Pandemic Mitigation Optimization

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1.1 SIRD Model with Variable-Cost Interventions

Parameters

- Suppose there are N total individuals, of whom I_0 are initially infected. There are m interventions to consider, each of which has (up to) n levels of intensity.
- Let A_{ijt} denote the fixed cost of implementing policy i at level j at time t.
- Let B_{ijt} denote the *switching* cost of implementing policy i at level j at time t (only incurred if policy not implemented in previous period).
- Let C_{ijt} denote the per-susceptible-individual cost of implementing poilcy i at level j at time t.
- Let $C_{infection}$ and C_{death} denote the costs associated with a single individual being infected in a given period, and a single individual losing their life due to disease, respectively.
- Let K_I correspond to the infection rate such that the number of new infections is proportional to K_I multiplied by the number of interactions between susceptible and infected individuals, modeled as the product of the sizes of those populations.
- Let K_R and K_D denote the proportion of infected individuals in each period who recover and die, respectively.
- Let P_{ijt} denote the factor by which new infections are decreased in period t as a result of implementing policy i at level j. In this model, these factors are independent of one another should multiple policies be implemented simultaneously.

Decision Variables

- Let
$$y_{ijt} = \begin{cases} 1 & : \text{policy } i \text{ is implemented at level } j \text{ in time period } t \\ 0 & : \text{otherwise} \end{cases} \tag{8a}$$
- Let
$$z_{ijt} = \begin{cases} 1 & : \begin{array}{l} \text{policy } i \text{ is implemented at level } j \text{ in time period } t, \\ \text{but not } t-1 \end{array} \\ 0 & : \text{otherwise} \end{cases} \tag{9a}$$

State Variables

- Let S_t, I_t, R_t, d_t, and D_t denote the population of individuals at time t who are <u>S</u>usceptible, <u>I</u>nfected, <u>R</u>ecovered, <u>d</u>ying (in the current period), and <u>D</u>ead (cumulatively), respectively. These values depend on the interventions applied.

- Let P_t denote the cumulative factor by which new infections are decreased between periods $t-1$ and t. That is,

$$P_t = \prod_{\substack{i,j \text{ s.t.} \\ \text{policy } i \text{ used} \\ \text{at level } j \\ \text{in period } t}} P_{ijt} \tag{6a}$$

Model Formulation: Disease Mitigation Optimization (DMO)

$$\begin{aligned}
&\underset{y,P,S,I,R,D,d}{\text{Minimize}} && \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}S_t y_{ijt} + \sum_{t=1}^{T} C_{infection}I_t + C_{death}d_t && && (0)\\
&\text{s.t.} \quad S_t && = S_{t-1} - K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} && \forall\ t \in \{2,\dots,T\} && (1)\\
& \quad I_t && = I_{t-1} + K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} - K_R \cdot I_{t-1} - K_D \cdot I_{t-1} && \forall\ t \in \{2,\dots,T\} && (2)\\
& \quad R_t && = R_{t-1} + K_R \cdot I_{t-1} && \forall\ t \in \{2,\dots,T\} && (3)\\
& \quad d_t && = K_D \cdot I_{t-1} && \forall\ t \in \{2,\dots,T\} && (4)\\
& \quad D_t && = D_{t-1} + d_t && \forall\ t \in \{2,\dots,T\} && (5)\\
& \quad P_t && = \prod_{i=1}^{m}\prod_{j=1}^{n}(1 - y_{ijt} + P_{ijt} \cdot y_{ijt}) && \forall\ t \in \{1,\dots,T\} && (6)\\
& \quad \sum_{j=1}^{n} y_{ijt} && \le 1 && \forall\ i,t && (7)\\
& \quad y_{ijt} && \in \{0,1\} && \forall\ i,j,t && (8)\\
& \quad z_{ijt} && \ge y_{ij(t)} - y_{ij(t-1)} \qquad (\text{let } y_{ij0} = 0\ \forall\ i,j) && \forall\ i,j,t && (9)\\
& \quad 0 \le z_{ijt} && \le 1 && \forall\ i,j,\ \forall\ t \ge 1 && (10)\\
& \quad I_1 && = I_0 && && \\
& \quad S_1 && = N - I_0 && && \\
& \quad D_1 && = 0 && && \\
& \quad R_1 && = 0 && && \\
& \quad d_1 && = 0 && &&
\end{aligned}$$

The objective function (0) is the sum of the costs of implementing the policy interventions in all periods and the costs associated with the resulting disease and death in all periods (due to lost productivity and resources).

The constraints (1),(2),(3),(4), and (5) model the SIRD compartment subpopulations as the disease progresses alongside the infection-reduction factors P_t at each period $t = 1,\dots,T$. The constraint (6) models the multiplicative effect of multiple interventions being applied in the same period, as described by (6a). Equation (7) enforces the logical constraint that at most one level from each policy be used in each period, and (8) ensures that the y_{ijt} variables correspond to the binary definition in (8a).

1.2 SIRD Model with Non-Variable-Cost Interventions

Replace the objective (0) in the mathematical program formulation of the **(DMO)** model with

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot S_t \cdot y_{ijt} + \sum_{t=1}^{T} C_{infection}\cdot I_t + C_{death}\cdot d_t \quad (0)$$

$$\Downarrow$$

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot N \cdot y_{ijt} + \sum_{t=1}^{T} C_{infection}\cdot I_t + C_{death}\cdot d_t \quad (11)$$

1.3 Parameters in Trials

All the numerical experiments performed utilized the following parameters:

- $N = 300000$
- $I_0 = 1000$
- $C_{infection} = 10000$
- $C_{death} = 10000000$
- $K_I = 6e-07$
- $K_R = 0.03$
- $K_D = 0.015$
- Policies:

Name	# Levels	P	A	B	C
1. "Movement"	2	[.95, .925]	[5e5, 1e6]	[1e6, 2e6]	[1e6, 2e6]
2. "Education (University level)"	2	[.995, .95]	[0, 0]	[0, 0]	[0, 0]
3. "Social Gatherings (in a house)"	4	[.995, .99, .975, .925]	[0, 0, 0, 0]	[0, 0, 0, 0]	[0, 0, 0, 0]
4. "Non-Food Service (bank,retail, etc)"	3	[.995, .95, .925]	[2.5e5, 5e5, 1e6]	[5e5, 1e6, 2e6]	[5e5, 1e6, 2e6]
5. "Restaurants"	2	[.95, .925]	[5e5, 1e6]	[1e6, 2e6]	[1e6, 2e6]
6. "Masking"	3	[.995, .95, .925]	[0, 0, 0]	[0, 0, 0]	[0, 0, 0]
7. "Mega Events"	3	[.995, .95, .925]	[2.5e5, 5e5, 1e6]	[5e5, 1e6, 2e6]	[5e5, 1e6, 2e6]
8. "Border Control"	2	[.95, .925]	[5e5, 1e6]	[1e6, 2e6]	[1e6, 2e6]
9. "Physical Distancing"	1	[.925]	[0]	[0]	[0]

Figure 1: *Policy parameters P_{ijt}, A_{ijt}, B_{ijt}, and C_{ijt} are in general dynamic (time-varying) in the full* **(DMO)** *model. In our numerical experiments, we use non-dynamic policies, and in each row of this table corresponding to policy i, parameters are listed for each level j.*

1.4 SIRD Model with Vaccination

The COVID-19 pandemic demonstrates that a vaccine, which alters disease progression, may become suddenly available within the horizon of intervention decision-making and disease progression. We model vaccination as a change in disease progression when a vaccine becomes available, beginning at time period T_{vax}. The value of T_{vax} is exogenous to the model, but can be treated via sensitivity analysis or optimization under uncertain parameters regarding the likelihood of possible vaccine development timelines. We do not treat the "waning" of immunity provided by vaccination and assume that vaccination provides a lasting but imperfect reduction in infection and death rates.

The model is modified via the following parameters:

- The period T_{vax} is when vaccination becomes available.
- The parameters S_t and I_t, which represent the number of susceptible and infected individuals in each period, are divided into the following compartments:
  - S_t^{vax}, S_t^{unvax}, and $S_t^{antivax}$ represent the populations of *vaccinated*, *unvaccinated but willing to be vaccinated*, and *vaccinated and unwilling to be vaccinated* susceptible individuals at time t.
  - Similarly, I_t^{vax} and I_t^{novax} represent the populations of *vaccinated* and *unvaccinated* infected individuals at time t. Here, the notation "novax" (rather than "unvax") reflects that this includes unvaccinated individuals who were previously either willing or unwilling to become vaccinated. Since this model assumes that infection and recovery lead to full immunity, there is no need to distinguish between willingness to become vaccinated among the infected.
  - The number of newly-vaccinated individuals in period t is v_t. Lowercase variable to denote "single-period" populations not constituting compartments, like d_t (contrasted with D_t).
  - The proportion of unvaccinated susceptible individuals willing to be vaccinated who become vaccinated in period t is $K_v(t)$. We exclusively model the case in which

$$K_v(t) = \begin{cases} K_v & : t \geq T_v \\ 0 & : \text{otherwise.} \end{cases} \tag{12}$$

    That is, fraction K_v of susceptible unvaccinated individuals (K_v of S_t^{unvax}) become vaccinated in each period after T_v, and none become vaccinated before that.
- The infection rate parameter K_I is represented by K_I^{vax} and K_I^{novax}, which represents the proportional infection rates of both vaccinated and unvaccinated individuals via the same mixing model.
- The recovery and death rate parameters K_R and K_D have vaccinated and unvaccinated variants K_R^{vax}, K_R^{novax}, K_D^{vax}, and K_D^{novax}.

Model Formulation: Disease Mitigation Optimization with Vaccination (DMOV)

The resulting model can be expressed as follows:

$$
\begin{array}{llll}
\underset{y,P,S^*,I^*,R,D,d,V}{\text{Minimize}} & & \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}S_t y_{ijt} + \sum_{t=1}^{T} C_{infection} I_t & + C_{death} d_t \\
\text{s.t.} & S_t & = S_t^{vax} + S_t^{unvax} + S_t^{antivax} & \forall\ t \in \{2,\dots,T\} \\
& I_t & = I_t^{vax} + I_t^{novax} & \forall\ t \in \{2,\dots,T\} \\
& S_t^{vax} & = S_{t-1}^{vax} - K_I^{vax} \cdot P_t \cdot S_{t-1}^{vax} \cdot I_{t-1} + v_t & \forall\ t \in \{2,\dots,T\} \\
& S_t^{unvax} & = S_{t-1}^{unvax} - K_I^{novax} \cdot P_t \cdot S_{t-1}^{unvax} \cdot I_{t-1} - v_t & \forall\ t \in \{2,\dots,T\} \\
& S_t^{antivax} & = S_{t-1}^{antivax} - K_I^{novax} \cdot P_t \cdot S_{t-1}^{antivax} \cdot I_{t-1} & \forall\ t \in \{2,\dots,T\} \\
& I_t^{vax} & = I_{t-1}^{vax} + K_I^{vax} \cdot P_t \cdot S_{t-1}^{vax} \cdot I_{t-1} - K_R^{vax} \cdot I_{t-1}^{vax} - K_D^{vax} \cdot I_{t-1}^{vax} & \forall\ t \in \{2,\dots,T\} \\
& I_t^{novax} & = I_{t-1}^{novax} + K_I^{novax} \cdot P_t \cdot (S_{t-1}^{unvax} + S_{t-1}^{antivax}) \cdot I_{t-1} - K_R^{novax} \cdot I_{t-1}^{novax} - K_D^{novax} \cdot I_{t-1}^{novax} & \forall\ t \in \{2,\dots,T\} \\
& R_t & = R_{t-1} + K_R^{vax} \cdot I_{t-1}^{vax} + K_R^{novax} \cdot I_{t-1}^{novax} & \forall\ t \in \{2,\dots,T\} \\
& v_t & = K_v(t) \cdot S_{t-1}^{unvax} & \forall\ t \in \{2,\dots,T\} \\
& d_t & = K_D^{vax} \cdot I_{t-1}^{vax} + K_D^{unvax} \cdot I_{t-1}^{unvax} & \forall\ t \in \{2,\dots,T\} \\
& D_t & = D_{t-1} + d_t & \forall\ t \in \{2,\dots,T\} \\
& P_t & = \displaystyle\prod_{i=1}^{m}\prod_{j=1}^{n} (1 - y_{ijt} + P_{ijt} \cdot y_{ijt}) & \forall\ t \in \{1,\dots,T\} \\
& \displaystyle\sum_{j=1}^{n} y_{ijt} & \leq 1 & \forall\ i,t \\
& y_{ijt} & \in \{0,1\} & \forall\ i,j,t \\
& z_{ijt} & \geq y_{ij(t)} - y_{ij(t-1)} \qquad (\text{let } y_{ij0} = 0\ \forall\ i,j) & \forall\ i,j,t \\
& 0 \leq z_{ijt} & \leq 1 & \forall\ i,j,\ \forall\ t \geq 1 \\
& I_1 = I_1^{novax} & = I_0 & \\
& S_1 & = N - I_0 & \\
& S_1^{vax}, S_1^{antivax} & = 0, S_0^{antivax} & \\
& D_1 & = 0 & \\
& R_1 & = 0 & \\
& d_1 & = 0 &
\end{array}
$$

2 Heuristics

2.1 Lagrangian Heuristic and Lower Bound

We can utilize a Lagrangian relaxation to the full problem **(DMO)** by relaxing constraints (6) and instead penalizing the objective function using dual multipliers. The relaxed problem decomposes into two computationally less expensive subproblems; by iteratively updating the multipliers using gradient ascent, the lower bound tightens. Furthermore, we can transform a solution to the relaxed problem into a feasible solution for the full problem, yielding a heuristic solution in its own right.

First, we focus on the variant of the model in which interventions do not have a variable cost depending on the size of the susceptible population (S_t), and instead have a "variable" cost proportional to the size of the entire population (N), as in (11) (i.e., the costs do not "vary" between time periods). This allows the Lagrangian minimization problem to be split into two subproblems.

Then, we transform constraint (6) using a logarithm:

$$\ln P_t = \sum_{i=1}^{m}\sum_{j=1}^{n} \ln(1 - y_{ijt} + P_{ijt}y_{ijt}),\ \forall\ t = 1,\ldots,T. \tag{6-log}$$

Next, we remove this constraint from **(DMO)** and augment the objective (11) via multipliers $\lambda_t, t = 1,\ldots,T$ to obtain a relaxed minimization problem:

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad [\text{Objective (11)}] + \sum_{t=1}^{T} \lambda_t \left(\sum_{i=1}^{m}\sum_{j=1}^{n} \ln(1 - y_{ijt} + P_{ijt}y_{ijt}) - \ln P_t \right) \tag{13}$$

$$\text{s.t.} \quad 0 \leq P_t \leq 1 \tag{14}$$

Constraints from **(DMO)** except for (6).

An optimal value to the augmented problem (13) is a lower bound to the optimal value of the full problem **(DMO)** . We add an extra constraint (14) to enforce the logical constraints that the policy effectiveness factors are between 0 and 1; note that in a numerical implementation, it may actually be preferable to precompute a reasonable lower bound on $\underline{P}_t \in (0, 1)$ and constrain $\underline{P}_t \leq P_t \leq 1$ because the logarithm in (13) is undefined for $P_t = 0$.

By iteratively solving the augmented problem (13) and then using subgradient ascent to update λ_t for all $t = 1,\ldots,T$, we obtain increasingly tighter lower bounds on the optimal value for the full problem **(DMO)** .

Note that the augmented problem (13) can be decomposed into two minimization problems with optimal values $L_1(\boldsymbol{\lambda})$ and $L_2(\boldsymbol{\lambda})$:

$L_1(\boldsymbol{\lambda})$ is the solution to

$$\underset{y}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}[A_{ijt}\cdot y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot N\cdot y_{ijt} + \lambda_t \ln(1 - y_{ijt} + P_{ijt}y_{ijt})]$$

$$\sum_{j=1}^{n} y_{ijt} \le 1 \qquad \forall\ i,t \tag{7}$$

$$y_{ijt} \in \{0,1\} \qquad \forall\ i,j,t \tag{8}$$

$$z_{ijt} \ge y_{ij(t)} - y_{ij(t-1)} \quad \forall\ i,j,t \tag{9}$$

$$0 \le z_{ijt} \le 1 \qquad \forall\ i,j,\ \forall\ t \ge 1 \tag{10}$$

This integer program can be solved by standard off-the-shelf software such as Gurobi.
Note: Nonlinear integer programs have only been solvable by Gurobi since November 2019. I wonder whether there is a more complete explanation of why this subproblem is in fact "easier" than the full problem.

$L_2(\boldsymbol{\lambda})$ is the solution to

$$\underset{P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{t=1}^{T}[C_{infection}\cdot I_t + C_{death}\cdot d_t - \lambda_t \ln P_t]$$

$$\text{s.t.} \quad S_t = S_{t-1} - K_I\cdot P_t\cdot S_{t-1}\cdot I_{t-1} \qquad \forall\ t\in\{2,\dots,T\} \tag{1}$$

$$I_t = I_{t-1} + K_I\cdot P_t\cdot S_{t-1}\cdot I_{t-1} - K_R\cdot I_{t-1} - K_D\cdot I_{t-1} \quad \forall\ t\in\{2,\dots,T\} \tag{2}$$

$$R_t = R_{t-1} + K_R\cdot I_{t-1} \qquad \forall\ t\in\{2,\dots,T\} \tag{3}$$

$$d_t = K_D\cdot I_{t-1} \qquad \forall\ t\in\{2,\dots,T\} \tag{4}$$

$$D_t = D_{t-1} + d_t \qquad \forall\ t\in\{2,\dots,T\} \tag{5}$$

$$P_t \le 1 \tag{14}$$

$$I_1 = I_0$$

$$S_1 = N - I_0$$

$$D_1 = 0$$

$$R_1 = 0$$

$$d_1 = 0$$

This problem has no integer constraints and can be solved by any nonlinear programming software.

To increase the tightness of the bound in the gradient-ascent step for the multipliers $\boldsymbol{\lambda}$, where $\boldsymbol{\lambda}^+$ represents the vector of multipliers at a subsequent iteration, we use the updating rule:

$$\boldsymbol{\lambda}^+ = \boldsymbol{\lambda} + \gamma\left(\nabla L_1(\boldsymbol{\lambda}) + \nabla L_2(\boldsymbol{\lambda})\right),$$

i.e.

$$\lambda_t^+ = \lambda_t + \gamma \cdot \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \ln(1 - y_{ijt} + P_{ijt} y_{ijt}) - \ln P_t \right). \tag{15}$$

To obtain a feasible solution to the full problem **(DMO)** after any iteration of this procedure, one can fix the values of $y_{ijt}, i = 1, \dots, m, j = 1, \dots, n, t = 1, \dots, T$ in **(DMO)** to those obtained in the decomposed minimizations, which immediately yields values of $P_t, t = 1, \dots, T$, which in turn gives values of the compartment subpopulations (S, I, R, d, and D) following basic bookkeeping.

This procedure can be iteratively performed indefinitely, and will yield a sequence of nondecreasing lower bounds to the full problem **(DMO)** . As a stopping criterion, one can terminate when the relative improvement between two iterations is less than a threshold ("improvement"), or when the relative optimality gap between the incumbent feasible solution and the greatest lower bound is less than a threshold ("optimality gap"). More formally, the heuristic corresponding to this procedure can be described as follows:

Initialize $\lambda_t \leftarrow 0$ for all t.
Do
    Minimize to obtain $L_1(\boldsymbol{\lambda})$ and $L_2(\boldsymbol{\lambda})$
    Update $\boldsymbol{\lambda}$ via gradient ascent as in (15)
    The value $L_1(\boldsymbol{\lambda}) + L_2(\boldsymbol{\lambda})$ gives a lower bound.
    The optimal values of y_{ijt} from $L_1(\boldsymbol{\lambda})$ substituted in the original problem **(DMO)**
    yield a feasible solution and thus an upper bound for that problem's optimal solution.
Repeat while stopping condition (improvement or optimality gap) is not met.

2.2 Index Policy

Index policies are natural heuristics and are likely to resemble intuitive decisionmaking in the absence of optimization tools. For comparison, we outline an index solution method and compare solutions produced in this manner and others in Section 3.1.

Consider a "block size" b, and the corresponding partition of time periods $\{1, \dots, T\}$ into blocks of size b. For simplicity, we assume b divides T evenly:

$$\begin{aligned}
B_1 &= (1, \dots, b) \\
B_2 &= (b+1, \dots, 2b) \\
&\vdots \\
B_{T/b} &= (T - b + 1, T - b + 2, \dots, T).
\end{aligned}$$

Let $T^{B_k} = \max\{t | t \in B_1 \cup B_2 \cup \dots \cup B_k\}$, i.e. the latest time period that appears in blocks $B_1, \dots, B_k$, and let $t_0^{B_k} = \min\{t | t \in B_k\}$. For every block except possibly the last block, $T^{B_k} = k \cdot b$; for every block, $t_0^{B_k} = (k-1)b + 1$.

A basic index policy is applied to all the periods in each block, one at a time, in order, using the index defined below in equation (16) for policy i at level j:

$$\text{index}(i,j,B) = P_{ijt} \times C_{ijt} \text{ (for some } t \text{ in block } B\text{[1].)} \tag{16}$$

For the following algorithm, we consider the $T_{horizon}$-period objective to be the objective obtained after $T_{horizon}$ periods, rather than the full T. We replace the objective of **(DMO)** (0) with the following:

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot S_t \cdot y_{ijt} + \sum_{t=1}^{T} C_{infection}\cdot I_t \qquad + C_{death}\cdot d_t \tag{0}$$

$$\Downarrow$$

$$\underset{y,P,S,I,R,D,d}{\text{Minimize}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T_{horizon}} A_{ijt}y_{ijt} + B_{ijt}z_{ijt} + C_{ijt}\cdot S_t \cdot y_{ijt} + \sum_{t=1}^{T_{horizon}} C_{infection}\cdot I_t + C_{death}\cdot d_t \tag{17}$$

We refer to the variant of **(DMO)** with only $T_{horizon}$ time periods as $(\mathbf{DMO})_{T_{horizon}}$. This algorithm iteratively focuses on $(\mathbf{DMO})_{T^B}$ for each block $B = B_1, B_2, \ldots$.

We use the term "fix" to mean that a variable's value is set, and the variable is no longer treated as a decision variable but a parameter of the problem.

The index policy can be described as follows:

```
For each block $B$ in $\{B_1, \ldots, B_{T/b}\}$ in order,
    Fix $y_{ijt} = 0$ for all $t \in B$; do not change values for previous blocks.
    Set chosenPolicies$\leftarrow \{\}$
    Do...
        Set oldObjectiveValue$\leftarrow$objective of $(\mathbf{DMO})_{T^B}$
        Calculate $(i^*, j^*) = \arg\min_{i,j}\{\text{index}(i,j,B) : \nexists j' s.t. (i,j') \in$ chosenPolicies$\}$
        For each policy-level pair $(i,j)$
            If $(i,j) \in$ chosenPolicies$\cup\{(i^*, j^*)\}$
                Fix $y_{ijt} = 1$ for all $t \in B$
            Else
                Fix $y_{ijt} = 0$ for all $t \in B$
        Set newObjectiveValue$\leftarrow$objective of $(\mathbf{DMO})_{T^B}$
        If newObjectiveValue < oldObjectiveValue
            Set chosenPolicies$\leftarrow$chosenPolicies$\cup\{(i^*, j^*)\}$
    ...repeat while newObjectiveValue < oldObjectiveValue
```

Note that this policy is both *greedy* with respect to policy decisions in each period in the sense

[1]The policy described here assumes that the cost and effectiveneess parameters, C_{ijt} and P_{ijt}, do not vary with respect to period t within blocks (note that in the numerical experiments in this paper, these parameters are not time-varying at all).

that the best indexed policies are chosen first, and also *myopic* with respect to time in the sense that only a small subset of time periods are considered in each iteration.

2.3 w-Period *Time-Greedy* Heuristic

The computational resources required to solve **(DMO)** to optimality will likely exceed what is available to decisionmakers when the number of policy options and the number of time periods are large. However, even with a large number of policy options, a small number of time periods may make the decision space small enough to solve to optimality even with an unsophisticated exhaustive search. Decisions made when considering a small number of time periods may be reasonable to use over a longer time horizon.

The 1-period *time-greedy* algorithm is the greedy policy in which decisions are made only considering one period at a time. In the w-period greedy heuristic, decisions are made only considering w periods at a time. After decisions have been made optimally over the first w periods, the decisions for period 1 are fixed, and the problem is solved for periods $2, \ldots, w+1$; on the l'th iteration, the horizon of optimization is $l, \ldots, l+w-1$. This continues until the horizon is $T-(w-1), \ldots, T$, for a total of $T-(w-1)$ iterations.

We refer to the variant of **(DMO)** with only $T_{horizon}$ time periods as $(\mathbf{DMO})_{T_{horizon}}$, as in (17) in Section 2.2, and use the term "fix" in the same way as in Section 2.2.

```
Initialize $y_{ijt} \leftarrow 0$ for all i,j,t.
For $T_0$ in $\{1, \ldots, T-w+1\}$
    For all t<$T_0$
        Fix $y_{ijt}$ to whatever value it currently holds for all i,j.
        Fix $P_t, S_t, I_t, R_t, D_t, d_t$ to whatever values they currently hold.
    For all t$\in \{T_0, \ldots, T_0+w-1\}$
        Unfix $y_{ijt}$ for all i,j. Unfix $P_t, S_t, I_t, R_t, D_t, d_t$.
    Solve $(\mathbf{DMO})_{T-w+1}$ with the un-fixed variables.
```

2.4 w-Period *Time-Greedy* Heuristic with T^+-period Lookahead

In the execution of the w-period time-greedy solution, and indeed any variant of **(DMO)** , the primary difficulty is optimally finding values of integer-constrained variables. It is trivial, in fact, to consider the disease progression over time periods subsequent to the w periods during which optimal interventions are being considered in the context of the w-period time-greedy heuristic. This motivates a lookahead heuristic, in which the quality of a decision is assessed not just on the disease-related and policy-related costs within a w-period interval, but additionally on the disease-related costs during T^+ subsequent time-periods, during which the decision-maker does not make *any* policy decisions (and so no interventions are chosen).

This should yield more aggressive policy decisions than the w-period time-greedy heuristic, as the disease-related costs associated with any policy intervention menu are higher, and thus it will be desirable to further decrease infections during the w-period window.

The difference between these two heuristics can be summarized as follows:

- In the w-period time-greedy heuristic, only w periods are considered at a time in terms of decisionmaking and disease prograssion.
- In the T^+-period lookahead variant, the decisionmaker's "hands are tied" (they are forced to use no intervention) after the w periods of decisionmaking, but they calculate and make decisions based on the costs associated with disease progression during an additional T^+ time periods.

The algorithm is as follows:

```
Initialize $y_{ijt} \leftarrow 0$ for all i,j,t.
For $T_0$ in $\{1,\dots,T-w+1\}$
    For all t<$T_0$
        Fix $y_{ijt}$ to whatever value it currently holds for all i,j.
        Fix $P_t, S_t, I_t, R_t, D_t, d_t$ to whatever values they currently hold.
    For all t$\in \{T_0,\dots,T_0+w-1\}$
        Unfix $y_{ijt}$ for all i,j. Unfix $P_t, S_t, I_t, R_t, D_t, d_t$.
    Solve $(\mathbf{DMO})_{T-w+1+T^+}$ with the un-fixed variables.
```

2.5 B-Policy ***Policy-Greedy*** Solution (implemented)

Much of the difficulty of solving **(DMO)** largely stems from the highly nonlinear constraint (6), which involves the product of $m \times n$ integer-constrained variables. On the other hand, if only a single policy with a single level is considered, the problem can be solved to optimality over a long time horizon quickly, with modest computational resources.

The following *policy-greedy* algorithm leverages this fact to make optimal decisions for only one policy at a time, fixing the plan for that policy while considering adding another, until either B policies are chosen or there is no improvement from adding any additional policy (at any level).

To articulate this, we introduce parameters P_t^0 for $t = 1,\dots,T$, and modify constraint (6) to

$$P_t = \prod_{i=1}^{m}\prod_{j=1}^{n}(1 - y_{ijt} + P_{ijt}\cdot y_{ijt}) \;\; \forall\, t \in \{1,\dots,T\} \tag{6}$$

$$\Downarrow$$

$$P_t = P_t^0\prod_{i=1}^{m}\prod_{j=1}^{n}(1 - y_{ijt} + P_{ijt}\cdot y_{ijt}) \;\; \forall\, t \in \{1,\dots,T\} \tag{18}$$

where P_t^0 (instead of simply "1") represents the factor by which the infection rate is decreased by decisionmaking *if no policies are implemented.*

```
Set USED← {}, OBJECTIVE← ∞
For b in {1,...,B}
    Set ITERATION_OBJECTIVE← ∞
    For each policy i
        If (i,j) ∉USED for any of j∈ {1,...,n}  # Policy i has not been used at any level
            For each level j
                Set m←1, n←1, and solve (DMO)  with only policy i at level j.
                Solve this problem
                Set SOLUTION_OBJECTIVE←objective value of this problem
                If SOLUTION_OBJECTIVE<ITERATION_OBJECTIVE
                    Set ITERATION_OBJECTIVE←SOLUTION_OBJECTIVE
    If ITERATION_OBJECTIVE<OBJECTIVE
        Set OBJECTIVE←ITERATION_OBJECTIVE
        Set USED←USED∪{(i,j)}
        Set P_t^0 ← P_t^0 × P_ijt for all t where policy i at level j is used in the solution
        that produced ITERATION_OBJECTIVE
    Else
        Terminate without adding any new policy
```

2.6 Index and Assortment Index Policy

2.7 Local Search

2.8 F-Factor Early Stopping Using BARON/Gurobi/DICOPT/BONMIN

The following heuristic requires a solution strategy (a "solver") for the **(DMO)** model formulated in Section 1.1 that can iteratively generate the following two quantities:

1. A sequence of feasible solutions with improving objective function values, referred to as "incumbent solutions" whose objective values serve as upper bounds for the problem, and

2. a sequence of increasing lower bounds for the problem, generated from any of the following:

    - continuous relaxation,
    - Lagrangian relaxation,
    - any other dualization or constraint relaxation.

This is, in fact, what most mathematical programming solvers aim to iteratively produce while solving a problem. We refer to a "solver" as a tool that achieves the two goals above. As the solvers compute, the percent difference between the lower and upper bounds - the "relative optimality gap" - shrinks. Mixed-integer programming tools typically do not prove optimality, but stop when this relative optimality gap falls below an acceptable threshold.

With an upper bound u and a lower bound l to the objective function, solving the problem to desired optimality factor F requires that

$$\frac{u-l}{u} < F.$$

Selecting a large value of F would amount to an "early-stopping" heuristic, and the solution may still be useful even though there is no reason to suspect that the generated solution is globally optimal.

```
Begin solving the (DMO) problem using a solver. For each iteration, do
    If (u-l)/u < F
        Stop
    Else
        Continue
```

2.9 Quadratic Policy Cost Approximation

The full **(DMO)** model considers atomic policy decisions and may be of great utility when there are too many possible policy decisions in each period for a policy-maker to evaluate them all via analysis or simulation. Unfortunately, in this context of a large number of policies (m) and levels (n), it becomes difficult to extract high-level insights from optimal decisions produced by such a model.

Furthermore, since we demonstrate that realistically large instances of the **(DMO)** model become intractible, effective high-level patterns in approximate solutions may be obscured by the "noise" of sub-optimal decisions.

An alternative, simplified model may be useful for analysis of the problem. Such a model, when fitted to real problem parameters, may also be useful to policy-makers for setting cost targets. One such simplification is to approximate all the costs associated with different policy assortments as smooth functions of an "effort level" that exactly corresponds to the policy "effectiveness" P_{ijt}.

Consider a policy assortment ω as a "policy vector"

$$\omega = \begin{bmatrix} j_{\omega 1} & \dots & j_{\omega n} \end{bmatrix},$$

where $j_{\omega i}$ is the level of policy i utilized in assortment ω. Denote the set of all possible policy assortments Ω. Note that in each period of the model **(DMO)** , exactly one policy assortment is utilized, and each policy assortment has associated "assortment parameters". That is, if assortment ω is used in period t, then the associated probability of becoming infected P_t can be called P_ω, and the associated setup and per-susceptible-individual costs A_ω and C_ω can be written as the sum of the costs of each atomic policy included in ω. The assotment $\mathbf{0}$ includes no policies, and has $P_{\mathbf{0}} = 1, A_{\mathbf{0}} = 0, C_{\mathbf{0}} = 0$.

Note that the switching costs of an assortment is not well-defined. The cost to switch from one assortment ω_1 to another ω_2 in the **(DMO)** model depends on which atomic policies are included in

ω_1 and ω_2. If they largely overlap, the switching cost will be small. Thus, an "assortment switching cost" B_ω is inappropriate conceptually unless it is ascribed a different meaning than in the **(DMO)** model[2].

Setting aside for the moment switching costs, consider that some policy assortments may be *dominated* by others. That is, for two assortments ω_1 and ω_2, it is possible that the infection probability factor $P_{\omega_1} < P_{\omega_2}$ and also the costs $A_{\omega_1} < A_{\omega_2}$ and $C_{\omega_1} < C_{\omega_2}$. If policy assortments are the unit of analysis, it may be desirable to consider only efficient assortments.

Given the policies described in Section 1.3, we demonstrate efficient policy assortments in Figure 2, along with a quadratic approximation of the costs associated with the efficient policies.

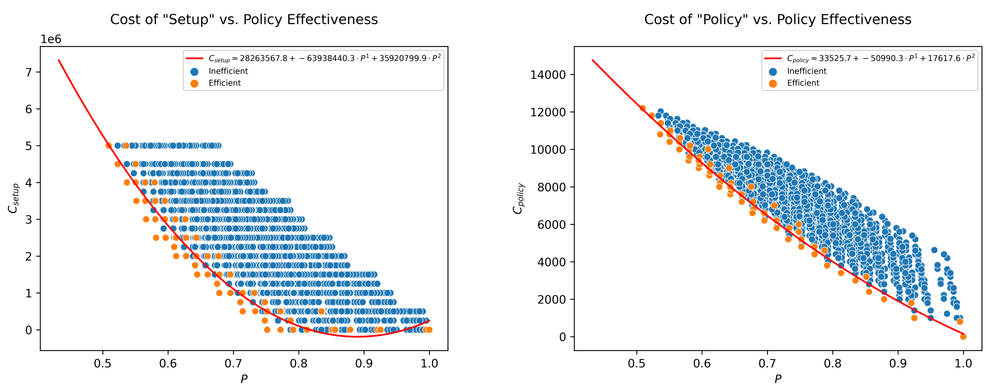


Figure 2: *From all possible assortments of the policies listed in Section 1.3, "efficient" assortments are approximated by a quadratic function of "effectiveness" P_t.*

Given the 9 atomic policies described in Section 1.3, each with between 1 and 4 levels, there are a total of $51,840$ possible policy assortments. Remarkably, only 61 of these assortments are non-dominated. For example, one of the "efficient" assortments in Figure 2 is "Movement (level 1/2) & Masking (level 2/3) & Physical Distancing (level 1/1)" parameters.

The **(DMO)** problem can in fact be solved as a bilinear program (rather than a nonlinear program including higher-order terms) by considering policy assortments as the unit of analysis, rather than atomic policies. Unfortunately, even when restricting to "efficient" assortments, this does not yield a significantly more tractable problem, as there are still likely to be a relatively high number of possible assortments and a long time horizon. However, a quadratic approximation of policy assortments may be tractable and may yield interpretable high-level output.

Suppose, as in Figure 2, the policy effectiveness of each assortment is calculated, and the setup and per-susceptible-individual costs of utilizing each assortment ω in period t are approximated as follows:

[2]It may be possible to include a policy regulation term that penalizes large period-to-period changes in policy, such as $\sum_{t=2}^{T}(P_t - P_{t-1})^2$.

- If assortment ω is used in period t, then the policy effectiveness in the period is

$$P_t = P^\omega = \prod_{\substack{\{(i,j):\text{policy } i,\\ \text{level } j \text{ in } \omega\}}} P_{ijt}. \tag{19}$$

  There is a "maximally effective" policy assortment consisting of every atomic policy at the highest possible level. This assortment is always non-dominated, and its effectiveness p_{min} serves as a lower bound for policy effectiveness: $p_{min} \leq P_t \leq 1$

- If assortment ω is used in period t, then the setup cost in the period is

$$A_t = A^\omega = \sum_{\substack{\{(i,j):\text{policy } i,\\ \text{level } j \text{ in } \omega\}}} A_{ijt} \tag{20}$$

  which we approximate as as "$A(P_t)$":

$$A_t \approx a_2 P_t^2 + a_1 P_t + a_0$$

- If assortment ω is used in period t, then the per-susceptible-individual cost in the period is

$$C_t = C^\omega = \sum_{\substack{\{(i,j):\text{policy } i,\\ \text{level } j \text{ in } \omega\}}} C_{ijt} \tag{21}$$

  which we approximate as "$C(P_t)$":

$$C_t \approx c_2 P_t^2 + c_1 P_t + c_0$$

The following mathematical program describes this approximation (note the absence of switching costs):

$$
\begin{aligned}
\underset{P,A,C,S,I,R,D,d}{\text{Minimize}} \quad & \sum_{t=1}^{T} A_t + C_t S_t + \sum_{t=1}^{T} C_{infection} I_t \quad + C_{death} d_t && (22) \\
\text{s.t.} \quad & S_t = S_{t-1} - K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} && \forall\, t \in \{2, \ldots, T\} \\
& I_t = I_{t-1} + K_I \cdot P_t \cdot S_{t-1} \cdot I_{t-1} - K_R \cdot I_{t-1} - K_D \cdot I_{t-1} && \forall\, t \in \{2, \ldots, T\} \\
& R_t = R_{t-1} + K_R \cdot I_{t-1} && \forall\, t \in \{2, \ldots, T\} \\
& d_t = K_D \cdot I_{t-1} && \forall\, t \in \{2, \ldots, T\} \\
& D_t = D_{t-1} + d_t && \forall\, t \in \{2, \ldots, T\} \\
& A_t = a_2 P_t^2 + a_1 P_t + a_0 && \forall\, t \in \{1, \ldots, T\} \\
& C_t = c_2 P_t^2 + c_1 P_t + c_0 && \forall\, t \in \{1, \ldots, T\} \\
& p_{min} \leq P_t \leq 1 && \forall\, t \in \{1, \ldots, T\} \\
& I_1 = I_0 \\
& S_1 = N - I_0 \\
& D_1 = 0 \\
& R_1 = 0 \\
& d_1 = 0
\end{aligned}
$$

A solution to problem (22) yields a natural heuristic solution method for the **(DMO)** mode (0) by finding policy assortments that yield similar cost and probability decisions. Specifically, for any value of (A_t, C_t, P_t), let $\omega(A_t, C_t, P_t)$ be the policy assortment that yields the "closest" values $(A^\omega, C^\omega, P^\omega)$ as defined by equations (19), (20), and (21) and some loss function L. Consider the loss function

$$L((A, C, P), \omega) = (P^\omega - P)^2 \tag{23}$$

and then the "closest" policy is defined as

$$\omega(A, C, P) = \underset{\omega \in \Omega}{\arg\min}\, L((A, C, P), \omega). \tag{24}$$

Finally, recalling that $j_{\omega i}$ is the level of policy i implied by assortment ω, define the mapping of a policy assortment ω to a binary decision of whether intervention i is used at level j as:

$$y_{ij}^{\omega} = \begin{cases} 1 & : j = j_{\omega i} \\ 0 & : \text{otherwise} \end{cases}. \tag{25}$$

The heuristic solution method utilizing solutions of the quadratically approximated problem (22) can then be written as follows:

Solve approximated problem (22) to obtain A_t, C_t, P_t for all t.
Calculate the ``closest'' assortment for each period $\omega_t = \omega(A_t, C_t, P_t)$
Use the policy decisions in the full **(DMO)** problem by substituting $y_{ijt} = y_{ij}^{\omega_t}$

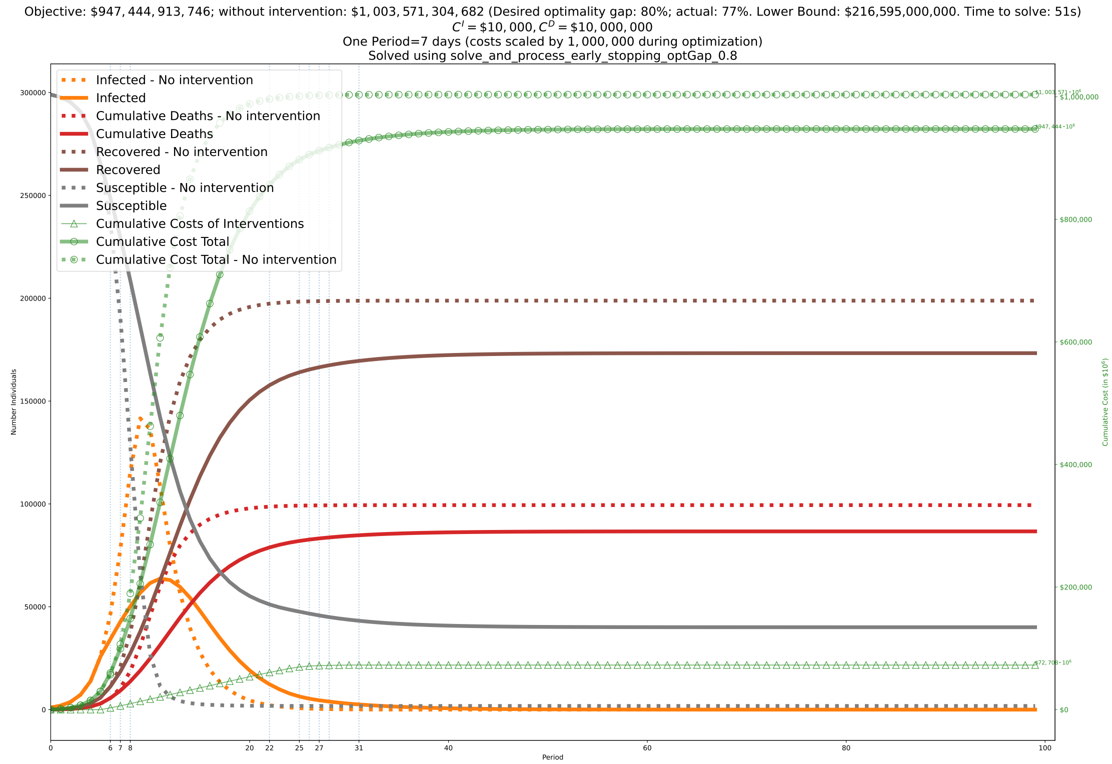
3 Results

3.1 Comparison of Heuristics

Here we present three solutions for the same **(DMO)** instance with $T = 100$ periods, calculated using different heuristics. The first is the "F-Factor Early Stopping" heuristic defined in Section 2.8 with $F = 0.8$; the second is the Lagrangian heuristic defined in Section 2.1; the third is the quadratic heuristic defined in Section 2.9.

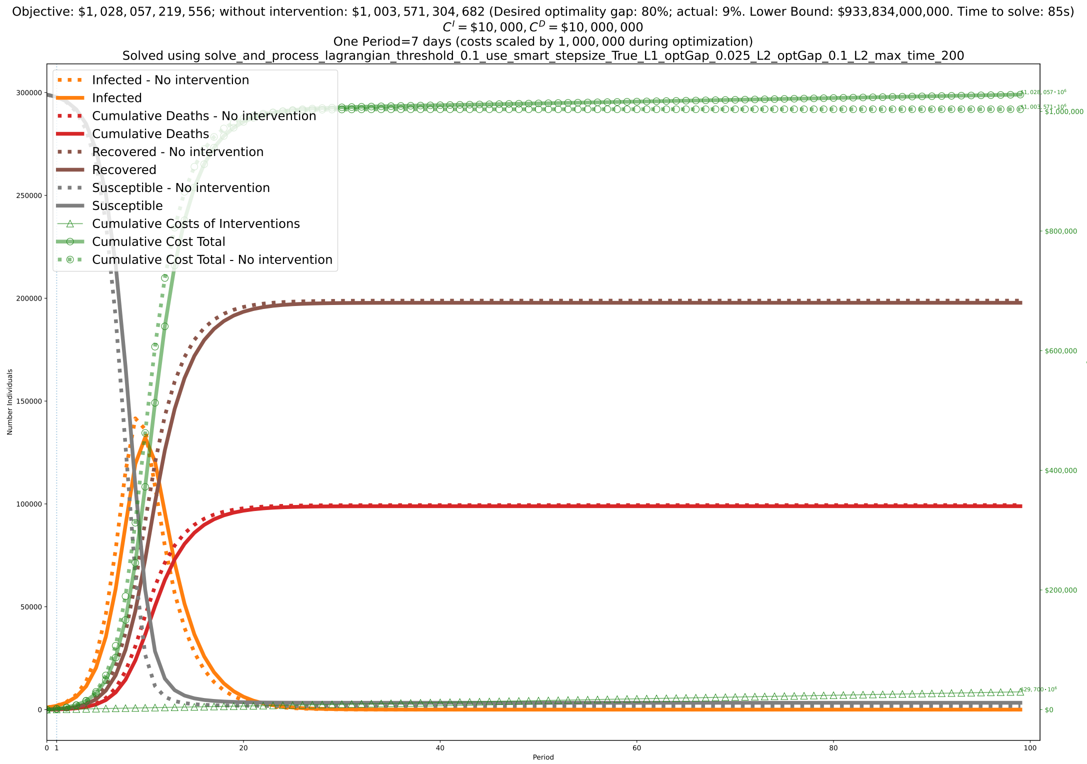
Note that the early-stopping heuristic in Figure 3a generates the best solution overall, the Lagrangian solution in Figure 3b, utilizing the Lagrangian lower-bound, has the tightest lower-bound[3], and the quadratic approximation in Figure 3c, which is solved by ignoring switching costs, has the most policy changes over the horizon.

[3] the listed lower bound of the quadratic approximation is not a true lower bound, but a lower bound of the quadratic approximation (22)



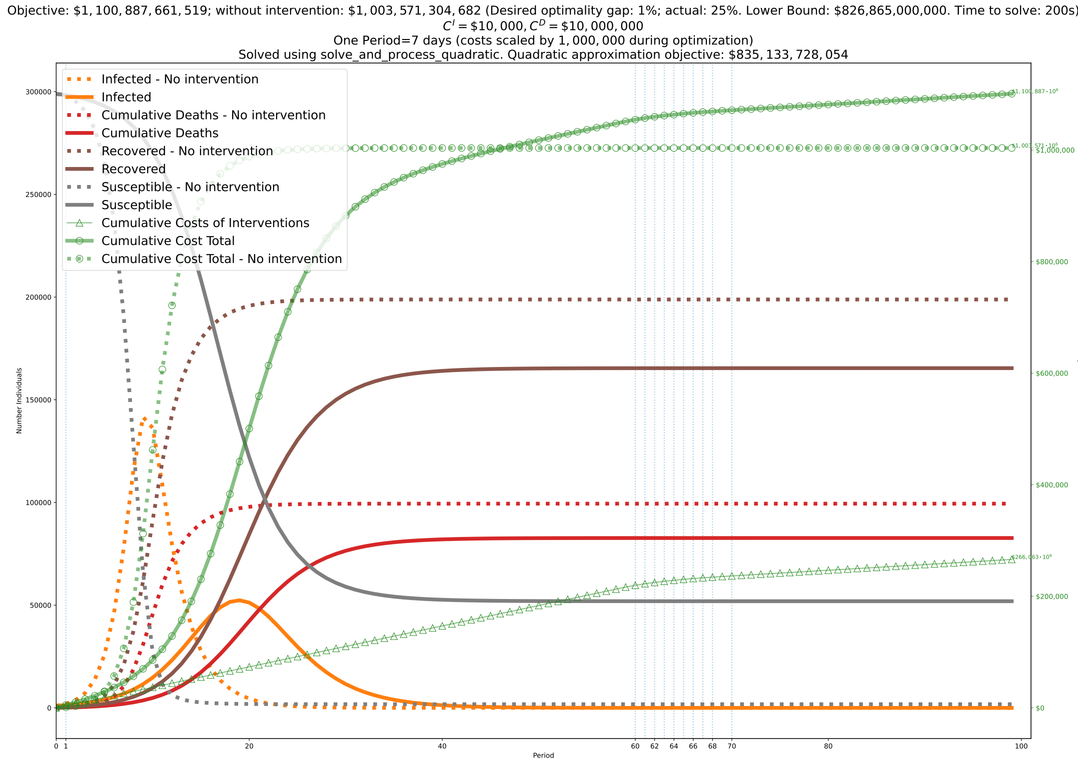
	0 -5	6 -6	7 -7	8 -21	22 -24	25 -25	26 -26	27 -27	28 -30	31 -99
0. Movement A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]		2	2	2	2					
1. Education (University level) A: $[0 ,0]·$10^2$ B: $[0 ,0]·$10^2$ C: $[10 ,14]·$10^2$ P: [.99 ,.95]				2						
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0 ,0]·$10^2$ C: $[8 ,10 ,12 ,14]·$10^2$ P: [.99 ,.99 ,.97 ,.93]		4	4	4	4	4	4	4		
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]		3	3	3	3	3				
4. Restaurants A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]			2	2	2	2	2			
5. Masking A: $[0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]		3	3	3	3	3	3			
6. Mega Events A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]		3	3	3	3	3				
7. Border Control A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]		2	2	2	2					
8. Physical Distancing A: $[0]·$10^2$ B: $[0]·$10^2$ C: $[10]·$10^2$ P: [.93]		1	1	1	1	1	1	1	1	
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$4.9e+09 $0.0 $4.9e+09 1.000	$3e+10 $2.8e+ $2.7e+ 0.579	$4e+10 $3.2e+ $3.6e+ 0.536	$5.3e+10 $3.7e+09 $4.9e+10 0.509	$1.6e+10 $3.2e+09 $1.3e+10 0.536	$1.1e+ $2.4e+ $8.3e+ 0.626	$8.3e+ $1.6e+ $6.8e+ 0.732	$6.3e+ $7.2e+ $5.6e+ 0.856	$4.5e+09 $3e+08 $4.2e+09 0.925	$3.2e+08 $0.0 $3.2e+08 1.000

(a) *Solution using* 0.8*-Factor Early Stopping heuristic with the BARON solver.*



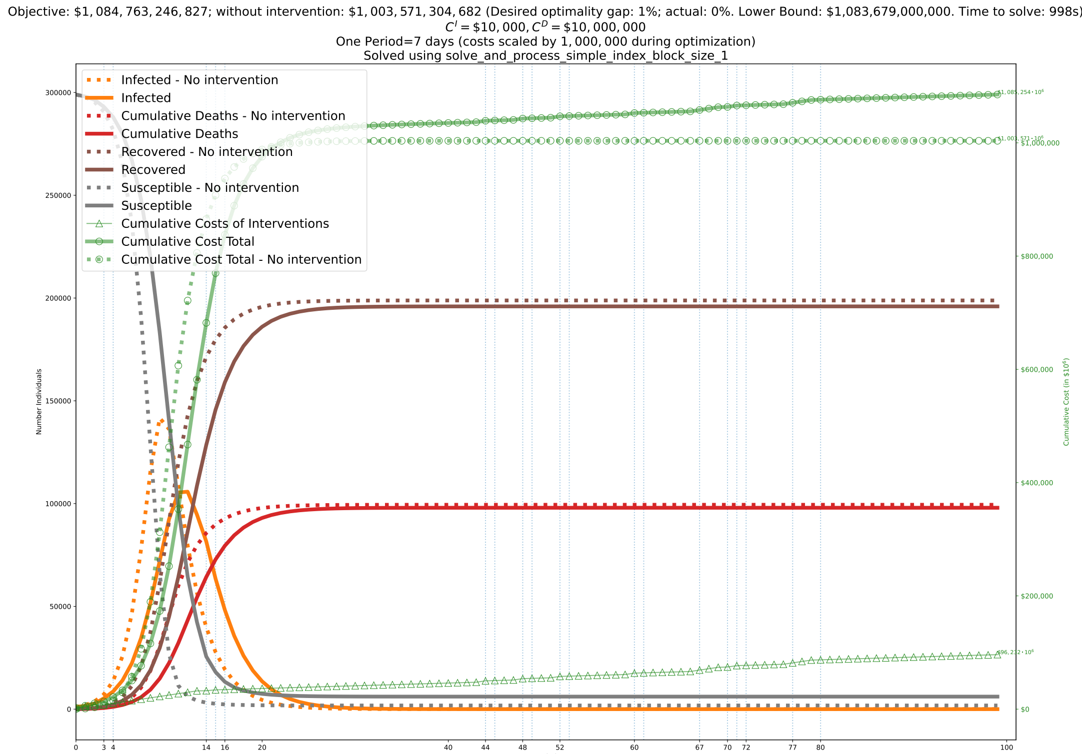
	1 -99
0. Movement A: $[5000 ,10000]·10^2 B: $[10000,20000]·10^2 C: $[10 ,14]·10^2 P: [.95 ,.93]	
1. Education (University level) A: $[0 ,0]·10^2 B: $[0 ,0]·10^2 C: $[10 ,14]·10^2 P: [.99 ,.95]	
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]·10^2 B: $[0 ,0 ,0 ,0]·10^2 C: $[8 ,10 ,12 ,14]·10^2 P: [.99 ,.99 ,.97 ,.93]	
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]·10^2 B: $[5000 ,10000,20000]·10^2 C: $[8 ,10 ,14]·10^2 P: [.99 ,.95 ,.93]	
4. Restaurants A: $[5000 ,10000]·10^2 B: $[10000,20000]·10^2 C: $[10 ,14]·10^2 P: [.95 ,.93]	
5. Masking A: $[0 ,0 ,0]·10^2 B: $[0 ,0 ,0]·10^2 C: $[8 ,10 ,14]·10^2 P: [.99 ,.95 ,.93]	
6. Mega Events A: $[2500 ,5000 ,10000]·10^2 B: $[5000 ,10000,20000]·10^2 C: $[8 ,10 ,14]·10^2 P: [.99 ,.95 ,.93]	
7. Border Control A: $[5000 ,10000]·10^2 B: $[10000,20000]·10^2 C: $[10 ,14]·10^2 P: [.95 ,.93]	
8. Physical Distancing A: $[0]·10^2 B: $[0]·10^2 C: $[10]·10^2 P: [.93]	1
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$1e+10 $3e+08 $1e+10 0.925

(b) *Solution using the Lagrangian heuristic.*



	1 -59	60 -60	61 -61	62 -62	63 -63	64 -64	65 -65	66 -66	67 -67	68 -69	70 -99
0. Movement A: $[5000 ,10000]$\cdot 10^2$ B: $[10000,20000]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	2	2	1	1	1	1	1	1			
1. Education (University level) A: $[0 ,0]$\cdot 10^2$ B: $[0 ,0]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.99 ,.95]	2	2					2				
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]$\cdot 10^2$ B: $[0 ,0 ,0 ,0]$\cdot 10^2$ C: $[8 ,10 ,12 ,14]$\cdot 10^2$ P: [.99 ,.99 ,.97 ,.93]	4	4	4	4	4	4	4	4	4	4	4
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]$\cdot 10^2$ B: $[5000 ,10000,20000]$\cdot 10^2$ C: $[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	3	3	2	2	2	2			1		
4. Restaurants A: $[5000 ,10000]$\cdot 10^2$ B: $[10000,20000]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	2	2	1	1	1						
5. Masking A: $[0 ,0 ,0]$\cdot 10^2$ B: $[0 ,0 ,0]$\cdot 10^2$ C: $[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	3	3	3	3	3	3	2	3	3	3	2
6. Mega Events A: $[2500 ,5000 ,10000]$\cdot 10^2$ B: $[5000 ,10000,20000]$\cdot 10^2$ C: $[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	3	3	3	2							
7. Border Control A: $[5000 ,10000]$\cdot 10^2$ B: $[10000,20000]$\cdot 10^2$ C: $[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	2		1								
8. Physical Distancing A: $[0]$\cdot 10^2$ B: $[0]$\cdot 10^2$ C: $[10]$\cdot 10^2$ P: [.93]	1	1	1	1	1	1	1	1	1	1	1
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$1.8e+10 $3.7e+09 $1.4e+10 0.509	$3.3e+ $3.2e+ $1.8e+ 0.550	$2.8e+ $2.8e+ $1.4e+ 0.596	$2.4e+ $2.3e+ $1.2e+ 0.645	$2.1e+ $2e+09 $9.7e+ 0.679	$1.7e+ $1.7e+ $8.1e+ 0.714	$1.7e+ $1.7e+ $6.8e+ 0.734	$1.4e+ $1.4e+ $5.8e+ 0.752	$1.4e+ $1.4e+ $4.9e+ 0.787	$1.1e+ $1.1e+ $3.9e+ 0.791	$1e+09 $1e+09 $7.4e+05 0.813

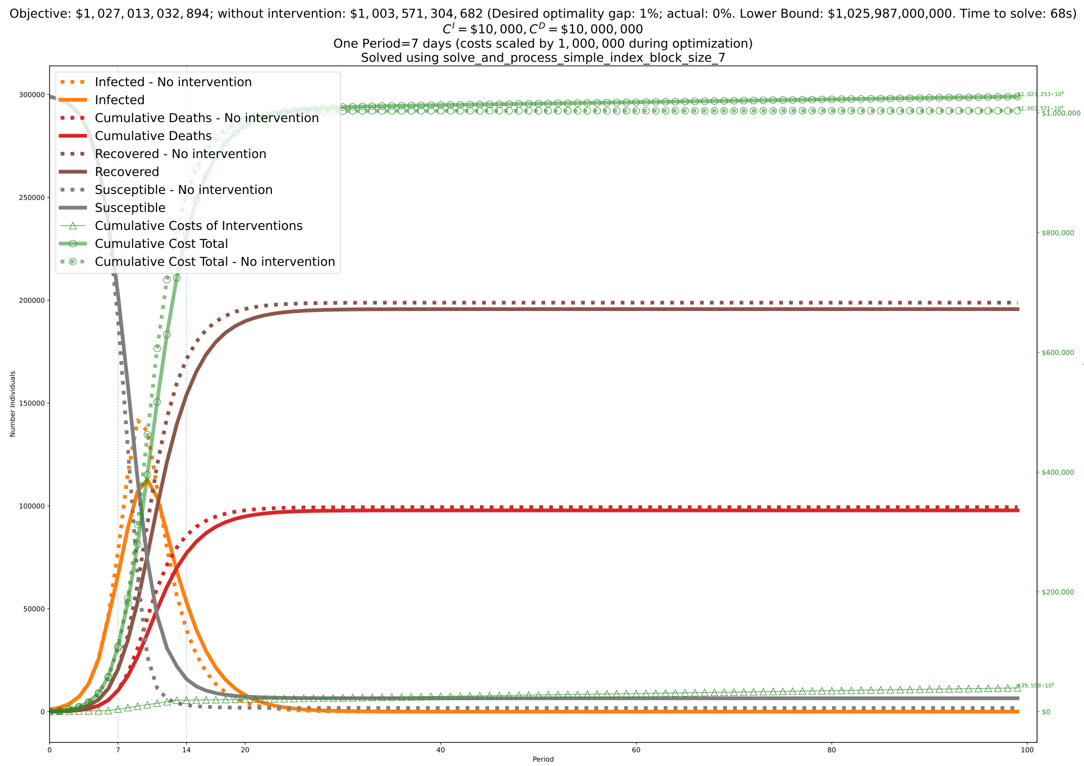
(c) *Solution using the quadratic approximation. Note that the lower bound listed is a lower bound on the quadratic approximation (22), not the full* **(DMO)** *problem.*



	0 -2	3 -3	4 -13	14 -14	15 -15	16 -43	44 -44	45 -47	48 -48	49 -51
0. Movement A: \$[5000 ,10000]$\cdot 10^2$ B: \$[10000,20000]$\cdot 10^2$ C: \$[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	1		1		1		1		1	
1. Education (University level) A: \$[0 ,0]$\cdot 10^2$ B: \$[0 ,0]$\cdot 10^2$ C: \$[10 ,14]$\cdot 10^2$ P: [.99 ,.95]	1		1				1		1	
2. Social Gatherings (in a house) A: \$[0 ,0 ,0 ,0]$\cdot 10^2$ B: \$[0 ,0 ,0 ,0]$\cdot 10^2$ C: \$[8 ,10 ,12 ,14]$\cdot 10^2$ P: [.99 ,.99 ,.97 ,.93]	1	1	1	1	1	1	1	1	1	1
3. Non-Food Service (bank,retail, etc) A: \$[2500 ,5000 ,10000]$\cdot 10^2$ B: \$[5000 ,10000,20000]$\cdot 10^2$ C: \$[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	1		1		1	1	1	1	1	1
4. Restaurants A: \$[5000 ,10000]$\cdot 10^2$ B: \$[10000,20000]$\cdot 10^2$ C: \$[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	1		1				1		1	
5. Masking A: \$[0 ,0 ,0]$\cdot 10^2$ B: \$[0 ,0 ,0]$\cdot 10^2$ C: \$[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	1		1		1		1		1	
6. Mega Events A: \$[2500 ,5000 ,10000]$\cdot 10^2$ B: \$[5000 ,10000,20000]$\cdot 10^2$ C: \$[8 ,10 ,14]$\cdot 10^2$ P: [.99 ,.95 ,.93]	1		1		1		1		1	
7. Border Control A: \$[5000 ,10000]$\cdot 10^2$ B: \$[10000,20000]$\cdot 10^2$ C: \$[10 ,14]$\cdot 10^2$ P: [.95 ,.93]	1		1				1		1	
8. Physical Distancing A: \$[0]$\cdot 10^2$ B: \$[0]$\cdot 10^2$ C: \$[10]$\cdot 10^2$ P: [.93]	1		1		1		1		1	
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	\$3.4e+09 \$2.5e+09 \$9.5e+08 0.773	\$3.2e+ \$2.4e+ \$2.9e+ 0.995	\$5.7e+10 \$2.5e+09 \$5.4e+10 0.773	\$1e+11 \$2.4e+ \$1e+11 0.995	\$8.8e+ \$1.6e+ \$8.6e+ 0.861	\$9.5e+09 \$4.8e+08 \$9e+09 0.990	\$2.5e+ \$2.5e+ \$5.6e+ 0.773	\$4.8e+08 \$4.8e+08 \$2.9e+06 0.990	\$2.5e+ \$2.5e+ \$1.4e+ 0.773	\$4.8e+08 \$4.8e+08 \$7.4e+05 0.990

	52 -52	53 -59	60 -60	61 -66	67 -69	70 -70	71 -71	72 -76	77 -79	80 -99
0. Movement	1		1		1		1		1	
1. Education (University level)	1		1		1		1		1	
2. Social Gatherings (in a house)	1	1	1	1	1	1	1	1	1	1
3. Non-Food Service (bank,retail, etc)	1	1	1	1	1	1	1	1	1	1
4. Restaurants	1		1		1		1		1	
5. Masking	1		1		1		1		1	
6. Mega Events	1		1		1		1		1	
7. Border Control	1		1		1		1		1	
8. Physical Distancing	1		1		1		1		1	
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	\$2.5e+ \$2.5e+ \$3.6e+ 0.773	\$4.8e+08 \$4.8e+08 \$1.1e+05 0.990	\$2.5e+ \$2.5e+ \$2.3e+ 0.773	\$4.8e+08 \$4.8e+08 \$6.9e+03 0.990	\$2.5e+09 \$2.5e+09 \$4.7 0.773	\$4.8e+ \$4.8e+ \$0.0 0.990	\$2.5e+ \$2.5e+ \$0.0 0.773	\$4.8e+08 \$4.8e+08 \$0.0 0.990	\$2.5e+09 \$2.5e+09 \$0.0 0.773	\$4.8e+08 \$4.8e+08 \$0.0 0.990

(d) *Solution using the index policy described in Section 2.2, using a block size of $b = 1$.*



	0 -6	7 -13	14 -99
0. Movement A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]		1	
1. Education (University level) A: $[0 ,0]·$10^2$ B: $[0 ,0]·$10^2$ C: $[10 ,14]·$10^2$ P: [.99 ,.95]		1	
2. Social Gatherings (in a house) A: $[0 ,0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0 ,0]·$10^2$ C: $[8 ,10 ,12 ,14]·$10^2$ P: [.99 ,.99 ,.97 ,.93]	1	1	1
3. Non-Food Service (bank,retail, etc) A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]		1	
4. Restaurants A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]		1	
5. Masking A: $[0 ,0 ,0]·$10^2$ B: $[0 ,0 ,0]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]		1	
6. Mega Events A: $[2500 ,5000 ,10000]·$10^2$ B: $[5000 ,10000,20000]·$10^2$ C: $[8 ,10 ,14]·$10^2$ P: [.99 ,.95 ,.93]		1	
7. Border Control A: $[5000 ,10000]·$10^2$ B: $[10000,20000]·$10^2$ C: $[10 ,14]·$10^2$ P: [.95 ,.93]		1	
8. Physical Distancing A: $[0]·$10^2$ B: $[0]·$10^2$ C: $[10]·$10^2$ P: [.93]		1	
Cost Per Period: TOTAL Cost Per Period: POLICY Cost Per Period: DISEASE Probability Factor	$8.3e+09 $2.4e+08 $8.1e+09 0.995	$9.5e+10 $2.5e+09 $9.3e+10 0.773	$3.5e+09 $2.4e+08 $3.3e+09 0.995

(e) *Solution using the index policy described in Section 2.2, using a block size of $b = 7$.*

3.2 Lagrangian Heuristic Lower Bound Improvement

The lower bound on the objective value of **(DMO)** obtained by optimizing the (decomposed) Lagrangian relaxation described in Section 2.1 seems relatively tight on several problem instances. The corresponding heuristic also performs quite well. On the other hand, the BARON solver is able to generate solutions within a few minutes to the full **(DMO)** problem that are extremely high quality, but it does not guarantee a reasonable level of optimality even after running for hours. In particular, the BARON solver generates lower bounds as part of its numerical optimization procedure, but these lower bounds are nowhere near the values it obtains. The bounds generated by the Lagrangian procedure prove that these solutions are nearly optimal. This yields a stopping condition for the BARON solver that guarantees a desired level of optimality.

The following trials were considered to evaluate the performance of the Lagrangian method:

Trial	T	cost multiplier	effect multiplier	m	n	nConstraints	nPolicies	nVariables
1	50	1.0	0.5	9	4	2545	51840	3900
2	50	1.0	0.6	9	4	2545	51840	3900
3	50	1.0	0.7	9	4	2545	51840	3900
4	50	1.0	0.8	9	4	2545	51840	3900
5	50	1.0	0.9	9	4	2545	51840	3900
6	50	1.0	1.0	9	4	2545	51840	3900
7	50	1.0	1.1	9	4	2545	51840	3900
8	50	1.0	1.2	9	4	2545	51840	3900
9	50	1.0	1.3	9	4	2545	51840	3900
10	50	1.0	1.4	9	4	2545	51840	3900
11	50	1.0	1.5	9	4	2545	51840	3900
12	50	0.5	1.0	9	4	2545	51840	3900
13	50	0.6	1.0	9	4	2545	51840	3900
14	50	0.7	1.0	9	4	2545	51840	3900
15	50	0.8	1.0	9	4	2545	51840	3900
16	50	0.9	1.0	9	4	2545	51840	3900
17	50	1.1	1.0	9	4	2545	51840	3900
18	50	1.2	1.0	9	4	2545	51840	3900
19	50	1.3	1.0	9	4	2545	51840	3900
20	50	1.4	1.0	9	4	2545	51840	3900
21	50	1.5	1.0	9	4	2545	51840	3900
22	100	1.0	0.5	9	4	5095	51840	7800
23	100	1.0	0.6	9	4	5095	51840	7800
24	100	1.0	0.7	9	4	5095	51840	7800
25	100	1.0	0.8	9	4	5095	51840	7800
26	100	1.0	0.9	9	4	5095	51840	7800
27	100	1.0	1.0	9	4	5095	51840	7800
28	100	1.0	1.1	9	4	5095	51840	7800
29	100	1.0	1.2	9	4	5095	51840	7800
30	100	1.0	1.3	9	4	5095	51840	7800
31	100	1.0	1.4	9	4	5095	51840	7800
32	100	1.0	1.5	9	4	5095	51840	7800
33	100	0.5	1.0	9	4	5095	51840	7800
34	100	0.6	1.0	9	4	5095	51840	7800
35	100	0.7	1.0	9	4	5095	51840	7800
36	100	0.8	1.0	9	4	5095	51840	7800
37	100	0.9	1.0	9	4	5095	51840	7800
38	100	1.1	1.0	9	4	5095	51840	7800
39	100	1.2	1.0	9	4	5095	51840	7800
40	100	1.3	1.0	9	4	5095	51840	7800
41	100	1.4	1.0	9	4	5095	51840	7800
42	100	1.5	1.0	9	4	5095	51840	7800
43	150	1.0	0.5	9	4	7645	51840	11700
44	150	1.0	0.6	9	4	7645	51840	11700
45	150	1.0	0.7	9	4	7645	51840	11700
46	150	1.0	0.8	9	4	7645	51840	11700
47	150	1.0	0.9	9	4	7645	51840	11700
48	150	1.0	1.0	9	4	7645	51840	11700
49	150	1.0	1.1	9	4	7645	51840	11700
50	150	1.0	1.2	9	4	7645	51840	11700
51	150	1.0	1.3	9	4	7645	51840	11700
52	150	1.0	1.4	9	4	7645	51840	11700
53	150	1.0	1.5	9	4	7645	51840	11700
54	150	0.5	1.0	9	4	7645	51840	11700
55	150	0.6	1.0	9	4	7645	51840	11700
56	150	0.7	1.0	9	4	7645	51840	11700
57	150	0.8	1.0	9	4	7645	51840	11700
58	150	0.9	1.0	9	4	7645	51840	11700
59	150	1.1	1.0	9	4	7645	51840	11700
60	150	1.2	1.0	9	4	7645	51840	11700
61	150	1.3	1.0	9	4	7645	51840	11700
62	150	1.4	1.0	9	4	7645	51840	11700
63	150	1.5	1.0	9	4	7645	51840	11700

(a) *Trials. The* `cost multiplier` *column multiplies all costs in the matrices* A, B, *and* C *(*setup, switching, *and* per-individual *costs) by the same factor. An entry* μ *in the* `effect multiplier` *column alters the intervention effectiveness as* $P_{ijt} \mapsto 1 - \mu \cdot (1 - P_{ijt})$ *for all* i, j, t.

Trial	no policy obj	solver obj	lagr heuristic obj	simple index blocksize 1 obj	simple index blocksize 7 obj	quadratic heuristic obj	lagr LB	solver LB	solver vs lagr lb gap	lagr vs lagr lb gap
1	1003570	1003408	1003570	1037173	1015196	1110367	961654	444081	0.043419	0.043588
2	1003570	999901	1003570	1105527	1015169	1102909	945001	351140	0.058095	0.061978
3	1003570	991941	1014902	1107608	1015141	1090845	983990	279286	0.008079	0.031415
4	1003570	980161	983218	1101752	1018725	1072228	968056	224625	0.012504	0.015662
5	1003570	965127	973238	1098135	1016878	1045926	911848	268957	0.058429	0.067324
6	1003570	946181	950248	1055382	1015010	1011130	924127	216591	0.023865	0.028267
7	1003570	922615	933689	1066852	1013141	968276	904137	223650	0.020438	0.032686
8	1003570	890871	897466	1022575	1011291	915973	881851	183291	0.010230	0.017708
9	1003570	843094	852618	1030038	1009478	850878	801300	197698	0.052158	0.064044
10	1003570	761036	766913	1070425	1007723	767153	735296	173682	0.035007	0.042999
11	1003570	649849	654125	1087891	1006042	654365	633927	138789	0.025116	0.031861
12	1003570	901317	930244	1010834	1001350	922533	890789	188454	0.011819	0.044293
13	1003570	911541	948232	1050540	1004082	940252	907164	194336	0.004825	0.045271
14	1003570	921155	946571	1023643	1006814	957971	915804	200036	0.005843	0.033596
15	1003570	930129	954897	1069797	1009546	975691	921255	205640	0.009633	0.036517
16	1003570	938381	956741	1052218	1012278	993410	926190	211217	0.013162	0.032986
17	1003570	953493	955877	1108700	1017743	1028850	942268	221925	0.011913	0.014443
18	1003570	960221	962801	1122946	1020475	1046571	951237	225047	0.009445	0.012157
19	1003570	966376	1021851	1114037	1018587	1064291	945983	230199	0.021558	0.080200
20	1003570	972198	979469	1102247	1019763	1082012	926299	235175	0.049551	0.057401
21	1003570	977641	982747	1141747	1020939	1099733	948020	239960	0.031245	0.036631
22	1003571	1003408	1003571	1061186	1027196	1127720	961662	444083	0.043410	0.043579
23	1003571	999904	1003571	1228629	1027169	1155374	945021	351143	0.058076	0.061956
24	1003571	991956	1003571	1230709	1027142	1145630	964979	279288	0.027956	0.039993
25	1003571	980221	1003571	1224854	1030726	1133770	960364	224626	0.020677	0.044990
26	1003571	965398	968947	1221237	1028880	1118546	947193	268971	0.019220	0.022966
27	1003571	947444	953342	1075636	1027013	1100887	934880	216595	0.013439	0.019748
28	1003571	927554	939339	1034417	1025144	1083693	888625	223583	0.043808	0.057071
29	1003571	908069	912756	1036022	1023294	1058986	899800	183142	0.009189	0.014399
30	1003571	891356	905198	1042040	1021482	1025793	872404	197119	0.021723	0.037590
31	1003571	876645	895689	1067327	1019727	976110	842901	186372	0.040033	0.062626
32	1003571	846988	896310	1213796	1018047	908013	834801	170979	0.014599	0.073681
33	1003571	903818	911961	1034292	1007353	960433	889297	188448	0.016330	0.025486
34	1003571	913842	919717	1124403	1011284	954720	905286	194325	0.009451	0.015940
35	1003571	923218	937126	1032044	1015216	1024216	886740	200034	0.041137	0.056822
36	1003571	931812	943920	1037312	1019148	1049067	898121	205630	0.037512	0.050994
37	1003571	939831	948139	1105014	1023080	1074879	924700	211194	0.016363	0.025347
38	1003571	954476	960878	1053120	1030945	1126646	937773	221925	0.017811	0.024638
39	1003571	960956	969301	1270667	1034877	1150722	940653	227191	0.021584	0.030455
40	1003571	966937	976852	1141517	1034188	1176112	943533	230205	0.024804	0.035313
41	1003571	972680	1003571	1184833	1036564	1202540	944757	235183	0.029556	0.062253
42	1003571	977981	1003571	1206736	1038940	1226912	944757	239970	0.035167	0.062253
43	1003571	1003408	1003571	1341440	1039196	1189874	961662	444042	0.043410	0.043579
44	1003571	999904	1003571	1351729	1039169	1206374	945021	351106	0.058076	0.061956
45	1003571	991956	1003571	1353809	1039142	1128547	954816	279257	0.038897	0.051062
46	1003571	980221	1003571	1347954	1042726	1184770	949752	224601	0.032082	0.056666
47	1003571	965399	1003571	1344337	1040880	1169546	928549	266958	0.039686	0.080795
48	1003571	947445	955634	1251489	1039013	1152133	916247	216548	0.034050	0.042987
49	1003571	927571	933491	1046417	1037144	1141654	903742	223534	0.026367	0.032917
50	1003571	908329	914037	1048022	1035294	1121826	890336	183102	0.020209	0.026620
51	1003571	892906	926871	1054040	1033482	1105064	884161	197041	0.009890	0.048305
52	1003571	881485	922966	1088577	1031727	1085665	871484	184622	0.011477	0.059074
53	1003571	872342	876923	1336896	1030047	1051158	858490	180112	0.016135	0.021471
54	1003571	903823	917708	1048614	1013353	997580	883151	188402	0.023407	0.039129
55	1003571	913845	939111	1198263	1018484	1028730	886446	194280	0.030909	0.059412
56	1003571	923220	939779	1040444	1023616	1059832	912176	199987	0.012108	0.030261
57	1003571	931811	935130	1046912	1028748	1090584	913021	204117	0.020580	0.024216
58	1003571	939832	945382	1081630	1033880	1121588	916247	209486	0.025741	0.031798
59	1003571	954476	965553	1381315	1044145	1182746	916247	219830	0.041724	0.053814
60	1003571	960955	975663	1418587	1049277	1212858	916247	227144	0.048796	0.064848
61	1003571	966938	983675	1157117	1049788	1243816	916247	230160	0.055325	0.073592
62	1003571	972680	990506	1325817	1053364	1273940	964737	235135	0.008234	0.026712
63	1003571	977981	998387	1260468	1056940	1211277	972599	239922	0.005534	0.026515

(b) *Accuracy results.* “lagr des L1 optGap*” : 0.03;* “lagr des L2 optGap*” : 0.07;* “lagr des optGap*” : 0.07;* “solver des optGap*” : 0.80;* “solver optGap*” : 0.80;* “lagr optGap*” : 0.00.*

Trial	solver timeToSolve	lagr timeToSolve	lagr timeToSolve L1 total	lagr timeToSolve L2 total
1	8.398634	5	0	1
2	8.973994	6	0	2
3	9.733989	19	3	9
4	8.982089	47	7	29
5	13.732315	70	5	54
6	13.609667	216	12	187
7	17.467292	694	10	670
8	23.765403	1356	11	1330
9	28.227027	946	7	928
10	28.890617	1031	7	1014
11	28.895946	1032	7	1015
12	15.408351	120	7	102
13	15.410255	127	7	110
14	14.860375	122	7	104
15	14.098879	128	7	110
16	13.730874	118	7	101
17	13.480146	210	12	181
18	16.466777	221	13	191
19	13.799418	195	10	171
20	14.160882	104	7	86
21	12.900917	143	7	125
22	26.435938	13	1	2
23	26.251833	15	1	4
24	39.744782	51	10	21
25	34.770565	69	10	38
26	42.216477	163	21	111
27	49.625622	232	20	182
28	65.323831	480	20	430
29	55.986864	1143	26	1076
30	103.364297	1084	22	1031
31	114.223831	1099	20	1048
32	125.471774	1651	30	1580
33	45.248591	223	20	172
34	44.999007	208	20	157
35	49.058455	196	15	151
36	45.373336	241	20	191
37	43.448731	216	21	164
38	51.251599	246	20	196
39	44.021137	248	20	198
40	47.395019	250	20	199
41	46.238122	142	9	112
42	50.300502	191	9	161
43	53.596585	26	1	3
44	59.828190	33	1	11
45	62.691758	87	20	25
46	60.002771	114	20	51
47	94.457674	149	21	86
48	110.508118	353	42	247
49	136.124612	785	43	679
50	116.871675	1148	43	1041
51	197.014625	2242	87	2047
52	191.785715	1397	57	1255
53	225.999437	1897	80	1710
54	84.762230	282	32	182
55	91.699221	360	43	253
56	99.982142	467	43	359
57	88.157391	314	45	206
58	85.643443	351	42	245
59	121.267694	350	43	244
60	101.093129	366	45	258
61	88.394640	354	43	246
62	119.876944	535	63	387
63	98.191028	602	68	449

(c) *Timing results*

Trial	no policy cost	no policy deaths	solver cost	solver deaths	solver disease cost	solver policy cost	lagr cost	lagr deaths	lagr disease cost	lagr policy cost
1	1.00e12	9.94e04	1.00e12	9.93e04	1.00e12	9.00e08	1.00e12	9.94e04	1.00e12	0.00e00
2	1.00e12	9.94e04	1.00e12	9.64e04	9.73e11	2.66e10	1.00e12	9.94e04	1.00e12	0.00e00
3	1.00e12	9.94e04	9.92e11	9.46e04	9.55e11	3.71e10	1.01e12	9.91e04	1.00e12	1.47e10
4	1.00e12	9.94e04	9.80e11	9.23e04	9.32e11	4.80e10	9.83e11	9.35e04	9.44e11	3.90e10
5	1.00e12	9.94e04	9.65e11	8.96e04	9.04e11	6.10e10	9.73e11	9.27e04	9.36e11	3.70e10
6	1.00e12	9.94e04	9.46e11	8.61e04	8.69e11	7.72e10	9.50e11	8.85e04	8.94e11	5.64e10
7	1.00e12	9.94e04	9.23e11	8.21e04	8.28e11	9.42e10	9.34e11	8.54e04	8.62e11	7.20e10
8	1.00e12	9.94e04	8.91e11	7.65e04	7.72e11	1.19e11	8.97e11	7.96e04	8.04e11	9.34e10
9	1.00e12	9.94e04	8.43e11	6.74e04	6.80e11	1.63e11	8.53e11	6.67e04	6.73e11	1.80e11
10	1.00e12	9.94e04	7.61e11	5.85e04	5.90e11	1.71e11	7.67e11	5.82e04	5.87e11	1.80e11
11	1.00e12	9.94e04	6.50e11	4.72e04	4.76e11	1.73e11	6.54e11	4.70e04	4.75e11	1.80e11
12	1.00e12	9.94e04	9.01e11	8.40e04	8.48e11	5.35e10	9.30e11	8.40e04	8.48e11	8.20e10
13	1.00e12	9.94e04	9.12e11	8.45e04	8.53e11	5.85e10	9.48e11	8.42e04	8.50e11	9.82e10
14	1.00e12	9.94e04	9.21e11	8.47e04	8.55e11	6.58e10	9.47e11	9.00e04	9.08e11	3.83e10
15	1.00e12	9.94e04	9.30e11	8.53e04	8.61e11	6.92e10	9.55e11	9.05e04	9.13e11	4.17e10
16	1.00e12	9.94e04	9.38e11	8.59e04	8.67e11	7.09e10	9.57e11	9.12e04	9.21e11	3.62e10
17	1.00e12	9.94e04	9.53e11	8.68e04	8.77e11	7.69e10	9.56e11	8.86e04	8.94e11	6.19e10
18	1.00e12	9.94e04	9.60e11	8.76e04	8.84e11	7.61e10	9.63e11	8.91e04	8.99e11	6.35e10
19	1.00e12	9.94e04	9.66e11	8.81e04	8.90e11	7.66e10	1.02e12	8.49e04	8.57e11	1.65e11
20	1.00e12	9.94e04	9.72e11	8.85e04	8.93e11	7.89e10	9.79e11	9.25e04	9.34e11	4.59e10
21	1.00e12	9.94e04	9.78e11	8.92e04	9.01e11	7.69e10	9.83e11	9.25e04	9.34e11	4.91e10
22	1.00e12	9.94e04	1.00e12	9.93e04	1.00e12	9.00e08	1.00e12	9.94e04	1.00e12	0.00e00
23	1.00e12	9.94e04	1.00e12	9.64e04	9.73e11	2.66e10	1.00e12	9.94e04	1.00e12	0.00e00
24	1.00e12	9.94e04	9.92e11	9.46e04	9.55e11	3.71e10	1.00e12	9.94e04	1.00e12	0.00e00
25	1.00e12	9.94e04	9.80e11	9.23e04	9.32e11	4.80e10	1.00e12	9.94e04	1.00e12	0.00e00
26	1.00e12	9.94e04	9.65e11	8.97e04	9.06e11	5.94e10	9.69e11	8.84e04	8.92e11	7.66e10
27	1.00e12	9.94e04	9.47e11	8.66e04	8.75e11	7.27e10	9.53e11	8.58e04	8.66e11	8.76e10
28	1.00e12	9.94e04	9.28e11	8.34e04	8.42e11	8.60e10	9.39e11	8.69e04	8.77e11	6.26e10
29	1.00e12	9.94e04	9.08e11	8.03e04	8.10e11	9.76e10	9.13e11	8.27e04	8.35e11	7.80e10
30	1.00e12	9.94e04	8.91e11	7.76e04	7.84e11	1.08e11	9.05e11	8.29e04	8.37e11	6.84e10
31	1.00e12	9.94e04	8.77e11	7.37e04	7.44e11	1.33e11	8.96e11	8.23e04	8.31e11	6.49e10
32	1.00e12	9.94e04	8.47e11	5.90e04	5.96e11	2.51e11	8.96e11	6.04e04	6.10e11	2.86e11
33	1.00e12	9.94e04	9.04e11	8.44e04	8.52e11	5.18e10	9.12e11	8.63e04	8.72e11	4.03e10
34	1.00e12	9.94e04	9.14e11	8.48e04	8.56e11	5.83e10	9.20e11	8.64e04	8.72e11	4.76e10
35	1.00e12	9.94e04	9.23e11	8.52e04	8.60e11	6.31e10	9.37e11	8.81e04	8.89e11	4.82e10
36	1.00e12	9.94e04	9.32e11	8.57e04	8.66e11	6.63e10	9.44e11	8.81e04	8.89e11	5.50e10
37	1.00e12	9.94e04	9.40e11	8.61e04	8.69e11	7.09e10	9.48e11	8.81e04	8.89e11	5.92e10
38	1.00e12	9.94e04	9.54e11	8.71e04	8.80e11	7.47e10	9.61e11	8.60e04	8.68e11	9.26e10
39	1.00e12	9.94e04	9.61e11	8.78e04	8.86e11	7.46e10	9.69e11	8.60e04	8.68e11	1.01e11
40	1.00e12	9.94e04	9.67e11	8.82e04	8.90e11	7.66e10	9.77e11	8.60e04	8.68e11	1.08e11
41	1.00e12	9.94e04	9.73e11	8.88e04	8.97e11	7.61e10	1.00e12	9.94e04	1.00e12	0.00e00
42	1.00e12	9.94e04	9.78e11	8.94e04	9.03e11	7.50e10	1.00e12	9.94e04	1.00e12	0.00e00
43	1.00e12	9.94e04	1.00e12	9.93e04	1.00e12	9.00e08	1.00e12	9.94e04	1.00e12	0.00e00
44	1.00e12	9.94e04	1.00e12	9.64e04	9.73e11	2.66e10	1.00e12	9.94e04	1.00e12	0.00e00
45	1.00e12	9.94e04	9.92e11	9.46e04	9.55e11	3.71e10	1.00e12	9.94e04	1.00e12	0.00e00
46	1.00e12	9.94e04	9.80e11	9.23e04	9.32e11	4.80e10	1.00e12	9.94e04	1.00e12	0.00e00
47	1.00e12	9.94e04	9.65e11	8.97e04	9.06e11	5.94e10	1.00e12	9.94e04	1.00e12	0.00e00
48	1.00e12	9.94e04	9.47e11	8.66e04	8.75e11	7.27e10	9.56e11	8.45e04	8.53e11	1.03e11
49	1.00e12	9.94e04	9.28e11	8.34e04	8.42e11	8.56e10	9.33e11	8.16e04	8.24e11	1.09e11
50	1.00e12	9.94e04	9.08e11	8.06e04	8.14e11	9.44e10	9.14e11	7.90e04	7.97e11	1.17e11
51	1.00e12	9.94e04	8.93e11	7.89e04	7.97e11	9.61e10	9.27e11	8.34e04	8.42e11	8.47e10
52	1.00e12	9.94e04	8.81e11	7.80e04	7.88e11	9.38e10	9.23e11	7.82e04	7.89e11	1.33e11
53	1.00e12	9.94e04	8.72e11	7.73e04	7.80e11	9.19e10	8.77e11	7.84e04	7.91e11	8.55e10
54	1.00e12	9.94e04	9.04e11	8.44e04	8.52e11	5.18e10	9.18e11	8.69e04	8.77e11	4.07e10
55	1.00e12	9.94e04	9.14e11	8.48e04	8.56e11	5.83e10	9.39e11	8.74e04	8.82e11	5.67e10
56	1.00e12	9.94e04	9.23e11	8.52e04	8.60e11	6.31e10	9.40e11	8.47e04	8.55e11	8.46e10
57	1.00e12	9.94e04	9.32e11	8.57e04	8.66e11	6.63e10	9.35e11	8.45e04	8.53e11	8.20e10
58	1.00e12	9.94e04	9.40e11	8.61e04	8.69e11	7.09e10	9.45e11	8.45e04	8.53e11	9.23e10
59	1.00e12	9.94e04	9.54e11	8.71e04	8.80e11	7.47e10	9.66e11	8.45e04	8.53e11	1.12e11
60	1.00e12	9.94e04	9.61e11	8.79e04	8.87e11	7.41e10	9.76e11	8.45e04	8.54e11	1.22e11
61	1.00e12	9.94e04	9.67e11	8.82e04	8.90e11	7.66e10	9.84e11	8.47e04	8.55e11	1.29e11
62	1.00e12	9.94e04	9.73e11	8.88e04	8.97e11	7.61e10	9.91e11	8.72e04	8.80e11	1.10e11
63	1.00e12	9.94e04	9.78e11	8.94e04	9.03e11	7.50e10	9.98e11	8.72e04	8.80e11	1.18e11

(d) *System outcome results*

3.2.1 Summaries of Results

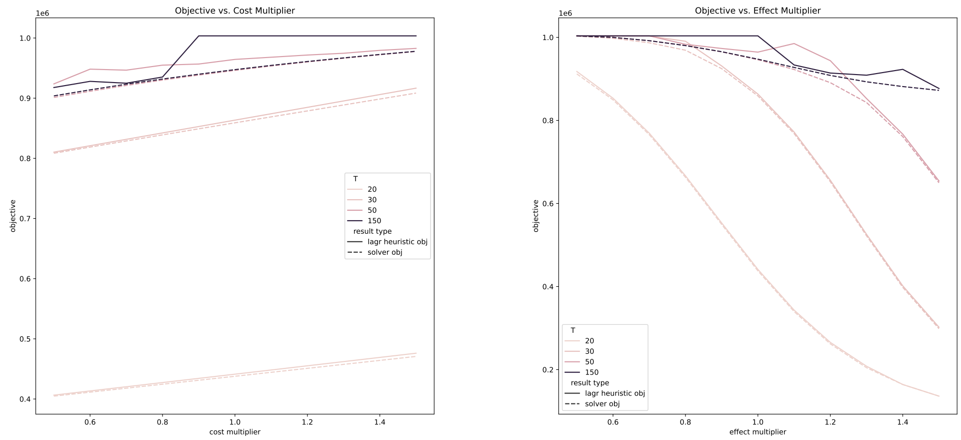


Figure 5: *Objective value vs. factor by which policy costs are all changed; objective vs. factor by which policy effectivenesses are all changed.*

3.3 Lagrangian Subproblem Quasiconvexity

The subproblem $L2(\lambda)$ defined in Section 2.1 has no integer constraints, but it still takes considerable time to solve with the BARON numerical solver. If the problem has suitable structure, such as quasiconvexity, a basic gradient descent algorithm might suffice for part of its solution. In particular, we examined whether $L2(\lambda)$ is quasiconvex in $P_t, t = 1, \ldots, T$.

A function $f(\boldsymbol{x})$ is quasiconvex if and only if for any two values $\boldsymbol{x}_1$, and $\boldsymbol{x}_2$ in its domain, the function of one variable $V(\theta) = f((1-\theta)\boldsymbol{x}_1 + \theta\boldsymbol{x}_2)$ is quasiconvex for values of $\theta \in [0, 1]$.

To probe this necessary and sufficient condition for quasiconvexity in P, we define the function $V_{P^1,P^2} : [0, 1] \to \mathbb{R}$ for any values P^1, P^2 where P^1_t and P^2_t are fixed for all $t = 1, \ldots, T$. This function is such that $V_{P^1,P^2}(\theta)$ is the optimal value of $L2(\lambda)$ where $P_t = (1-\theta)P^1_t + \theta P^2_t$ for all $t = 1, \ldots, T$.

The following plots illustrate the value of $V_{P^1,P^2}(\theta)$ for $\theta \in [0, 1]$. If the curves appear quasiconvex, then a necessary condition is met for $L2(\lambda)$ being quasiconvex in P. In each trial, the values $P^1_t, P^2_t, t = 1, \ldots, T$ were generated in the interval $[0, 1]$ uniformly randomly[4].

[4]In fact, for this model P_t cannot be equal to 0, because of its definition in the full model as a product of "intervention effectiveness" factors. So, the lowest value it can possibly take on is the product of the effectiveness factors of *all* possible interventions. Both in the full model and in this investigation of quasiconvexity, values of P_t are constrained to be in the interval $[P_{lb}, 1]$, where P_{lb} is this small value. Since the logarithm of P_t is part of the model, the domain becomes effectively open (the objective is undefined at 0), this adjustment improves performance of the solver by giving a closed domain without loss of generality of solutions. The restriction of the domain for this quasiconvexity test is also without loss of generality, because for the purposes of this model, only values of P_t in the constrained interval are relevant.

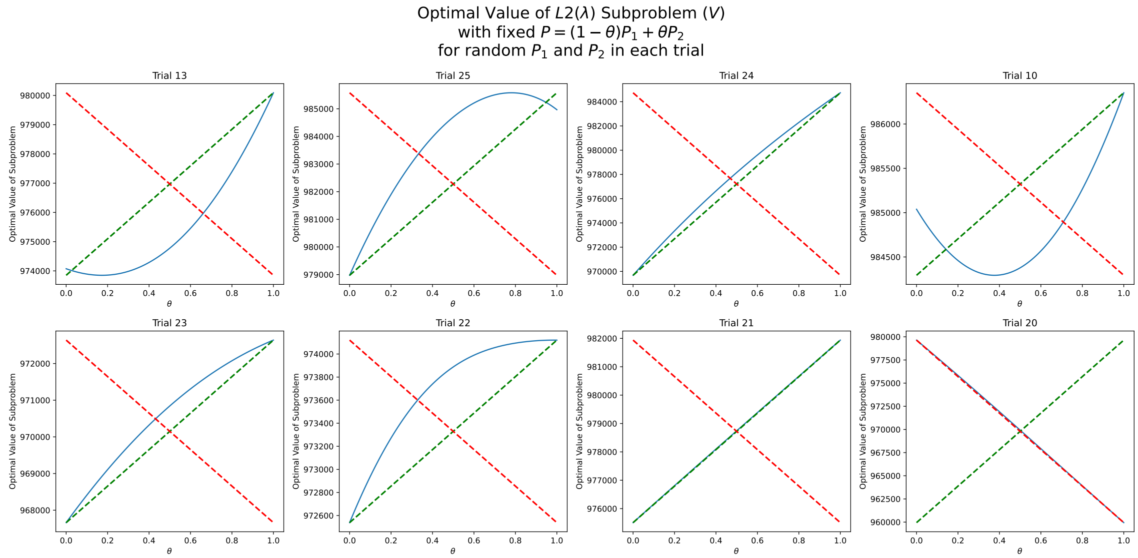


Figure 6: *Optimal value of $L2(\lambda)$ with P fixed to values that vary along a line; that is, $V_{P^1,P^2}(\theta)$ vs θ for several randomly-selected values of P^1 and P^2.*

As can be seen in Figure 6, the function V appears to be neither quasiconvex nor quasiconcave.

To probe the possibility that the function V is not jointly quasiconvex in the P_t variables, but is componentwise quasiconvex in each $P_t, t = 1, \ldots, P$, we can investigate whether the function $V_{P^1,P^2}(\theta)$ is quasiconvex for any P^1, P^2 such that the line segment connecting P^1 and P^2 is parallel to an axis P_t (for some t), i.e. by repeating the same experiment but only varying one P_t at a time.

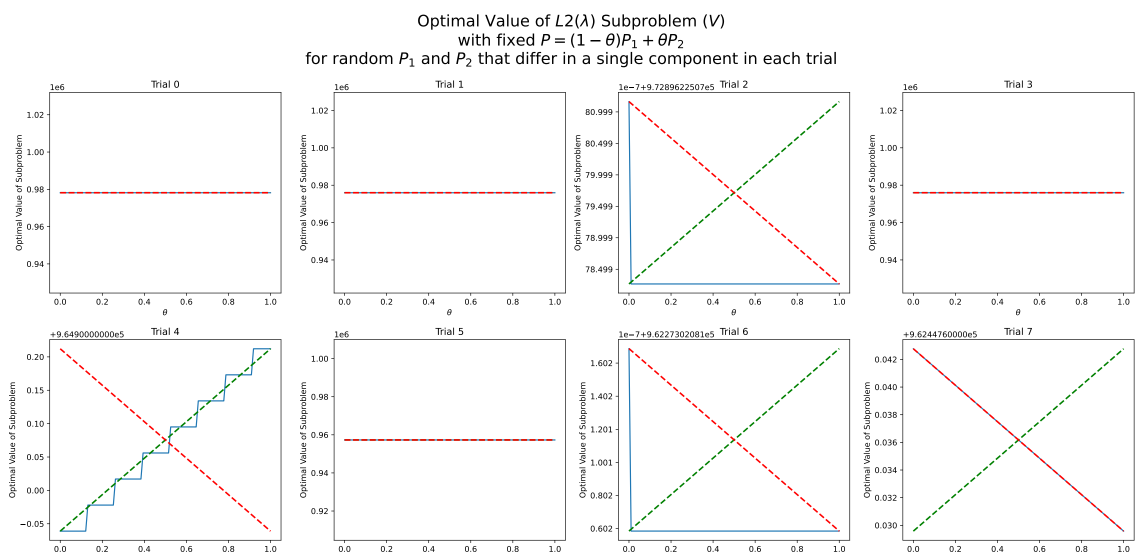


Figure 7: *The same experiment was repeated as is illustrated in Figure 6, but by only varying a single P_t component at a time.*

Based on the plots in Figure 7, it is inconclusive whether V is componentwise quasiconcave or quasiconvex. The variation in the objective of the $L2(\lambda)$ subproblem may be small with respect to any one component P_t.

4 Conclusion

References