Image Enhancement Sample Exam Problems with Solutions

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1. Propose a method that uses variable size spatial filters to reduce background noise without blurring the image significantly.

   Solution

   Noise is more visible within background areas (areas with slowly varying intensity) because the **local** Signal-to-Noise Ratio within these areas is lower. This is because the power of the local image is lower. In image regions of high detail, such as edge regions, we don’t want to do any lowpass filtering or we want to do mild filtering. This is because we don’t want to ruin the sharp edges and fine details of the image. We can afford to do this since noise is less visible withing regions of high detail because **local** Signal-to-Noise Ratio within these areas is higher.

   Large spatial masks can be used within flat or slowly varying areas, whereas small masks can be used within high activity areas. The idea behind this approach is that noise is more visible in the first type of areas. Local variance can classify each pixel into each of the corresponding areas.

2. Why are bandpass filters useful in image processing? Justify your answer. Propose a method to obtain a bandpass filtered version of an image using spatial masks.

   Solution

   Bandpass filters are useful for removing noise without eliminating completely the background information. A bandpass filter can be implemented by spatial masks as follows. The original image $f(x,y)$ is first convolved with a lowpass spatial mask of size $N_1 \times N_1$ to produce an output $g_1(x,y)$. Then it is convolved with a lowpass spatial mask of size $N_2 \times N_2$ with $N_2 > N_1$ to produce an output $g_2(x,y)$. Note that the image $g_2(x,y)$ will be blurrier compared to the image $g_1(x,y)$. The cutoff frequency of the Discrete Space Fourier Transform (DSFT) of $g_2(x,y)$ will be lower compared to the cutoff frequency of the DSFT of $g_1(x,y)$. (Recall that wide in time/frequency yields narrow in frequency/time). Therefore, if we use as final output the difference $g_1(x,y) - g_2(x,y)$, we can deduct that the DSFT of this difference will exhibit a bandpass type of behavior.

3. Propose a pair of spatial masks that detects edges in an image along the directions $\pm 45^{o}$.

Solution

It can easily be deducted that a pair of masks which detect edges at 45 and -45 degrees could be the following.

$$\begin{matrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{matrix}$$

$$\begin{matrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{matrix}$$

Note that there many other alternatives.

4. Propose a method for spatially adaptive noise reduction in images, that exploits jointly the following two observations:
   a. Noise is typically less visible to human viewers in bright areas than in dark areas.
   b. Noise is typically less visible to human viewers in image regions of high detail, than in image regions of low detail.

Solution

Since dark areas are represented by small intensities, we can calculate the local mean around each pixel to decide whether this belongs to a dark or to a bright area. We denote the mean around a pixel with coordinates (x, y) as $m_{(x,y),W}$. The subscript W denotes the neighborhood used to calculate this mean. If this is a rectangular window of size $(2W+1)(2W+1)$, symmetrically placed around the pixel of interest, then

$$m_{(x,y),W} = \frac{1}{(2W+1)^2} \sum_{i=-W}^{i=W} \sum_{j=-W}^{j=W} f(x+i, y+j)$$

Similarly, high detail regions are represented by large variances. We can calculate the local variance around each pixel to decide whether this belongs to a high detail or to a low detail area. We denote the variance around a pixel with coordinates (x, y) as $\sigma^2_{(x,y),W}$.

$$\sigma^2_{(x,y),W} = \frac{1}{(2W+1)^2} \sum_{i=-W}^{i=W} \sum_{j=-W}^{j=W} f(x+i, y+j)^2 - m^2_{(x,y),W}$$

A possible approach is to implement weaker filtering in both high detail and bright areas. High detail areas can be identified from the local variance $\sigma^2_{(x,y),W}$. More specifically, a large $\sigma^2_{(x,y),W}$ indicates a high activity area. Bright areas can be identified from the local mean $m_{(x,y),W}$. More specifically, a large $m_{(x,y),W}$ indicates a bright area. Therefore, we can normalize $\sigma^2_{(x,y),W}$ and $m_{(x,y),W}$ so that they occupy the same range of values (for example from 0 to 1) to obtain $\tilde{\sigma}^2_{(x,y),W}$ and $\widetilde{m}_{(x,y),W}$ and then we can consider the sum $\tilde{\sigma}^2_{(x,y),W} + \widetilde{m}_{(x,y),W}$. If the sum is greater than a pre-defined threshold, a weaker lowpass filter (small spatial mask) can be locally applied. If the sum is lower than the same pre-defined threshold, a stronger lowpass filter (large spatial mask) can be locally applied. Note that there many other alternatives.

6. An old movie is to be restored. Frames of the movie contain black spots which we wish to remove by median filtering. A black spot is a single pixel or a tiny patch of pixels which have 0 values and are located among non-zero pixels. **Figure 1** below shows a representative example of a small region of a frame of the movie.
   (i) Apply median filtering using a 3×3 window to the image of **Figure 1** below. Assume that pixel values are zero outside the image.

12	11	10	12	15
12	11	0	11	14
13	12	12	12	10
12	10	13	12	11

Figure 1

   (ii) An alternative method of processing applies first a median filter within a local window of 3 columns and 1 row and then applies to the previous result, a median filter within a local window of 1 column and 3 rows. What is the result if this method is applied to the above image?
   (iii) If the image contains rectangular objects with sharp corners, which of the two methods given in (i) and (ii) above is best? Explain your answer.
   (iv) If the black spots in the pixel frames are 2×2 pixels in size, which of the methods is best? Explain your answer.
   (v) Even in regions without black spots median filtering will change the values of many pixels. Propose a method based on a 3×3 window in which pixels are left unchanged unless there is a black spot within the 3×3 window.

Solution

(i)

0	**10**	**10**	**10**	**0**
11	**12**	**11**	**12**	**11**
11	**12**	**12**	**12**	**11**
0	**12**	**12**	**11**	**0**

(ii)

11	**11**	**11**	**11**	**11**
11	**11**	**11**	**12**	**11**
11	**12**	**12**	**12**	**11**
10	**12**	**12**	**12**	**10**

(iii)

For an image containing rectangular objects the second method is better. The first method rounds off corners because when the 3×3 window is centred at a corner pixel, 4 pixels within the window are within the object and 5 on the background, hence the output pixel is given the value of the background. For the second method, where we use 1×3 and 3×1 spatial masks, when the window is centred a corner pixel most pixels (2 out of 3) are located within the object and hence, the background has no effect on the median filtering.

(iv)

Now the first method is better. For a window centred on one of the black pixels, 4 out of 9 pixels within the window cover the black spot, whereas a majority, 5 out of 9 are on the background, hence the output value is the background. On the contrary, for the second method a majority of window pixels are one the black spot and so the output is the same as the input and the black spot is not removed.

(v)

Check for instance whether the smallest value in the window is less than 20% of the median value, if it is apply median filtering and replace the centre pixel with the median value. Otherwise just copy the input value to the output value.

7. We wish to set a local threshold for pixels of an image $f(x, y)$, in order to find pixels which are significantly brighter than their surroundings. We create an output $g(x, y)$ such that

$$g(x,y) = \begin{cases} 1, & f(x,y) > \mathrm{mean}_{5\times5}\{f(x,y)\} + T \\ 0, & \text{otherwise} \end{cases}$$

where $\mathrm{mean}_{5\times5}\{f(x,y)\}$ is the local mean around the pixel (x, y), averaged over a 5×5 pixel region, and T is an input parameter. Explain how you would implement the above process using standard linear convolution followed by thresholding. What is the convolution filter kernel that should be used?

Solution

$f(x,y) = f(x,y) * \delta(x,y)$ and $\mathrm{mean}\{f(x,y)\} = f(x,y) * w(x,y)$. Therefore,

$$g(x,y) = \begin{cases} 1, & f(x,y) * (\delta(x,y) - w(x,y)) > T \\ 0, & \text{otherwise} \end{cases}$$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$\delta(x,y)$

$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$
$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$
$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$
$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$
$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$

$w(x,y)$

$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$
$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$
$-\frac{1}{25}$	$-\frac{1}{25}$	$\frac{24}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$
$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$
$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$	$-\frac{1}{25}$

$$\delta(x, y) - w(x, y)$$

8. State which one of the following filters is nonlinear: high boost filter, weighted averaging filter, Sobel filter, median filter. Justify your answer with an example.

   Solution

   Trivial. The answer is median filter. This can be demonstrated by simple example, i.e., ***median*** $\{0,1,0\}$+***median*** $\{2,0,0\} = 0 + 0 = 0$. This is not the same as the median of $\{2,1,0\}$ which is 1.