1. Elementary Algebra

1.1)

$$\frac{(2^3)^2}{2^2 \cdot 2^8} = \frac{2^6}{2^{10}} = \frac{1}{2^4} = \boxed{2^{-4}}$$

1.2)

$$12^2 \cdot 3^x \cdot 2^{2x} = 12^{-2}$$
$$(3\cdot 2^2)^2 \cdot 3^x \cdot 2^{2x} = (3\cdot 2^2)^{-2}$$
$$3^{2+x} \cdot 2^{4+2x} = 3^{-2} \cdot 2^{-4}$$
$$3^{2+x} \cdot 2^{2(2+x)} = \cancel{3}\,12^{-2}$$
$$12^{(2+x)} = 12^{-2}$$
$$2+x = -2$$
$$\boxed{x = -4}$$

Control: $\cancel{1 \cdot 12^2}$

$$12^2 \cdot 3^{-4} \cdot 2^{-8} = $$
$$= 12^2 \cdot 12^{-4} = 12^{-2}$$

1.3) → Tried to solve this before typo

$$x - 1 \cdot y^{-1} = 5$$
$$x - \frac{1}{y} = 5$$
$$x = 5 + \frac{1}{y}$$

$$x^3 \cdot y^3 = \left(5 + \frac{1}{y}\right)^3 \cdot y^3 =$$
$$= \left(5^3 + 3\cdot 5^2 \cdot \frac{1}{y} + 3 \cdot 5 \cdot \frac{1}{y^2} + \frac{1}{y^3}\right) \cdot y^3 =$$
$$= 125y^3 + 75y^2 + 15y + 1$$

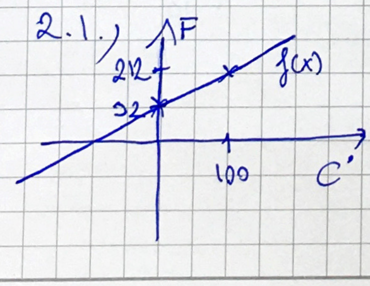
1.4)

$$\frac{\sqrt{3^{10}}}{\sqrt{9^3}} = \frac{3^{\frac{10}{2}}}{3^{\frac{6}{2}}} = \boxed{3^2}$$

1.5) (a) = T (b) = T (c) = F (d) = F

2. Functions of one variable

2.1.)



$$f(x) = 32 + \frac{212-32}{100} x$$

$$f(x) = \frac{18}{10} x + 32$$

↳ supposing $x = y$, then

$$x = \frac{18}{10} x + 32 \Longrightarrow \boxed{x = -40}$$

2.2) $f(y) = 3y + 3$, if $f(y) = 54$

$3y + 3 = 54$

$y = \frac{51}{3}$

$\boxed{y = 17}$

2.3)

$10^{4x^2 - 16x + 3} = 1000$

$10^{4x^2 - 16x + 3} = 10^3$

$4x^2 - 16x + 3 = 3$

$4x(x-4) = 0$

$\boxed{x_1 = 0}$ $\cancel{x_2}$

$\boxed{x_2 = 4}$

2.4) $g_Y = 3,2\%$ $g_{target} = 200\%$

$x \cdot 1,032^y = 2x$

$1,032^y = 2$

$y = \log_{1,032} 2$

$\boxed{y = 22,0056}$

Answer:

The economy will be close to doubling it's GDP by the 22nd year, but will eventually exceed that in the 23rd year.

2.5) $\ln\left(\frac{1}{e^{-5}}\right) = \ln(e^5) = \boxed{5}$

3. Calculus

3.1) $\sum_{i=0}^{\infty}\left(\frac{1}{13^i} + 0{,}25^i\right) = \sum_{i=0}^{\infty}\frac{1}{13^i} + \sum_{i=0}^{\infty}\frac{1}{4^i} = \frac{1}{1-\frac{1}{13}} + \frac{1}{1-\frac{1}{4}} =$

$= \boxed{2,4167}$

3.2) $\lim_{x \to 4} \frac{2x-8}{2}$ if $x=4$, then $\frac{2 \cdot 4 - 8}{2} = \boxed{0}$

3.3) $f(x) = x^3 - 4$ $P = (-1; -5)$

$f'(x) = 3x^2$

slope at $x = -1 \rightarrow 3 \cdot (-1)^2 = \boxed{3}$

3.4)

$$\frac{d}{dx} \frac{2x^2 + x}{x - 32} = \frac{(4x+1) \cdot (x-32) - (2x^2+x) \cdot 1}{(x-32)^2} = \frac{4x^2 - 128x + \cancel{x} - 32 - 2x^2 - \cancel{x}}{(x-32)^2} =$$

$f'(x) = 4x + 1$

$f'(\cancel{x}) = 1$

$$= \boxed{\frac{2(2x^2 - 64x - 16)}{(x-32)^2}}$$

3.5) $\frac{d^2}{dx^2} 4x^{-3} + 4 = \frac{d}{dx} -12 \cdot x^{-4} = \boxed{48 \cdot x^{-5}}$

3.6) Check if $f(x)$ function is continous at -2. $f(x) = \frac{1}{x+2}$

A: $f(x)$ function is cont. if $\boxed{f(x) = \lim_{a \to x} f(a)}$ $\not\Longleftrightarrow$ $f(-2) = \frac{1}{-2+2}$ ↯, since -2 is not in the Domain of $f(x)$

~~3.7)~~ $\Longrightarrow \lim_{a \to -2} \frac{1}{a-2} = \frac{1}{0} = \infty$

~~$f(x) = \frac{\ln x}{x}$, local min, max, inflection~~

~~$f'(x) = 0$ $f''(x) = 0$~~

$\boxed{\text{Therefore } f(x) \text{ is not continous at } -2.}$

3.8) $f(x;y) = x^3 \cdot y^2$.

$f(2;3) = 2^3 \cdot 3^2 = 8 \cdot 9 = \boxed{72}$

3.9) $f(x,y) = \ln(x - 2y)$ the function is defined where

$x - 2y > 0$

$\boxed{x > 2y}$

3.10) $\frac{d^2}{dx^2} x^5 + x^2y^3 = \frac{d}{dx} 5x^4 + 2xy^3 = \boxed{20x^3 + 2y^3}$

~~3.10~~ 3.11) $f(x;y) = \sqrt{xy} - 0,25x - 0,25y$

$\frac{d}{dx} f(x;y) = \frac{1}{2} \cdot x^{-\frac{1}{2}} \cdot y^{\frac{1}{2}} - 0,25 = \frac{\sqrt{y}}{2\sqrt{x}} - 0,25$

$\frac{d}{dy} f(x;y) = \frac{\sqrt{x}}{2\sqrt{y}} - 0,25$

$\frac{d}{dx} f(x;y) = 0$

$\frac{\sqrt{y}}{2\sqrt{x}} - \frac{1}{4} = 0$

$\sqrt{y} = \frac{1}{4} 2\sqrt{x}$

$\boxed{y = \frac{x}{4}}$

$\frac{d}{dy} f(x;y) = \frac{\sqrt{x}}{2\sqrt{y}} - \frac{1}{4} = 0$

$\frac{\sqrt{x}}{2\sqrt{\frac{x}{4}}} = \frac{1}{4}$

$\sqrt{x} = \frac{\sqrt{x}}{4}$

$\boxed{x = 0}$ $\boxed{y = 0}$

Answer = The equation has route at $x=0; y=0$, but that can't be local max; min as ~~it is~~ $x=0; y=0$ is not yielding comprehensible results at $\mathbb{R}$.

4. Linear algebra

4.1)

$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ $\boxed{A \cdot B} = \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$

4.2)

$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$ $\boxed{A \cdot B} = \begin{bmatrix} 19 & 11 \\ 10 & 11 \end{bmatrix}$

4.3)

$A = \begin{bmatrix} 3.3 & 5.1 & 4.7 \\ 2 & 6.1 & 1.23 \\ 4 & 5.76 & 0 \end{bmatrix}$ $\boxed{A^T} = \begin{bmatrix} 3.3 & 2 & 4 \\ 5.1 & 6.1 & 5.76 \\ 4.7 & 1.23 & 0 \end{bmatrix}$

4.4)

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $D = \cancel{2 \cdot 5 + 3 \cdot 4 = 22}\ 2 \cdot 5 - 3 \cdot 4 = -2$

5) Probability theory

5.1) $\Omega = \{$HHHH, HHHT, HHTH, HHTT, HTHT, HTTH, HTTT, TTTT, ~~HTTT~~, THHH, TTHH, HTHH, TTTH, THTT, TTHT, THTH, THHT$\}$

$|\Omega| = 16$

5.2) A = person ~~tests positive~~ is drug user
B = person ~~is drug user~~ tests positive

$P(B|A) = 99\%$
$P(A) = 1\%$
$P(B) = 1\% \cdot 99\% + 99\% \cdot 0{,}5\%$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} = \frac{99\% \cdot 1\%}{1\% \cdot 99\% + 99\% \cdot 0{,}5\%} = \frac{2}{3}$$

5.3) x = roll of a die

$$EX = \frac{1}{6} \cdot 1 + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} \dots 6 \cdot \frac{1}{3} = 3{,}5$$

y = sum of roll of 2 dice

$$y = 2 \cdot EX = 2 \cdot 3{,}5 = \boxed{7}$$

1.3) $x^{-1} \cdot y^{-1} = 5$ $\qquad x^{3} \cdot y^{3} = (xy)^{3} = \left(\frac{1}{5}\right)^{3} = \boxed{\frac{1}{125}}$

$(x \cdot y)^{-1} = 5$

$$\boxed{xy = \frac{1}{5}}$$

3.7) $\cancel{x \neq 0}$ $\qquad \underline{x > 0}$

$$f(x) = \frac{\ln x}{x}$$

$$f'(x) = \frac{\frac{1}{x} \cdot x - 1 \cdot \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$f''(x) = \frac{-\frac{1}{x} \cdot x^2 - 2x \cdot (1 - \ln x)}{x^4} = \frac{-x - 2x + 2x \ln x}{x^4} = \frac{2 \ln x - 3}{x^3}$$

Local max; min:

$f'(x) = 0$

$\frac{1 - \ln x}{x^2} = 0$ /since $x \neq 0$

$\ln x = 1$

$x = e$

$f'(x)$: + | 1 | e | 3 | −

f(x) has local maxima at $x = e$

Inflation:

$f''(x) = 0$

$\frac{2 \ln x - 3}{x^3} = 0$

$\ln x = \frac{3}{2}$

$x = e^{\frac{3}{2}}$

$f''(x)$: − | 1 | e | 5 | +

f(x) has inflation point at $x = e^{\frac{3}{2}}$, where the function changes from concave to convex

3.12) max x^3y^3 , constrain: $x+y=2$

$\mathcal{L}(x,y,\lambda) = f(x,y) - \lambda(g(x,y)-c) = x^3y^3 - \lambda(x+y-2)$

$\frac{d}{dx}\mathcal{L}(x,y,\lambda) = 3x^2y^3 - \lambda \rightarrow \frac{d}{dx}\mathcal{L}(x,y,\lambda) = 0$

$\frac{d}{dy}\mathcal{L}(x,y,\lambda) = 3y^2x^3 - \lambda \rightarrow \frac{d}{dy}\mathcal{L}(x,y,\lambda) = 0$

$\frac{d}{d\lambda}\mathcal{L}(x,y,\lambda) = -(x+y-2) \rightarrow \frac{d}{d\lambda}\mathcal{L}(x,y,\lambda) = 0$

$$\left.\begin{array}{l} 3x^2y^3 = \lambda \\ 3y^2x^3 = \lambda \end{array}\right\} \rightarrow x = y$$

$$\boxed{y = 1}$$
$$\boxed{x = 1}$$

$-(x+y-2) = 0$

$-(y+y-2) = 0$

$-2y = -2$

$y = 1$