Web

March 08 2024

Contents

1. Page 1

The American electoral system and its adaptation to the European Union

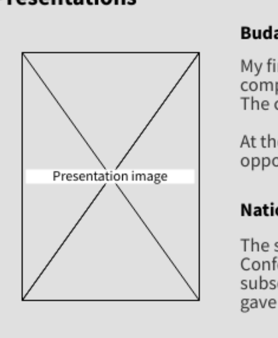
About the paper

[illegible]

Abstract

[illegible]

Presentations



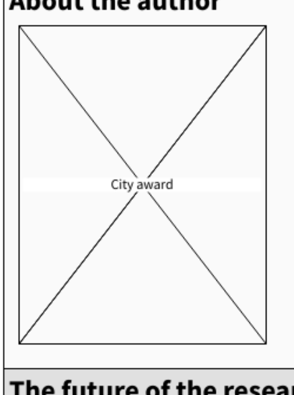
Budapest University of Technology and Economics

[illegible]

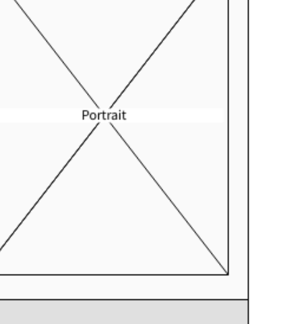
National Conference of Students' Research Societies

[illegible]

About the author



[illegible]



The future of the research

[illegible]

The context of the apportionment systems

United States

The history

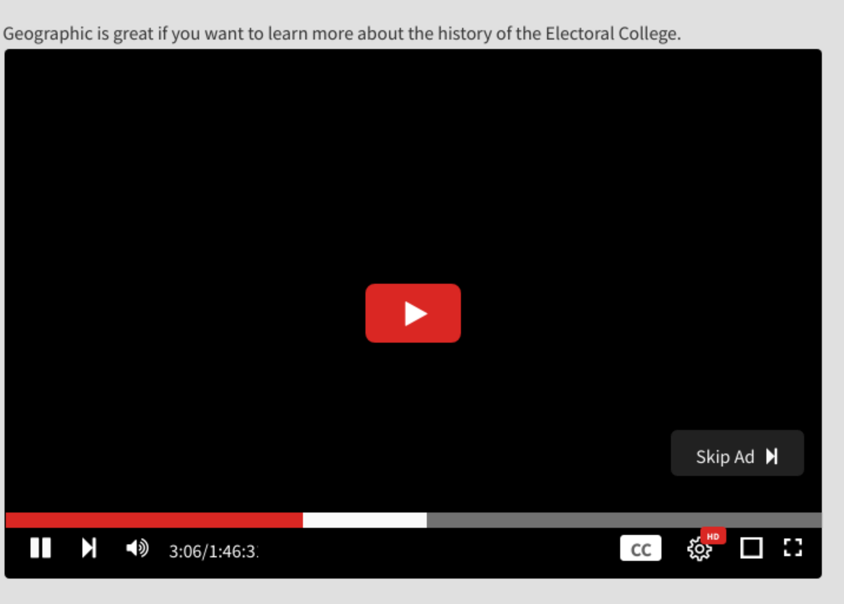
After the foundation of the United States in 1776, the Founding Fathers needed to establish a method for choosing the officeholders in the different branches of government.

The keyword when looking at the history American electoral system is "compromise". The founders needed to resolve the conflict between the smaller and larger states. The small states wanted each state to be equally represented in the decision-making, while the large states wanted representation to be proportional to the population. The result of this compromise was a bicameral legislature. In the Senate, every state has two senators, thus giving the smaller states a lot of power in the legislative process. In the House of Representatives, each state has a certain number of representatives proportional to its population, with each state having at least one seat in the House. The US constitution requires that the seats assigned to each state be proportional to its population, but it doesn't specify any method to use for apportionment.

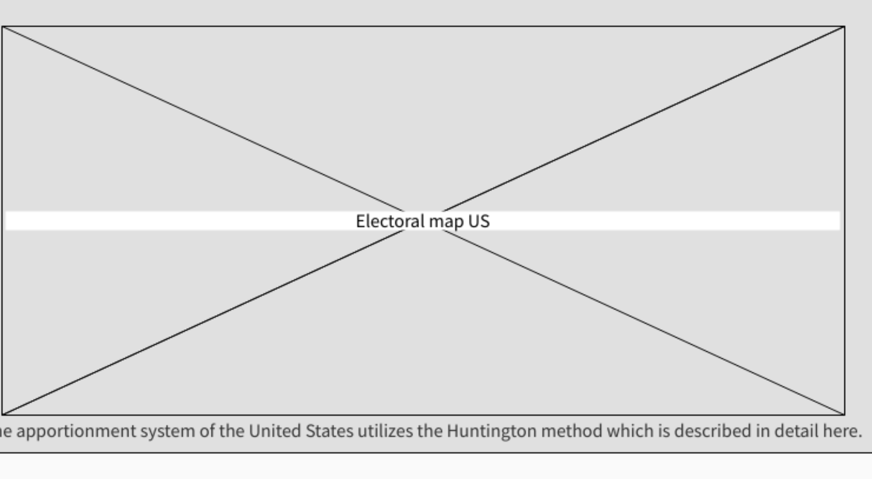
The method for electing the executive is derived from the method for electing the legislature. The President is not directly elected by the people, rather by the Electoral College. According to this system, each state is assigned a certain number of electors, who will vote according to the will of the voters. Each state is assigned the same number of electors as seats in the Congress (representatives + senators). Most states award all of their electors to the candidate who received the most votes in their state. To win the election, a candidate needs the votes of a majority of all electors.

Video

I think this video by the National Geographic is great if you want to learn more about the history of the Electoral College.



The system

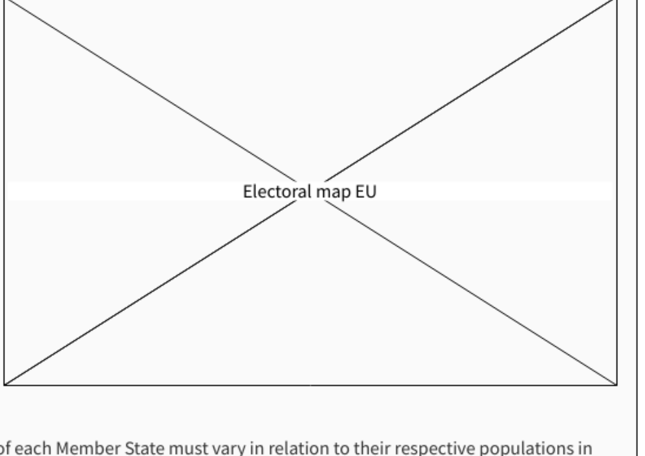


The apportionment system of the United States utilizes the Huntington method which is described in detail here.

European Union

The history

The President of the European Commission is the most powerful official in the European Union. This position is comparable to prime ministers in parliamentary democracies or presidents in presidential republics.



The election of the President is a contentious issue after every European election. To resolve this issue, the Spitzenkandidaten system was developed. According to this system, the person nominated by the party with the most seats in the European Parliament will become the President of the European Commission. This system was last used in 2014, when Jean-Claude Juncker of the European People's Party was elected President, after the centre-right party won a plurality of seats in the European Parliament.

The equivalent document of the US Constitution for the European Union which establishes the branches of government is the Treaty of Lisbon. It establishes the European Parliament as the primary legislative institution of the EU. Like the US Constitution, the Treaty does not specify the apportionment method, but it does set some guidelines. The document establishes the 705-seats of the European Parliament. It gives every member state at least six, and at most 96 seats in the Parliament.

The Treaty also requires any apportionment method to adhere to the principle of degressive proportionality. The guidelines for the current method of apportionment were described in the Cambridge compromise, which was finalised at the Cambridge Apportionment Meeting (CAM). (see \cite[Page1]{Grimmett2012}, \cite{Grimmett2017})

The system

Degressive proportionality means "the ratio between the population and the number of seats of each Member State must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State." (this is known as the Lamassoure-Severin definition, see \cite[Page 2]{Grimmett2012})

The following requirements were devised at the Cambridge Apportionment Meeting:

- "no smaller State shall receive more seats than a larger State,"
- "the ratio population/seats shall increase as population increases."

At the time, it seemed impossible to design a system that satisfies both requirements under the hypotheticals tested. This resulted in the modification of the Lamassoure-Severin definition. After the amendment, it defined degressive proportionality as follows: "the ratio between the population and the number of seats of each Member State before rounding to whole numbers must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State."

After the definition was amended, the Cambridge Apportionment Meeting developed the base + prop method for apportionment.

Base + prop method

The base + prop method for apportionment consists of two stages, as written in \cite[Page 2-3]{Grimmett2012}:

- The allocation of a fixed (base) number of states to each member state.
- The allocation of the remaining states in proportion to the member state's population (subject to rounding and capping at the maximum).

To achieve the desired size for the Parliament, a house-size divisor is introduced. Using the given base b, maximum M and divisor d, we can define the following allocation function $A_d: [0, \infty) \to [0, \infty)$:
$$
A_d (p) = \min \left\{ b + \frac{p}{d}, M \right\}
$$
We shall assign the seat share $A_d (p)$ to the member state with population p. After that, we need to round this to an integer, and adjust the divisor d, so that the sum of the seat numbers equals the desired Parliament size.

The Cambridge Apportionment Meeting recommended the use of base $b = 5$, and the use of upwards rounding.

References

[Gri12] G. R. Grimmett, European apportionment via the cambridge compromise, Mathematical Social Sciences 63 (2012), no. 2, 68–73, Around the Cambridge Compromise: Apportionment in Theory and Practice.

[Gri17] G. Grimmett, F. Pukelsheim, V. R. González, W. Słomczyński, K. Życzkowski, The Composition of the European Parliament, (2017)

The Huntington method

The current method of apportionment was developed by an American mathematician, Edward V. Huntington in the early 20th century. The method was described in \cite{Huntington1928}, \cite{Huntington1928}, \cite{Sullivan1972} and \cite{Sullivan1982}.

The requirements for the system

Huntington determined, that given two states, which have populations A and B respectively, have a and b seats in the House, the perfect system would give a result where

$$
\frac{A}{a} = \frac{B}{b} \text{.}
$$

As it would be basically impossible to satisfy this equation in a representative democracy, Huntington's goal was to make this inequality as small as possible.

The absolute and the relative difference

A crucial question in developing the system was whether to use the absolute or the relative difference for the apportionment process. For example, if there are two districts, one with a population of 1,000 and another one with 1,001 people, the absolute difference here is 1, and the relative difference is 0.1\%, but if two districts have a population of 10 and 11, the absolute difference would still be 1, but the relative difference would be 10\%. Due to this reason, Huntington recommended the use of the relative difference instead of the absolute one. We can achieve the relative difference between states A and B by calculating $\frac{\frac{A}{a}-\frac{B}{b}}{\frac{B}{b}}$.

The first test for the method of equal proportions

Huntington defined the first test for his method (known as the method of equal proportions or the method of the geometric mean, which we will be abbreviating as MGM) as follows (see \cite[Page 3]{Huntington1928}) :

"If the relative difference between the congressional districts, $\frac{A}{a}$ and $\frac{B}{b}$, belonging to any two states can be reduced by a transfer of a representative from one state to the other, then this transfer should be made."

We can write the following "inequality" \footnote{Please note that this is not a "real inequality". We are writing this as an inequality to transform these fractions into the desired form.} for the test:
$$
\frac{\frac{A}{a + 1} - \frac{B}{b}}{\frac{B}{b}} < \frac{\frac{B}{b + 1} - \frac{A}{a}}{\frac{A}{a}}
$$
We rearrange this inequality to get the multiplier that can be used for successive applications of the test: \footnote{We know that the population of a state and the number of representatives assigned to it will be a positive integer, so the direction of the inequality will not change, and we can take the square root of these numbers on the set of real numbers.}
$$
\frac{\frac{A}{a + 1} - \frac{B}{b}}{\frac{B}{b}} < \frac{\frac{B}{b + 1} - \frac{A}{a}}{\frac{A}{a}} \Rightarrow \nonumber \\
\left(\frac{A}{a} \right) \left(\frac{A}{a + 1} - \frac{B}{b} \right) < \left(\frac{B}{b} \right) \left(\frac{B}{b + 1} - \frac{A}{a} \right) \Rightarrow \nonumber \\
\frac{A^2}{a\left(a+1\right) } - \frac{AB}{ab} < \frac{B^2}{b \left(b+1 \right) } - \frac{AB}{ab} \Rightarrow \nonumber \\
\frac{A^2}{a \left(a + 1 \right) } < \frac{B^2}{b \left(b + 1 \right) } \Rightarrow \nonumber \\
\frac{A}{\sqrt{a \left(a + 1 \right) }} < \frac{B}{\sqrt{b \left(b + 1 \right) }}
$$

The second test for the method of equal proportions

This is similar to the first one, but instead of the ratio $\frac{A}{a}$, it uses $\frac{a}{A}$. Here, we divide the number of representatives by the population. We can think of this as the "individual share" each person living in the state gets of the representative. In perfect apportionment, the equality would look like this:
$$
\frac{a}{A} = \frac{b}{B}
$$
Huntington defined the second test as follows(see \cite[Page 4]{Huntington1928}:

"If the relative difference between the two "individual shares", $\frac{a}{A}$ and $\frac{b}{B}$, belonging to any two states, can be reduced by a transfer of a representative from one state to the other, then this transfer should be made."

We can see that the equality in \eqref{E:second_test} is the reciprocal of the one in \eqref{E:requirement}. If we perform transformations as seen in \eqref{E:transform_first_test}, we will arrive at the following inequality (which is also the reciprocal of the one in \eqref{E:transform_first_test}):
$$
\frac{\sqrt{a \left(a + 1 \right) }}{A} < \frac{\sqrt{b \left(b + 1 \right) }}{B}
$$

Working rule for the method of equal proportions

In \eqref{E:transform_first_test}, we transformed the inequality into the following:
$$
\frac{A}{\sqrt{a \left(a + 1 \right) }} < \frac{B}{\sqrt{b \left(b + 1 \right) }}
$$
As we are dividing the population of a state, we can interpret it as multiplying the population of state A by the following fraction:
$$
\frac{1}{\sqrt{a \left(a + 1 \right)}}
$$
Let's start by writing down the notations we will be using:
- n: the number of states
- r: the number of representatives that we need to assign
- A: the population of state A
- a: the number of representatives assigned to state A

The first thing we need to do is to write down each state, its population and the number of representatives assigned to it. At the start, each state will be assigned one representative. Now we see that we need to perform the procedure $n-r$ times.

Every time we assign a representative, we need to multiply the population of each state by
$$
\frac{1}{\sqrt{a \left(a + 1 \right) }} \text{,}
$$
where a will be the number of representatives the state currently holds (after each iteration). The state that will be assigned the representative will be the state with the highest population after performing the multiplication.

Alternative methods for apportionment

We can achieve alternative methods for apportionment by slightly modifying the tests defined by Huntington for the method of the geometric mean. By using the absolute difference instead of the relative difference, we can arrive at the method of the harmonic mean and the method of the arithmetic mean. These methods have been described in \cite{Huntington1928}, \cite{Huntington1928} and \cite{Sullivan1972}.

Method of the arithmetic mean

We can achieve the method of the arithmetic mean (which we will be abbreviating as MAM) by taking the absolute difference between the individual shares of two states. Huntington defined the test for the method of the arithmetic mean (or the method of major fractions) as follows (see \cite[Page 7]{Huntington1928}):

"If the absolute difference between the two "individual shares", $\frac{a}{A}$ and $\frac{b}{B}$, can be reduced by a transfer of a representative from one state to the other, then this transfer should be made (except that no state shall be left without at least one representative)."

We can write the following inequality using this test (using the same notations as before):
$$
\frac{a + 1}{A} - \frac{b}{B} < \frac{b + 1}{B} - \frac{a}{A}
$$
To achieve the working rule for the method of the arithmetic mean, we need to transform the inequality like so:
$$
\frac{a + 1}{A} - \frac{b}{B} < \frac{b + 1}{B} - \frac{a}{A} \Rightarrow \\
\frac{a + 1}{A} + \frac{a}{A} < \frac{b + 1}{B} + \frac{b}{B} \Rightarrow \\
\frac{2a + 1}{A} < \frac{2b + 1}{B}
$$
Taking the reciprocals, we get:
$$
\frac{A}{2a+1} > \frac{B}{2b+1} \Rightarrow \nonumber \\
\frac{A}{a + \frac{1}{2}} > \frac{B}{b + \frac{1}{2}}
$$
From this result, we can see that the multiplier for state A for the method of the arithmetic mean will be:
$$
\frac{1}{a + \frac{1}{2}}
$$
With this multiplier, we can arrive at the working rule for this method, which is similar to that of the method of the geometric mean. We need to perform the same procedure as we did for the method of equal proportions, with the only difference being the multiplier. Instead of using \eqref{E:multiplier}, we shall use \eqref{E:arithmetic_multiplier}.

Method of the harmonic mean

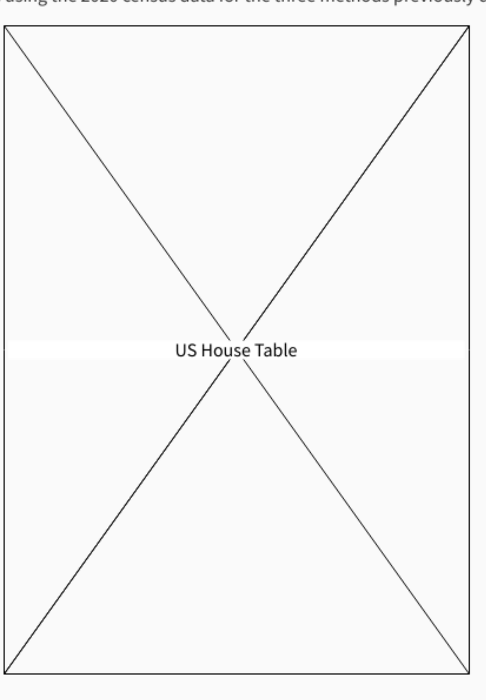
We can achieve the method of the harmonic mean (which we will be abbreviating as MHM) by taking the absolute difference between the population of two districts. Huntington defined the test for the method of the harmonic mean as follows (see \cite[Page 7]{Huntington1928}):

"If the absolute difference between the two congressional districts $\frac{A}{a}$ and $\frac{B}{b}$, can be reduced by a transfer of a representative from one state to the other, then this transfer should be made."

We can write the following inequality using this test (using the same notations as before):
$$
\frac{A}{a+1} - \frac{B}{b} < \frac{B}{b+1} - \frac{A}{a}
$$
To achieve the working rule for the method of the arithmetic mean, we need to transform the inequality like so:
$$
\frac{A}{a+1} - \frac{B}{b} < \frac{B}{b+1} - \frac{A}{a} \Rightarrow \nonumber \\
\frac{A}{a+1} + \frac{A}{a} < \frac{B}{b+1} + \frac{B}{b} \Rightarrow \nonumber \\
A \left(\frac{1}{a+1} + \frac{1}{a} \right) < B \left(\frac{1}{b+1} + \frac{1}{b} \right) \Rightarrow \nonumber \\
A \left(\frac{a}{a(a+1)} + \frac{a+1}{a(a+1)} \right) < B \left(\frac{b}{b(b+1)} + \frac{b+1}{b(b+1)} \right) \Rightarrow \nonumber \\
A \left(\frac{2a+1}{a(a+1)} \right) < B \left(\frac{2b+1}{b(b+1)} \right) \Rightarrow \nonumber \\
\frac{A}{\frac{a(a+1)}{2a+1}} < \frac{B}{\frac{b(b+1)}{2b+1}} \Rightarrow \nonumber \\
\frac{A}{2 \cdot \frac{a(a+1)}{2a+1}} < \frac{B}{2 \cdot \frac{a(a+1)}{2b+1}}
$$
From this result, we can see that the multiplier for state A for the method of the harmonic mean will be:
$$
\frac{1}{2 \cdot \frac{a(a+1)}{2a+1}}
$$
With this multiplier, we can arrive at the working rule for this method, which is similar to that of the method of the geometric mean. We need to perform the same procedure as we did for the method of equal proportions, with the only difference being the multiplier. Instead of using \eqref{E:multiplier}, we shall use \eqref{E:harmonic_multiplier}.

Composition of the US House

The following table shows the apportionment results using the 2020 census data for the three methods previously described.



References

[Hun21] E. V. Huntington, A new method of apportionment of representatives, Quarterly Publications of the American Statistical Association 17 (1921), no. 135, 859–870.

[Hun28] E. V. Huntington, The apportionment of representatives in Congress, Trans. Amer. Math. Soc. 30 (1928), no. 1, 85–110. MR 1501423

[Sul72] J. J. Sullivan, The election of a president, The Mathematics Teacher 65 (1972), no. 6, 493–501.

[Sul82] J. J. Sullivan, Apportionment—a decennial problem, The Mathematics Teacher 75 (1982), no. 1, 20–25.

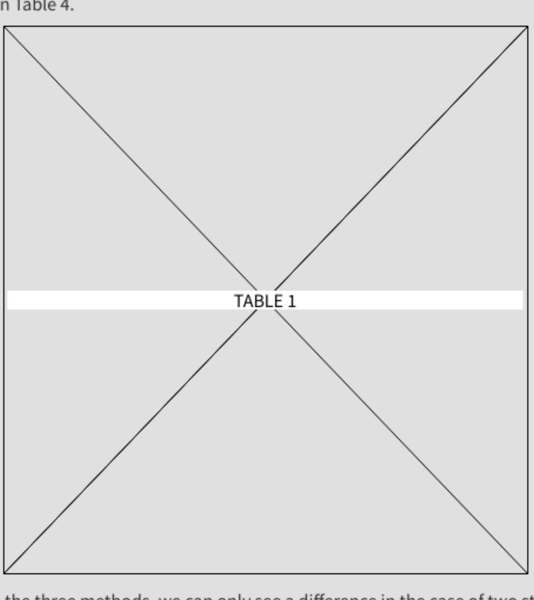
Adaptation of the Huntington method for the European Union

There are two approaches we can take when adapting Huntington's method.

The first is to completely ignore the principle of degressive proportionality and give every state near equal representation in the European Parliament. Taking this approach would lead us to a Parliament which looks more like the US House of Representatives.
The second approach is to keep the principle of degressive proportionality but grant every member state an m minimum number of seats, and allocate the remaining seats using Huntington's method subject to a maximum of 96 seats. After a member state reaches 96 seats, it shall be assigned no more seats. These guidelines were established [Gri12, Page 3].

Ignoring degressive proportionality

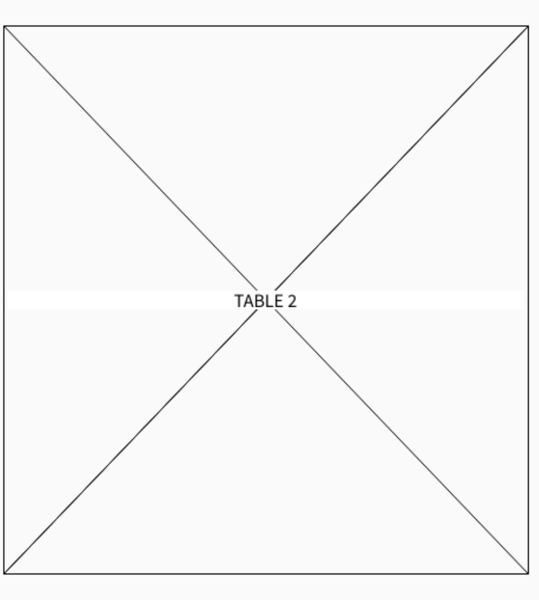
If we wish to ignore the principle of degressive proportionality, we will get a result, where every member state has roughly equal representation. This will give us a result that gives the more populous member states (such as Germany and France) a lot more power when compared to the current system. The results, if we applied the different methods using the Python program, can be found in Table 4.



When comparing the apportionment data between the three methods, we can only see a difference in the case of two states. The method of the arithmetic mean assigns one seat fewer to Cyprus, and one more to Germany than the other two methods. Cyprus is one of the least populous states in the European Union, and Germany is the most populous. This also highlights that the method of the arithmetic mean favours larger states over smaller states.
As we no longer use the principle of degressive proportionality, we can see the effects of the removal of the cap on the number of representatives. Germany and France have crossed the 96 seat-threshold currently used during apportionment, with Italy coming close at 95 seats.
The minimum number of seats assigned to a state has also been lowered to one. This also causes significant differences and gives less power to the smaller states.
From these results, it is clear why the principle of degressive proportionality is crucial in the balance between the power of a member state, and the power of the people living in the European Union. Using this system, the four most populous states alone would have a sizeable 402-seat (or 403-seat) majority.

Keeping degressive proportionality

We will use a version of the Python code modified according to the guidelines in [Gri12, Page 3]. The code along with a full, line-by-line explanation can be found in section 4.2.
Using census data from 2011 from across the European Union, we have arrived at the following composition using the different methods (see the table below).
Using this data, we can see that all three methods generated the same results. When analysing the difference between our apportionment and the official apportionment, we can notice a few major differences.
As this simulation respects degressive proportionality, we can say that the least populous member states benefited greatly from this apportionment. We also see that there are more states with the minimum number of seats than under the current system.
The only member state that reached the maximum of 96 seats under the current system is Germany. Germany reached this threshold under our system as well, making Germany less represented than the smaller states.
We can also see that a larger number of states reached 96 seats. These states were already populous, so we can say that this system benefits the larger states as well.
Another significant statement that we can make after looking the data is that the medium-sized states were the biggest losers of our system. They lost seats at the expense of larger states.



References

[Gri12] G. R. Grimmett, European apportionment via the cambridge compromise, Mathematical Social Sciences 63 (2012), no. 2, 68–73, Around the Cambridge Compromise: Apportionment in Theory and Practice.

Contact

If you have some questions about my research or you would like to share your opinion with me, I would be more than happy to hear from you. If you have found a mistake on this website, I would be very grateful if you could share it with me. You can use the contact form below to send me a message.

Name

Email

Type of message

- Question
- Opinion
- Report a mistake
- Other

Message

I would like to receive emails with further details about the research

Submit