(9) Como visto, podemos nos referenciar a estados na esfera de bloch usando a seguinte equação:

$$|\upsilon\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\,\mathrm{sen}\left(\frac{\theta}{2}\right)|1\rangle$$

Como queremos gerar estados antipodais, somamos $\frac{\pi}{2}$ ao valor do ângulo θ.

$$|\upsilon^{A}\rangle = \cos\left(\frac{\theta+\pi}{2}\right)|0\rangle + e^{i\varphi}\,\mathrm{sen}\left(\frac{\theta+\pi}{2}\right)|1\rangle$$

E, para descobrir se pontos antipodais são ortogonais:

$$\boxed{\langle\upsilon|\upsilon^{A}\rangle = 0} \quad \therefore \quad \boxed{\langle\upsilon|\cdot|\upsilon^{A}\rangle = 0}$$

Logo, pegando a representação vetorial de $|\upsilon\rangle$ e pegando seu conjugado complexo:

$$|\upsilon\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\,\mathrm{sen}\left(\frac{\theta}{2}\right)|1\rangle \Rightarrow \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\varphi}\,\mathrm{sen}\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$\therefore \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & e^{-i\varphi}\,\mathrm{sen}\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Pegando a representação vetorial de $|\upsilon^{A}\rangle$:

$$|\upsilon^{A}\rangle = \cos\left(\frac{\theta+\pi}{2}\right)|0\rangle + e^{i\varphi}\cdot\mathrm{sen}\left(\frac{\theta+\pi}{2}\right)|1\rangle \Rightarrow \begin{pmatrix} \cos\left(\frac{\theta+\pi}{2}\right) \\ e^{i\varphi}\,\mathrm{sen}\left(\frac{\theta+\pi}{2}\right) \end{pmatrix}$$

Realizando o produto interno dos vetores:

$$\begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & e^{-i\varphi}\,\mathrm{sen}\left(\frac{\theta}{2}\right) \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\frac{\theta+\pi}{2}\right) \\ e^{i\varphi}\,\mathrm{sen}\left(\frac{\theta+\pi}{2}\right) \end{pmatrix}$$

$$\Rightarrow \cos\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta+\pi}{2}\right) + e^{-i\varphi}\cdot e^{i\varphi}\cdot\mathrm{sen}\left(\frac{\theta}{2}\right)\cdot\mathrm{sen}\left(\frac{\theta+\pi}{2}\right)$$

$$\therefore \cos\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta+\pi}{2}\right) + \mathrm{sen}\left(\frac{\theta}{2}\right)\cdot\mathrm{sen}\left(\frac{\theta+\pi}{2}\right)$$

$\cos(a-b) = \cos(a)\cos(b) + \mathrm{sen}(a)\cdot\mathrm{sen}(b)$

$$\therefore \cos\left(\frac{\theta}{2} - \frac{\theta+\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right)$$

$$\boxed{\Rightarrow 0}$$

Q.E.D.