Homework 2

BMEGEMMNWCM, Continuum Mechanics

We use the 2nd-order incompressible Ogden's isotropic hyperelastic constitutive model to represent the mechanical behavior of an incompressible rubber-like material. The strain energy potential of the model is defined as

$$W = \sum_{k=1}^{2} \frac{2\mu_k}{\alpha_k^2} \left(\lambda_1^{\alpha_k} + \lambda_2^{\alpha_k} + \lambda_3^{\alpha_k} - 3\right).$$

The model contains four independent material parameters $(\mu_1, \mu_2, \alpha_1, \alpha_2)$. We have performed the following experimental tests on the material: Uniaxial extension, Equibiaxial extension, Planar Extension. The measured data are given in tabular form.

TASKS:

General: Determine the material parameters of the 2nd-order Ogden's model by performing a parameter-fitting task. Use the "*Root Mean Squared Relative Error*" with the engineering stress to define the quality function for all experiments. Use *constrained minimization* with constraint $\mu_1 + \mu_2 > 0$ in order to ensure positive ground-state Young's modulus.

- **Task 1.** Plot the experimental data points for each test in the same coordinate system. Use stretch for the horizontal axis and the nominal stress for the vertical axis. The plot range for stretch must be $\lambda = 1 \dots 4$. The plot range for the nominal stress must be $P = 0 \dots P_{\max}$, where $P_{\max}$ is the maximum measured nominal stress in the equibiaxial test. You can connect the data point with lines, but use dots to indicate the particular data points.

- **Task 2.** Obtain the material parameters by fitting the model only to the uniaxial data. Consequently, in the calculation of the quality function use only the uniaxial data. The parameter-fitting is acceptable if $Q_U < 5\ \%$. Report the values of the fitted material parameters and Q_U.

- **Task 3.** Compute the quality functions (numerical values) of the fitted model (in Task 2) for the equibiaxial and the planar extensions. Report the values!

- **Task 4.** Visualize the model accuracy by plotting the model solutions for the stresses in each test using the fitted material parameters. Use separate figures for the tests. Use the plot range defined in Task 1. Write a few sentences (your personal conclusions and observation) about the accuracy of the fitted model.

Task 5. Obtain the material parameters by fitting the model simultaneously to all measurements. Consequently, the quality function is the mean of the individual quality values: $Q = (Q_U + Q_B + Q_P)/3$. The parameter-fitting is acceptable if $Q < 5\ \%$. Report the values of the fitted parameters and Q, Q_U, Q_B, Q_P.

- **Task 6.** Visualize the model accuracy by plotting the model solutions for the stresses in each test using the fitted material parameters in Task 5. Use separate figures for the tests. Use the plot range defined in Task 1. Write a few sentences (your personal conclusions and observation) about the accuracy of the fitted model.

- **Task 7.** The homogeneous displacement field in the material is given with the components of the material displacement field. The displacement components are given in [mm] if the coordinates are substituted in [mm]. The material is in a plane stress state, i.e. $\sigma_z = \tau_{xz} = \tau_{yz} = 0$. Determine the unknown scalar k included in the displacement field using the incompressibility constraint!

- **Task 8.** Determine the principal stretches and the normalized principal Eulerian directions.

- **Task 9.** Compute the components of the matrix of the Cauchy stress tensor using the fitted material parameters in Task 5. Determine the Mises equivalent stress.

Your data can be downloaded from the following link:

`https://www.mm.bme.hu/~kossa/NEPTUN-HW2.pdf`

The string **NEPTUN** has to be replaced with the Student's NEPTUN code. The link above is case-sensitive! Use uppercase letters for the file-name! Students must solve the problem corresponding to their NEPTUN's code!