Probability Theory and Statistics

FOR COMPUTER SCIENCE

Tutorial 2. - 19th September 2019

14. We throw two dice. What is the probability that both of them being 6, if we know that at least one is 6?

15. We flip a coin as many times as the result of a dice throw. What is the probability of never resulting heads?

16. Suppose you are given 100 coins. You are told that 99 of them are normal, and one is fake with heads on both sides. Suppose you reach into the bag, picking out a coin uniformly at random, flip it 10 times, and get 10 heads. What is the probability that the coin you chose is the fake coin, conditioned on the above event?

17. Suppose a student taking an exam. He knows the correct answer with probability p. If he doesn't know it, takes a chance, and gets the answer right with probability $\frac{1}{5}$. What is the conditional probability that he knew the answer, if he answered correctly?

18. Odysseus reaches a road fork containing 3 directions forward. One goes to Athens, the other to Sparta, and the last one to Mycenae. People of Athens are merchants, they like to deceive wanderers, and tell the truth only every 3rd time. Myceneans lie slightly less, every second time. Spartans tell always the truth, thanks to the strict education. Odysseus chooses uniformly random (using a dice) among the directions and continues his journey. Arriving to a city, he asks a man that $2 \cdot 2 =?$, and gets the correct answer. What is the (conditional) probability that he arrived to Athens?

19. For which $n > 1$ positive integers are the following events independent?

a) A: flipping a coin n times it falls both heads and tails, B: it falls at most once tails.

b) A: flipping a coin n times it falls both heads and tails, B: the first lands on heads.

20. Sharing problem (unfinished game): Consider a tournament involving 2 players, A and B playing the same game repetitively. Each game can be won by A and B with the same probability, and these games are independent. If someone has won 4 games, the tournament is over, and gets the prize money. Suppose that the tournament is discontinued before any player has won 4 games. How should the money be shared in order to distribute it proportionally to the chances of winning, if the last standing was $2 : 1$?

21. How large is the probability that on the Hungarian lottery the largest drawn number is k? (All numbers are $1, 2, \ldots, 90$, and 5 numbers are drawn.)

22. `[HW]` Suppose that $A_1, \ldots, A_n$ are arbitrary events. Prove the following inequality:

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - n + 1.$$

23. `[HW]` A and B are playing table tennis. Each rally is independently won by A with probability $1/3$, or by B with probability $2/3$. The game is over if someone has at least 21 points, and at least 2 points more than the other player. Suppose the standing is $A = 19 : 20 = B$. What is the probability of winning the game by A?