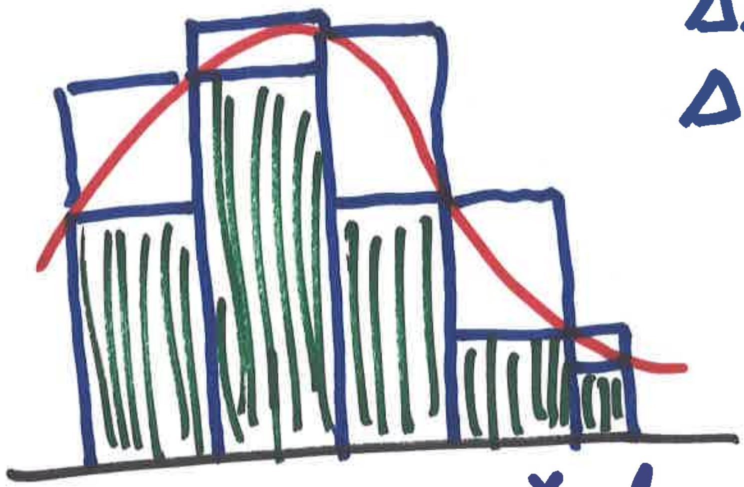
INTEGRALNO IZRAČUNAVAMO I DEFINIRAMO VELIČINA

Δx - RAZDIOBA $[a, b\rangle$:

$\Delta x_i = \Delta x$

$\Delta x_n = \Delta x' = b - x_n$



$D_f(\Delta x) \leq G_f(\Delta x)$

$$D_f(\Delta x) := m_1 \Delta x + m_2 \Delta x + \dots + m_n \Delta x'$$

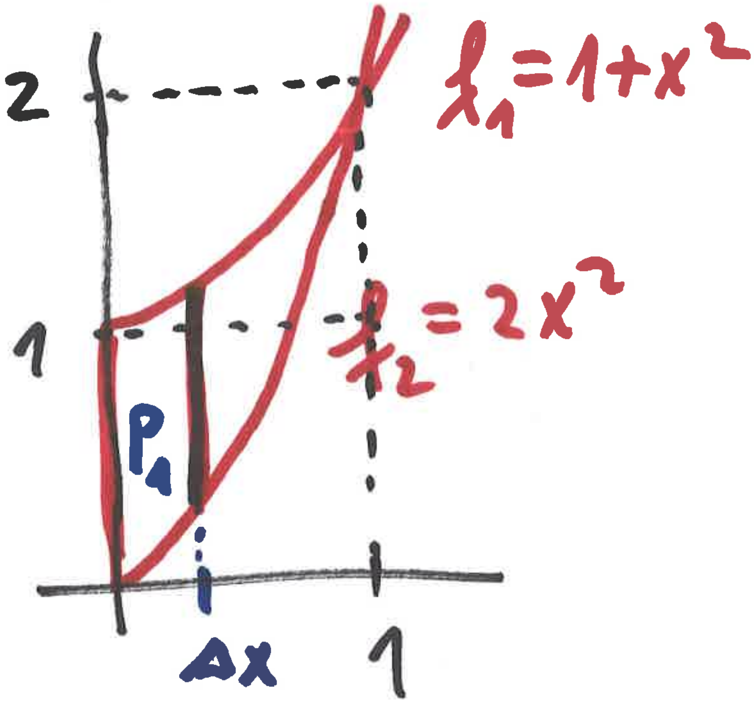
$$G_f(\Delta x) := M_1 \Delta x + M_2 \Delta x + \dots + M_n \Delta x'$$

$$\int_a^b f(x)\,dx := \lim_{\Delta x \to 0} D_f(\Delta x) = \lim_{\Delta x \to 0} G_f(\Delta x)$$

$V = \int_a^b f(x)\,dx$ AKO JE $(\forall \Delta x)$

$$m_i \Delta x \leq V_i \leq M_i \Delta x$$

$$\& \ \sum V_i = V$$



KAKO OPRAVDATI:

$$P = \int_0^1 \underbrace{(f_1 - f_2)}_{f}\, dx \; ?$$

$$D_f(\Delta x) = (1 - \Delta x^2)\Delta x + \ldots$$

$$G_f(\Delta x) = 1 \cdot \Delta x + \ldots$$

$$\boxed{P = \int_0^1 f\, dx \quad \text{AKO JE } (\forall \Delta x) \quad m_i \Delta x \le P_i \le M_i \Delta x}$$

NI ZA JEDAN Δx NIJE

$$(1 - \Delta x^2)\Delta x \le P_1 \le 1 \cdot \Delta x$$

IZRAČUNAVANJE

TEOREM (DUHAMEL 1856):

AKO JE FUNKC. f NEPR. NA $[a,b]$ I AKO JE VELIČINA V ZBROJ VELIČINA V_i TAKVIH DA JE

$$\left| f(x_i)\Delta x_i - V_i \right| \le g(\Delta x)\cdot \Delta x_i$$

ZA FUNKCIJU $g(\Delta x)$ TAKVU DA JE

$$\lim_{\Delta x \to 0} g(\Delta x) = 0$$

ONDA JE $V = \int_a^b f(x)\,dx$.

DOKAZ:

$$\left| \sum_1^n f(x_i)\Delta x_i - V \right| \le \sum_1^n \left| f(x_i)\Delta x_i - V_i \right|$$

$$\le \sum_1^n g(\Delta x)\Delta x_i = g(\Delta x)(b-a) \to 0$$

$$\Longrightarrow V = \int_a^b f(x)\,dx$$

KOD DUHAMELA JE PRETPOSTAVLJENO POSTOJANJE VELIČINE V, A DOKAZUJE SE KAKO SE ONA IZRAČUNAVA ODGOVARAJUĆIM INTEGRALOM.

AKO VELIČINU V TEK KANIMO DEFINIRATI KAO INTEGRAL:

TEOREM (OSGUD 1903, TAYLOR 1955)

AKO JE f NEPR. NA $[a,b]$ I AKO SVAKOM INTERVALU $[x_i, x_{i+1}]$ IZ Δx-RAZDIOBE MOŽEMO PRIDR. $J_i(\Delta x)$ TAKAV

$$|J_i(\Delta x)| \leq g(\Delta x) \cdot \Delta x \ \& \ f(x_i)\Delta x_i \in J_i(\Delta x)$$

$$\& \lim_{\Delta x \to 0} g(\Delta x) = 0$$

ONDA ZA SVAKI IZBOR $V_i(\Delta x) \in J_i(\Delta x)$

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} V_i(\Delta x) = \int_a^b f(x)\,dx$$

(PA STOGA $V := \int_a^b f(x)\,dx$)

DOKAZ:

(ZA $\Delta x \leq 1$)

$$G(\Delta x) := \left| \sum_{1}^{n} f(x_i)\Delta x_i - \int_a^b f(x)\,dx \right|$$

$$\boxed{\left| \sum_{1}^{n} V_i(\Delta x) - \int_a^b f(x)\,dx \right|} \leq$$

$$\left| \sum_{1}^{n} V_i(\Delta x) - \sum_{1}^{n} f(x_i)\Delta x_i + \sum_{1}^{n} f(x_i)\Delta x_i - \int_a^b f(x)\,dx \right|$$

$$\leq \sum_{1}^{n} \left| V_i(\Delta x) - f(x_i)\Delta x_i \right| + G(\Delta x) \leq$$

OBA U $J_i(\Delta x)$

$$\leq \sum_{1}^{n} g(\Delta x)\cdot \Delta x + G(\Delta x) \leq$$

$$\leq (b+1-a)\Delta x + G(\Delta x) \xrightarrow[\Delta x \to 0]{} 0$$

GDJE JE TU N-L FORMULA ?

ONA JE POSLJEDICA DUHAMELA :

$$F'(x)=f(x) \text{ NA } [a,b] \Longrightarrow$$

$$\Delta F_i := F(x_{i+1})-F(x_i) = f(c_i)\,\Delta x_i \Longrightarrow$$

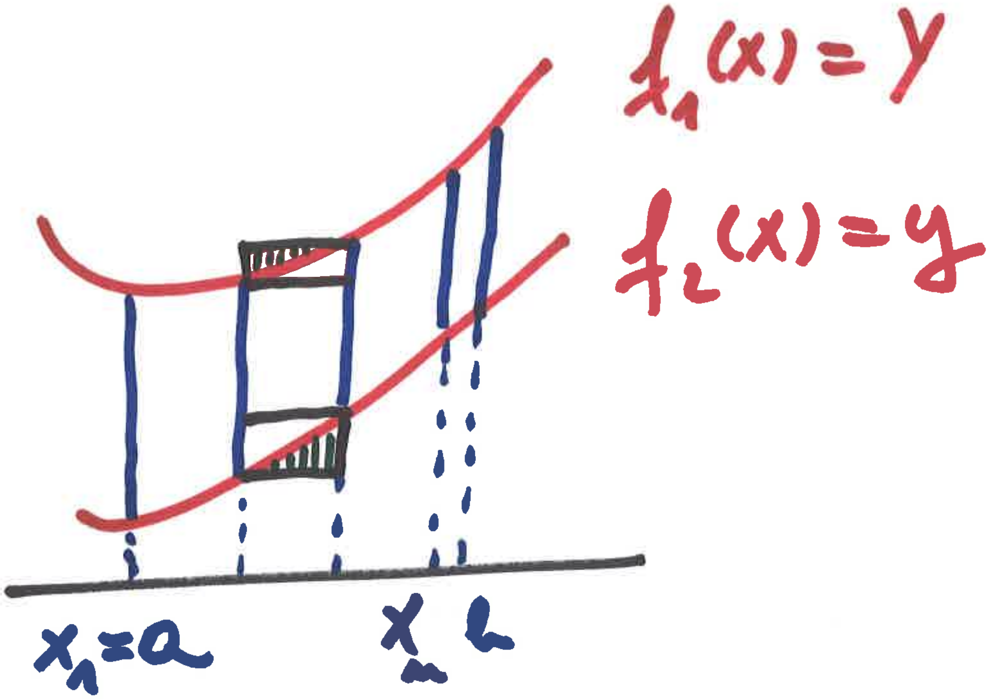
$$\underbrace{F(b)-F(a)}_{\text{VELIČINA V}} = \sum_1^n \Delta F_i = \sum_1^n f(c_i)\Delta x_i$$

$$\& \ |f(c_i)-f(x_i)|\,\Delta x_i \le g_f(\Delta x)\cdot \Delta x_i$$

$$g_f(\Delta x) := \max_i (M_i - m_i) \quad \left(\begin{matrix}\text{MAX \& MIN} \\ \text{NA } \Delta x_i\end{matrix}\right)$$

$$\& \lim_{\Delta x \to 0} g_f(\Delta x) = 0 \xrightarrow{\text{DUHAMEL}}$$

$$\boxed{\int_a^b f(x)\,dx = F(b)-F(a)}$$

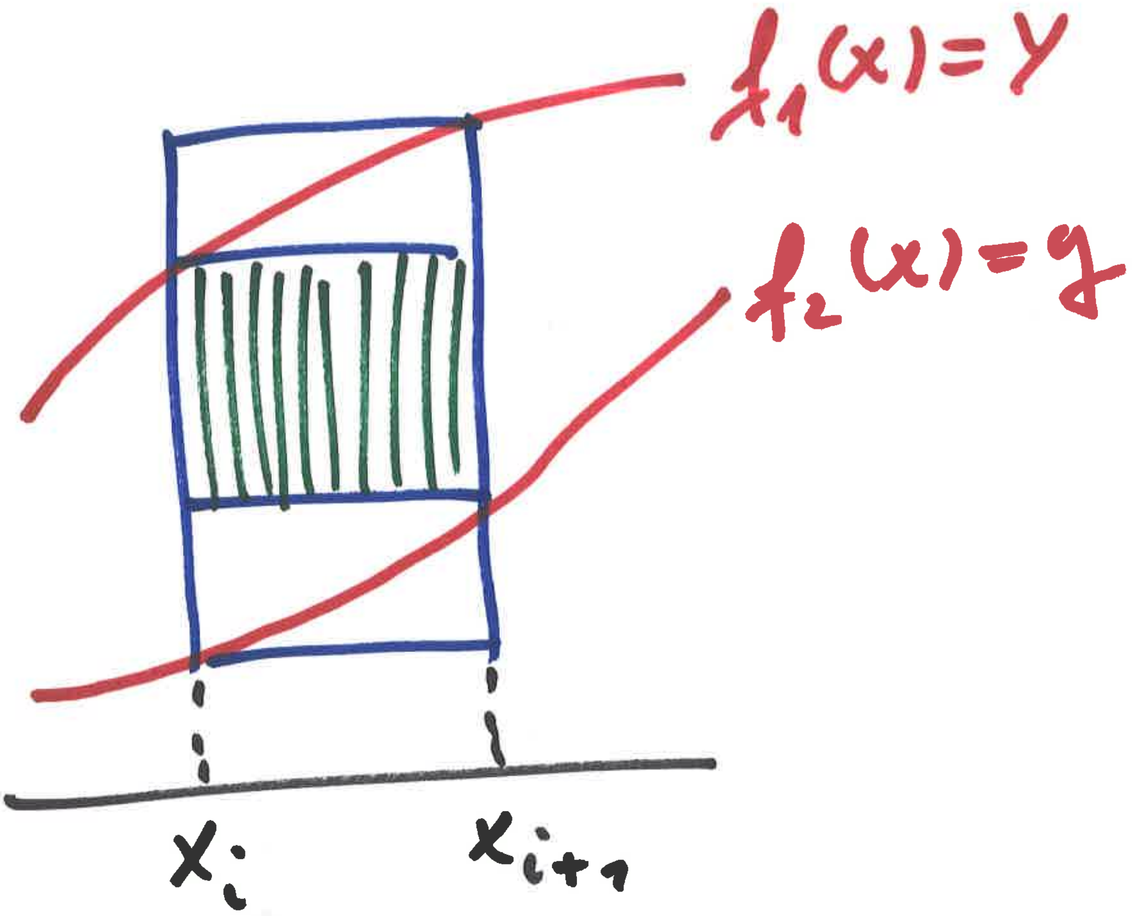


$$\left|f(x_i)\Delta x_i - P_i\right| \leq \left((\overline{Y}_i - \underline{Y}_i) + (\overline{y}_i - \underline{y}_i)\right)\Delta x_i$$

$$\leq \underbrace{\left(\max_i(\overline{Y}_i - \underline{Y}_i) + \max_i(\overline{y}_i - \underline{y}_i)\right)\Delta x_i}_{g(\Delta x)}$$

I OČITO :

$$\lim_{\Delta x \to 0} g(\Delta x) = 0$$



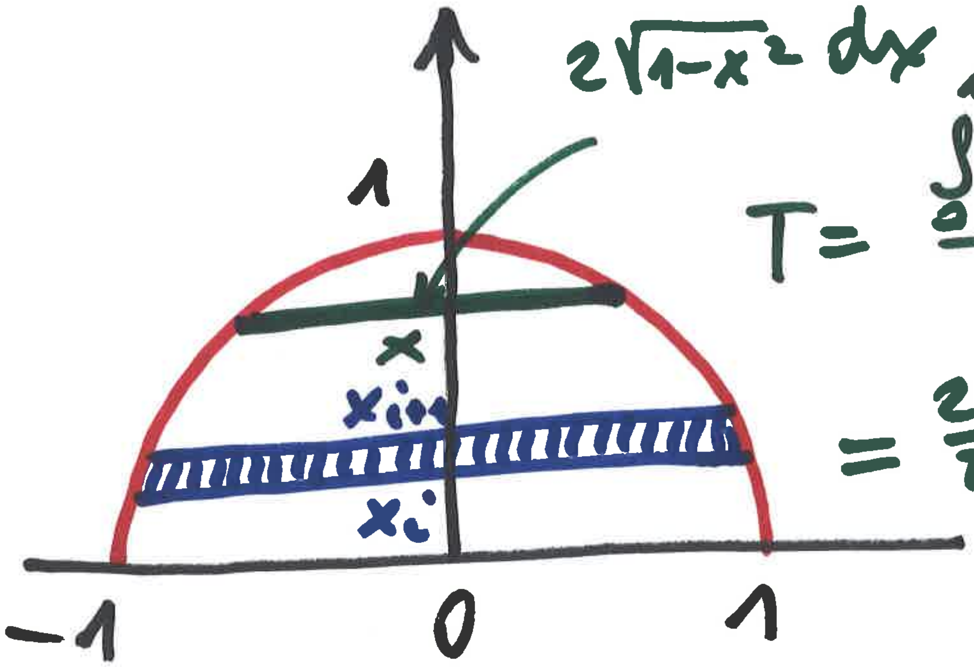
$$\Delta x_i \to J_i(\Delta x) = \left[(\underline{Y}_i - \bar{y}_i)\Delta x_i , (\bar{Y}_i - \underline{y}_i)\Delta x_i\right]$$

$$f(x_i)\Delta x_i \in J_i(\Delta x)$$

$$|J_i(\Delta x)| \le (\underbrace{g_Y(\Delta x) + g_y(\Delta x)}_{\xrightarrow[\Delta x \to 0]{} 0})\Delta x$$

PA STOGA: $P = \int_a^b (Y - y)\,dx$

TEŽIŠTE POLUKRUGA:



$$T = \frac{\int_0^1 x\, 2\sqrt{1-x^2}\,dx}{\pi/2} = \frac{2/3}{\pi/2} = \frac{4}{3\pi}$$

i-ti MOMENT

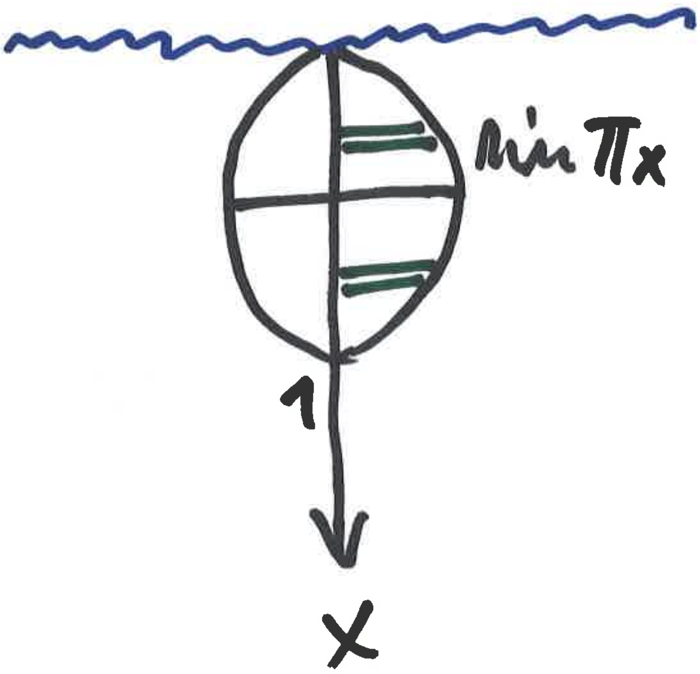
$$\underbrace{x_i\sqrt{1-x_{i+1}^2}\,\Delta x_i}_{\text{NIJE } D_i} \le M_i \le \underbrace{x_{i+1}\sqrt{1-x_i^2}\,\Delta x_i}_{\text{NIJE } G_i}$$

NITI SE MOMENT M_i MOŽE SMJESTITI IZMEĐU NEKOG D_i I G_i !!!

JER $x\uparrow \Leftrightarrow \sqrt{1-x^2}\downarrow$

HARVARD: POMOĆU DONJIH I GORNJIH SUMA OBJASNITE DA JE SILA NA PROZOR U BAZENU

$$F = \int_0^1 w\, x \sin \pi x \, dx$$



U GORNJOJ POLOVICI PROZORA TO MOŽEMO ALI U DONJOJ NE!!

GORE: $x_i \cdot \sin \pi x_i \cdot \Delta x_i \le F_i \le x_{i+1} \sin \pi x_{i+1} \Delta x_i$

DOLE: $x_i \cdot \sin \pi x_{i+1} \Delta x_i \le F_i \le x_{i+1} \sin \pi x_i \Delta x_i$

GORE: $x_i \uparrow \Rightarrow \sin \pi x_i \uparrow$

DOLE: $x_i \uparrow \Rightarrow \sin \pi x_i \downarrow$

(PETER LOEB)