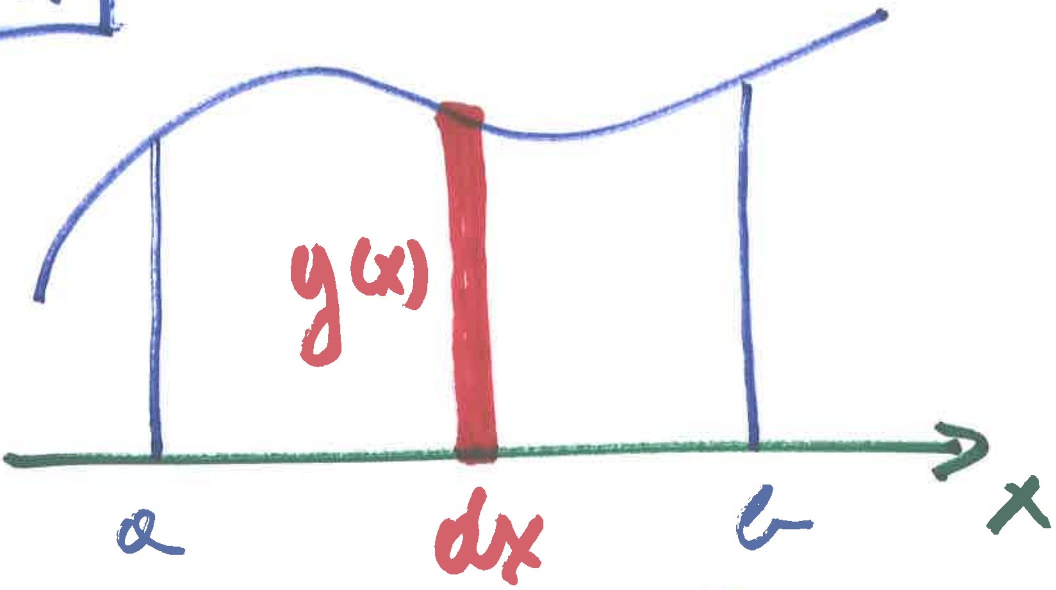
DULJINE, POVRŠINE, VOLUMENI (KAO INTEGRALI)

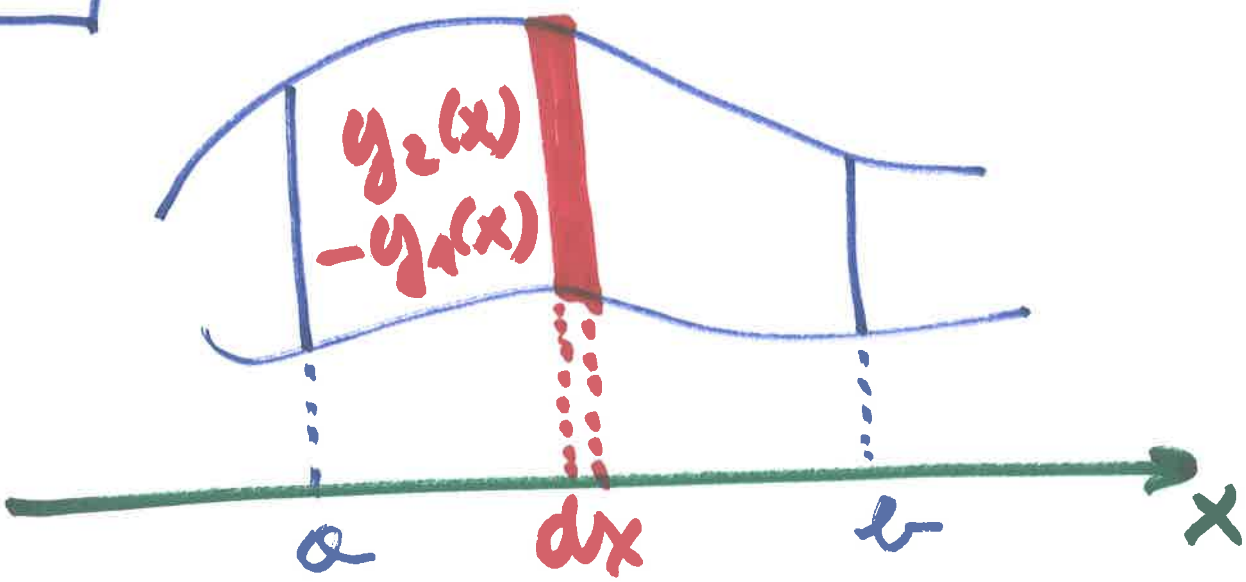
Z. ŠIKIĆ

Pr. 1



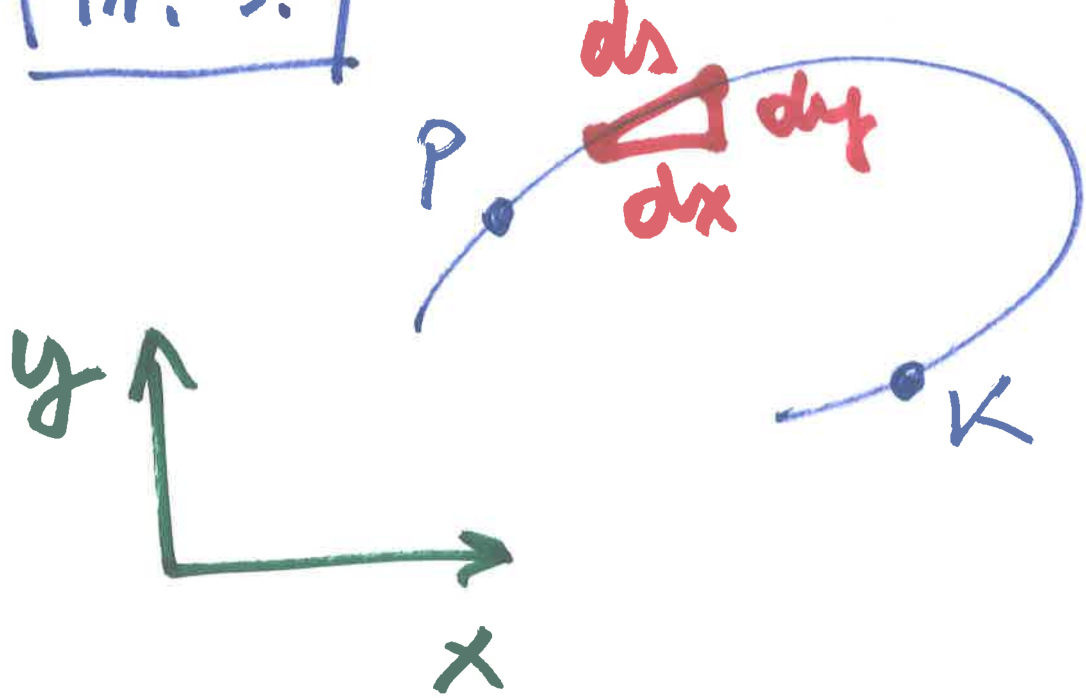
$$P[a,b] = \int_a^b dP = \int_a^b y\,dx$$

Pr. 2.



$$P[a,b] = \int_a^b dP = \int_a^b (y_2 - y_1)\,dx$$

Pr. 3.



$$L_P^K = \int_P^K ds = \int_P^K \sqrt{dx^2 + dy^2}$$

$$= \int_{t_P}^{t_K} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

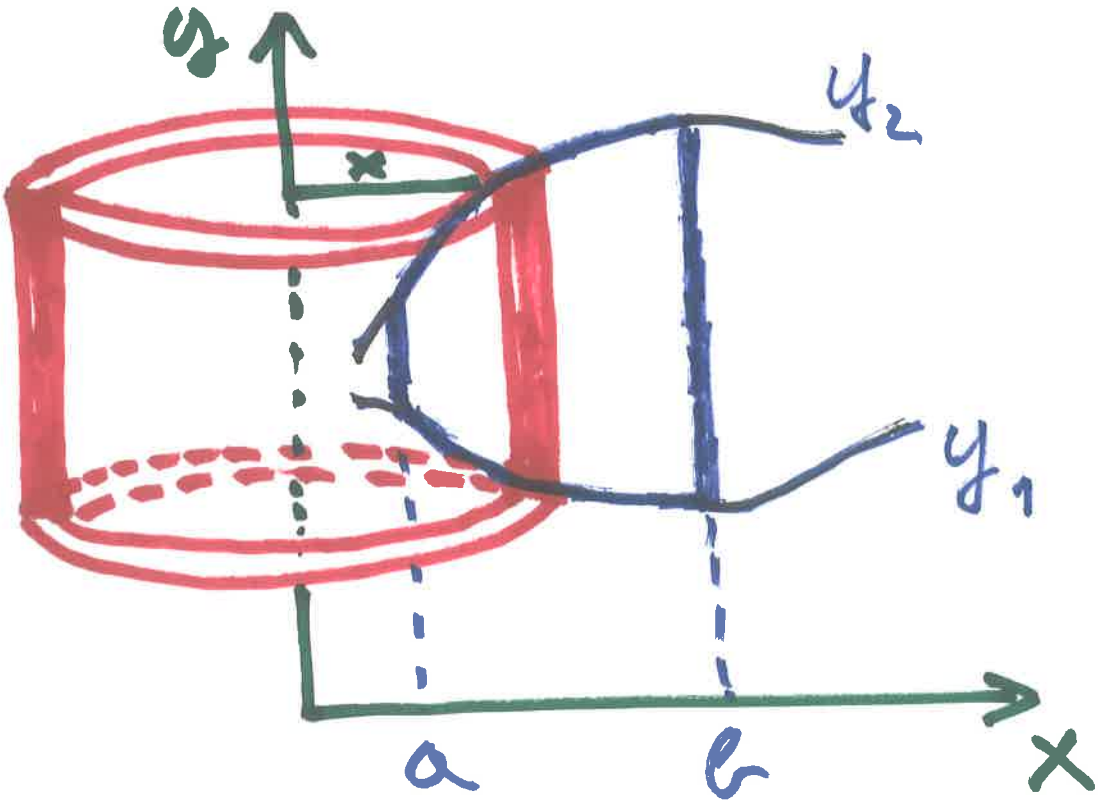
$$= \int_{x_P}^{x_K} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Pr. 4.

$$P_{VIJENCA} = (x+dx)^2\pi - x^2\pi$$

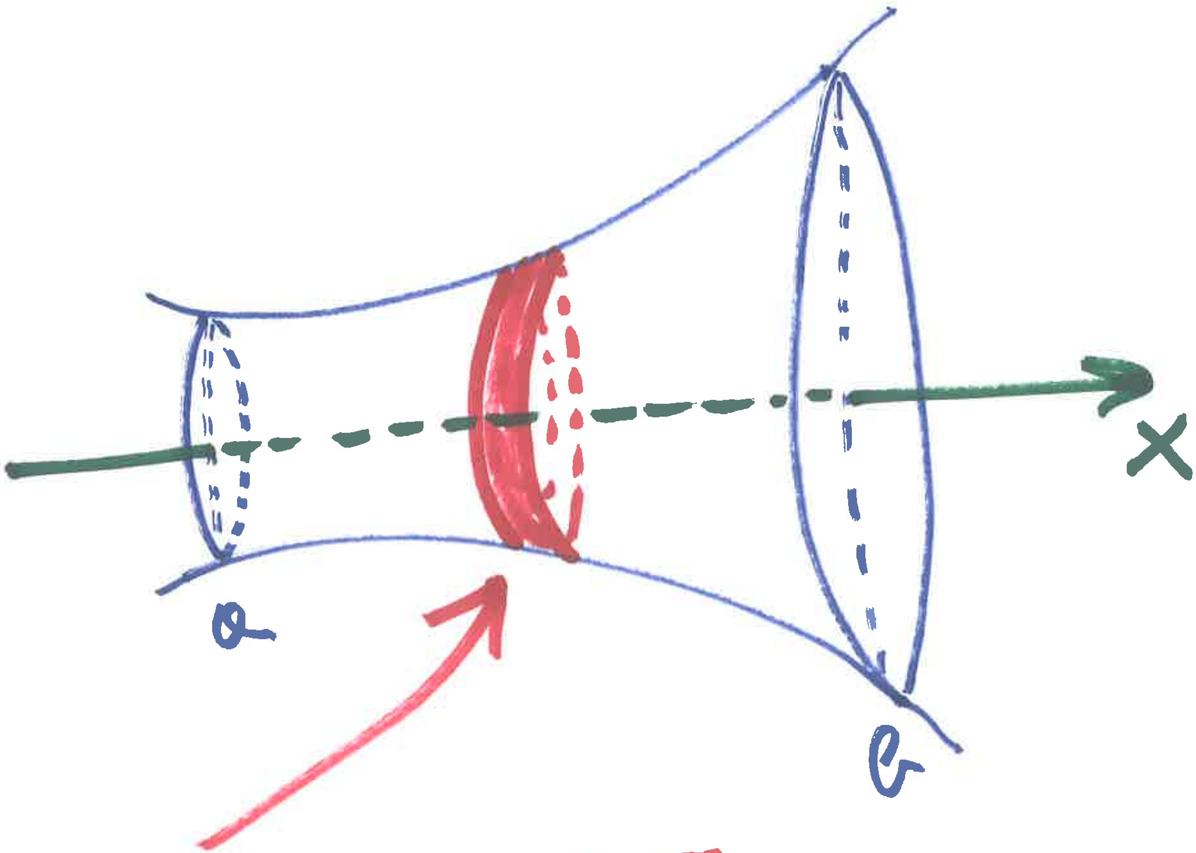
$$= 2\pi x\,dx + \cancel{dx^2\pi}$$

$$\text{ILI} \quad = 2\pi\left(x+\frac{dx}{2}\right)dx$$



$$V[a,b] = \int_a^b dV = \int_a^b (y_2-y_1)\,2\pi x\,dx$$

$\boxed{\text{Pr. J.}}$



$$P_{\text{KRNJ. STOŠCA}} = 2\pi \bar{r} s$$

$$P_{[a,b]} = \int_a^b dP = \int_a^b 2\pi y \sqrt{dx^2 + dy^2}$$

$$= \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

TEOREM

AKO JE $V[x_1, x_2]$ DEFINIRANO ZA SVE $x_1, x_2 \in [a, b]$ I AKO JE

(1) $V[a,x] + V[x, x+\Delta x] = V[a, x+\Delta x]$

(2) POSTOJE $\delta(x, \Delta x)$ I $\gamma(x, \Delta x)$ TAKVI DA $\delta, \gamma \xrightarrow[\Delta x \to 0]{} y(x)$ I

$$\Delta x \, \delta \leq V[x, x+\Delta x] \leq \Delta x \, \gamma$$

ONDA JE

$$\boxed{V[a,b] = \int_a^b y \, dx}$$

DAKLE, DA BI IZRAČUNALI $V[a,b]$
DOVOLJNO JE PROVJERITI
(1) ŠTO JE TRIVIJALNO I
(2) ŠTO JE KLJUČNO.

DOKAZ

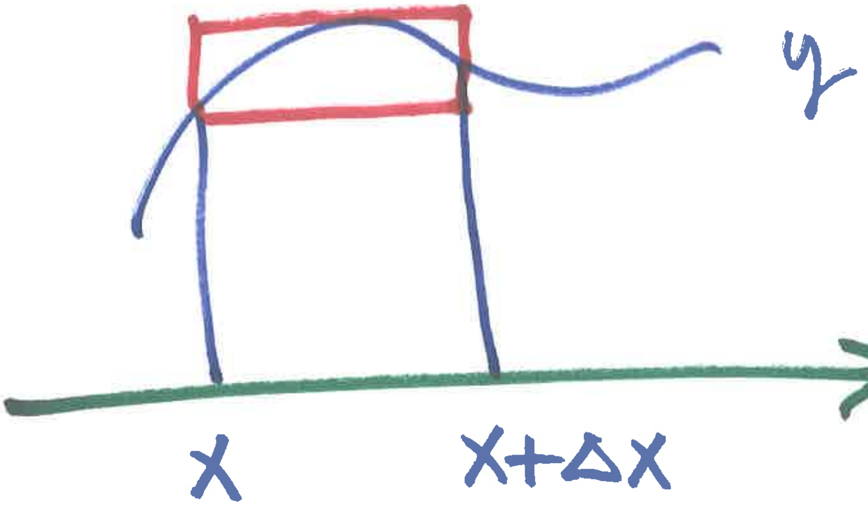
$$\frac{V[a,x+\Delta x]-V[a,x]}{\Delta x} \overset{(1)}{=} \frac{V[x,x+\Delta x]}{\Delta x}$$

$$\xrightarrow[\Delta x \to 0]{(2)} y(x) \quad \text{TJ.} \quad V' = y \quad \overset{N.L.}{\Longrightarrow}$$

$$\boxed{\int_a^b y\,dx = V[a,b] - V[a,a]}$$

$$\boxed{= V[a,b]}$$

Pr. 1.



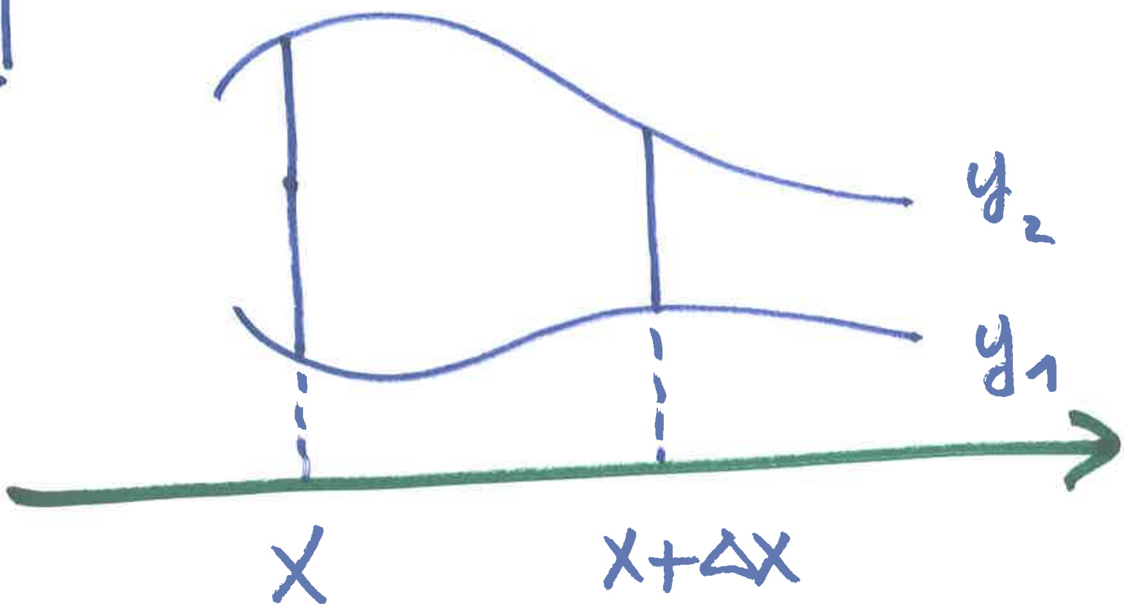
(PRETP. O "SADRŽAVAMU")

$$\Delta x \underbrace{\min_{[x,x+\Delta x]} y}_{\delta(x,\Delta x)} \leq P[x,x+\Delta x] \leq \Delta x \underbrace{\max_{[x,x+\Delta x]} y}_{\gamma(x,\Delta x)}$$

$$\delta(x,\Delta x) \xrightarrow[\Delta x \to 0]{} y(x) \xleftarrow[\Delta x \to 0]{} \gamma(x,\Delta x)$$

$$\Rightarrow \boxed{P[a,b] = \int_a^b y\,dx}$$

$\boxed{\text{Pr. 2.}}$



(PRETPOSTAVKA "CAVALIERI")

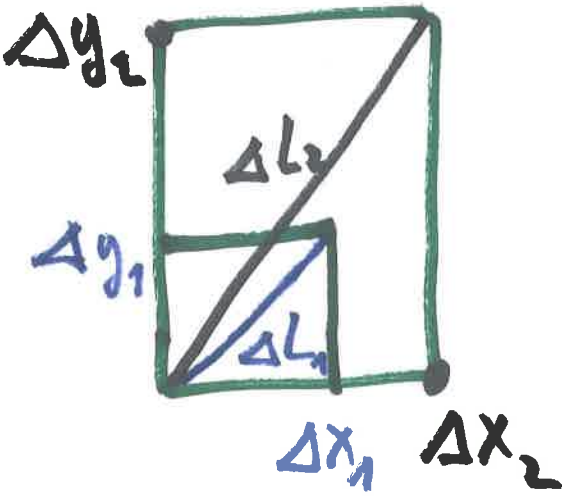
$$\underbrace{\Delta x \min(y_2-y_1)}_{\delta} \leq P[x, x+\Delta x] \leq \underbrace{\Delta x \max(y_2-y_1)}_{\gamma}$$

$$\delta \xrightarrow[\Delta x \to 0]{} (y_2-y_1) \xleftarrow[\Delta x \to 0]{} \gamma$$

$$\Rightarrow \boxed{P[a,b] = \int_a^b (y_2-y_1)\,dx}$$

Pr. 3. PRETPOSTAVKA O L:

$$\left.\begin{aligned} \frac{dx_1}{dt} &\le \frac{dx_2}{dt} \\ \frac{dy_1}{dt} &\le \frac{dy_2}{dt} \end{aligned}\right\} \text{ NA } [t, t+\Delta t] \implies$$



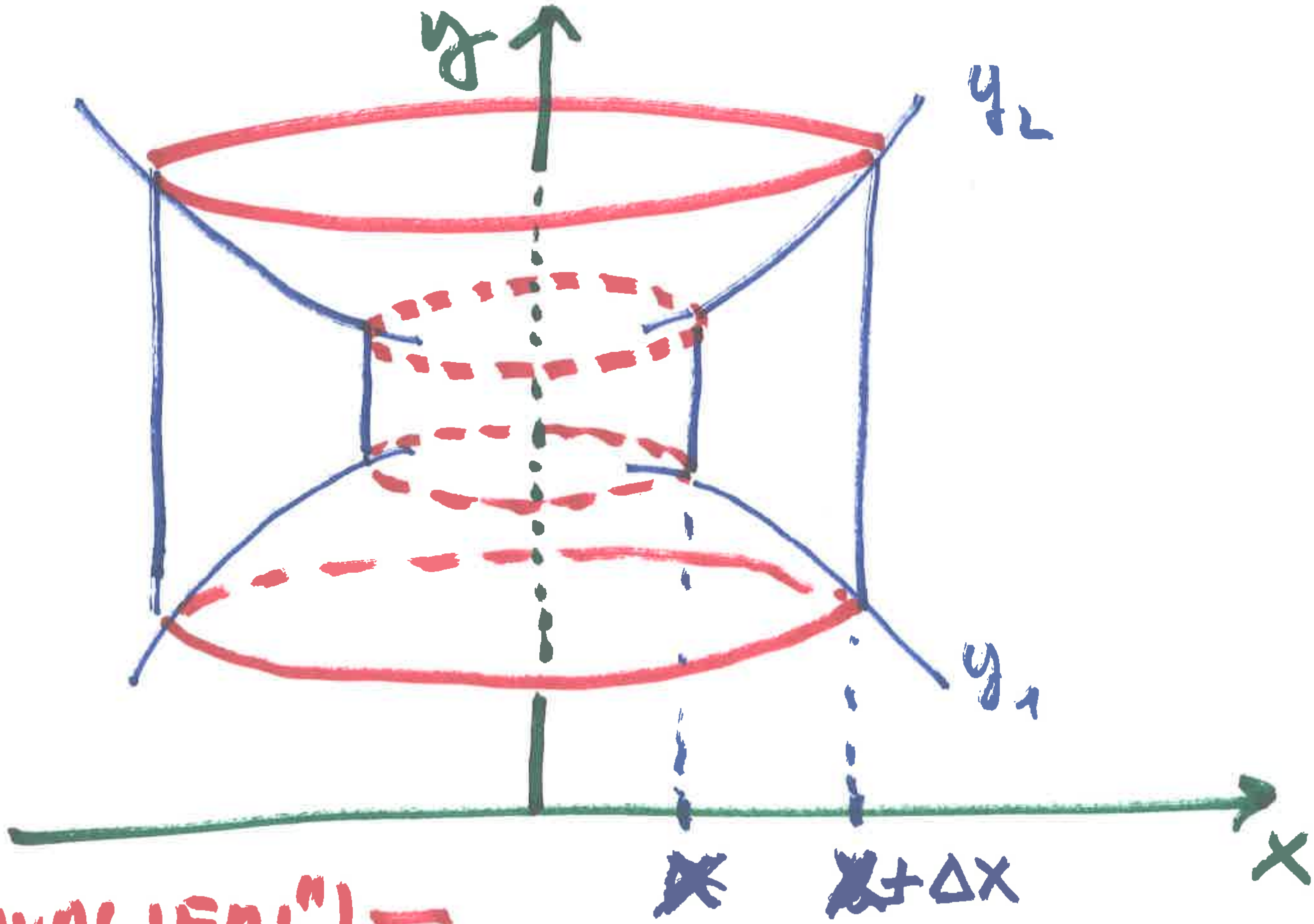
$$L_1[t, t+\Delta t] \le L_2[t, t+\Delta t]$$

$$\implies$$

$$\Delta t \underbrace{\sqrt{\min\left(\frac{dx}{dt}\right)^2 + \min\left(\frac{dy}{dt}\right)^2}}_{\delta} \le L[t, t+\Delta t]$$

$$\le \Delta t \underbrace{\sqrt{\max\left(\frac{dx}{dt}\right)^2 + \max\left(\frac{dy}{dt}\right)^2}}_{\gamma} \implies$$

$$\boxed{L[t_p, t_k] = \int_{t_p}^{t_k} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt}$$



(P. "CAVALIERI")

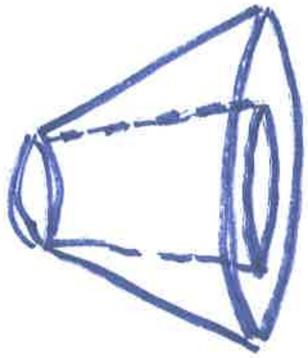
$$\overbrace{(\min y_2 - \max y_1)\, 2\pi \left(x+\frac{\Delta x}{2}\right) \Delta x}^{\delta} \leq$$

$$\leq V[x, x+\Delta x] \leq$$

$$\leq \underbrace{(\max y_2 - \min y_1)\, 2\pi \left(x+\frac{\Delta x}{2}\right) \Delta x}_{\gamma}$$

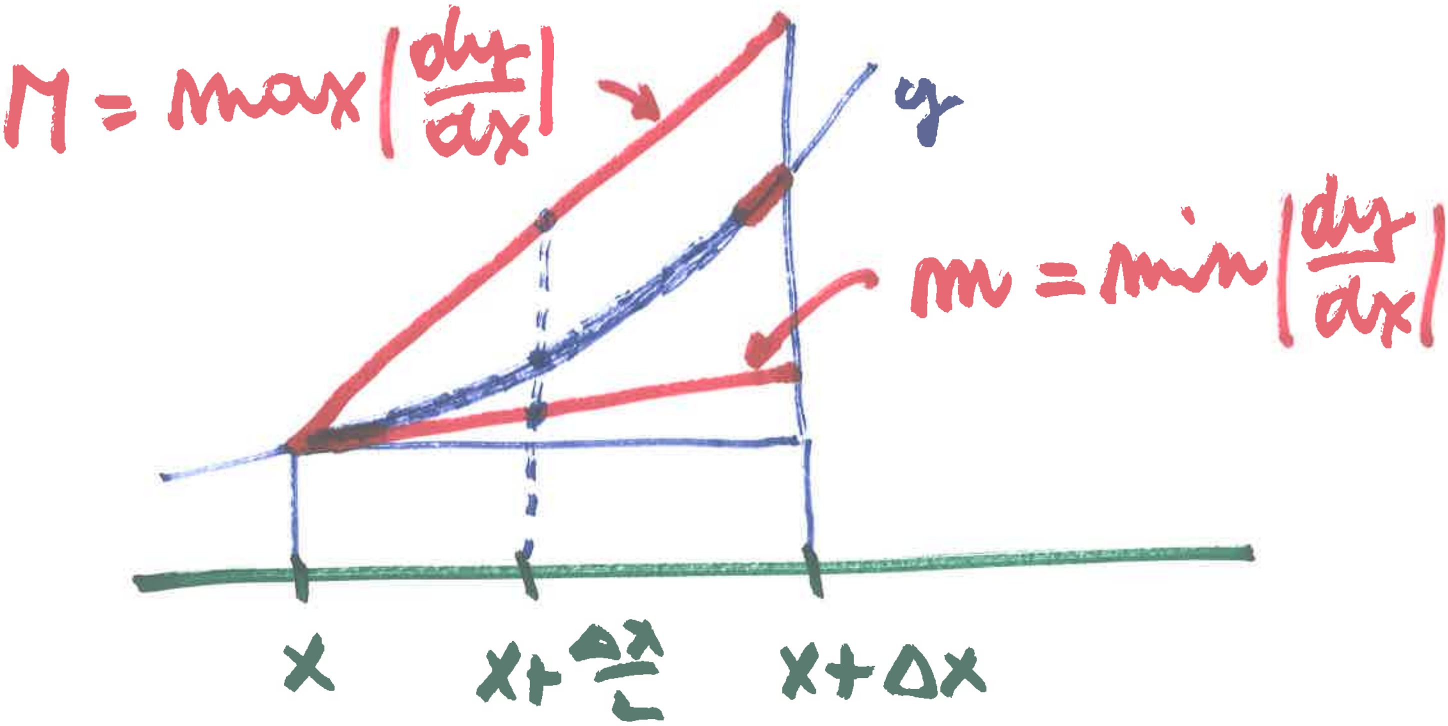
$$\Rightarrow \boxed{V[a,b] = \int_a^b (y_2 - y_1)\, 2\pi x \, dx}$$

$\boxed{\text{Pr. 5.}}$ PRETPOSTAVKA O P:



VEĆI NAGIB VEĆA ROTACIJSKA POVRŠINA:

$$2\pi \bar{r} s$$



$$2\pi \underline{y}\sqrt{\Delta x^2 + m^2\Delta x^2} \le P[x, x+\Delta x]$$

$$\le 2\pi \bar{y}\sqrt{\Delta x^2 + M^2\Delta x^2} \Longrightarrow$$

$$\boxed{P[a,b] = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}$$