

Homework 12

Instructions

This homework contains **5** concepts and **3** programming questions. In MS word or a similar text editor, write down the problem number and your answer for each problem. Combine all answers for concept questions in a single PDF file. Export/print the Jupyter notebook as a PDF file including the code you implemented and the outputs of the program. Make sure all plots and outputs are visible in the PDF.

Combine all answers into a single PDF named `andrewID_hw12.pdf` and submit it to Gradescope before the due date. Refer to the syllabus for late homework policy. Please assign each question a page by using the “Assign Questions and Pages” feature in Gradescope.

Here is a breakdown of the points for programming questions:

| Name | Points |
|--------------|------------|
| Concept 1 | 6 |
| Concept 2 | 6 |
| Concept 3 | 6 |
| Concept 4 | 6 |
| Concept 5 | 6 |
| M12-L1-P1 | 15 |
| M12-L2-P1 | 25 |
| M12-HW1 | 50 |
| Total | 120 |
| Bonus | 6 |

Problem 1 (2 points)

What would the dimension of the covariance matrix be for the following data:

(Choose one)

1. 2 x 2
2. 6 x 6
3. 12 x 12
4. 20 x 20

| x_1 | x_2 | x_3 | x_4 | x_5 | x_6 |
|--------|-------|-------|-------|-------|-------|
| -7.55 | 5.85 | 11.88 | 1.99 | 6.39 | 3.05 |
| -10.93 | 6.56 | 8.96 | -0.89 | 7.43 | 4.07 |
| -9.44 | 6.37 | 9.86 | -0.62 | 7.73 | 2.88 |
| -1.83 | 0.53 | 8.55 | -6.21 | -8.05 | 5.13 |
| 6.38 | 0.47 | -6.72 | 2.71 | -5.24 | -2.11 |
| 7.85 | -0.17 | -8.48 | 1.40 | -7.62 | -3.71 |
| 9.17 | 0.70 | -7.45 | 2.09 | -6.13 | -4.66 |
| 0.76 | 1.97 | 8.46 | -5.47 | -7.57 | 3.33 |
| -11.58 | 6.13 | 9.34 | 0.21 | 9.00 | 3.03 |
| -8.41 | 5.29 | 10.13 | -0.97 | 7.48 | 5.11 |
| -7.87 | 5.48 | 10.50 | 1.71 | 6.04 | 3.79 |
| -0.84 | 0.23 | 7.99 | -6.91 | -7.59 | 3.11 |
| 1.06 | -0.56 | 7.47 | -7.12 | -6.31 | 3.82 |
| 7.43 | 1.26 | -8.13 | 1.30 | -5.78 | -6.79 |
| 0.59 | 0.88 | 7.85 | -6.20 | -8.18 | 3.94 |
| 7.35 | 1.04 | -5.98 | 1.61 | -5.69 | -5.54 |
| 1.01 | 1.40 | 9.87 | -5.62 | -7.74 | 4.08 |
| 8.47 | 2.80 | -7.24 | 0.93 | -5.39 | -4.60 |
| 8.00 | 1.39 | -6.57 | 0.53 | -2.77 | -7.12 |
| -10.92 | 7.00 | 8.96 | -1.30 | 6.90 | 4.82 |

Problem 2 (2 points)

Provided the following eigenvalues and eigenvectors e_1 and e_2 , what are the values i, j, k , that comprise the unit normalized third eigenvector, e_3 ?

(Text entry for each i, j, k)

$$\lambda_1 = 16$$

$$\lambda_2 = 4$$

$$\lambda_3 = 0$$

$$e_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$e_3 = \begin{bmatrix} i & j & k \end{bmatrix}$$

Problem 3 (2 points)

The eigenvalues of the covariance matrix from the data in the first concept question are included below. Which components should be used to explain at least 80% of the variance in the data?

$$\lambda_1 = 160.30$$

$$\lambda_2 = 44.31$$

$$\lambda_3 = 1.86$$

$$\lambda_4 = 1.47$$

$$\lambda_5 = 0.62$$

$$\lambda_6 = 0.49$$

Multiple choice (select all that apply)

- PC1
- PC2
- PC3
- PC4
- PC5
- PC6

Problem 4 (2 points)

What should the dimension of the covariance matrix be for the following data:

(Choose one)

1. 2 x 2
2. 6 x 6
3. 10x10
4. 20 x 20

| x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | x_7 | x_8 | x_9 | x_{10} |
|--------|--------|-------|-------|-------|-------|-------|-------|-------|----------|
| -9.25 | 2.84 | -9.38 | 0.66 | 5.71 | -2.23 | 8.76 | -5.37 | -2.56 | 1.25 |
| -10.24 | 3.23 | -8.34 | -0.70 | 5.53 | -2.72 | 8.70 | -4.77 | -2.61 | 0.44 |
| 2.36 | -10.36 | 5.22 | -2.26 | 7.44 | -4.88 | -4.87 | 1.83 | -8.76 | -7.48 |
| -7.84 | 5.72 | -2.35 | 8.14 | -6.54 | 10.40 | -2.19 | -2.51 | -3.84 | -1.19 |
| -7.51 | 5.07 | -2.21 | 6.73 | -7.42 | 8.83 | -4.00 | -2.65 | -3.57 | -0.89 |
| 0.49 | -8.68 | 4.84 | 0.05 | 6.40 | -4.71 | -4.96 | 2.05 | -7.59 | -6.18 |

Problem 5 (2 points)

Select the following statements about t-SNE which are true:

(Multiple choice, select all that apply)

1. t-SNE can be used to project unseen high dimensional data into a reduced feature space
2. t-SNE preserves global structure and distances between data points by computing pairwise similarities
3. Like PCA, t-SNE is a linear dimensionality reduction technique that is used to reduce high dimensional data to a low dimensional feature space
4. t-SNE is a non-linear dimensionality technique that can learn embeddings of manifolds