Probability Theory and Statistics
FOR COMPUTE SCIENCE

Tutorial 9. - 14th November 2019

74. Flip a fair coin 3 times. Let X be the number of heads in the first 2 flips and let Y be the number of heads on the last 2 flips (so there is overlap on the middle flip). Compute $\text{cov}(X, Y)$.

75. Consider the following joint density function of X and Y: $f(x, y) = 4xy$, if $0 < x < 1$ and $0 < y < 1$ and 0 otherwise.

(a) Prove that this is indeed a density function.

(b) Find the marginal densities.

(c) Find $P(Y < 0.5)$.

(d) Find the joint cummulative distribution function.

(e) Find $P(0 < X \leq 0.5, 0.25 \leq Y \leq 0.5))$.

76. The bivariate distribution of X and Y is described by the following table:

		Y		
	P(X,Y)	5	10	15
	1	1/9	1/9	0
X	2	1/6	2/9	1/6
	3	0	0	2/9

(a) Show that the table indeed defines a distribution.

(b) Compute the marginal distributions.

(c) Show that X and Y are not independent.

(d) Compute $P(X < 15)$.

77. Consider the following joint density function of X and Y: $f(x, y) = c(x + 3y)$, if $0 < x < 1$ and $0 < y < 1$ and 0 otherwise.

(a) Find c.

(b) Find the marginal densities.

(c) Find the joint cummulative distribution function.

78. The compression force of a certain type of concrete is modeled by a normal random variable with expectation μ and variance σ^2. The measurement unit is the psi (pound per square inch). The empirical variance σ^{*2} is equal to 1000. An empirical mean of 3250 psi has been observed from a sample of 12 measurements. Give a confidence interval on level 95% and on level 99% and compare them. What would be different if we knew that the variance was 1000?

79. The weight of grapes produced per vine has been measured on 10 vines selected at random in a vineyard. The results in kilograms are the following: 2.4, 3.4, 3.6, 4.1, 4.3, 4.7, 5.4, 5.9, 6.5, 6.9. The weight of grapes produced by each vine is modeled by a normal distribution. Find the empirical mean and variance of the sample. Give a 95% confidence interval for σ^2 and μ.