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Modeling Neural Connectivity in a Point-Process Analogue of Kalman Filter*

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***Abstract*— A neural encoding model describes how single neuron tunes to external stimuli as well as its connectivity with other neurons. The connectivity illustrates the neuronal interaction within populations in response to the shared latent brain states. Understanding these interactions is crucial to computationally predict the neural activity, which elucidates the neural encoding mechanism A computational analysis on the neural connectivity also facilitates developing more point process decoding model to interpret movement state from neural spike observations for brain machine interfaces (BMI). Most of the previous point process models only consider single neural tuning property and assumes conditional independence among multiple neurons. The connectivity among neurons is not considered in such a Bayesian approach to derive the state. In this work, we propose a point-process analogue of Kalman Filter to model the neural connectivity in a closed-form Bayesian filter. Neural connectivity corrects the posterior of the state given the multi-dimension observation, and a Gaussian distribution is used to approximate the updated posterior distribution. We validate the proposed method on simulation data and compared with traditional point process filtering with conditional independent assumption. The result shows that our method models the neural connectivity information and the single neuronal tuning property at the same time and achieves a better performance of the state decoding.**

***Clinical Relevance*— This paper proposes a closed-form derivation of a point process filter based on Gaussian approximations. It can model both single neuronal tuning property and the neural connectivity, which is potential to understanding the neural connectivity computationally.**

I. Introduction

Neural encoding represents how neurons modulate underlying states and external stimuli in environment. The neuronal activity tunes to the information at multiple spatial scales, typically in single neuron and in neural population [1], [2]. Single neuronal tuning property describes how an individual neuron encodes the stimuli or movement parameters such as preferred direction, modulation depth. Neural connectivity illustrates the interaction within neural ensembles in response to the shared latent brain states. Besides single neuronal tuning, it is also crucial for elucidating computational neural encoding models. On the one hand, neural connectivity has influence on neural encoding. It has been found that spatially correlated spiking can strongly drive responses across sub cortical regions [3]. Valente et al. also found that correlations in neural populations can enhance the performance of discrimination tasks [4]. Considering both single neuron and neural ensembles in neural encoding can help computational analysis of the dynamical nature of brain function.

Modeling neural connectivity also contributes to more accurate decoding in brain machine interface (BMI) [5]. BMI builds a closed-loop control system that links between neural activities and control commands on external devices, which aims to help restore lost functions of paralyzed patients [6]. Costa et al. studied how task-relevant neural populations coordinate and demonstrated that the shared latent states decomposed from the neural connectivity could contribute to better decoding the arm trajectory of monkey performing a center-out task [7]. Modeling how neurons interact to process information can help interpret the neural activity patterns in the observation space for BMI decoding [8].

In BMI system, point process model extracts state information from inter-spike intervals observation in a Bayesian framework. However, the model assumes that neurons encode information in a conditionally independent manner regardless of the neural connectivity [9]. It is not general in the real scenario. In the previous work, a generalized linear model (GLM) was proposed to project the external stimuli, self-spiking history and interneuron coupling items to a scalar which modulates the spike train in a tuning function [10]. The multiple components in the tuning function are assumed independent to form the neural population patterns in GLM, where the external stimuli may be easily dominated by the interneuron coupling items. Latent variable models (LVM) are also exploited to extract the shared latent states of neural connectivity from multi-neuron spike trains [1], [7]. But the decomposed latent states by LVM contain neural connectivity encoded information that is not only related to motion intentions but also noise or other stimuli in the environment.

In this paper, we propose a point-process analogue of Kalman Filter that takes account of the neural connectivity in the Bayesian framework. The proposed method derives the

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conditional probability of kinematics given the multiple neural spike observation in a closed form. The modeling on the neural connectivity corrects the shape of the posterior distribution, and a Gaussian distribution is used to approximate the first- and second-order statistics of the updated posterior distribution in the recursive approach. And Gaussian assumptions are introduced to simplify the recursive evolution of states. We test the proposed method on simulation data of a rat two-lever discrimination task. Compared with the traditional point-process analogue of Kalman Filter (PPKF) [11], the decoding performance of the rat movement is evaluated to validate the improvement when the neural connectivity is considered. The rest of this paper is organized as follows. Section II presents the derivation and details of the proposed method. Simulation and results are shown in Section III. Conclusion and future work are discussed in Section IV.

II. Methodology

A. Deriving Neural Connectivity in a Bayesian Filter with Point Process Observation

The point process filter estimates a posterior distribution of the state given the observation of spike trains. N_k is defined as the total number of neuronal spikes up to t_k, where t_k is time index in sequence $\{t_k\}_{k=0}^{K}$ with a constant time interval $\Delta t = t_k - t_{k-1}$. And $\Delta N_k = N_k - N_{k-1}$ means the number of the observed spikes in the interval $(t_{k-1}, t_k]$. The chosen small enough interval Δt (~10 msec) can guarantee that the most intervals have no more than one spike. The conditional intensity function characterizes a spike train using the inhomogeneous Poisson process as

$$\lambda_k(x_k, \theta_k, z_{k-1}) = \frac{\Pr(\Delta N_k = 1 | x_k, \theta_k, z_{k-1})}{\Delta t} \tag{1}$$

where θ_k is the single neuronal tuning property parameters at time t_k and the states x_k are the movements in this paper. $z_k = [x_{1:k}, N_{1:k}]$ contains the movement history and the spike observations from the start to time t_k. The state evolves over time and is described by a linear system model as

$$x_k = F x_{k-1} + r_k \tag{2}$$

where F is the state transition matrix and r_k is zero-mean Gaussian noise with covariance R. The conditional intensity function is assumed to be a known fixed nonlinear function as

$$\lambda_k(x_k, \theta_k) = f(x_k, \theta_k) \tag{3}$$

where $f(x_k, \theta_k)$ is an exponential neuronal tuning function modulating x_k with fixed parameters θ_k in our method. The posterior density of kinematics can be decomposed according to the Bayes' theorem as

$$p(x_k | \Delta N_k, z_{k-1}) = \frac{p(x_k | z_{k-1}) p(\Delta N_k | x_k, z_{k-1})}{p(\Delta N_k | z_{k-1})} \tag{4}$$

where $p(x_k|z_{k-1})$ is the prior estimation of the movements from the system model. The denominator $p(\Delta N_k|z_{k-1})$ can be seen as the normalization item since it is not related to the kinematics. And $p(\Delta N_k|x_k, z_{k-1})$ is the likelihood of the observed spikes within $(t_{k-1}, t_k]$, which can be simplified as $p(\Delta N_k|x_k)$ under the assumption of instantaneous Markov process. By introducing an intermediate variable, the vector of neuronal firing probabilities $\boldsymbol{\lambda}_k^*$, it can be transformed as:

$$p(\Delta N_k | x_k) = \frac{p(\boldsymbol{\lambda}_k^* | x_k) p(\Delta N_k | \boldsymbol{\lambda}_k^*, x_k)}{p(\boldsymbol{\lambda}_k^* | \Delta N_k, x_k)} \tag{5}$$

The second term in the numerator $p(\Delta N_k|\boldsymbol{\lambda}_k^*, x_k)$ is the likelihood based on the tuning function. Therefore, the $\boldsymbol{\lambda}_k^*$ here can be determined by (3) (represented as $\boldsymbol{\lambda}_k^+$). The first term means the neural patterns density encoded from kinematics and can be transformed following Bayes' theorem:

$$p(\boldsymbol{\lambda}_k^* | x_k) = \frac{p(x_k | \boldsymbol{\lambda}_k^*) p(\boldsymbol{\lambda}_k^*)}{p(x_k)} \tag{6}$$

where $p(x_k|\boldsymbol{\lambda}_k^*)$ is the probability density of kinematics given the firing probability patterns. This $\boldsymbol{\lambda}_k^*$ is directly estimated from the spike trains (represented as $\boldsymbol{\lambda}_k^-$). This probability density describes the decoding of kinematics given the neural population firing. The interaction items among population given the firing observation $\boldsymbol{\lambda}_k^-$ in $p(x_k|\boldsymbol{\lambda}_k^-)$ can represent the neural connectivity. Therefore, the information of the neural connectivity can be implemented in this item. $p(\boldsymbol{\lambda}_k^*)$ is the marginal probability of the firing probability regardless of x_k. $p(x_k)$ is the marginal probability of movements. If we use $\boldsymbol{\lambda}_k^-$ in the denominator in (5), the denominator can be simplified as $p(\boldsymbol{\lambda}_k^-|\Delta N_k)$. As $\boldsymbol{\lambda}_k^-$ can be obtained from ΔN_k without using kinematic information, we further simplify $p(\boldsymbol{\lambda}_k^-|\Delta N_k)$ to $p(\boldsymbol{\lambda}_k^-)$. Equation (5) and (6) can be combined to obtain:

$$p(\Delta N_k | x_k) \propto \frac{p(x_k | \boldsymbol{\lambda}_k^-) p(\Delta N_k | \boldsymbol{\lambda}_k^+, x_k)}{p(x_k)} \tag{7}$$

In our method, Gaussian approximations on $p(x_k|\boldsymbol{\lambda}_k^-)$ and $p(x_k)$ are applied. The spike train likelihood can be obtained:

$$p(\Delta N_k | x_k) \propto \frac{N(x_k; f(\boldsymbol{\lambda}_k^-), Q) p(\Delta N_k | \boldsymbol{\lambda}_k^+, x_k)}{N(x_k; 0, S)} \tag{8}$$

where $N(x_k; f(\boldsymbol{\lambda}_k^-), Q)$ represents the Gaussian distribution $p(x_k|\boldsymbol{\lambda}_k^-)$ with expectation $f(\boldsymbol{\lambda}_k^-)$ and covariance Q. When a nonlinear function $f(\boldsymbol{\lambda}_k^-)$ is applied to multiple neuron observations to combine them and their coupling items with some coefficients, the information of neural connectivity can be involved. In this paper, we use a polynomial approximation to access the expectation of kinematics given the neuronal population observation $\boldsymbol{\lambda}_k^-$ as

$$f(\boldsymbol{\lambda}_k^-) = \boldsymbol{\omega}^T \boldsymbol{\Lambda}_k \tag{9}$$

where $\boldsymbol{\omega}$ is the weight vector and obtained by least square method. $\boldsymbol{\Lambda}_k$ includes single neural firing probability, the interaction terms $\lambda_k^{(i)}\lambda_k^{(j)}$ between pair of neurons and $\lambda_k^{(i)}\lambda_k^{(j)}\lambda_k^{(m)}$ among three neurons. i, j, m are index of neurons. In this way, the neural connectivity information is incorporated. Here we also include a bias item as

$$\boldsymbol{\Lambda}_k = \left\{1, \left[\lambda_k^{(i)}\right], \left[\lambda_k^{(i)}\lambda_k^{(j)}\right], \left[\lambda_k^{(i)}\lambda_k^{(j)}\lambda_k^{(m)}\right]\right\} \quad i \neq j \neq m \tag{10}$$

$N(x_k; 0, S)$ in (8) is the zero-mean Gaussian distribution $p(x_k)$ with covariance S. Covariance Q and S are estimated through the residual error. And $p(\Delta N_k|\boldsymbol{\lambda}_k^+, x_k)$ is obtained through the single neuronal tuning property according to conditionally independent Poisson processes as

$$p(\Delta N_k | \boldsymbol{\lambda}_k^+, x_k) = \prod_{i=1}^{L} \frac{(\lambda_k^{(i)}\Delta t)^{\Delta N_k^{(i)}} \exp(-\lambda_k^{(i)}\Delta t)}{\Delta N_k^{(i)}!} \tag{11}$$

where L is the number of neurons and i is the neuron index. In summary, the posterior estimation of the kinematics given point process observation is as follows

$$p(x_k|\Delta N_k, z_{k-1}) \propto \frac{p(x_k|z_{k-1})N(x_k; f(\boldsymbol{\lambda}_k^-), Q)}{N(x_k; 0, S)} \times \prod_{i=1}^{L} \frac{\left(\lambda_k^{(i)}\Delta t\right)^{\Delta N_k^{(i)}} \exp\left(-\lambda_k^{(i)}\Delta t\right)}{\Delta N_k^{(i)}!} \quad (12)$$

B. Point-Process Analogue of Kalman Filter with Neural Connectivity

Our proposed method derives the closed form representation of the posterior from prior recursively overtime. Both prior and the posterior density at each time instance are obtained by Gaussian approximations. With the Chapman-Kolmogorov equation, the prior density follows

$$p(x_k|z_{k-1}) = \int p(x_k|x_{k-1}, z_{k-1}) p(x_{k-1}|\Delta N_{k-1}, z_{k-2})\, dx_{k-1} \quad (13)$$

where the Gaussian distribution $p(x_k|z_{k-1})$, with expectation $x_{k|k-1}$ and covariance $P_{k|k-1}$, can be transmitted by a linear evolution model from the posterior estimation of the previous state $x_{k-1|k-1}$ and $P_{k-1|k-1}$ as

$$x_{k|k-1} = F x_{k-1|k-1} \quad (14)$$

$$P_{k|k-1} = F P_{k-1|k-1} F^T + R \quad (15)$$

where the state transition matrix F is trained by the least squares and covariance R is estimated through the residual error of the linear approximation. The posterior distribution $p(x_k|\Delta N_k, z_{k-1})$ in (12) can be obtained by a Gaussian distribution with expectation $x_{k|k}$ and covariance $P_{k|k}$ as

$$p(x_k|\Delta N_k, z_{k-1}) \propto \exp\left(-\frac{1}{2}(x_k - x_{k|k})^T (P_{k|k})^{-1} (x_k - x_{k|k})\right) \quad (16)$$

Following similar derivations in PPKF [11], $P_{k|k}$ evolves as

$$P_{k|k}^{-1} = P_{k|k}^{-1} + Q^{-1} - S^{-1} + \sum_{i=1}^{L}\left[\left(\frac{\partial log \lambda_k^{(i)}}{\partial x_k}\right)^T \lambda_k \Delta t \left(\frac{\partial log \lambda_k^{(i)}}{\partial x_k}\right) - (\Delta N_k^{(i)} - \lambda_k^{(i)}\Delta t)\frac{\partial^2 log \lambda_k^{(i)}}{\partial x_k (\partial x_k)^T}\right]_{x_{k|k-1}} \quad (17)$$

And the expectation $x_{k|k}$ of the state posterior density, which is the output of the estimation, can be represented as

$$x_{k|k} = x_{k|k-1} + P_{k|k} S^{-1} x_{k|k-1} + P_{k|k} Q^{-1}[f(\lambda_k) - x_{k|k-1}] + P_{k|k} \sum_{i=1}^{L}\left[\left(\frac{\partial log \lambda_k^{(i)}}{\partial x_k}\right)^T (\Delta N_k^{(i)} - \lambda_k^{(i)}\Delta t)\right]_{x_{k|k-1}} \quad (18)$$

III. Result

In this section, the proposed method is tested in a simulation compared with PPKF. A 2-dimensional trajectory and the corresponding spike trains of three neurons are generated to simulate that a rat performs a two-lever discrimination task. In this task, the rats were required to discriminate the cue audio, then press and hold one of the two levers correspondingly for water reward. The state $x_k = [p_k^{(x)}, p_k^{(y)}]$ where $p_k^{(x)}$ and $p_k^{(y)}$ are the positions in the x and y directions at time index k respectively. 50 trails (25 trials for the high-lever pressing and 25 trials for the low-lever pressing) are randomly produced over time, and each trial contains 250 data samples recording the trajectory from the resting state ($x_k = [0,0]$) to the lever-pressing state ($x_k = [1,1]$ for high lever, and $x_k = [1,-1]$ for low lever) and back to the resting state. Both dimensions added with zero-mean Gaussian noise with 0.1 variance are shown in Fig. 1.

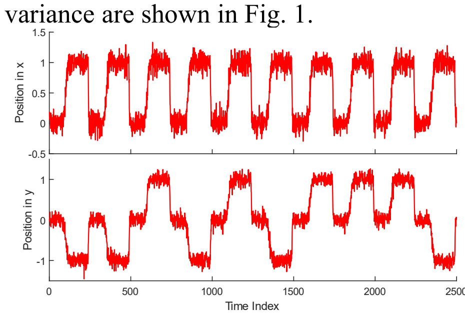

Figure 1. The simulated movement. The horizontal axis is the time index. The vertical axes are the position in x and y respectively.

We then generate the neuron spike trains through the following encoding model. The neuron i's firing probability $\lambda_k^{(i)}$ consists of the single neuronal modulation $\lambda_k^{(s_i)}$ and the pair-wise neural interaction $\lambda_k^{(c_i)}$, which represents the existence of the neural connectivity encoding [3], [4].

$$\lambda_k^{(i)} = \lambda_k^{(s_i)} + \lambda_k^{(c_i)} \quad (19)$$

where $\lambda_k^{(s_i)}$ is the firing probability sourcing from the single neuron modulating the movements by the single neuronal tuning functions as following:

$$\begin{cases} \lambda_k^{(s_1)} = \exp\left(-1.2 + 0.24 p_k^{(x)} - 0.18 p_k^{(y)}\right) \\ \lambda_k^{(s_2)} = \exp\left(-1.1 + 0.40 p_k^{(x)}\right) \\ \lambda_k^{(s_3)} = \exp\left(-1.14 + 0.21 p_k^{(x)} - 0.214 p_k^{(y)}\right) \end{cases} \quad (20)$$

In terms of the single neuronal tuning function, neuron 1, neuron 2 and neuron 3 have higher firing probability corresponding to low lever pressing, both levers pressing and high lever pressing, respectively. $\lambda_k^{(c_i)}$ is the component in the firing probability modulated by neural connectivity.

For $\lambda_k^{(c_i)}$, it is assumed that there are some latent neural connectivity states driving neurons to response interactively. $\gamma_k^{(ij)}$ represents the pair-wise states shared by neuron i and j at time t_k. $\lambda_k^{(c_i)}$ is modulated by the latent states defined as

$$\lambda_k^{(c_i)} = \sum_{j \neq i} \gamma_k^{(ij)} \lambda_k^{(s_j)} \quad (21)$$

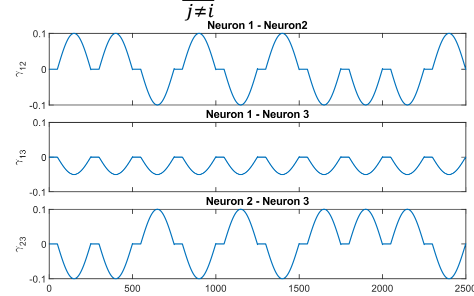

Figure 2. Latent neural connectivity states between neurons in the first 10 training trials. The horizontal axis is the time index, and the vertical axis is the latent connectivity parameters.

As shown in Fig. 2, the latent neural connectivity states produce more active coordination from the start of the reaching to the end of the leaving and contribute to additional correlated information of the neural encoding. And the connectivity in resting state is weak. In our simulation, the latent neural connectivity states are set as sinusoid curves with 0.05 amplitude for $\gamma_k^{(13)}$ and 0.1 amplitude for the other two states within each trial. The sign of values is related to the neuron's tuning property and which lever is pressed. During the latent connectivity states, $\gamma_k^{(12)}$ is positive for lower-lever trial and negative for high-lever trial. While $\gamma_k^{(23)}$ is converse for the two types of trials. There is always a mutually weakening state between neuron 1 and neuron 3.

The spike trains are generated through a Bernoulli stochastic process according to the firing probability. The firing probability and the spike trains of the three simulated neurons in testing trials are shown in Fig. 3. The firing probability of each neuron is shown as the orange curve, and spikes are illustrated as the blue bars correspondingly. 40 trials are used for training. 10 trials are for testing.

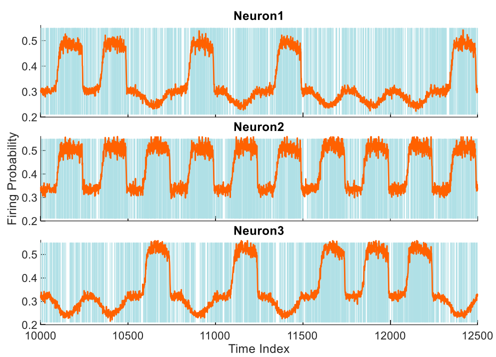

Figure 3. The firing probability and spike trains of the three simulated neurons. The horizontal axis is the time index, and the vertical axis is the firing probability. The orange lines represent the firing probability. Each blue bar represents a spike.

Fig. 4 shows the movement decoding results on a segment of the testing data. The red lines represent the ground truth of the trajectory. The green lines and the black lines are the decoding results of PPKF and the proposed method respectively. The mean square error (MSE) on the position in x between the ground truth and the result of PPKF is 0.1575 while that of our method is 0.1063. For the position in y, the MSE of PPKF is 0.1804 and that of our method is 0.1566. We can see that during the lever pressing stage, our method can reach more accurate and refined pressing positions in almost every trials. While PPKF fails to reach it especially in the y position. The MSE of our method in pressing stages on the position y is 0.0564 while that of PPKF is 0.1941. And the improvement is also shown in the resting states especially at the 1st, 5th, and 7th trials. These improvements demonstrate that introducing neural connectivity information can provide effective elements for the computational model. Therefore, considering neural connectivity in the point-process Bayesian methods can achieve a better decoding performance.

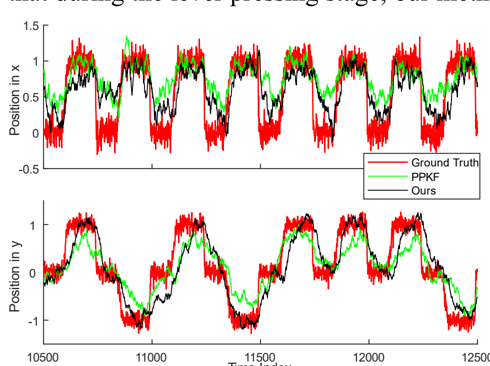

Figure 4. A segment of the decoding results. The horizontal axis is the time index, the vertical axis is the position in x and the position in y. The red line is the ground truth. The green line is the PPKF's result, and the black line is the result obtained by our method.

IV. Conclusion

A neural encoding model demonstrates how neurons responds to external stimuli and interacts with other neurons. Previous point process filters assume neuronal conditionally independent encoding, which may cause deviation of the state estimation in BMI. In this paper, we model neural connectivity in a point-process analogue of Kalman Filter. The closed-form derivation based on Gaussian approximations in the Bayesian filter corrects the posterior of the state given the multiple neural spikes. In this way, our method can combine single neuronal tuning property and neural connectivity information for state estimation. The simulation results indicate that considering neural connectivity improves the decoding performance of the point process filter. In the future, we will improve the method and validate it on real data to track time-varying neural properties.

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