Problem Set

September 19, 2019

1 Assignment 1

Ordinary least squares (OLS) is one of the oldest and simplest tools available to the econometrician. If we have a linear relationship between random variables X and Y, represented by:

$$Y = X\beta + \epsilon,$$

OLS provides the following estimates for the slope coefficients

$$\hat{\beta} = (X'X)^{-1}X'Y.$$

In this exercise, consider the stochastic equation model

$$y = 10x + 5 + \epsilon,$$

$$\epsilon \sim N(0, 1).$$

Note that this equation can equivalently be written in matrix form as follows:

$$y = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix} + \epsilon.$$

To be consistent with our formula for the OLS estimator, x and y are column vectors of identical length containing observations generated by the above process, 1 is a column vector of ones the same length as x and y, and ϵ is column vector of residuals the same length as x and y.

Create an M-File that starts by generating 100 artificial observations of this process. To do this, let x take on the values 0.01, 0.02,...,1.00, draw corresponding values for ϵ at random from the standard normal distribution, and compute the implied vector y. You should now have a column vector containing values of x, a column vector containing values of y, and additionally should create an equally sized column vector consisting of ones. Concatenate the vector of ones with the vector x to form the matrix X referred to in the single equation model above. Now compute the OLS estimator $\hat{\beta}$ using the formula listed above. The result should be a vector where the first element corresponds to the intercept parameter and the second element corresponds to the slope.

Plot the artificial observations along with the OLS regression line $y = X\hat{\beta}$ in a single figure with an appropriate title, a legend, and axes labels. Finally, compute the 100 OLS residuals defined as $e = y - X\hat{\beta}$ and display them in a histogram. Remember to document your code.

1.1 Extra

Consider now the following equation

$$y = 10x + 30z + 5 + \epsilon,$$

Repeat the steps above, where x takes on the values 0.01, 0.02,...,1.00 and z takes on the values 1.01, 1.02,...,2.00 and $X = [1xz]$.

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What do you observe? Did you expect this?

2 Assignment 2

Consider a competitive firm that produces according to a Cobb-Douglas production technology. The firm uses as inputs capital, k, and labor, l, and takes their market prices r and w, respectively, as given. The profit function, assuming the relative price of the firm's output is 1, is then:

$$\Pi(k, l) = Ak^{\theta}l^{1-\theta} - rk - wl.$$

where θ and A are known constants. Assume that the firm's capital is fixed at a constant k = K. The firm's profit is then only a function of l:

$$\Pi(l) = AK^{\theta}l^{1-\theta} - rK - wl.$$

The first order condition for maximizing the firm's profit with respect to l implies that the firm should choose l such that its marginal product of labor is equal to the wage, or

$$(1 - \theta)Al^{-\theta}K^{\theta} = w.$$

(a) Assume that $A = 30$, $K = 20$, $\theta = 1/3$, $r = 1$, and $w = 10$. Create an in-line function that takes l as an argument and returns the difference between the marginal product of labor and the wage. With the above parameters write a script that plots the output of the function you created in Part for various values of l, along with the corresponding value of Π.

(b) Compute the profit maximizing value of l in two different ways:

1. Using `fzero` and then `fsolve` to find the zero of the first order condition. Is the solution unique?
2. By defining an anonymous function that returns the negative of the profit for any value of ; and then using fminunc to find the minimum of this new function.

(c) Create a function that calculate the optimal amount of labor, along with maximum value of profits as a function of the parameter w. Create a plot to show how do optimal labor and profits respond to w.

3 Assignment 3:

Plot the integrand function $H(x) = \int_0^x f(z)dz$, for x in $[0, \pi]$, where:

- $f(z) = 3z^2 + 2z$
- $f(z) = sin(z) + exp(-z^2)$

1. Calculate the integral analytically (you can use built in functions in MATLAB).
2. Calculate the integral numerically, using one of the quadrature formulas, you may use the algorithm we used in class.
3. Evaluate the difference of the two approaches.

Hint: `help erf`

4 Assignment 4:

Let time be denoted as $t = 0, 1, 2, ..., T$, where T is a finite number. Given an initial level of capital $\overline{k}_0$, the benevolent social planner chooses sequences of capital $\{k_t\}_{t=1}^T$ and consumption $\{c_t\}_{t=0}^T$ to maximize the discounted present value of a representative agent's utility subject to a sequence of resource constraints:

$$\max_{\{k_t\}_{t=1}^T, \{c_t\}_{t=0}^T} \sum_{t=0}^{T} \beta^t \log(c_t)$$

subject to

$$k_{t+1} = Ak_t^\alpha + (1-\delta)k_t - c_t \text{ for } t = 0, 1, 2, 3..., T$$

and $\overline{k}_{T+1}$, the terminal value of capital, is also given.

The first order necessary conditions for this problem reduce to the following set of equations:

$$0 = [Ak_{t+1}^\alpha + (1-\delta)k_{t+1} - k_{t+2}] - \beta[Ak_t^\alpha + (1-\delta)k_t - k_{t+1}]\left(\alpha Ak_{t+1}^{\alpha-1} + (1-\delta)\right), \forall t = 0, 1, 2..., T$$

We can think of these first order conditions as a list of second order difference equations of the form:

$$0 = g(\overline{k}_0, k_1, k_2)$$

$$0 = g(k_t, k_{t+1}, k_{t+2}), \forall T - 1 > t > 0$$

$$0 = g(k_{T-1}, k_T, \overline{k}_{T+1})$$

This is a system of T equations in T unknowns $\{k_t\}_{t=1}^T$. Denote $\{k_t^*\}_{t=1}^T$ as the solution to this system of equations.

We will solve now for the dynamic path of capital.

Steps:

- Set the time horizon to be $T = 100$.
- Set $\overline{k}_{T+1} = k_{ss}$ and $k_0 = 0.1k_{ss}$.

- Implement the following solution method known as the "shooting" algorithm:
  - Guess a value k_1^0, for the value of capital in the 1st period k_1.
  - Create a function $k_{t+2} = f(k_{t+1}, k_t)$, which is the solution of:

$$0 = [Ak_{t+1}^{\alpha} + (1-\delta)k_{t+1} - k_{t+2}] - \beta[Ak_t^{\alpha} + (1-\delta)k_t - k_{t+1}]\left(\alpha Ak_{t+1}^{\alpha-1} + (1-\delta)\right)$$

  - Solve for k_2^0 to k_{t+1}^0 using this function.
  - Given that we have a solution for the value of our capital in the last period, compare k_{T+1}^0 with $\overline{k}_{T+1}$. If the two are reasonably close, you are done. If not, make a new guess for k_1 as follows: If the difference $k_{T+1}^0 > \overline{k}_{T+1}$, pick $k_1^1 < k_1^0$. Otherwise, pick $k_1^1 > k_1^0$.
  - Iterate on this procedure until $k_{T+1}^0 - \overline{k}_{T+1} \approx 0$.

  Tips:

- use a while loop,
- if you are having trouble converging still, try using a **slow updating procedure** for k_1.
- Repeat this exercise for $T = 100, 500, 1000$. Check any problems that may appear when you solve for the solution in this way?
- Use the difference $k_{T+1}^0 - \overline{k}_{T+1}$, between the terminal value of your guess and the steady state to update your guess, in a single line of code.

5 Due Date: September 22nd

You may skip any extra exercises indicated in the problem set.

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