Introduction to Machine Learning
Fall 2018

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Homework 1
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Notice, to get the full credits, please present your solutions step by step.

Exercise 1: 2pts

Show that $(1-\epsilon)^m \leq e^{-m\epsilon}$, where $m \in \mathbb{N}$.

Solution: There may be another condition lost,the $\epsilon \in [-\infty, 1]$,otherwise if $\epsilon = 2$ and $m = 2$,the inequality is broken.

And with my condition ,the question will be easy ,we all know that

$$e^x \leq (1-x), \forall x \in [-\infty, 1]$$

This is easy to prove.We can make the fuction

$$f(x) = e^{-x} - 1 + x$$

$f(x) = 0$ when $x = 0$,and $f^{'}(x) = 1 - e^{-x}$,and we can know

$$f^{'}(x) \begin{cases} \leq 0 & \text{if } x \leq 0 \\ > 0 & \text{if } x > 0 \end{cases} \tag{1}$$

So

$$0 \text{ s } f(x)_{min} \text{ when } x \in [-\infty, 1] \tag{1}$$

and for m = 0 , m = 1 ,the inequality is ok,and we can assume that the inequality is ok for all $m \leq N$,and for $m = N + 1$,

$$(1-\epsilon)^{N+1} = (1-\epsilon)^N * (1-\epsilon) \leq e^{-(N\epsilon)} * (1-\epsilon)$$

then use (1) we have proved,

$$e^{-(N\epsilon)} * (1-\epsilon) \leq e^{-(N\epsilon)} * e^{-\epsilon} = e^{-(N+1)\epsilon}$$

The inequality is ok for N+1,so we prove it!

■

Exercise 2: Markov inequality 2pts

Let X be a nonnegative randome variable on $\mathbb{R}$. Then, for all $t > 0$, show that

$$\mathbf{P}(X \geq t) \leq \frac{\mathbf{E}[X]}{t}.$$

Solution: For noneegatave randome variable, we assume $f(x)$ is the probablity distribution function of the x; for any $t \geq 0$

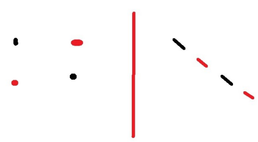
$$E(X) = \int_0^{\infty} xf(x)dx \geq \int_t^{\infty} xf(x)dx \geq t\int_t^{\infty} f(x)dx = tP(x \geq t)$$

This exactly the what we need. ■

Exercise 3: VC-dimension 2pts

Assume that the instance space $X = \mathbb{R}^2$ and the hypothesis space H be the set of all linear threshold functions defined on $\mathbb{R}^2$. Find $VC(H)$ and prove it.

Solution: $VC(H) = 3$,and we know that three points can be shatted if points $\in R^2$(there are all cases in the ppt), and we can construct samples that 4 poinst can't be shatted.



The red and black points represent two categories. ■

Exercise 4: Learning intervals 4pts

Let the target concept class be $C = \{[a, b] : a < b, a, b \in \mathbb{R}\}$ and the hypotheses class $H = C$, and the version space be $VS_{H,D}$. Each $c \in C$ labels the points inside the interval positive and the others negative. A consistent learner will pick a consistent hypothesis—if any—$h \in H$ according to a set of i.i.d. samples $\{(x_1, c(x_1)), (x_2, c(x_2), \ldots, (x_m, c(x_m)\}$ that obey an unknown distribution $\mathcal{D}$. Please find

$$\mathbf{P}[\exists\, h \in VS_{H,D} \text{ and } error_{\mathcal{D}}(h) > \epsilon],$$

and the corresponding sample complexity.

Solution: The learner can be sample,we make the the min negative points as the a' and the max positive point as b'.

We should find a upper bound of the probablity because calculate it directly is very difficult and I don't know the bound I find is tight or not .

we use B represent $\exists\, h \in VS_{H,D}$ and $error_{\mathcal{D}}(h) > \epsilon$,and we should find a event E and P(E) $\geq$ P(B). and error(D) can be define as the Integration of distribution functions between [a,a'] and [b',b]

How to define E ? This is very difficult to describe it clearly . if we limit Integration of distribution functions between [a,a'] equal to $\frac{\epsilon}{2}$ and [b',b] as same. we define E as none of train points hit the interval. So the $P(B) \leq 2(1 - \frac{\epsilon}{2})^m \leq 2e^{-\frac{m\epsilon}{2}} \leq \delta$ (because it must be at least one interval is miss) So the complexity of ample $m \geq 2\frac{ln\frac{\delta}{2}}{\epsilon}$ ■