Imperial College London

Digital Image Processing

Image Transforms

The 2D Discrete Cosine Transform

DR TANIA STATHAKI

READER (ASSOCIATE PROFFESOR) IN SIGNAL PROCESSING

IMPERIAL COLLEGE LONDON

Imperial College London

What is this lecture about?

- **Welcome back to the Digital Image Processing lecture!**
- In this lecture we will learn about one of the so-called Discrete Cosine Transforms (DCTs).
- From the above bullet point you might already have suspected that the DCT is not a single transform but a family of transforms.
- We will call the one that we will see here, DCT. In various textbooks different versions of the DCT have names such as Type I DCT, Type II DCT etc. Let us ignore these names and call the transform just DCT.
- We will start with the one-dimensional Discrete Cosine Transform (1D-DCT) and show how this transform can be extended into two dimensions.
- The 1D-DCT is also a member of the family of **unitary transforms**.

One-dimensional Discrete Cosine Transform (1D-DCT)

- The one-dimensional Discrete Cosine Transform (DCT) is defined as:

$$C(u) = a(u)\sum_{x=0}^{N-1} f(x)\cos\left[\frac{(2x+1)u\pi}{2N}\right],\ 0 \le u \le N-1$$

$$a(u) = \begin{cases} \sqrt{\frac{1}{N}} & u = 0 \\ \sqrt{\frac{2}{N}} & u = 1, \dots, N-1 \end{cases}$$

- The inverse transform is:

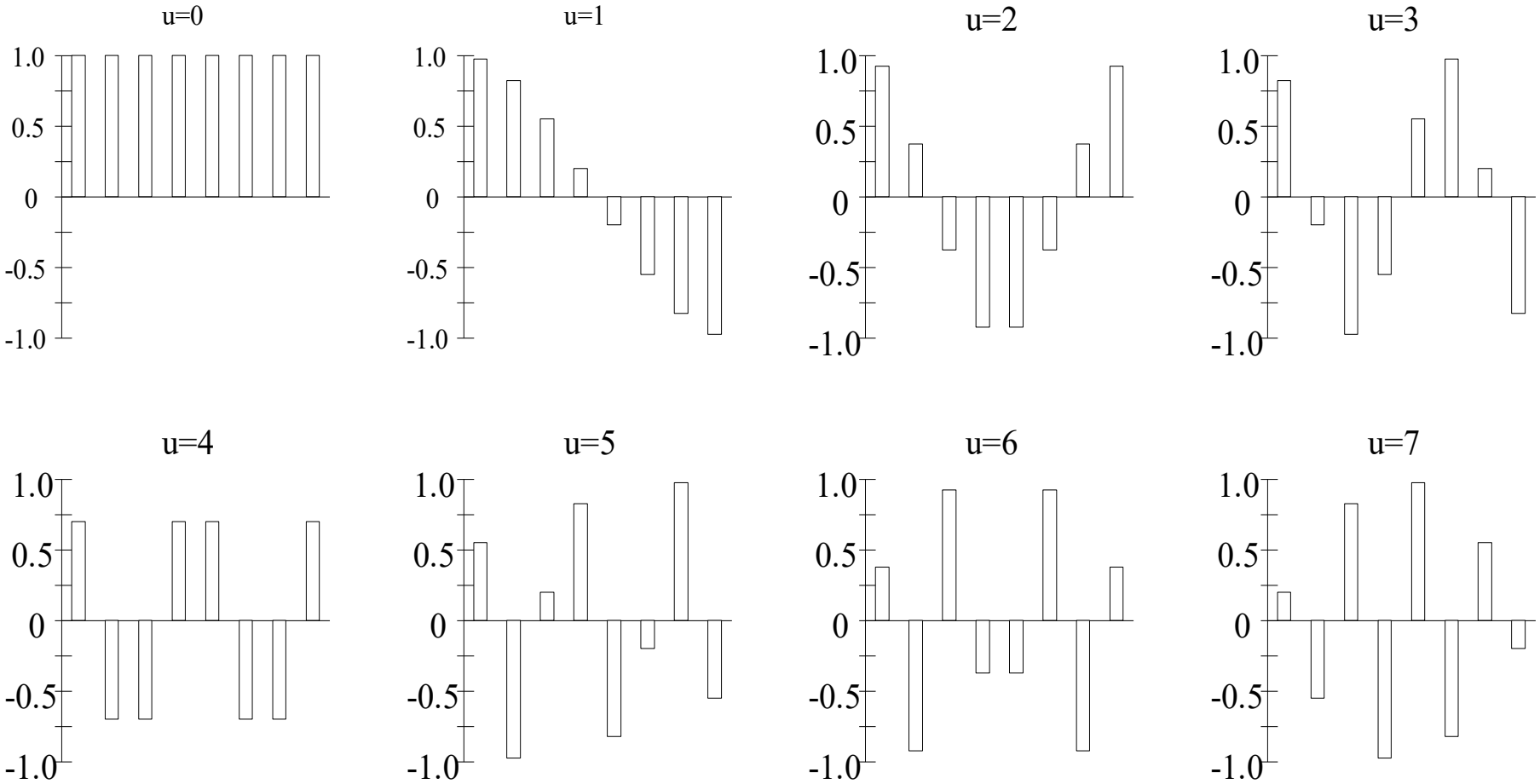
$$f(x) = \sum_{u=0}^{N-1} a(u)C(u)\cos\left[\frac{(2x+1)u\pi}{2N}\right]$$

- It can be shown that DCT is a unitary transform.
- The difference with DFT is that the signal is projected onto real sinewaves instead of complex. It is a real transform.

Imperial College London

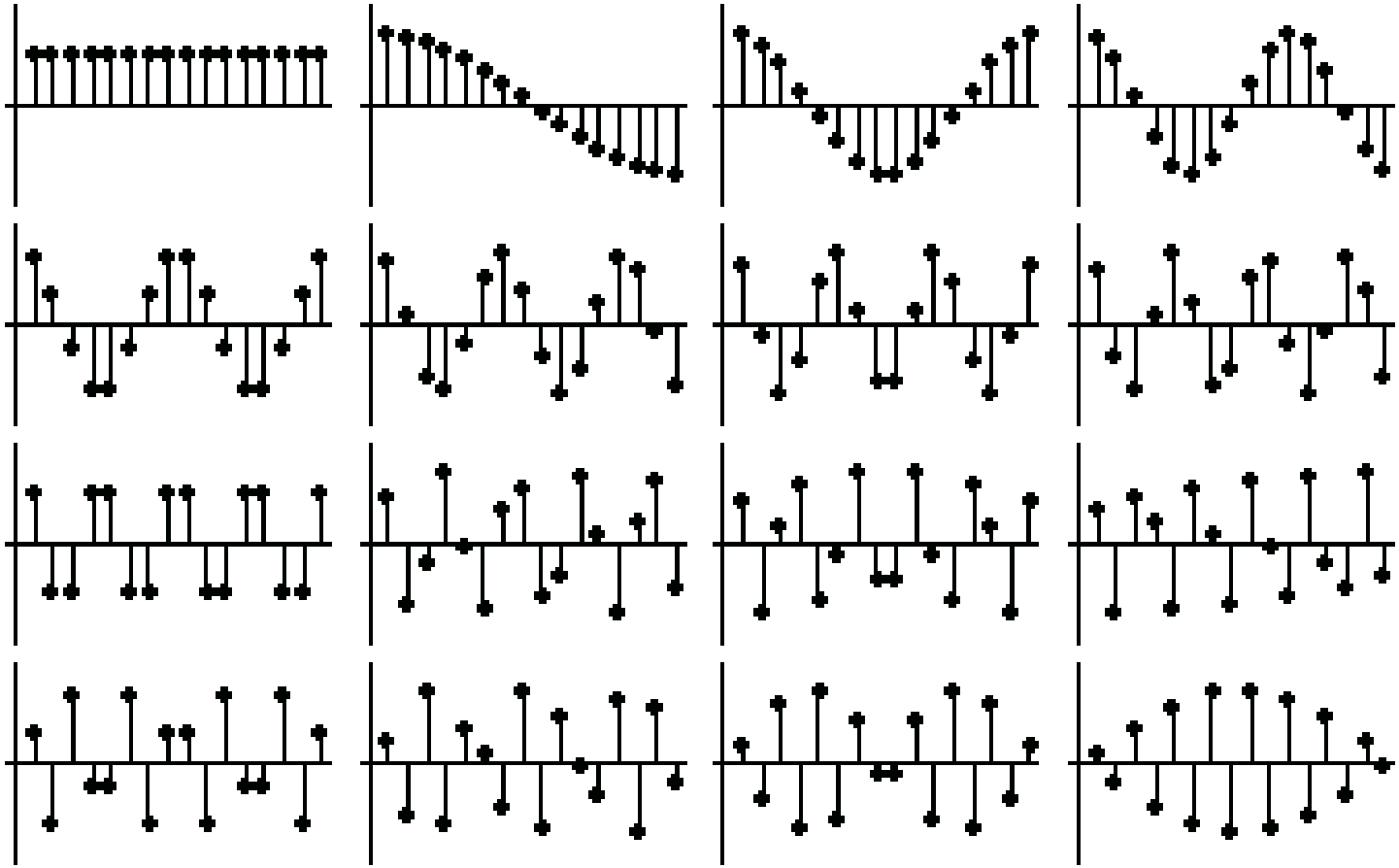
1-D Basis Functions N=8

- For a signal $f(x)$ with 8 samples, the rows of the 8×8 transformation matrix of the DCT are depicted below.
- **Values have been normalised from 0 to 1.**



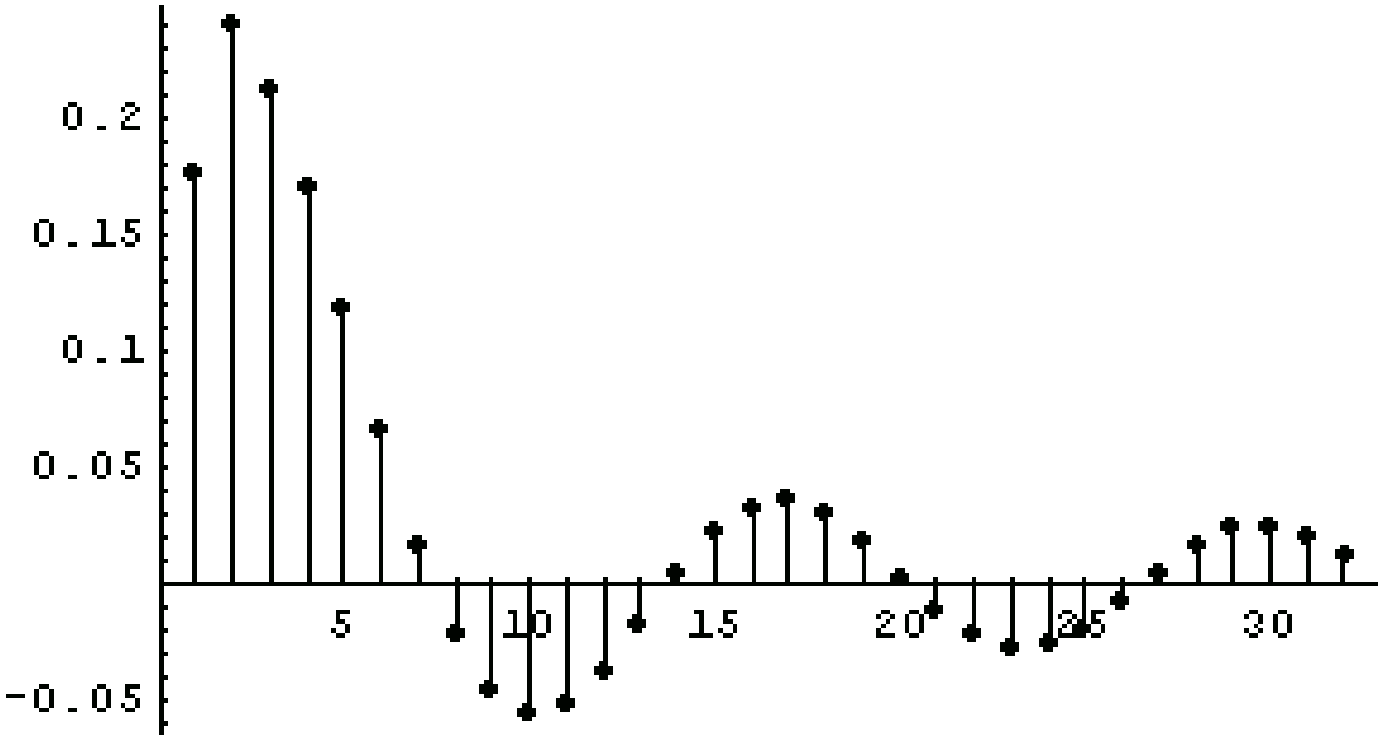
1-D Basis Functions $N=16$

- For a signal $f(x)$ with 16 samples, the rows of the 16×16 transformation matrix of the DCT are depicted below. **(My goal here is to depict their shapes rather than their values)**.



Example: One-dimensional Discrete Cosine Transform (DCT)

- Consider the signal $x[n] = \begin{cases} \frac{1}{5} & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$
- Its DCT is shown in the figure below.



Two-dimensional Discrete Cosine Transform (2D-DCT)

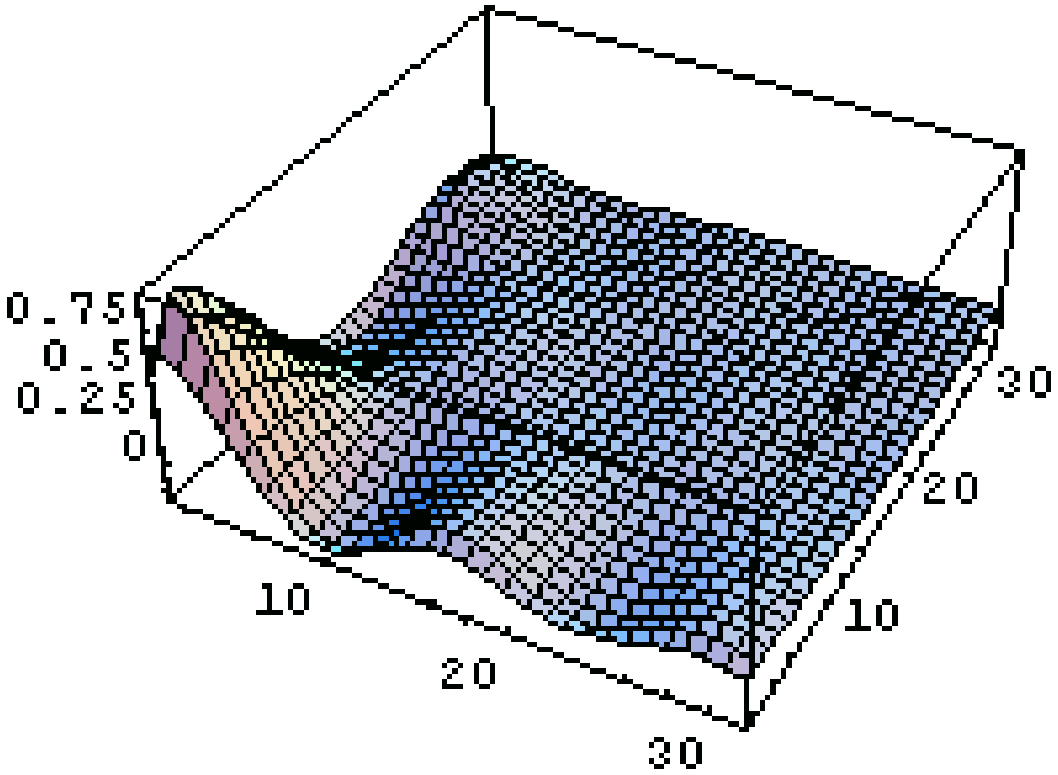
- Consider an image $f(x,y)$ of size $M \times N$.
- The two-dimensional Discrete Cosine Transform (DCT) is defined as:
  $C(u,v) = a(u)a(v)\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\frac{(2x+1)u\pi}{2M}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$,
  $0 \le u \le M-1,\ 0 \le v \le N-1$
- The inverse transform is:
  $$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} a(u)a(v)C(u,v)\cos\left[\frac{(2x+1)u\pi}{2M}\right]\cos\left[\frac{(2y+1)v\pi}{2N}\right]$$
  $0 \le x \le M-1,\ 0 \le y \le N-1$
- $a(u)$ is defined as previously.

Example: Two-dimensional Discrete Cosine Transform (DCT)

- Consider the two-dimensional signal

$$f(x,y) = \begin{cases} 1 & 0 \le x \le 2, 0 \le y \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

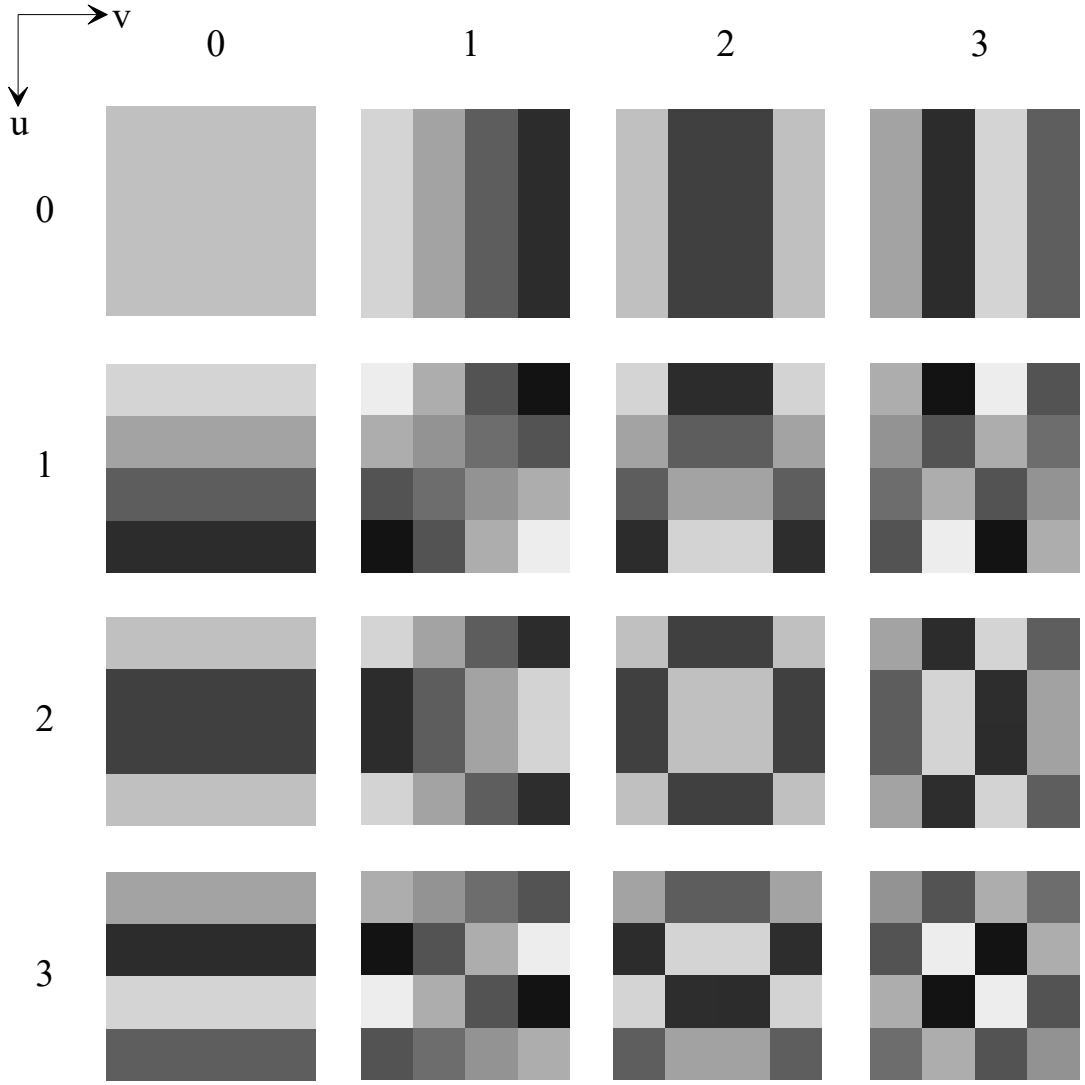
- Its DCT is shown in the figure below.



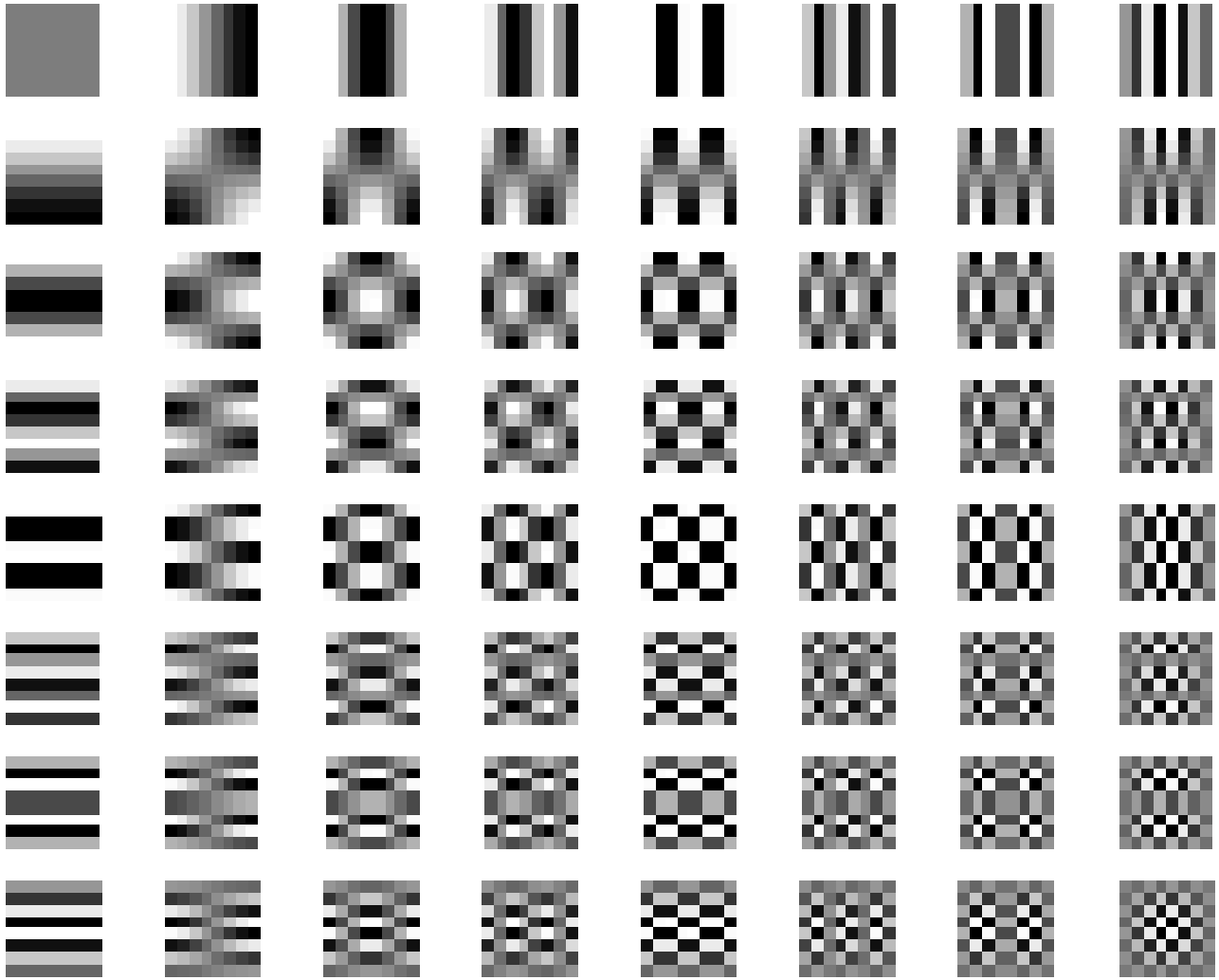
Advantages of the Discrete Cosine Transform

- The DCT is a real transform.
- The DCT has excellent energy compaction properties.
- There are fast algorithms to compute the DCT similar to the FFT.

How to visualise 2D Basis Functions $N = 4$

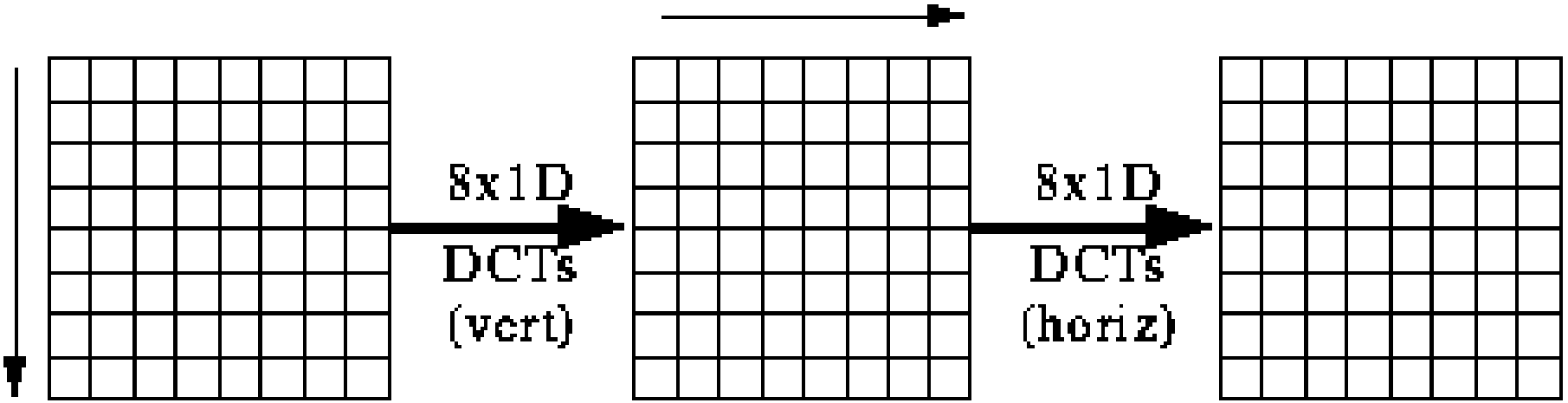


2-D Basis Functions $N = 8$



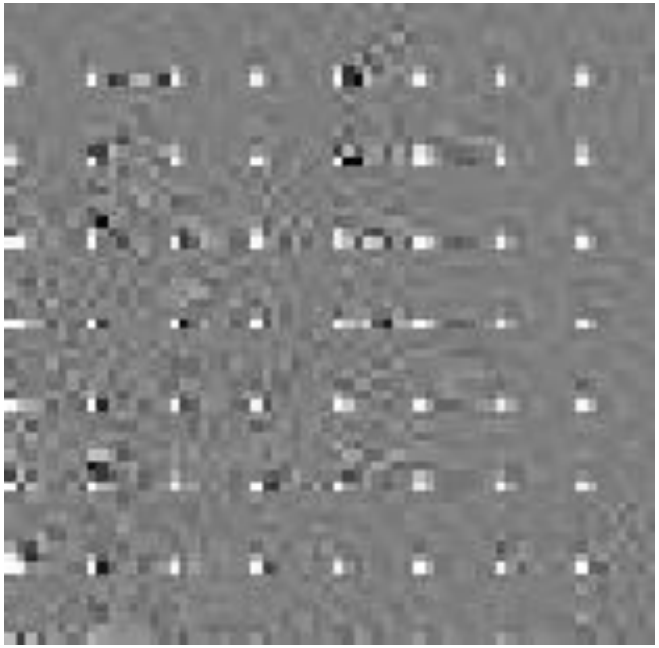
Separability of DCT

- The implementation 2D-DCT requires the sequential implementation of the corresponding one-dimensional transform row-by-row and then column-by-column (or the inverse), as with the case of 2D-DFT.



Example: 8×8 Block DCT

- The image below left is divided in patches (blocks) of size 8×8 pixels.
- The 2D-DCT is applied in each block.
- The result is depicted in the image below right.
- This is a standard way to use 2D-DCT in Image Compression Standards (JPEG). Details will be presented in Part 4 of the course.



Example: Energy Compaction

- Observe the excellent compaction property of 2D-DCT.
- Lena is shown on the left and its 2D-DCT on the right.