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① Simplify $\frac{x^{32}}{x^9 \cdot x^2} \cdot \frac{x^7}{x^2} = \frac{x^{32}}{x^{11}} \cdot \frac{x^7}{x^2} = x^{32-11} \cdot x^{7-2} = x^{21} \cdot x^5 = \boxed{x^{26}}$

② $8^2 \cdot 4^x \cdot 2^x = 8^4$

$(2^3)^2 \quad (2^2)^x \quad (2^3)^4$

$2^6 \cdot 2^{2x} \cdot 2^x = 2^{12} \rightarrow 2^{6+2x+x} = 2^{12}$

$6 + 3x = 12 \mid -6$

$3x = 6 \mid :3$

$\boxed{x = 2}$

③ If $\frac{x}{y} = 3$; $x^{-4} y^4 = ?$

$\frac{x}{y} = x \cdot \frac{1}{y} = x \cdot y^{-1}$

$x \cdot y^{-1} = 3 \Rightarrow (x \cdot y^{-1})^{-4} = 3^{-4} \Rightarrow x^{-4} \cdot y^4 = \frac{1}{3^4}$

$\hookrightarrow \boxed{x^{-4} \cdot y^4 = \frac{1}{81}}$

④ $\frac{\sqrt{4^{15}}}{\sqrt{16^7} \rightarrow \sqrt{(4^2)^7}}$ $\Longrightarrow \frac{\sqrt{4^{15}}}{\sqrt{4^{14}}} = \sqrt{\frac{4^{15}}{4^{14}}} = \sqrt{4^{15-14}} = \sqrt{4} = \boxed{2}$

⑤ TRUE OR FALSE

a) $x + (y+z) = (y+x) + z \rightarrow$ TRUE

b) $y(x+z) = xy + zy \rightarrow$ TRUE

c) $x^{y+z} = x^z + x^y \rightarrow$ FALSE ; $x^{y+z} = x^y \cdot x^z$

d) $\frac{x^z}{x^y} = x^{y-z} \rightarrow$ FALSE ; $\frac{x^z}{x^y} = x^{z-y}$

⑥ $\ln(x) \geq e$

$x \geq e^e$

$e^{\ln x} = x$

$\ln x = e$

⑦ $0°C = 32°F$

$100°C = 212°F$

$C° = (F - 32) \cdot \frac{5}{9}$

$\boxed{-40°}$ is the same in °C & °F

⑧ $f(x) = 3x - 12 \quad f(y) = 0$

$f(x) = y \quad 3x - 12 = 0 \mid +12$

$3x = 12 \mid :3$

$\boxed{x = 4} \Rightarrow \boxed{y = 4}$

(9) $9^{x^2-6x+2} = 81$ $\quad 9^{x^2-6x+2} = 9^2$

$81 = 9^2$

$\Rightarrow x^2-6x+2 = 2 \mid -2$

$x^2-6x = 0$

$\frac{6 \pm \sqrt{36-4\cdot 0}}{2}$ (6)

$x_1: \frac{6+6}{2} = 6$

$x_2: \frac{6-6}{2} = 0$

$\boxed{x_1 = 6}$

$\boxed{x_2 = 0}$

(10) Annual GDP growth is 3%. → how long until it triples?

$1{,}03^t = 3 \mid \ln$

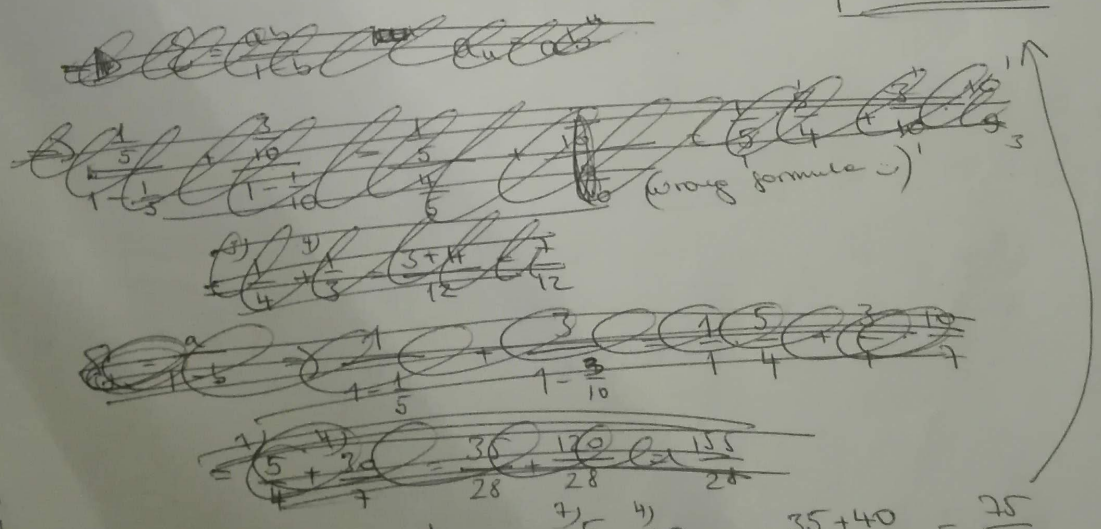
$\ln 1{,}03^t = \ln 3 \Rightarrow t \cdot \ln 1{,}03 = \ln 3 \mid :$

$t = \frac{\ln 3}{\ln 1{,}03}$ $\quad \boxed{t \approx 37{,}16700097}$

(11) $\log_\pi\left(\frac{1}{\pi^5}\right) = \log_\pi 1 - \log_\pi \pi^5 = 0 - 5 = \boxed{-5}$

(12) $\sum_{i=0}^{\infty}\left(\frac{1}{5^i} + 0.3^i\right) = \sum_{i=0}^{\infty}\frac{1}{5^i} + \sum_{i=0}^{\infty}\left(\frac{3}{10}\right)^i$ ← $a_2 = 3$, $b_2 = \frac{1}{10}$

$\left(\frac{1}{5}\right)^i$ ← $a_1 = 1$, $b_1 = \frac{1}{5}$ $\quad \boxed{\approx 2.67}$



$\sum_{i=0}^{\infty} = \frac{a}{1-b} \Rightarrow \frac{1}{1-\frac{1}{5}} + \frac{1}{1-\frac{3}{10}} = \frac{5}{4} + \frac{10}{7} = \frac{35+40}{28} = \frac{75}{28}$

(13.) $\lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to} \frac{(x+5)(x-5)}{x-5} = \lim_{x\to 5} x+5$

$\Rightarrow \lim_{x\to 5} 5+5 = \boxed{10}$

(14.) $f(x) = x^3 - 4$ at $(-2, -12)$

$f'(x) = 3x^2 - 0 = 3x^2$

$f(-2) = 3\cdot(-2)^2 = 12$

$f(-12) = 3\cdot(-12)^2 = 432$

(15.) $f(x) = \frac{x^5+3}{x^2-1}$ $\quad f'(x) = \frac{(5x^4+0)(x^2-1) - (2x-0)(x^5+3)}{(x^2-1)^2}$

$\Rightarrow \frac{5x^6 - 5x^4 - 2x^6 - 6x}{(x^2-1)^2} = \frac{3x^6 - 5x^4 - 6x}{(x^2-1)^2} = \boxed{\frac{3x^6 - 5x^4 - 6x}{x^4 - 2x^2 - 1}}$

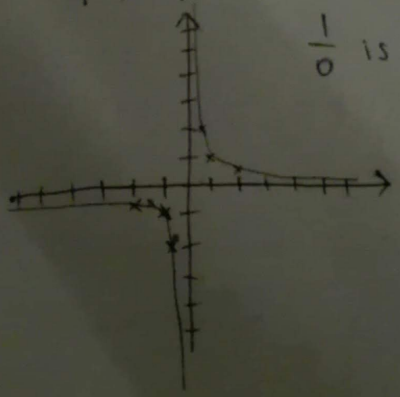
(Simplified $\Rightarrow \frac{\frac{1}{x^4}(3x^6 - 5x^4 - 6x)}{\frac{1}{x^4}(x^4 - 2x^2 - 1)} = \frac{3x^2 - 5 - \frac{6}{x^3}}{1 - \frac{2}{x^2} - \frac{1}{x^4}}$)

(16.) $f(x) = x^9 + 3$

$f'(x) = 9x^8 + 0 = 9x^8$

$f''(x) = 8\cdot 9x^7 = \boxed{72x^7}$

(17.) $f(x) = \frac{1}{x}$ → is it continuous at 0? Why?



$\frac{1}{0}$ is not defined → $\boxed{\frac{1}{x} \text{ is not continuous at } 0.}$

(18.) $f(x) = 4x^3 - 12x$ Find local min/max/inflection points

$f'(x) = 12x^2 - 12$

$12x^2 - 12 = 0 \mid +12 \quad 12x^2 = 12 \mid :12 \quad x^2 = 1 \mid \sqrt{\ }$ $\boxed{x = \pm 1}$

$f''(x) = 24x - 0 = 24x$

$f(1) = 4 - 12 = -8$

$f(-1) = -4 + 12 = 8$

$f''(1) = 24$ ← convex, minima $(1; -8)$

$f''(-1) = -24$ ← concave, maxima $(-1; 8)$

(19.) $f(x,y) = x^3 - y^2 \quad f(2,3) = ?$

↳ $2^3 - 3^2 = 8 - 9 = \boxed{-1}$

(20.) $f(x,y) = \ln(x - 3y)$ Domain?

↑ can be negative

$x - 3y > 0 \mid +3y \quad x > 3y$

(21.) $\frac{\partial}{\partial x}\left(x^5 y^7 + \frac{x^2}{y^3}\right) = \boxed{5x^4 \cdot y^7 + \frac{2x}{y^3}}$

(22.) $f(x,y) = \sqrt{xy} - x - y$ Find local min/max

$= \sqrt{x} \cdot \sqrt{y} - x - y = x^{\frac{1}{2}} \cdot y^{\frac{1}{2}} - x - y$

$\frac{\partial f(x,y)}{\partial x} = \frac{1}{2} x^{-\frac{1}{2}} \cdot y^{\frac{1}{2}} - 1 - 0 = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} \cdot \sqrt{y} - 1 = \frac{1}{2\sqrt{x}} \cdot \sqrt{y} - 1$

$\frac{\partial f(x,y)}{\partial y} = \sqrt{x} \cdot \frac{1}{2} y^{-\frac{1}{2}} - 0 - 1 = \sqrt{x} \cdot \frac{1}{2\sqrt{y}} - 1$

$\frac{1}{2\sqrt{x}} \cdot \sqrt{y} - 1 = 0 \mid^2 \quad \frac{1}{4x} \cdot y - 1 = 0 \mid \cdot 4x$

~~$4xy - 4x = 0 \mid +4x \quad 4xy = 4x \mid :4x$~~

~~$y = 1$~~

~~$\sqrt{x} \cdot \frac{1}{2\sqrt{y}} - 1 = 0 \mid^2 \quad x \cdot \frac{1}{4y} - 1 = 0 \mid$~~

$y - 4x = 0 \mid +4x$ $\boxed{y = 4x}$ $\boxed{x = \frac{y}{4}}$

~~$\frac{1}{\sqrt{4x} \cdot 2\sqrt{4x}} - 1 = 0 \mid^2 \quad \frac{1}{4x \cdot 16x} - 1 = 0$~~

(23) $\max x^2y^2$ s.t. $2x+y=9$

$f(x,y) = x^2y^2 \quad g(x,y) = 2x+y-9$

$\mathcal{L} = f(x,y) - \lambda \cdot (g(x,y)) = x^2y^2 - \lambda(2x+y-9)$

$\frac{\partial \mathcal{L}}{\partial x} = 2x \cdot y^2 - \lambda \cdot 2 \quad 2xy^2 = 2\lambda \;/:2 \quad xy^2 = \lambda$

$\frac{\partial \mathcal{L}}{\partial y} = x^2 \cdot 2y - \lambda \cdot 1 \quad 2x^2y = \lambda$

$\Rightarrow 2x^2y = xy^2 \;/:y \quad 2x^2 = xy \;/:x \quad 2x = y$

$x = \frac{y}{2}$

$\frac{\partial \mathcal{L}}{\partial \lambda} = 2x+y-9=0 \quad 2x+y=9 \quad y+y=9 \quad 2y=9 \;/:2 \quad \boxed{y=4.5}$

$\boxed{x = \frac{4.5}{2} = 2.25}$

(24) $A = \begin{pmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{pmatrix} \quad B \cdot A = ?$

	2 5
	2 1
	7 6
1 0 1	9 11
9 1 5	55 76

$\boxed{B \cdot A = \begin{pmatrix} 9 & 11 \\ 55 & 76 \end{pmatrix}}$

(25) $A = \begin{pmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{pmatrix} \quad A \cdot B = ?$

	8 4 0
	2 1 2
5 3	46 23 6
0 1	2 1 2
1 2	12 6 4

$\boxed{A \cdot B = \begin{pmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{pmatrix}}$

(26) $A = \begin{pmatrix} e & 93 & 4.7 \\ 2 & 6.1 & 4.22 \\ 4 & \pi & 0 \end{pmatrix} \quad A^T = ?$

$\boxed{A^T = \begin{pmatrix} e & 2 & 4 \\ 93 & 6.1 & \pi \\ 4.7 & 4.22 & 0 \end{pmatrix}}$

(27) What's the determinant of A?

$A = \begin{pmatrix} 2 & 6 \\ 2 & 8 \end{pmatrix} \quad \det A = 2 \cdot 8 - 2 \cdot 6 = 16 - 12 = \boxed{4}$

(28.) Toss dice 2x

$\Omega = ?$

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

this is Ω are all the possible combinations

(29.) 0,1 % uses drugs
98 % right positive
2 % false negative
99,7 % right negative
0,3 % false positive

$P(A)$ = a person is a drug user

$P(B)$ = a person tests positive

$P(A|B) = ?$

$P(A) = 0,1\,\% = 0,001$

$P(B) = \underbrace{98\,\% \cdot 0,1\,\%}_{9,8\,\%} + \underbrace{99,7\,\% \cdot 0,3\,\%}_{29,91\,\%}$

$= 39,71\,\% = 0,3971$

$P(A|B) = \frac{P(A \times B)}{P(B)}$ $\quad P(A \times B) = 0,1 \cdot 98 = 9,8\,\% = 0,098$

$$\boxed{P(A|B) = \frac{9,8}{39,71} \approx 0.246789}$$

(30.) Toss dice 20 times

How many times did you end up with a 5?

Chances of $5 = \frac{1}{6}$

$$20 \cdot \frac{1}{6} = \frac{20}{6} = \frac{10}{3} \boxed{\approx 3.33}$$