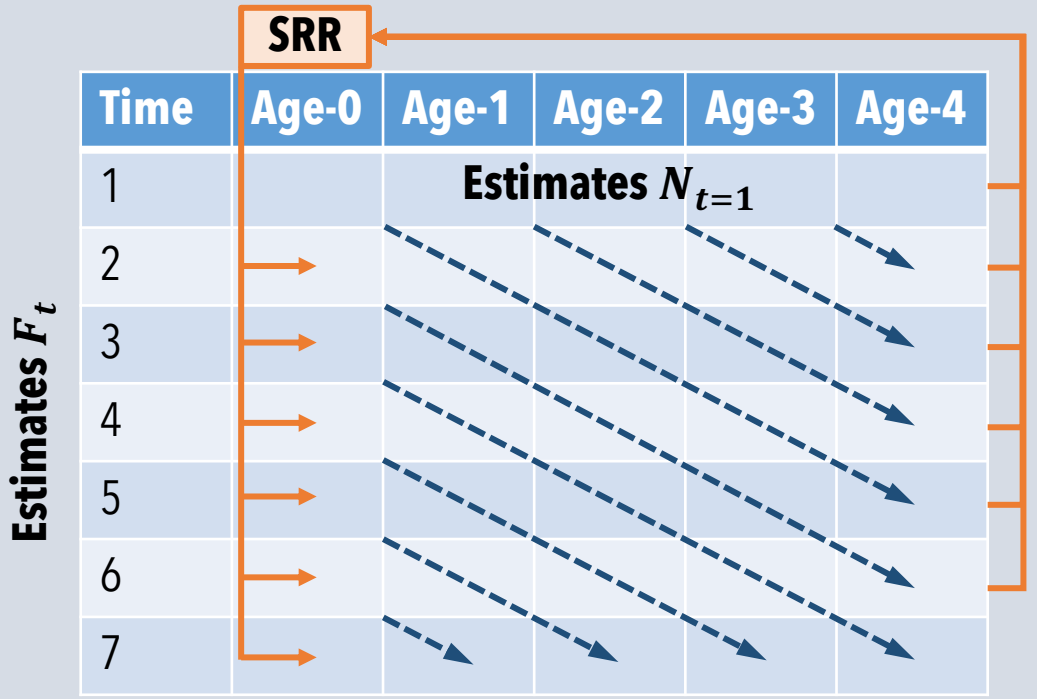
Catch at Age Data

Time	Age-0	Age-1	Age-2	...	Age-A
1					
2					

Virtual Population Analysis

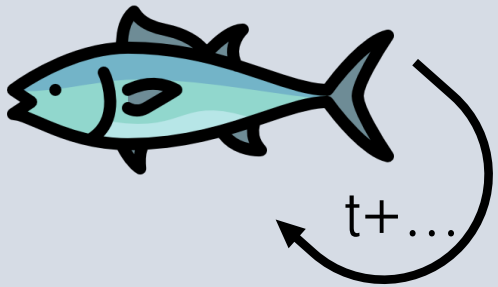
Time	Age-0	Age-1	Age-2	Age-3	Age-4
1					
2					
3					
4					
5					
6					
7	Estimates F_T for uncompleted cohorts				

Statistical Catch at Age Analysis



SCA STRUCTURE

Numbers dynamic model



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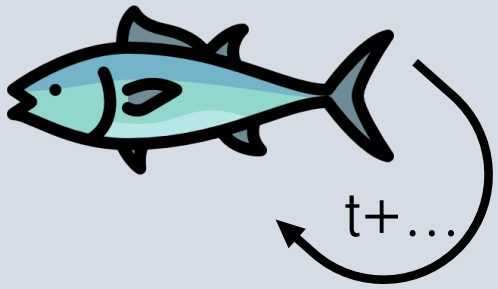
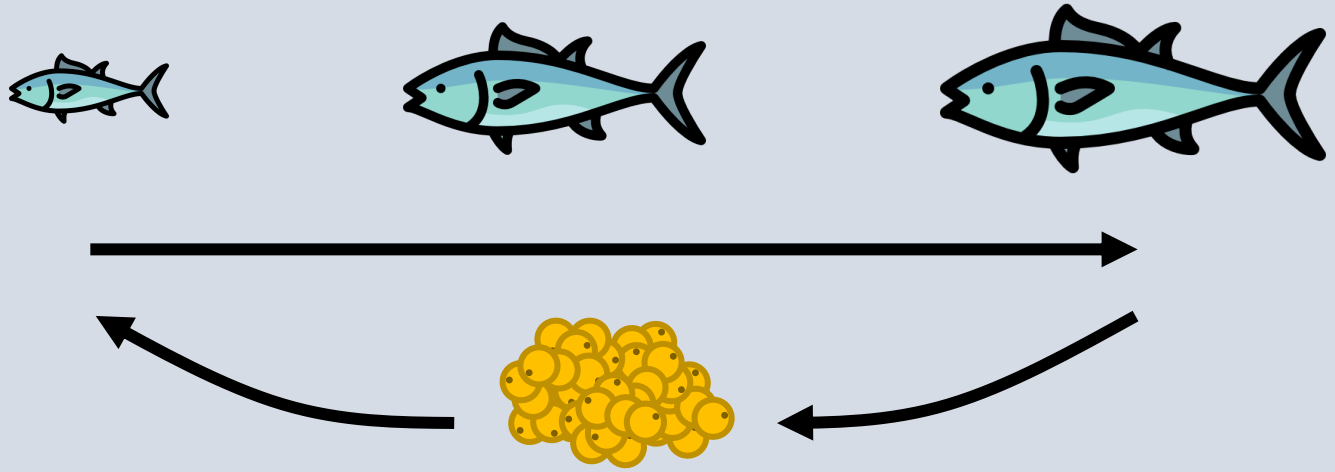


Statistical Catch at Age model



COMPARISON

Numbers dynamic model

Leading parameters:

M, F_{CUR} >> constant mortality

κ , R_0 >> recruitment

σ >> likelihood

N_t >> abundance

C_t >> catch

θ	Age-0	Age-1	Age-2	…	Age-A
Length					
Weight					
Fecundity					
Survivorship					
Vulnerability					

Time	Age-0	Age-1	Age-2	…	Age-A
1					
2					
3					
4					
5					

Time	Age-0	Age-1	Age-2	…	Age-A
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1					
2					
3					
4					
5					

Time	Age-0	Age-1	Age-2	…	Age-A
1					
2					
3					
4					
5					

Excel Example
– Sheet ‘LH’ –

SCA STRUCTURE

LEADING PARAMETERS

Recruitment: h, R_0
Fishing Mortality: $\bar{F}$
Variability:
$\sigma_C, \sigma_F, \sigma_{CPUE}$

VBGF: L_∞, L_0, k
L-W: a, b
Fecundity: W_{mat}
Natural Mortality: M
Vulnerability: L_{50}, σ_{50}

θ	Age-0	...	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-0	...	Age-A
1			
2	**Observed Catch at time,age**		
3			
4			
5			

Time	Age-0	...	Age-A
1			
2	**Numbers at time,age**		
3			
4			
5			

Time	Age-0	...	Age-A
1			
2	**Total Mortality at time,age**		
3			
4			
5			

Time	Age-0	...	Age-A
1			
2	**Predicted Catch at time,age**		
3			
4			
5			

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Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-0	...	Age-A
1			
2			
3			
4			
5			

Observed Catch at time,age

Time	Age-0	...	Age-A
1			
2			
3			
4			
5			

Numbers at time,age

Time	Age-0	...	Age-A
1			
2			
3			
4			
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Total Mortality at time,age

Time	Age-0	...	Age-A
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2			
3			
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Predicted Catch at time,age

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θ	Age-0	…	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-A
$C_{t,a}$	

Time	Age-0	…	Age-A
$N_{t+1,a} = \begin{cases} \frac{aE_t}{1+bE_t} & N_{a=1} \\ N_{t,a}e^{-Z_{t,a}} & N_{a>1} \end{cases}$			

Time	$\sigma_{F,t}$	F_t	Z_t
$\sigma_{F,t} \sim N(0, \sigma_F)$			
$Z_{t,a} = M +$			
$(\bar{F} + \sigma_{F,t})v_a$			

Time	Age-0	…	Age-A
$\hat{C}_{t,a} = \left(\frac{F_{t,a}}{Z_{t,a}}\right)*$			
$N_{t,a}(1 - e^{-Z_{t,a}})$			

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θ	Age-0	…	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-A
$C_{t,a}$	

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L-W: a, b
Fecundity: W_{mat}
Natural Mortality: M
Vulnerability: L_{50}, σ_{50}

θ	Age-0	…	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-A
$C_{t,a}$	

Time	Age-0	…	Age-A
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Time	Age-0	…	Age-A
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Excel Example
– Sheet ‘Data’ –

SCA STRUCTURE

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θ	Age-0	…	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-A
$C_{t,a}$	

Time	Age-0	…	Age-A
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Time	$\sigma_{F,t}$	F_t	Z_t
$\sigma_{F,t} \sim N(0, \sigma_F)$			
$Z_{t,a} = M + (\bar{F} + \sigma_{F,t})v_a$			

Time	Age-0	…	Age-A
$\hat{C}_{t,a} = \left(\frac{F_{t,a}}{Z_{t,a}}\right) * N_{t,a}(1 - e^{-Z_{t,a}})$			

Excel Example
– Sheet ‘Mortality’ –

SCA STRUCTURE

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Fishing Mortality: $\bar{F}$
Variability:
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VBGF: L_∞, L_0, k
L-W: a, b
Fecundity: W_{mat}
Natural Mortality: M
Vulnerability: L_{50}, σ_{50}

θ	Age-0	…	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-A
$C_{t,a}$	

Time	Age-0	…	Age-A
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Time	Age-0	…	Age-A
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$N_{t,a}(1 - e^{-Z_{t,a}})$			

Excel Example
– Sheet ‘Abundance’ –

SCA STRUCTURE

LEADING PARAMETERS

Recruitment: h, R_0
Fishing Mortality: $\bar{F}$
Variability:
$\sigma_C, \sigma_F, \sigma_{CPUE}$

VBGF: L_∞, L_0, k
L-W: a, b
Fecundity: W_{mat}
Natural Mortality: M
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θ	Age-0	…	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-A
$C_{t,a}$	

Time	Age-0	…	Age-A
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Time	Age-0	…	Age-A
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Excel Example
– Sheet ‘CC Likelihood’ –

SCA STRUCTURE

LEADING PARAMETERS

Recruitment: h, R_0
Fishing Mortality: $\bar{F}$
Variability: σ_C, σ_F

VBGF: L_∞, L_0, k
L-W: a, b
Fecundity: W_{mat}
Natural Mortality: M
Vulnerability: L_{50}, σ_{50}

θ	Age-0	…	Age-A
Length			
Weight			
Fecundity			
Survivorship			
Vulnerability			

Time	Age-0	…	Age-A
1			
2	**Observed Catch at time,age**		
3			
4			
5			

Time	Age-0	…	Age-A
1			
2	**Numbers at time,age**		
3			
4			
5			

Time	Age-0	…	Age-A
1			
2	**Total Mortality at time,age**		
3			
4			
5			

Time	Age-0	…	Age-A
1			
2	**Predicted Catch at time,age**		
3			
4			
5			

WHAT'S NEXT?

LEADING PARAMETERS

Recruitment: h, R_0

Natural Mortality: M

Vulnerability: L_{50}, σ_{50}

Age-varying mortality: e.g., Lorenzen mortality
Time-varying mortality

Dome-shaped vulnerability: double logistic, Gaussian
Estimating vulnerability

Estimating recruitment deviations

Time	Age-0	…	Age-A

Subsamples & proportions
Catch at length

Time	Age-A

Alternative parameterizations

Time	Age-A

Alternative likelihoods

Time	Age-A

Other facets

Deriving reference points
Uncertainty in parameters
CPUE ⊗ Abundance
Move to R-ADMB

BEFORE NEXT CLASS…

–Review the Excel sheet to thoroughly understand the–
SCA structure
–Review the YPR-Shiny module if you need a refresher–
on the life history parameters

IN NEXT CLASS…

We will build the SCA and
estimate its parameters from
scratch in Excel