In [1]:
```python
import pandas as pd
import numpy as np
import os
import yfinance as yf
from datetime import timedelta
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
```

In [2]:
```python
SPX_Prices = yf.download('SPY', start='2002-01-01', end='2024-01-01', interval = "1d")
SPX_Prices
```

```
[*********************100%%**********************]  1 of 1 completed
```

Out[2]:

Date	Open	High	Low	Close	Adj Close	Volume
2002-01-02	115.110001	115.750000	113.809998	115.529999	76.324333	18651900
2002-01-03	115.650002	116.949997	115.540001	116.839996	77.189743	15743000
2002-01-04	117.169998	117.980003	116.550003	117.620003	77.705055	20140700
2002-01-07	117.699997	117.989998	116.559998	116.790001	77.156708	13106500
2002-01-08	116.790001	117.059998	115.970001	116.519997	76.978333	12683700
...	...	...	...	...	...	...
2023-12-22	473.859985	475.380005	471.700012	473.649994	473.649994	67126600
2023-12-26	474.070007	476.579987	473.989990	475.649994	475.649994	55387000
2023-12-27	475.440002	476.660004	474.890015	476.510010	476.510010	68000300
2023-12-28	476.880005	477.549988	476.260010	476.690002	476.690002	77158100
2023-12-29	476.489990	477.029999	473.299988	475.309998	475.309998	122234100

5537 rows × 6 columns

In [3]:
```python
VIX = yf.download('^VIX', start='2002-01-01', end='2024-01-01', interval = "1d")
VIX
```

```
[*********************100%%**********************]  1 of 1 completed
```

Out[3]:

Date	Open	High	Low	Close	Adj Close	Volume
2002-01-02	23.780001	24.200001	22.709999	22.709999	22.709999	0
2002-01-03	22.219999	22.430000	21.330000	21.340000	21.340000	0
2002-01-04	20.969999	21.530001	20.400000	20.450001	20.450001	0
2002-01-07	21.410000	22.150000	21.350000	21.940001	21.940001	0
2002-01-08	21.629999	22.290001	21.280001	21.830000	21.830000	0
...	...	...	...	...	...	...
2023-12-22	13.720000	13.960000	13.000000	13.030000	13.030000	0
2023-12-26	13.770000	13.800000	12.960000	12.990000	12.990000	0
2023-12-27	13.020000	13.040000	12.370000	12.430000	12.430000	0
2023-12-28	12.440000	12.650000	12.380000	12.470000	12.470000	0

2023-12-29	12.550000	13.190000	12.360000	12.450000	12.450000	0

5537 rows × 6 columns

In [4]:
```
def garman_klass_daily_variance(data):
    """
    Calculate daily Garman-Klass variance for given price data.
    """
    log_hl = np.log(data['High'] / data['Low'])
    log_co = np.log(data['Close'] / data['Open'])
    daily_variance = 0.5 * log_hl**2 - (2 * np.log(2) - 1) * log_co**2
    return daily_variance
```

In [5]:
```
def rolling_volatility(data, rolling_window):
    """
    Calculate annualized Garman-Klass volatility over a given period
    """
    Daily_Volatility = garman_klass_daily_variance(data)
    Rolling_Vol = np.sqrt((Daily_Volatility.rolling(rolling_window).mean())*252)
    return Rolling_Vol
```

In [6]:
```
daily_vol = rolling_volatility(SPX_Prices, 1)
weekly_vol = rolling_volatility(SPX_Prices, 5)
monthly_vol = rolling_volatility(SPX_Prices, 21)
quartely_vol = rolling_volatility(SPX_Prices, 63)

# Here we remove the NaN values from the volatility metrics
weekly_vol_nan = (weekly_vol[~np.isnan(weekly_vol)])
monthly_vol_nan = (monthly_vol[~np.isnan(monthly_vol)])
quartely_vol_nan = (quartely_vol[~np.isnan(quartely_vol)])
# We go as far back as the minimum of the amount of valid data points
far_back = min(len(weekly_vol_nan) , len(monthly_vol_nan) , len(quartely_vol_nan))
# Adjust the data to be the same size
weekly_vol_nan = weekly_vol_nan[-far_back:]
monthly_vol_nan = monthly_vol_nan[-far_back:]
quartely_vol_nan = quartely_vol_nan[-far_back:]
daily_vol_nan = daily_vol[-far_back:]
volatilities = pd.DataFrame(
    data = {"daily_vol" : daily_vol_nan,
            "weekly_vol" : weekly_vol_nan,
            "monthly_vol" : monthly_vol_nan,
            "quartely_vol" : quartely_vol_nan
           },
    index = (SPX_Prices.index))

volatilities = volatilities.dropna()
volatilities
```

Out[6]:

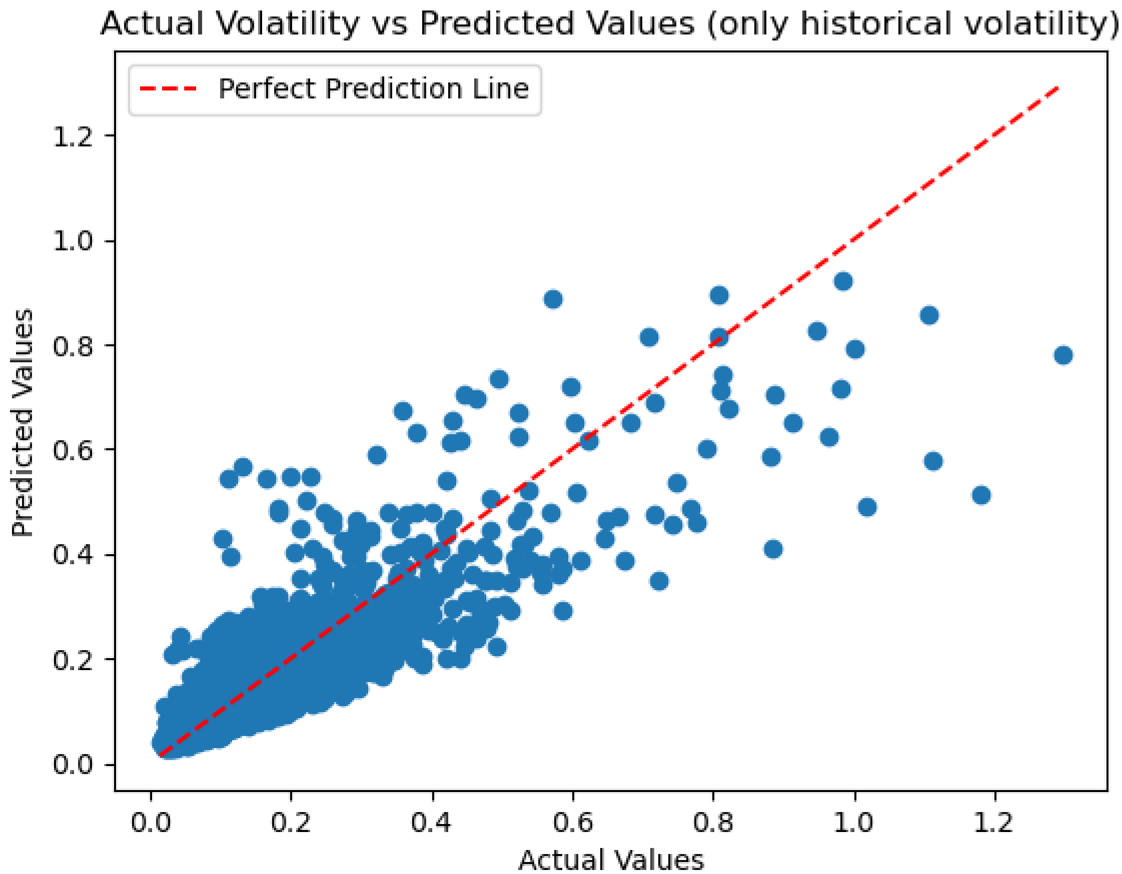
	daily_vol	weekly_vol	monthly_vol	quartely_vol
Date				
2002-04-03	0.188741	0.141114	0.129831	0.156948
2002-04-04	0.116253	0.141567	0.128777	0.155872
2002-04-05	0.137383	0.145083	0.128437	0.156408
2002-04-08	0.111350	0.136978	0.125265	0.156159
2002-04-09	0.085117	0.132406	0.123316	0.155873
...	...	...	...	...
2023-12-22	0.087123	0.095185	0.080046	0.101953

2023-12-26	0.051613	0.092716	0.080736	0.101349
2023-12-27	0.035382	0.092868	0.080752	0.099901
2023-12-28	0.030107	0.065317	0.079735	0.098773
2023-12-29	0.084652	0.062576	0.079709	0.098149

5475 rows × 4 columns

In [7]:

```
### This is our regression model with only historical volatility as our independent vari

# Data with three independent variables (X1, X2, X3) and one dependent variable (Y)
All_Vols = {'X1': weekly_vol_nan,
        'X2': monthly_vol_nan,
        'X3': quartely_vol_nan,
        'Y': daily_vol_nan}

df = pd.DataFrame(All_Vols)

# Separate independent variables (features) and dependent variable
X = df[['X1', 'X2', 'X3']]
y = df['Y']

# Create and fit the multiple regression model
all_preds = LinearRegression()
all_preds.fit(X, y)

# Predictions
y_pred = all_preds.predict(X)

# Plotting the actual vs predicted values
plt.scatter(y, y_pred)
plt.plot([min(y), max(y)], [min(y), max(y)], linestyle='--', color='red', label='Perfect
plt.xlabel('Actual Values')
plt.ylabel('Predicted Values')
plt.title('Actual Volatility vs Predicted Values (only historical volatility)')
plt.legend()
plt.show()

# Print the coefficients (slope) and intercept
print('Coefficients (Slope):', all_preds.coef_)
print('Intercept:', all_preds.intercept_)
```




```
Coefficients (Slope): [ 0.95530808 -0.0420618   0.01360054]
Intercept: 0.003314003394721113
```

In [11]:

```python
# Import the Real GDP data from FRED
GDP = pd.read_csv('/Users/henryhartwell/Downloads/GDPC1.csv')
# Make the data frame the same size as the other data
GDP_df = pd.DataFrame(GDP[-far_back:])
# make the data frame a series so it can be added to the other data
GDP_series = pd.Series(GDP_df['GDPC1'])
# Change the indicies to the same as the other data
GDP_series.index = df.index
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[11], line 8
      6 GDP_series = pd.Series(GDP_df['GDPC1'])
      7 # Change the indicies to the same as the other data
----> 8 GDP_series.index = df.index

File ~/anaconda3/lib/python3.11/site-packages/pandas/core/generic.py:6002, in NDFrame.__setattr__(self, name, value)
   6000 try:
   6001     object.__getattribute__(self, name)
-> 6002     return object.__setattr__(self, name, value)
   6003 except AttributeError:
   6004     pass

File ~/anaconda3/lib/python3.11/site-packages/pandas/_libs/properties.pyx:69, in pandas._libs.properties.AxisProperty.__set__()

File ~/anaconda3/lib/python3.11/site-packages/pandas/core/generic.py:730, in NDFrame._set_axis(self, axis, labels)
    725 """
    726 This is called from the cython code when we set the `index` attribute
    727 directly, e.g. `series.index = [1, 2, 3]`.
    728 """
```

```
    729 labels = ensure_index(labels)
--> 730 self._mgr.set_axis(axis, labels)
    731 self._clear_item_cache()

File ~/anaconda3/lib/python3.11/site-packages/pandas/core/internals/managers.py:225, in
BaseBlockManager.set_axis(self, axis, new_labels)
    223 def set_axis(self, axis: AxisInt, new_labels: Index) -> None:
    224     # Caller is responsible for ensuring we have an Index object.
--> 225     self._validate_set_axis(axis, new_labels)
    226     self.axes[axis] = new_labels

File ~/anaconda3/lib/python3.11/site-packages/pandas/core/internals/base.py:70, in DataM
anager._validate_set_axis(self, axis, new_labels)
     67     pass
     69 elif new_len != old_len:
---> 70     raise ValueError(
     71         f"Length mismatch: Expected axis has {old_len} elements, new "
     72         f"values have {new_len} elements"
     73     )

ValueError: Length mismatch: Expected axis has 88 elements, new values have 5475 element
s
```

In [9]:

```
### This is our regression model with historical volatility and Real GDP as our independ

# Data with four independent variables (X1, X2, X3, X4) and one dependent variable (Y)
All_Vols = {'X1': weekly_vol_nan,
        'X2': monthly_vol_nan,
        'X3': quartely_vol_nan,
        'X4': GDP_series,
        'Y': daily_vol_nan}

df = pd.DataFrame(All_Vols)

# Separate independent variables (features) and dependent variable
X = df[['X1', 'X2', 'X3', 'X4']]
y = df['Y']

# Create and fit the multiple regression model
all_preds = LinearRegression()
all_preds.fit(X, y)

# Predictions
y_pred = all_preds.predict(X)

# Plotting the actual vs predicted values
plt.scatter(y, y_pred)
plt.plot([min(y), max(y)], [min(y), max(y)], linestyle='--', color='red', label='Perfect
plt.xlabel('Actual Values')
plt.ylabel('Predicted Values')
plt.title('Actual Volatility vs Predicted Values (with Real GDP)')
plt.legend()
plt.show()

# Print the coefficients (slope) and intercept
print('Coefficients (Slope):', all_preds.coef_)
print('Intercept:', all_preds.intercept_)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In[9], line 7
      1 ### This is our regression model with historical volatility and Real GDP as our
independent variables ###
      2
```

```
      3 # Data with four independent variables (X1, X2, X3, X4) and one dependent variab
le (Y)
      4 All_Vols = {'X1': weekly_vol_nan,
      5          'X2': monthly_vol_nan,
      6          'X3': quartely_vol_nan,
----> 7          'X4': GDP_series,
      8          'Y': daily_vol_nan}
     10 df = pd.DataFrame(All_Vols)
     12 # Separate independent variables (features) and dependent variable

NameError: name 'GDP_series' is not defined
```

In [ ]:

In [ ]: