Tutorial 5. - 10$^{\text{th}}$ October 2019

47. The distribution of random variable X is the following: $P(X = i) = \frac{1}{2N+1}$ $(i = 0, \pm 1, \pm 2, \ldots, \pm N)$. Calculate the standard deviation of X.

48. Suppose that $0 < Y < 3$ is a random variable with distribution function $F(x) = cx^3$ on interval $[0, 3]$. Find c and $P(-1 < Y < 1)$.

49. Do the following expressions determine distribution functions?

a) $F(x) = \begin{cases} 1 - \left(\frac{c}{x}\right)^a & if \ x > c \\ 0 & \text{otherwise} \end{cases}$ (a and c positive real numbers)

b) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{[x]}{2} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$

50. For which value of c do we get a distribution function, if $F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ cx^3 & \text{if } 0 < x \le 3 \\ 1 & \text{if } 3 < x \end{cases}$. Find the

probability $P(-1 < X < 1)$, and the density function, respectively.

51. Let X be a random variable with density function
$f(x) = \begin{cases} cx^4 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$.
 (a) c=? (b) Find the distribution function of X. (c) P(X<-0.5)=? (d) P(X<0.5)=?
 (e) P(X<1.5)=?

52. Suppose that the results of university students on IQ tests follow normal distribution with expected value 105 and variance 10. How large is the probability that someone reaches 120 points?

53. The heights of the students in a class are the following (in cm):
 180 163 150
 157 165 165
 174 191 172
 165 168 186
 Analyze the height data using mean, standard deviation and boxplot (quartiles).

54. [HW] Let the density function of random variable X be $f(x) = \begin{cases} \frac{c}{x^4}, & \text{if } x > 1 \\ 0 & \text{else} \end{cases}$. Find constant c, the value of $E(X)$ and $Var(X)$, and the distribution function, respectively.

55. [HW] Random variable X has exponential distribution with parameter λ. Find the density function of $-\log(X)$.