1. (a) The image of the square shown on the left of **Figure 1** below is represented by a function $f(x, y)$. The gray level $f(x, y)$ shows the value at a given point (x, y), with black being 1 and white being 0. Next to the square image $f(x, y)$ is a plot of the magnitude of its Fourier transform $F(u, v)$.

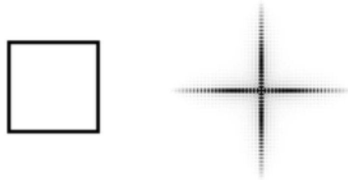


Figure 1

Let $a > 1$ be a fixed constant, and let A be the matrix

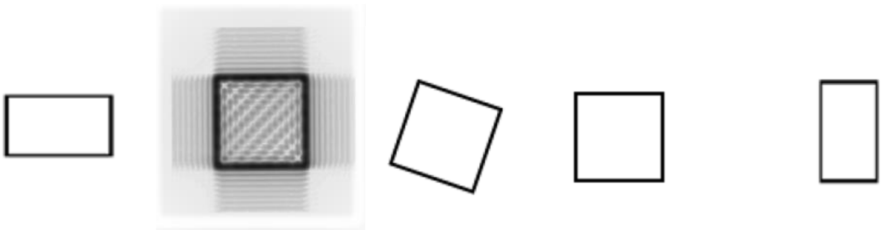
$$A = \begin{bmatrix} \cos\left(\frac{\pi}{6}\right) & -\sin\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) \end{bmatrix}$$

Consider the following five transformations of $f(x, y)$.

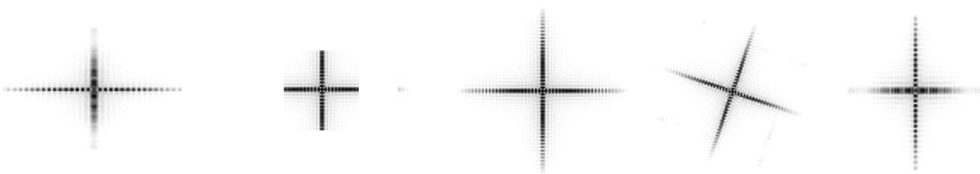
1. $f(ax, y)$
2. $f(x, ay)$
3. $f(x + a, y)$
4. $f(x_1, y_1)$ with $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}$
5. $f(x, y) * [\mathrm{sinc}(ax) \cdot \mathrm{sinc}(ay)]$

The sign "$*$" indicates convolution. Match the transformation 1 – 5 of $f(x, y)$ with the corresponding plot (i) – (v) shown below and with the plot of the corresponding Fourier transform (A) – (E) shown below. Give explanations. Each correct matching counts 2 marks (10 marks in total).

(i) (ii) (iii) (iv) (v)



(A) (B) (C) (D) (E)



SOLUTION

1. The function $f(ax, y)$ shrinks the square in the x −direction and so that's figure (v). For the Fourier transform, $\mathcal{F}\big(f(ax, y)\big) = \left(\frac{1}{a}\right) F(\frac{u}{a}, v)$ where u, v are the frequency components. Therefore, the Fourier transform is stretched in the u direction and the whole figure is likewise stretched. This matches with figure (A).

2. The function $f(x, ay)$ shrinks the square in the y −direction and so that's figure (i). For the Fourier transform, $\mathcal{F}\big(f(x, ay)\big) = \left(\frac{1}{a}\right) F(u, \frac{v}{a})$ where u, v are the frequency components. Therefore, the Fourier transform is stretched in the v direction and the whole figure is likewise stretched. This matches with figure (E).

3. The function $f(x + a, y)$ is a shift in the x −direction by an amount a to the left. Therefore, the square hasn't changed shape, just location relative to where it was before. There is only a phase change in the Fourier transform, which has magnitude 1, thus the plot is the same as for $\mathcal{F}\big(f(x, y)\big)$. The matches are with (iv) and (C).

4. $f(x_1, y_1)$ with $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}$. The matrix A causes the rotation of vector $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ by $-\pi/6$. When an image is rotated in space its Fourier transform is also rotated. Therefore, the matches are with (iii) and (D). It is implied from the pair of images (iii), (D) that the horizontal axis is y.

5. The matching that is left is (ii) and (B). Convolving in space with a two-dimensional sinc function yields in multiplying in frequency domain with the Fourier transform of a two-dimensional sinc function which is a two-dimensional rectangular pulse. This will eliminate all frequencies outside this pulse as it shown in (B).

(b) Consider the population of vectors $\underline{f}$ of the form

$$\underline{f}(x, y) = \begin{bmatrix} f_1(x, y) \\ f_2(x, y) \\ f_3(x, y) \end{bmatrix}.$$

Each component $f_i(x, y),\ i = 1, 2, 3$ represents an image of size $M \times M$ where M is even. The population arises from the formation of the vectors $\underline{f}$ across the entire collection of pixels (x, y). The three images are defined as follows:

$$f_1(x, y) = \begin{cases} r_1 & 1 \le x \le \dfrac{M}{2}, 1 \le y \le M \\ r_2 & \dfrac{M}{2} < x \le M, 1 \le y \le M \end{cases}$$

$$f_2(x, y) = r_3,\ 1 \le x \le M,\ 1 \le y \le M$$

$$f_3(x, y) = r_4,\ 1 \le x \le M,\ 1 \le y \le M$$

Consider now a population of random vectors of the form

$$\underline{g}(x, y) = \begin{bmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{bmatrix}$$

where the vectors $\underline{g}$ are the Karhunen-Loeve (KL) transforms of the vectors $\underline{f}$.

(i) Find the images $g_1(x,y)$, $g_2(x,y)$ and $g_3(x,y)$ using the Karhunen-Loeve (KL) transform. **[7]**

SOLUTION

$$f_1(x,y)=\begin{cases} r_1 & 1\le x\le \dfrac{M}{2}, 1\le y\le M \\ r_2 & \dfrac{M}{2}< x\le M, 1\le y\le M \end{cases}$$

Mean value of $f_1(x,y)$ is $m_1=\frac{r_1}{2}+\frac{r_2}{2}$. Zero-mean version of $f_1(x,y)$ is

$$f_1(x,y)-m_1=\begin{cases} \dfrac{r_1}{2}-\dfrac{r_2}{2} & 1\le x\le \dfrac{M}{2}, 1\le y\le M \\ \dfrac{r_2}{2}-\dfrac{r_1}{2} & \dfrac{M}{2}< x\le M, 1\le y\le M \end{cases} \quad 1$$

Mean value of $f_2(x,y)$ is r_3. Zero-mean version of $f_2(x,y)$ is $f_2(x,y)-m_2=0$.
Mean value of $f_3(x,y)$ is r_4. Zero-mean version of $f_3(x,y)$ is $f_3(x,y)-m_3=0$.
Variance of $f_1(x,y)-m_1$ is $\frac{1}{2}\frac{1}{4}(r_1-r_2)^2+\frac{1}{2}\frac{1}{4}(r_1-r_2)^2=\frac{1}{4}(r_1-r_2)^2$.
Variance of $f_2(x,y)-m_2$ is 0.
Variance of $f_3(x,y)-m_3$ is 0.
Covariance between $f_1(x,y)-m_1$ and $f_2(x,y)-m_2$ is 0. Therefore, the covariance matrix is $\begin{bmatrix} \frac{1}{4}(r_1-r_2)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ with eigenvalues $\frac{1}{4}(r_1-r_2)^2$ and 0. Therefore, by using the Karhunen Loeve transform we produce three new images, with two of them being 0 and the other being $f_1(x,y)-m_1$.

(ii) Comment on whether you could obtain the result of (b)-(i) above using intuition rather than by explicit calculation. **[2]**

SOLUTION

The above result is expected since two of the given images are constant and therefore they don’t carry any information. This means that there is only one principal component in the given set.

2. (a) A two-dimensional filter $h(m,n)$ of size 3×3 is given below, where the centre position corresponds to $m = n = 0$:

$$h(m,n) = \begin{bmatrix} -1 & 2 & -1 \\ -2 & 4 & -2 \\ -1 & 2 & -1 \end{bmatrix}$$

(i) Is the filter separable? If so, give the one-dimensional filters in horizontal and vertical directions. **[4]**

SOLUTION

The given response can be written as follows.

$$\begin{bmatrix} -1 & 2 & -1 \\ -2 & 4 & -2 \\ -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}$$

Therefore, the system is separable. The vertical filter is a lowpass filter and the horizontal filter is a high pass filter.

(ii) Determine the two-dimensional Discrete Time Fourier Transform

$$H(u,v) = \sum_{m=-1}^{m=1} \sum_{n=-1}^{n=1} h(m,n) e^{j\frac{(um+vn)}{3}}$$

of the filter and sketch the one-dimensional profiles $H(u,0)$, $H(u,1/2)$, $H(0,v)$, and $H(1/2,v)$. **[4]**

SOLUTION

$H(u,0) = \sum_{m=-1}^{m=1} \sum_{n=-1}^{n=1} h(m,n) e^{j\frac{um}{3}} = h(0,-1) + h(0,0) + h(0,1) + h(-1,-1)e^{j\frac{-u}{3}} + h(-1,0)e^{j\frac{-u}{3}} + h(-1,1)e^{j\frac{-u}{3}} + h(1,-1)e^{j\frac{u}{3}} + h(1,0)e^{j\frac{u}{3}} + h(1,1)e^{j\frac{u}{3}} = -2 + 4 - 2 - e^{j\frac{-u}{3}} + 2e^{j\frac{-u}{3}} - e^{j\frac{-u}{3}} - e^{j\frac{u}{3}} + 2e^{j\frac{u}{3}} - e^{j\frac{u}{3}} = 0.$

$$\boldsymbol{H\left(u,\frac{1}{2}\right) = \sum_{m=-1}^{m=1} \sum_{n=-1}^{n=1} h(m,n) e^{j\frac{um}{3}} e^{\frac{jn}{6}}}$$

$$\boldsymbol{= h(0,-1)e^{\frac{-j}{6}} + h(0,0) + h(0,1)e^{\frac{j}{6}} + h(-1,-1)e^{j\frac{-u}{3}}e^{\frac{-j}{6}} + h(-1,0)e^{j\frac{-u}{3}} + h(-1,1)e^{j\frac{-u}{3}}e^{\frac{j}{6}} + h(1,-1)e^{j\frac{u}{3}}e^{\frac{-j}{6}} + h(1,0)e^{j\frac{u}{3}} + h(1,1)e^{j\frac{u}{3}}e^{\frac{j}{6}}}$$

$$\boldsymbol{= -2e^{\frac{-j}{6}} + 4 - 2e^{\frac{j}{6}} - e^{j\frac{-u}{3}}e^{\frac{-j}{6}} + 2e^{j\frac{-u}{3}} - e^{j\frac{-u}{3}}e^{\frac{j}{6}} - e^{j\frac{u}{3}}e^{\frac{-j}{6}} + 2e^{j\frac{u}{3}} - e^{j\frac{u}{3}}e^{\frac{j}{6}}}$$

$$\boldsymbol{= -4\cos\left(\frac{1}{6}\right) + 4 - 2e^{j\frac{-u}{3}}\cos\left(\frac{1}{6}\right) + 2e^{j\frac{-u}{3}} - 2e^{j\frac{u}{3}}\cos\left(\frac{1}{6}\right) + 2e^{j\frac{u}{3}}}$$

$$\boldsymbol{= -4\cos\left(\frac{1}{6}\right) + 4 - 4\cos\left(\frac{1}{6}\right)\cos\left(\frac{u}{3}\right) + 4\cos\left(\frac{u}{3}\right)}$$

$$\boldsymbol{= 4\left(1 - \cos\left(\frac{1}{6}\right)\right)\left(1 + \cos\left(\frac{u}{3}\right)\right)}$$

$$H(0,v) = \sum_{m=-1}^{m=1}\sum_{n=-1}^{n=1} h(m,n)e^{j\frac{vn}{3}}$$

$$= h(0,-1)e^{j\frac{-v}{3}} + h(0,0) + h(0,1)e^{j\frac{v}{3}} + h(-1,-1)e^{j\frac{-v}{3}}$$
$$+ h(-1,0) + h(-1,1)e^{j\frac{v}{3}} + h(1,-1)e^{j\frac{-v}{3}} + h(1,0) + h(1,1)e^{j\frac{v}{3}}$$
$$= -2e^{j\frac{-v}{3}} + 4 - 2e^{j\frac{v}{3}} - e^{j\frac{-v}{3}} + 2 - e^{j\frac{v}{3}} - e^{j\frac{-v}{3}} + 2 - e^{j\frac{v}{3}}$$
$$= 8 - 8\cos\left(\frac{v}{3}\right)$$

$$H\left(\frac{1}{2},v\right) = \sum_{m=-1}^{m=1}\sum_{n=-1}^{n=1} h(m,n)e^{j\frac{(um+vn)}{3}} = \sum_{m=-1}^{m=1}\sum_{n=-1}^{n=1} h(m,n)e^{j\frac{\left(\frac{m}{2}+vn\right)}{3}}$$

$$= h(0,-1)e^{-j\frac{v}{3}} + h(0,0) + h(0,1)e^{j\frac{v}{3}} + h(-1,-1)e^{-j\frac{1}{6}}e^{-j\frac{v}{3}}$$
$$+ h(-1,0)e^{-j\frac{1}{6}} + +h(-1,1)e^{-j\frac{1}{6}}e^{j\frac{v}{3}} + h(1,-1)e^{j\frac{1}{6}}e^{-j\frac{v}{3}}$$
$$+ h(1,0)e^{j\frac{1}{6}} + +h(1,1)e^{j\frac{1}{6}}e^{j\frac{v}{3}}$$
$$= -2e^{-j\frac{v}{3}} + 4 - 2e^{j\frac{v}{3}} - e^{-j\frac{1}{6}}e^{-j\frac{v}{3}} + 2e^{-j\frac{1}{6}} - e^{-j\frac{1}{6}}e^{j\frac{v}{3}} - e^{j\frac{1}{6}}e^{-j\frac{v}{3}}$$
$$+ 2e^{j\frac{1}{6}} - e^{j\frac{1}{6}}e^{j\frac{v}{3}}$$
$$= 4 - 4\cos\left(\frac{v}{3}\right) - 2e^{-j\frac{1}{6}}\cos\left(\frac{v}{3}\right) - 2e^{j\frac{1}{6}}\cos\left(\frac{v}{3}\right) + 4\cos\left(\frac{1}{6}\right)$$
$$= 4\left(1 + \cos\left(\frac{1}{6}\right)\right)\left(1 - \cos\left(\frac{v}{3}\right)\right)$$

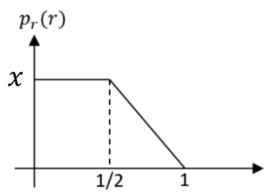
(iii) What is the functionality of the filter? Explain your reasoning. **[4]**

SOLUTION

The vertical filter is a lowpass filter and the horizontal filter is a high pass filter.

(b) Two images have grey level probability density function (or histogram normalized by the number of pixels) $p_r(r)$ illustrated in **Figure 2** below. For each image, find the transformation function that will help to equalize the histogram.

(i)



[6]

SOLUTION

The parameter x shown in the Figure above is calculated so that the area under the pdf is 1. It is straightforward to show that $\frac{x}{2} + \frac{x}{4} = 1$ and therefore, $x = 4/3$.

We will assume that the image to be processed has a continuous intensity r that lies within the interval [0,1]. Let $p_r(r)$ denote the probability density function (pdf) of the variable r. We now apply the following transformation function to the intensity:

$$s = T(r) = \int_0^r p_r(w)dw, 0 \le r \le 1$$

For $0 \le r \le 1/2$ we have $s = T(r) = \int_0^r \frac{4}{3} dw = \frac{4}{3} r$

The line with the negative slope is described by the equation:

$y = ax + b$

$\frac{4}{3} = \frac{a}{2} + b$ and $0 = a + b$

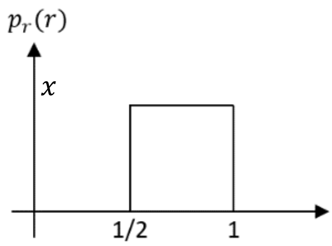
$\frac{4}{3} = \frac{a}{2} - a \Rightarrow \frac{4}{3} = -\frac{a}{2} \Rightarrow a = -\frac{8}{3}$ and $b = \frac{8}{3}$

For ½ $\le r \le 1$ we have $p_r(r) = -\frac{8}{3}r + \frac{8}{3}$

$$s = T(r) = \frac{2}{3} + \int_{\frac{1}{2}}^{r} \left(-\frac{8}{3}w + \frac{8}{3}\right) dw = -4\frac{r^2}{3} + \frac{8r}{3} - \frac{1}{3}$$

(ii)

[2]



The parameter x shown in the Figure above is calculated so that the area under the pdf is 1. It is straightforward to show that $x = 2$.

For $0 \le r \le 1/2$ we have $s = T(r) = 0$

For ½ $\le r \le 1$ we have

$$s = T(r) = \int_{\frac{1}{2}}^{r} 2dw = 2(r - \frac{1}{2})$$

3. We are given the degraded version g of an image f such that in lexicographic ordering

$$g = Hf + n$$

where H is the degradation matrix which is assumed to be block-circulant and n is the noise term which is assumed to be zero-mean, white and independent of the image f. All images involved have size $N \times N$ after extension and zero-padding.

(a) (i) Describe how inverse filtering can be used to restore the degraded image above. **[3]**

SOLUTION

The objective is to minimize $J(\mathbf{f}) = \|\mathbf{n}(\mathbf{f})\|^2 = \|\mathbf{y} - \mathbf{Hf}\|^2$.

We set the first derivative of the cost function equal to zero $\frac{\partial J(\mathbf{f})}{\partial \mathbf{f}} = 0 \Rightarrow -2\mathbf{H}^{\mathbf{T}}(\mathbf{y} - \mathbf{Hf}) = \mathbf{0}$.

By solving that system we obtain $\mathbf{H}^{\mathbf{T}}\mathbf{Hf} = \mathbf{H}^{\mathbf{T}}\mathbf{y}$.

If $M = N$ and $\mathbf{H}^{-1}$ exists then $\mathbf{f} = \mathbf{H}^{\mathbf{-1}}\mathbf{y}$.

According to the previous analysis if $\mathbf{H}$ (and therefore $\mathbf{H}^{\mathbf{-1}}$) is block circulant the above problem can be solved as a set of $M \times N$ scalar problems as follows

$$F(u,v) = \frac{H^*(u,v)Y(u,v)}{|H(u,v)|^2} \Rightarrow f(i,j) = \Im^{-1}\left[\frac{H^*(u,v)Y(u,v)}{|H(u,v)|^2}\right] = \frac{Y(u,v)}{H(u,v)}$$

(ii) Given knowledge of the exact degradation function, under what assumption can we perfectly restore the image? (2 points in total) How can we avoid erratic behaviour when the assumption is not met? (2 points in total) **[4]**

SOLUTION

In the presence of external noise we have that

$$\hat{F}(u,v) = \frac{H^*(u,v)\big(Y(u,v) - N(u,v)\big)}{|H(u,v)|^2} = \frac{H^*(u,v)Y(u,v)}{|H(u,v)|^2} - \frac{H^*(u,v)N(u,v)}{|H(u,v)|^2} \Rightarrow$$

$$\hat{F}(u,v) = F(u,v) - \frac{N(u,v)}{H(u,v)}$$

If $H(u,v)$ becomes very small, the term $N(u,v)$ dominates the result. Therefore, only under the assumption that the image is noiseless we can perfectly restore it.

In order to avoid erratic behaviour we carry out the restoration process in a limited neighborhood about the origin where $H(u,v)$ is not very small. This procedure is called pseudoinverse filtering. In that case we set:

$$\hat{F}(u,v) = \begin{cases} \dfrac{H^*(u,v)Y(u,v)}{|H(u,v)|^2} & H(u,v) \neq 0 \\ 0 & H(u,v) = 0 \end{cases}$$

or

$$\hat{F}(u,v) = \begin{cases} \dfrac{H^*(u,v)Y(u,v)}{|H(u,v)|^2} & |H(u,v)| \geq \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

In general, the noise may very well possess large components at high frequencies (u,v), while $H(u,v)$ and $Y(u,v)$ normally will be dominated by low frequency components. The parameter ε (called **threshold**) is a small number chosen by the user.

(iii) In a particular scenario the degradation process can be modelled as a linear filter with the two-dimensional impulse response given below:

$$h(x,y) = \begin{cases} 0.25 & x \pm 1, y = 0 \\ 0.5 & x = 0, y = 0 \\ 0 & \text{elsewhere} \end{cases}$$

Estimate the frequency pairs for which Inverse Filtering cannot be applied. **[4]**

SOLUTION

Inverse Filtering cannot be applied if $H(u,v) = 0$.

$$H(u,v) = \frac{1}{NN} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} h(x,y) e^{-j2\pi(ux/N+vy/N)} = \frac{0.25}{NN} (e^{-j2\pi u/N} + e^{j2\pi u/N} + 2)$$
$$= \frac{0.5}{NN} [\cos(2\pi u/N) + 1]$$

The values of (u,v) for which $H(u,v)$ cannot be estimated are found by:
$\cos(2\pi u/N) + 1 = 0 \Rightarrow \cos(2\pi u/N) = -1 \Rightarrow u = \pi$

(b) (i) Give the equation of Wiener filter in frequency domain. It is often referred to as the least square error filter. Explain the statement. **[3]**

SOLUTION

The objective is to minimize the following function (least squares error filter)

$$E\{(\mathbf{f} - \hat{\mathbf{f}})^{\mathbf{T}} (\mathbf{f} - \hat{\mathbf{f}})\}$$

with $\hat{\mathbf{f}}$ the estimated original image and $\mathbf{f}$ the original image.

To do so the following conditions should hold:

1. $E\{\hat{\mathbf{f}}\} = E\{\mathbf{f}\} \Rightarrow E\{\mathbf{f}\} = \mathbf{W}E\{\mathbf{y}\}$

2. the error must be orthogonal to the observation about the mean $E\{(\hat{\mathbf{f}} - \mathbf{f})(\mathbf{y} - E\{\mathbf{y}\})^{\mathbf{T}}\} = 0$

In frequency domain

$$W(u,v) = \frac{S_{ff}(u,v)H^*(u,v)}{S_{ff}(u,v)\left|H(u,v)\right|^2 + S_{nn}(u,v)}, \ \hat{F}(u,v) = \frac{S_{ff}(u,v)H^*(u,v)}{S_{ff}(u,v)\left|H(u,v)\right|^2 + S_{nn}(u,v)}Y(u,v)$$

(ii) Define the terms in the equation. **[3]**

SOLUTION

S_{xx} denotes the power spectral density of signal x

$Y(u,v)$ is the two-dimensional Fourier transform of the output

$H(u,v)$ denotes the frequency response of the degradation system.

(iii) Explain under which conditions the Wiener filter is equal to the inverse filtering procedure. **[3]**

SOLUTION

In the absence of any blur, $H(u,v) = 1$ and

$$W(u,v) = \frac{S_{ff}(u,v)}{S_{ff}(u,v) + S_{nn}(u,v)} = \frac{(SNR)}{(SNR)+1}$$

(a) $(SNR) >> 1 \Rightarrow W(u,v) \cong 1$

(b) $(SNR) << 1 \Rightarrow W(u,v) \cong (SNR)$

(SNR) is high in low spatial frequencies and low in high spatial frequencies so $W(u,v)$ can be implemented with a lowpass (smoothing) filter.

If $S_{nn}(u,v) = 0 \Rightarrow W(u,v) = \dfrac{1}{H(u,v)}$ which is the inverse filter

If $S_{nn}(u,v) \to 0$

$$\lim_{S_{nn}\to 0} W(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & H(u,v) \neq 0 \\ 0 & H(u,v) = 0 \end{cases}$$

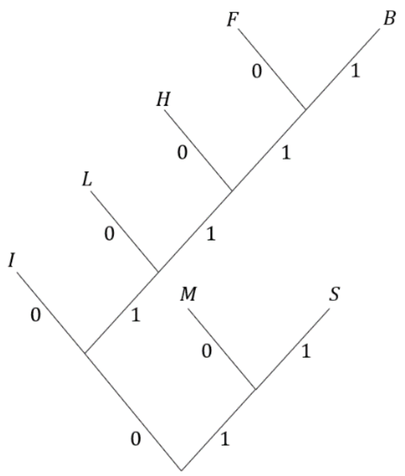
which is the pseudoinverse filter.

4. (a) (i) Draw the Huffman tree corresponding to the encoding table in **Figure 4.1** below. The first column indicates the list of symbols, the second column their frequency and the third column their Huffman codeword. **[5]**

symbol	frequency	encoding
B	2	01111
F	1	01110
H	3	0110
I	?	00
L	5	010
M	15	10
S	15	11

Figure 4.1

SOLUTION



(ii) State the range of frequency values that are possible for character I.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 **[5]**

***H*, *F* and *B* have a total frequency of 6 and are combined with *L* which has a frequency of 5. The next symbol is *I*. The frequency of *I* is at least 6 since for lower frequencies, *I* would be combined with *L*. The two next symbols are *M* and *S* with frequency 15 and they are combined together. Therefore, the frequency of *I* cannot be higher than 15. Therefore, the frequency of *I* is between 6 and 15 included.**

(b) A set of images is to be compressed by a lossless method. Each pixel of each image has a value in the range (0-3), i.e., 2 bits/pixel. An image from this set is given in **Figure 4.2** below; in this image the occurrence of pixels of different values is typical of the set of images as a whole.

3	3	3	2
2	3	3	3
3	2	2	2
2	1	1	0

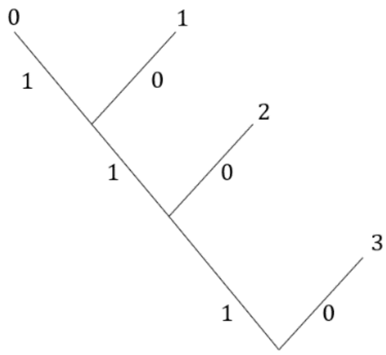
Figure 4.2

What is the degree of compression achievable using the following methods?

(i) Using Huffman coding of the pixel values. **[5]**

SOLUTION

symbol	frequency	probability	encoding
0	1	0.0625	111
1	2	0.125	110
2	6	0.375	10
3	7	0.4375	0



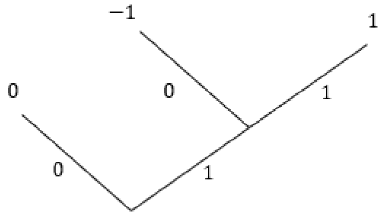
The average number of bits per pixel is:
$l_{\text{avg}} = 1 \cdot \frac{7}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{3}{16} + 3 \cdot \frac{1}{16} = \frac{28}{16} = 1.75\text{bits/pixel}.$
The entropy of the source is:
$H(s) = -\sum_{i=1}^{4} p_i \log_2(p_i) = 0.0625 \cdot 4 + 0.125 \cdot 3 + 0.375 \cdot 1.415 + 0.4375 \cdot 1.192 = 0.25 + 0.375 + 0.53 + 0.5215$ =1.68.
The compression ratio is $\frac{2}{1.75} = 1.14$.

(ii) Using differences between pixels in the following fashion: a pixel value is replaced by the difference between itself and the value of the pixel in the same row and the adjacent to the left column (horizontal scan). The top left pixel is ignored. Then, applying Huffman code to these differences. **[5]**

SOLUTION

$$\begin{array}{cccc} & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 \end{array}$$

symbol	frequency	probability	encoding
0	10	10//15=2/3	0
1	1	1/15	11
-1	4	4/15	01



The number of bits required to transmit the image is $1 \cdot 10 + 2 \cdot 1 + 2 \cdot 4 = 20$ but we must add 2 bits to transmit the value of the top-left corner. The plain message has 16 pixels with 2 bits per pixel, i.e., 32 bits. The compression ratio is, therefore, $\frac{32}{22} = 1.45$. Therefore, the second method is preferable.