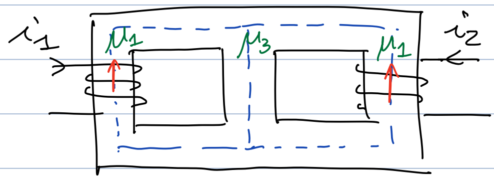
TD5 : Inductance propre et mutuelle

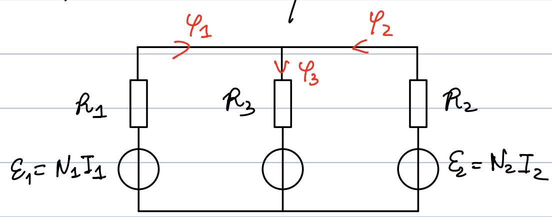
1.1)



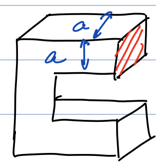
1.2)

a) ligne de champ moyenne (moy)
le matériau linéaire (μ)

b) schéma équivalent



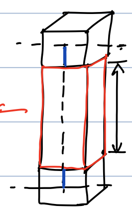
c) $\mathcal{R} = \int \frac{1}{\mu(x)} \cdot \frac{dl}{S(x)}$ (H^{-1})



$S = a^2$

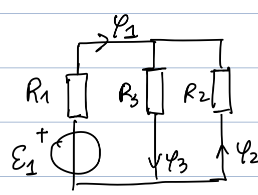
- $\mathcal{R}_1 = \mathcal{R}_2 = \frac{1}{\mu_0 \mu_{R1}} \cdot \frac{3(b+a)}{a^2} = \frac{1}{4\pi \cdot 10^{-7} \cdot 2000} \cdot \frac{3(0{,}14+0{,}05)}{0{,}05^2} = 90718\ H^{-1}$
$= 9{,}1 \cdot 10^4\ H^{-1}$
- $\mathcal{R}_3 = \left\{ \frac{1}{\mu_0 \mu_{R3}} \cdot \frac{(b \cancel{+a})}{a^2} \right\} + \left\{ \frac{1}{\mu_0 \mu_{R1}} \cdot \frac{a}{a^2} \right\}$

⚠ matériau différent !



$= \frac{1}{4\pi \cdot 10^{-7} \cdot 100} \cdot \frac{0{,}14}{0.05^2} + \frac{1}{4\pi \cdot 10^{-7} \cdot 2000} \cdot \frac{1}{0{,}05} = 453552\ H^{-1}$
$= 45{,}4 \cdot 10^4\ H^{-1}$

1.3).



$$\varphi_1 = \frac{\mathcal{E}_1 \rightarrow N_1 I_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$N\phi = Li$
$R\phi = Ni$ $\quad L = \frac{N^2}{R}$

$$\downarrow$$

$$L_1 = \frac{N_1 \varphi_1}{I_1} = N_1^2 \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = 60\ mH$$

idem. $$L_2 = N_2^2 \frac{R_1 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = 240\ mH$$

1.4) $$M_{21} = \frac{N_2 \varphi_{2 \leftarrow 1}}{I_1}$$

$$\varphi_2 = -\varphi_1 \cdot \frac{R_3}{R_2 + R_3} = -(N_1 I_1) \cdot \frac{R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$\Rightarrow M_{21} = -N_1 N_2 \cdot \frac{R_3}{R_1R_2 + R_1R_3 + R_2R_3} = M_{12} \quad \text{Réciprocité}$$

$$M = -100\,\mathrm{mH}$$

le coefficient de couplage : $k = \frac{M}{\sqrt{L_1 L_2}} = -0{,}83$ (moyen).

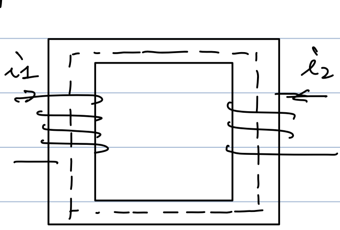
$0 < |k| < 1$

$\uparrow$ pas couplé $\qquad \uparrow$ parfaitement couplé

$$k^2 = \frac{M^2}{L_1L_2} = \frac{\dfrac{(N_1N_2)^2 \cdot R_3^2}{\Sigma^2}}{\dfrac{N_1^2N_2^2 \times (R_1+R_3)(R_2+R_3)}{\Sigma^2}} = \frac{R_3^2}{(R_1+R_3)(R_2+R_3)}$$

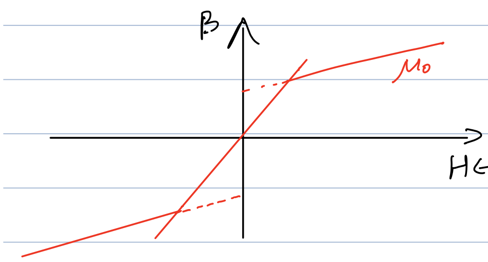
$$= \frac{1}{\left(1+\frac{R_1}{R_3}\right)\left(1+\frac{R_2}{R_3}\right)}$$

$$\mu_{R_3} \downarrow_{0} \Rightarrow R_3 \uparrow^{+\infty} \Rightarrow k \to 1$$



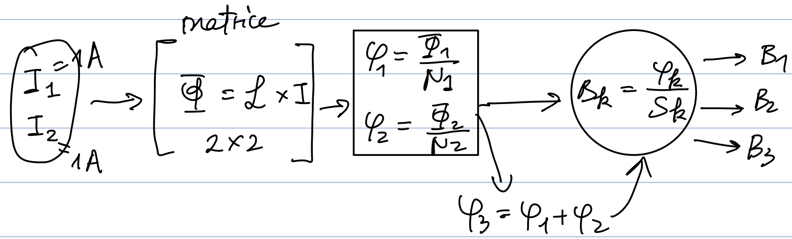
parfaitement couplé

1,5)



$$L_1 = \frac{N_1 \varphi_{1\leftarrow 1}}{I_1} \qquad M = \frac{N_1 \varphi_{1\leftarrow 2}}{I_2}$$

$$\begin{cases} \Phi_1 = N_1\varphi_1 = (L_1 I_1) + (M I_2) \\ \Phi_2 = N_2\varphi_2 = (M I_1) + (L_2 I_2) \end{cases}$$



$L_1 = 60\,\mathrm{mH}$

$L_2 = 240\,\mathrm{mH}$

$M = -100\,\mathrm{mH}$.

$$\begin{bmatrix} I_1 = 1A \\ I_2 = 1A \end{bmatrix} \rightarrow \begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2\end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2\end{bmatrix} = \begin{bmatrix} L_1+M = -40\,mWb \\ M+L_2 = +140\,mWb\end{bmatrix} \rightarrow \begin{bmatrix}\varphi_1 = \frac{\Phi_1}{100} = -0{,}4\,mWb \\ \varphi_2 = \frac{\Phi_2}{200} = +0{,}7\,mWb\end{bmatrix}$$

$$\hookrightarrow \varphi_3 = \varphi_1+\varphi_2 = +0.3\,mWb$$

$$S = a^2 = 25\,10^{-4}\,m^2$$

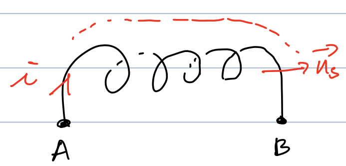
$$\longrightarrow \begin{bmatrix} B_1 = -0{,}16\,T \\ B_2 = +0{,}28\,T \\ B_3 = +0{,}12\,T \end{bmatrix}$$

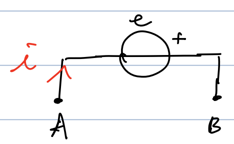
2,1)

Loi de Faraday

M.F. : $\vec{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$e = -\frac{d\Phi_B}{dt}$$





En conv. récepteur :

$$\begin{bmatrix}V_1\\V_2\end{bmatrix} = +\frac{d}{dt}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} = \begin{bmatrix} L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} \\ M\frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}\end{bmatrix}$$

$$\frac{d}{dt} \Leftrightarrow j\omega$$

$$\begin{bmatrix}V_1\\V_2\end{bmatrix} = j\omega \begin{bmatrix}L_1 & M\\ M & L_2\end{bmatrix}\cdot\begin{bmatrix}I_1\\ \underset{0}{\cancel{I_2}}\end{bmatrix}$$

à vide

2,2)

$$\Rightarrow \begin{bmatrix}V_1\\V_2\end{bmatrix} = j\omega\begin{bmatrix}L_1 I_1\\ M I_1\end{bmatrix}$$

$$L_1 = \frac{\overset{100}{|V_1|}}{\underset{100\pi}{\omega}\,\underset{3,2}{|I_1|}} = 100\,mH$$

$$\left(\frac{M}{L_1}\right) = \left(\frac{U_2}{V_1}\right) \Rightarrow |M| = L_1 \cdot \frac{|V_2|}{|V_1|}$$

$$M = -135\,mH.$$

$-180°$ déphasage