Dans cette partie, nous allons travailler sur les simulations numériques du processus de Bienaymé-Galton-Watson. Nous allons étudier la probabilité d'extinction dans les cas critique, surcritique et sous-critique quand le temps tends vers l'infini. Dans la suite, toutes les populations suivent la loi de Poisson de paramètre a.

1) Cas sous-critique:

On rappelle que dans ce cas, l'espérance doit être plus petite que 1. Et nous allons d'abord simuler les populations qui suivent le loi de Poisson de paramètre a.

```
Sumpoisson<- function(x)
{
  Sumpoisson <- rpois(1,x*a)
}
Sumpoisson
```

```
## function(x)
## {
##   Sumpoisson <- rpois(1,x*a)
## }
```

L'algorithme ci-dessus nous permet de retourner le nombre totale d'individus d'une génération. On regarde un exemple ci-dessous:

```
a=0.5
print("pour l'espérance 0.5,on trouve que la somme de ces 10 individus aléatoire")
```

```
## [1] "pour l'espérance 0.5,on trouve que la somme de ces 10 individus aléatoire"
```

```
print(paste0("qui suit la loi de Poisson est égale à " ,Sumpoisson(10)))
```

```
## [1] "qui suit la loi de Poisson est égale à 4"
```

Avec l'algorithme précédent, on peut maintenant s'intéresser au processus de BGW dans le cas sous-critique. On rappelle que pour vérifier si la chaîne s'éteint, le dernier élément doit être 0 (c'est à dire que la dernière génération est vide).

```
k=10000 #nombre de simulation
n=10000  #nombre de génération


b=0 #compteur pour calculer le nombre de d'extinction
t=0 #temps
for (j in 1:k){
x=1 # Le premier individu
  for (i in 2:n){
    x<-c(x,Sumpoisson(x[length(x)])) # ajouter un individu qui suit une loi de Poisson à la fin de x
    if (x[length(x)]==0){ #si le dernier individu de x n'a pas la génération
      b=b+1
      t=t+i
      break
    }
    else{
    if(i==n){
      t=t+n
    }
    }
}
#print(x)
```

```
}
prob=b/k# la probabilité d'extinction de la population
print(paste0("la probabilité empirique d'extinction est:",prob))
```

```
## [1] "la probabilité empirique d'extinction est:1"
```

```
exp_ext = t/k# l'espérance de temps d'extinction
print(paste0("l'espérance de temps d'extinction est: ",exp_ext, " dans le cas sous-critique"))
```

```
## [1] "l'espérance de temps d'extinction est: 2.7316 dans le cas sous-critique"
```

```
print("Ce qui correspond bien au théorème de la référence. ")
```

```
## [1] "Ce qui correspond bien au théorème de la référence. "
```

Grace à la commande(print(x)), on voit que toutes les populations s'éteignent après peu de générations.

2) Cas critique

On rappelle que dans ce cas, l'espérance doit être égale à 1. C'est à dire que le paramètre a doit être égal à 1.

```
a=1
print("pour l'espérance 1,on trouve que la somme de ces 10 individus aléatoire")
```

```
## [1] "pour l'espérance 1,on trouve que la somme de ces 10 individus aléatoire"
```

```
print(paste0("qui suit la loi de Poisson est égale à " ,Sumpoisson(10)))
```

```
## [1] "qui suit la loi de Poisson est égale à 3"
```

```
k=10000 #nombre de simulation
n=10000#nombre de génération


b=0#compteur pour calculer le nombre de d'extinction
t=0
for (j in 1:k){
x=1 # Le premier individu
  for (i in 2:n){
    x<-c(x,Sumpoisson(x[length(x)])) # ajouter un individu qui suit une loi de Poisson à la fin de x
    if (x[length(x)]==0){ #si le dernier individu de x n'a pas la génération
      b=b+1
      t=t+i
      break
    }
    else{
    if(i==n){
      t=t+n
    }
    }
}
#print(x)
}
prob=b/k #la probabilité d'extinction de la population
print(paste0("la probabilité empirique d'extinction est:",prob))
```

```
## [1] "la probabilité empirique d'extinction est:1"
```

```
exp_ext = t/k# l'espérance de temps d'extinction
print(paste0("l'espérance de temps d'extinction est: ",exp_ext, " dans le cas critique"))
```

```
## [1] "l'espérance de temps d'extinction est: 15.3721 dans le cas critique"
```

```
print("Ce qui correspond bien au théorème de la référence. ")
```

```
## [1] "Ce qui correspond bien au théorème de la référence. "
```

Grâce à la commande(print(x)), on peut constater que toutes les populations s'éteignent mais cette fois à des générations plus tardives que précédemment. On peut conclure que pour une espérance plus grande, le temps d'e s'éteintdre'extinction de la population est plus long. Ainsi, pour n est plus grand, on trouvera le temps d'extinction tends vers l'infinie. De plus, on observe que la probabilité empirique d'extinction est 1 dans le cas critique. Ce qui correspond bien au théorème de la référence.

3) cas surcritique

```
a=3
print("pour l'espérance 3,on trouve que la somme de ces 10 individus aléatoire")
```

```
## [1] "pour l'espérance 3,on trouve que la somme de ces 10 individus aléatoire"
```

```
print(paste0("qui suit la loi de Poisson est égale à " ,Sumpoisson(10)))
```

```
## [1] "qui suit la loi de Poisson est égale à 34"
```

```
k=50#nombre de simulation
n=50#nombre de génération


b=0#compteur pour calculer le nombre de d'extinction
t=0
for (j in 1:k){
x=1 # Le premier individu
  for (i in 2:n){
    x<-c(x,Sumpoisson(x[length(x)])) # ajouter un individu qui suit une loi de Poisson à la fin de x
    if (x[length(x)]==0){ #si le dernier individu de x n'a pas la génération
      b=b+1
      t=t+i
      break
    }
}
print(x)
}
```

```
##  [1] 1.000000e+00 2.000000e+00 4.000000e+00 1.000000e+01 2.800000e+01
##  [6] 7.700000e+01 2.060000e+02 6.270000e+02 1.889000e+03 5.611000e+03
## [11] 1.676400e+04 5.047500e+04 1.506190e+05 4.518630e+05 1.357496e+06
## [16] 4.069196e+06 1.220661e+07 3.661848e+07 1.098496e+08 3.295408e+08
## [21] 9.886386e+08 2.965883e+09 8.897614e+09 2.669296e+10 8.007938e+10
## [26] 2.402377e+11 7.207137e+11 2.162139e+12 6.486417e+12 1.945925e+13
## [31] 5.837776e+13 1.751333e+14 5.253999e+14 1.576200e+15 4.728599e+15
## [36] 1.418580e+16 4.255739e+16 1.276722e+17 3.830165e+17 1.149050e+18
## [41] 3.447149e+18 1.034145e+19 3.102434e+19 9.307302e+19 2.792191e+20
## [46] 8.376572e+20 2.512971e+21 7.538914e+21 2.261674e+22 6.785023e+22
##  [1] 1.000000e+00 6.000000e+00 1.800000e+01 5.700000e+01 1.760000e+02
##  [6] 5.370000e+02 1.617000e+03 4.871000e+03 1.454400e+04 4.354400e+04
## [11] 1.304320e+05 3.909060e+05 1.172644e+06 3.518203e+06 1.055447e+07
```

```
## [16] 3.166125e+07 9.500243e+07 2.849891e+08 8.549456e+08 2.564840e+09
## [21] 7.694628e+09 2.308392e+10 6.925162e+10 2.077547e+11 6.232634e+11
## [26] 1.869791e+12 5.609371e+12 1.682811e+13 5.048433e+13 1.514530e+14
## [31] 4.543591e+14 1.363077e+15 4.089232e+15 1.226770e+16 3.680309e+16
## [36] 1.104093e+17 3.312278e+17 9.936833e+17 2.981050e+18 8.943150e+18
## [41] 2.682945e+19 8.048835e+19 2.414651e+20 7.243952e+20 2.173185e+21
## [46] 6.519556e+21 1.955867e+22 5.867601e+22 1.760280e+23 5.280841e+23
##  [1] 1.000000e+00 3.000000e+00 1.100000e+01 2.700000e+01 6.600000e+01
##  [6] 2.090000e+02 6.400000e+02 1.964000e+03 5.833000e+03 1.730700e+04
## [11] 5.165300e+04 1.549970e+05 4.649230e+05 1.395469e+06 4.185885e+06
## [16] 1.255782e+07 3.767789e+07 1.130442e+08 3.391320e+08 1.017390e+09
## [21] 3.052190e+09 9.156548e+09 2.746942e+10 8.240814e+10 2.472244e+11
## [26] 7.416746e+11 2.225024e+12 6.675072e+12 2.002522e+13 6.007566e+13
## [31] 1.802270e+14 5.406810e+14 1.622043e+15 4.866129e+15 1.459839e+16
## [36] 4.379516e+16 1.313855e+17 3.941564e+17 1.182469e+18 3.547408e+18
## [41] 1.064222e+19 3.192667e+19 9.578001e+19 2.873400e+20 8.620201e+20
## [46] 2.586060e+21 7.758181e+21 2.327454e+22 6.982363e+22 2.094709e+23
##  [1] 1.000000e+00 2.000000e+00 2.000000e+00 7.000000e+00 1.800000e+01
##  [6] 4.900000e+01 1.390000e+02 4.260000e+02 1.317000e+03 4.042000e+03
## [11] 1.206100e+04 3.607000e+04 1.084950e+05 3.238960e+05 9.740590e+05
## [16] 2.919724e+06 8.759849e+06 2.628372e+07 7.884873e+07 2.365208e+08
## [21] 7.095477e+08 2.128625e+09 6.385832e+09 1.915742e+10 5.747236e+10
## [26] 1.724166e+11 5.172492e+11 1.551748e+12 4.655243e+12 1.396573e+13
## [31] 4.189720e+13 1.256916e+14 3.770748e+14 1.131224e+15 3.393673e+15
## [36] 1.018102e+16 3.054306e+16 9.162918e+16 2.748875e+17 8.246626e+17
## [41] 2.473988e+18 7.421963e+18 2.226589e+19 6.679767e+19 2.003930e+20
## [46] 6.011790e+20 1.803537e+21 5.410611e+21 1.623183e+22 4.869550e+22
##  [1] 1.000000e+00 3.000000e+00 1.000000e+01 3.200000e+01 1.000000e+02
##  [6] 2.990000e+02 9.050000e+02 2.754000e+03 8.359000e+03 2.508700e+04
## [11] 7.527700e+04 2.263270e+05 6.797470e+05 2.040812e+06 6.127766e+06
## [16] 1.838808e+07 5.515960e+07 1.654901e+08 4.964586e+08 1.489410e+09
## [21] 4.468175e+09 1.340464e+10 4.021390e+10 1.206415e+11 3.619233e+11
## [26] 1.085771e+12 3.257317e+12 9.771955e+12 2.931587e+13 8.794760e+13
## [31] 2.638428e+14 7.915284e+14 2.374585e+15 7.123756e+15 2.137127e+16
## [36] 6.411380e+16 1.923414e+17 5.770242e+17 1.731073e+18 5.193218e+18
## [41] 1.557965e+19 4.673896e+19 1.402169e+20 4.206506e+20 1.261952e+21
## [46] 3.785856e+21 1.135757e+22 3.407270e+22 1.022181e+23 3.066543e+23
##  [1] 1.000000e+00 2.000000e+00 1.000000e+01 3.400000e+01 9.600000e+01
##  [6] 2.900000e+02 8.600000e+02 2.489000e+03 7.578000e+03 2.249200e+04
## [11] 6.759600e+04 2.027270e+05 6.085310e+05 1.827018e+06 5.480866e+06
## [16] 1.643408e+07 4.930383e+07 1.479167e+08 4.437743e+08 1.331377e+09
## [21] 3.994075e+09 1.198205e+10 3.594619e+10 1.078389e+11 3.235162e+11
## [26] 9.705502e+11 2.911654e+12 8.734962e+12 2.620488e+13 7.861464e+13
## [31] 2.358439e+14 7.075318e+14 2.122595e+15 6.367786e+15 1.910336e+16
## [36] 5.731008e+16 1.719302e+17 5.157907e+17 1.547372e+18 4.642116e+18
## [41] 1.392635e+19 4.177905e+19 1.253371e+20 3.760114e+20 1.128034e+21
## [46] 3.384103e+21 1.015231e+22 3.045692e+22 9.137077e+22 2.741123e+23
##  [1] 1.000000e+00 5.000000e+00 8.000000e+00 2.900000e+01 9.100000e+01
##  [6] 2.770000e+02 8.570000e+02 2.525000e+03 7.497000e+03 2.247300e+04
## [11] 6.709800e+04 2.015290e+05 6.035550e+05 1.806978e+06 5.417803e+06
## [16] 1.625911e+07 4.877475e+07 1.463163e+08 4.389690e+08 1.316922e+09
## [21] 3.950659e+09 1.185196e+10 3.555595e+10 1.066675e+11 3.200033e+11
## [26] 9.600101e+11 2.880028e+12 8.640088e+12 2.592026e+13 7.776079e+13
## [31] 2.332824e+14 6.998472e+14 2.099542e+15 6.298624e+15 1.889587e+16
```

```
## [36] 5.668762e+16 1.700629e+17 5.101886e+17 1.530566e+18 4.591697e+18
## [41] 1.377509e+19 4.132528e+19 1.239758e+20 3.719275e+20 1.115782e+21
## [46] 3.347347e+21 1.004204e+22 3.012613e+22 9.037838e+22 2.711351e+23
##  [1] 1.000000e+00 4.000000e+00 6.000000e+00 2.100000e+01 6.600000e+01
##  [6] 1.860000e+02 5.580000e+02 1.695000e+03 5.099000e+03 1.533900e+04
## [11] 4.593100e+04 1.379570e+05 4.144820e+05 1.243718e+06 3.729734e+06
## [16] 1.118660e+07 3.355013e+07 1.006363e+08 3.018914e+08 9.057307e+08
## [21] 2.717211e+09 8.151672e+09 2.445475e+10 7.336358e+10 2.200920e+11
## [26] 6.602752e+11 1.980826e+12 5.942478e+12 1.782743e+13 5.348230e+13
## [31] 1.604469e+14 4.813407e+14 1.444022e+15 4.332066e+15 1.299620e+16
## [36] 3.898859e+16 1.169658e+17 3.508973e+17 1.052692e+18 3.158076e+18
## [41] 9.474228e+18 2.842269e+19 8.526806e+19 2.558042e+20 7.674125e+20
## [46] 2.302238e+21 6.906713e+21 2.072014e+22 6.216041e+22 1.864812e+23
##  [1] 1.000000e+00 6.000000e+00 1.700000e+01 5.200000e+01 1.390000e+02
##  [6] 4.410000e+02 1.281000e+03 3.880000e+03 1.169500e+04 3.514800e+04
## [11] 1.058530e+05 3.174360e+05 9.520300e+05 2.857287e+06 8.566816e+06
## [16] 2.569230e+07 7.705979e+07 2.311685e+08 6.934892e+08 2.080438e+09
## [21] 6.241356e+09 1.872419e+10 5.617248e+10 1.685168e+11 5.055508e+11
## [26] 1.516651e+12 4.549954e+12 1.364987e+13 4.094961e+13 1.228488e+14
## [31] 3.685465e+14 1.105639e+15 3.316918e+15 9.950755e+15 2.985226e+16
## [36] 8.955679e+16 2.686704e+17 8.060111e+17 2.418033e+18 7.254100e+18
## [41] 2.176230e+19 6.528690e+19 1.958607e+20 5.875821e+20 1.762746e+21
## [46] 5.288239e+21 1.586472e+22 4.759415e+22 1.427825e+23 4.283474e+23
##  [1] 1.000000e+00 2.000000e+00 8.000000e+00 3.300000e+01 9.800000e+01
##  [6] 2.700000e+02 7.610000e+02 2.283000e+03 6.842000e+03 2.047800e+04
## [11] 6.149200e+04 1.846030e+05 5.527230e+05 1.656970e+06 4.969939e+06
## [16] 1.491316e+07 4.474715e+07 1.342517e+08 4.027355e+08 1.208133e+09
## [21] 3.624283e+09 1.087267e+10 3.261790e+10 9.785391e+10 2.935614e+11
## [26] 8.806831e+11 2.642049e+12 7.926148e+12 2.377844e+13 7.133534e+13
## [31] 2.140060e+14 6.420180e+14 1.926054e+15 5.778162e+15 1.733448e+16
## [36] 5.200345e+16 1.560104e+17 4.680311e+17 1.404093e+18 4.212280e+18
## [41] 1.263684e+19 3.791052e+19 1.137316e+20 3.411947e+20 1.023584e+21
## [46] 3.070752e+21 9.212256e+21 2.763677e+22 8.291030e+22 2.487309e+23
##  [1] 1.000000e+00 2.000000e+00 5.000000e+00 1.400000e+01 5.000000e+01
##  [6] 1.340000e+02 4.190000e+02 1.245000e+03 3.770000e+03 1.139500e+04
## [11] 3.407900e+04 1.019630e+05 3.059680e+05 9.173030e+05 2.752525e+06
## [16] 8.254292e+06 2.475773e+07 7.425620e+07 2.227767e+08 6.683563e+08
## [21] 2.005044e+09 6.015201e+09 1.804521e+10 5.413581e+10 1.624078e+11
## [26] 4.872243e+11 1.461672e+12 4.385012e+12 1.315503e+13 3.946510e+13
## [31] 1.183953e+14 3.551859e+14 1.065558e+15 3.196673e+15 9.590019e+15
## [36] 2.877006e+16 8.631017e+16 2.589305e+17 7.767915e+17 2.330375e+18
## [41] 6.991124e+18 2.097337e+19 6.292011e+19 1.887603e+20 5.662810e+20
## [46] 1.698843e+21 5.096529e+21 1.528959e+22 4.586876e+22 1.376063e+23
##  [1] 1.000000e+00 1.000000e+00 1.000000e+00 2.000000e+00 8.000000e+00
##  [6] 3.600000e+01 1.040000e+02 3.140000e+02 8.930000e+02 2.619000e+03
## [11] 7.737000e+03 2.304600e+04 6.896800e+04 2.071090e+05 6.206710e+05
## [16] 1.862179e+06 5.584601e+06 1.674773e+07 5.023454e+07 1.507007e+08
## [21] 4.521295e+08 1.356411e+09 4.069247e+09 1.220772e+10 3.662295e+10
## [26] 1.098691e+11 3.296077e+11 9.888218e+11 2.966466e+12 8.899402e+12
## [31] 2.669821e+13 8.009464e+13 2.402839e+14 7.208518e+14 2.162555e+15
## [36] 6.487666e+15 1.946300e+16 5.838899e+16 1.751670e+17 5.255009e+17
## [41] 1.576503e+18 4.729508e+18 1.418853e+19 4.256558e+19 1.276967e+20
## [46] 3.830902e+20 1.149271e+21 3.447812e+21 1.034343e+22 3.103030e+22
##  [1] 1.000000e+00 2.000000e+00 6.000000e+00 1.800000e+01 4.500000e+01
```

```
##  [6] 1.380000e+02 4.330000e+02 1.344000e+03 4.104000e+03 1.229100e+04
## [11] 3.686200e+04 1.109600e+05 3.328960e+05 9.972850e+05 2.992182e+06
## [16] 8.972127e+06 2.690606e+07 8.071238e+07 2.421082e+08 7.263256e+08
## [21] 2.178941e+09 6.536713e+09 1.961025e+10 5.883047e+10 1.764915e+11
## [26] 5.294752e+11 1.588426e+12 4.765283e+12 1.429585e+13 4.288754e+13
## [31] 1.286626e+14 3.859878e+14 1.157963e+15 3.473890e+15 1.042167e+16
## [36] 3.126501e+16 9.379504e+16 2.813851e+17 8.441553e+17 2.532466e+18
## [41] 7.597398e+18 2.279219e+19 6.837658e+19 2.051297e+20 6.153892e+20
## [46] 1.846168e+21 5.538503e+21 1.661551e+22 4.984653e+22 1.495396e+23
##  [1] 1.000000e+00 5.000000e+00 1.300000e+01 4.200000e+01 1.220000e+02
##  [6] 3.600000e+02 1.025000e+03 3.054000e+03 9.152000e+03 2.723400e+04
## [11] 8.154700e+04 2.453470e+05 7.346820e+05 2.203984e+06 6.611270e+06
## [16] 1.983759e+07 5.950460e+07 1.785150e+08 5.355184e+08 1.606524e+09
## [21] 4.819620e+09 1.445899e+10 4.337700e+10 1.301310e+11 3.903935e+11
## [26] 1.171182e+12 3.513548e+12 1.054064e+13 3.162193e+13 9.486579e+13
## [31] 2.845974e+14 8.537922e+14 2.561377e+15 7.684130e+15 2.305239e+16
## [36] 6.915717e+16 2.074715e+17 6.224145e+17 1.867244e+18 5.601731e+18
## [41] 1.680519e+19 5.041557e+19 1.512467e+20 4.537402e+20 1.361221e+21
## [46] 4.083662e+21 1.225098e+22 3.675295e+22 1.102589e+23 3.307766e+23
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## [21] 5.418555e+08 1.625605e+09 4.876775e+09 1.463024e+10 4.389073e+10
## [26] 1.316718e+11 3.950157e+11 1.185047e+12 3.555142e+12 1.066543e+13
## [31] 3.199627e+13 9.598881e+13 2.879664e+14 8.638993e+14 2.591698e+15
## [36] 7.775094e+15 2.332528e+16 6.997585e+16 2.099275e+17 6.297826e+17
## [41] 1.889348e+18 5.668044e+18 1.700413e+19 5.101239e+19 1.530372e+20
## [46] 4.591115e+20 1.377335e+21 4.132004e+21 1.239601e+22 3.718803e+22
##  [1] 1.000000e+00 3.000000e+00 7.000000e+00 1.100000e+01 3.300000e+01
##  [6] 1.060000e+02 3.280000e+02 1.025000e+03 3.029000e+03 9.104000e+03
## [11] 2.691500e+04 8.039100e+04 2.422220e+05 7.260050e+05 2.178502e+06
## [16] 6.534248e+06 1.960607e+07 5.882904e+07 1.765040e+08 5.295093e+08
## [21] 1.588577e+09 4.765707e+09 1.429705e+10 4.289142e+10 1.286743e+11
## [26] 3.860220e+11 1.158064e+12 3.474192e+12 1.042257e+13 3.126772e+13
```

```
## [31] 9.380314e+13 2.814094e+14 8.442282e+14 2.532685e+15 7.598054e+15
## [36] 2.279416e+16 6.838248e+16 2.051474e+17 6.154423e+17 1.846327e+18
## [41] 5.538981e+18 1.661694e+19 4.985083e+19 1.495525e+20 4.486575e+20
## [46] 1.345972e+21 4.037917e+21 1.211375e+22 3.634125e+22 1.090238e+23
##  [1] 1.000000e+00 3.000000e+00 5.000000e+00 1.300000e+01 3.900000e+01
##  [6] 1.110000e+02 3.730000e+02 1.077000e+03 3.222000e+03 9.622000e+03
## [11] 2.893100e+04 8.711400e+04 2.616580e+05 7.850700e+05 2.356419e+06
## [16] 7.069972e+06 2.121623e+07 6.365133e+07 1.909557e+08 5.728965e+08
## [21] 1.718602e+09 5.155674e+09 1.546697e+10 4.640079e+10 1.392021e+11
## [26] 4.176065e+11 1.252819e+12 3.758457e+12 1.127537e+13 3.382611e+13
## [31] 1.014783e+14 3.044350e+14 9.133051e+14 2.739915e+15 8.219746e+15
## [36] 2.465924e+16 7.397771e+16 2.219331e+17 6.657994e+17 1.997398e+18
## [41] 5.992195e+18 1.797658e+19 5.392975e+19 1.617893e+20 4.853678e+20
## [46] 1.456103e+21 4.368310e+21 1.310493e+22 3.931479e+22 1.179444e+23
##  [1] 1.000000e+00 2.000000e+00 5.000000e+00 1.500000e+01 5.800000e+01
##  [6] 1.780000e+02 5.210000e+02 1.550000e+03 4.689000e+03 1.387900e+04
## [11] 4.180100e+04 1.255940e+05 3.777870e+05 1.133510e+06 3.398659e+06
## [16] 1.019267e+07 3.058442e+07 9.175335e+07 2.752695e+08 8.258121e+08
## [21] 2.477458e+09 7.432423e+09 2.229736e+10 6.689217e+10 2.006767e+11
## [26] 6.020293e+11 1.806090e+12 5.418274e+12 1.625483e+13 4.876448e+13
## [31] 1.462934e+14 4.388803e+14 1.316641e+15 3.949923e+15 1.184977e+16
## [36] 3.554931e+16 1.066479e+17 3.199438e+17 9.598313e+17 2.879494e+18
## [41] 8.638481e+18 2.591544e+19 7.774633e+19 2.332390e+20 6.997170e+20
## [46] 2.099151e+21 6.297453e+21 1.889236e+22 5.667708e+22 1.700312e+23
##  [1] 1.000000e+00 2.000000e+00 3.000000e+00 6.000000e+00 2.500000e+01
##  [6] 9.200000e+01 2.940000e+02 8.690000e+02 2.634000e+03 7.916000e+03
## [11] 2.395900e+04 7.134300e+04 2.147180e+05 6.442400e+05 1.932421e+06
## [16] 5.795458e+06 1.738240e+07 5.213979e+07 1.564175e+08 4.692638e+08
## [21] 1.407812e+09 4.223433e+09 1.267022e+10 3.801068e+10 1.140317e+11
## [26] 3.420949e+11 1.026284e+12 3.078855e+12 9.236572e+12 2.770971e+13
## [31] 8.312914e+13 2.493874e+14 7.481622e+14 2.244487e+15 6.733459e+15
## [36] 2.020038e+16 6.060114e+16 1.818034e+17 5.454102e+17 1.636231e+18
## [41] 4.908692e+18 1.472608e+19 4.417823e+19 1.325347e+20 3.976041e+20
## [46] 1.192812e+21 3.578436e+21 1.073531e+22 3.220593e+22 9.661778e+22
##  [1] 1.000000e+00 2.000000e+00 7.000000e+00 2.100000e+01 6.100000e+01
##  [6] 1.860000e+02 5.150000e+02 1.516000e+03 4.641000e+03 1.379800e+04
## [11] 4.152600e+04 1.242430e+05 3.727880e+05 1.119311e+06 3.359648e+06
## [16] 1.008139e+07 3.023488e+07 9.070804e+07 2.721124e+08 8.163533e+08
## [21] 2.449024e+09 7.347217e+09 2.204139e+10 6.612405e+10 1.983715e+11
## [26] 5.951148e+11 1.785343e+12 5.356031e+12 1.606810e+13 4.820428e+13
## [31] 1.446128e+14 4.338385e+14 1.301515e+15 3.904546e+15 1.171364e+16
## [36] 3.514092e+16 1.054228e+17 3.162683e+17 9.488048e+17 2.846414e+18
## [41] 8.539243e+18 2.561773e+19 7.685319e+19 2.305596e+20 6.916787e+20
## [46] 2.075036e+21 6.225108e+21 1.867532e+22 5.602597e+22 1.680779e+23
##  [1] 1.000000e+00 5.000000e+00 1.200000e+01 3.500000e+01 1.080000e+02
##  [6] 3.250000e+02 9.860000e+02 3.042000e+03 9.254000e+03 2.777200e+04
## [11] 8.313900e+04 2.500230e+05 7.487380e+05 2.247908e+06 6.741274e+06
## [16] 2.021911e+07 6.065612e+07 1.819668e+08 5.458841e+08 1.637659e+09
## [21] 4.913002e+09 1.473876e+10 4.421616e+10 1.326483e+11 3.979438e+11
## [26] 1.193831e+12 3.581492e+12 1.074448e+13 3.223343e+13 9.670031e+13
## [31] 2.901009e+14 8.703027e+14 2.610908e+15 7.832724e+15 2.349817e+16
## [36] 7.049452e+16 2.114836e+17 6.344507e+17 1.903352e+18 5.710056e+18
## [41] 1.713017e+19 5.139050e+19 1.541715e+20 4.625145e+20 1.387544e+21
## [46] 4.162631e+21 1.248789e+22 3.746368e+22 1.123910e+23 3.371731e+23
```

```
##  [1] 1.000000e+00 3.000000e+00 8.000000e+00 2.600000e+01 7.200000e+01
##  [6] 2.200000e+02 6.180000e+02 1.888000e+03 5.663000e+03 1.697100e+04
## [11] 5.099300e+04 1.526130e+05 4.585130e+05 1.375589e+06 4.126642e+06
## [16] 1.237992e+07 3.714135e+07 1.114252e+08 3.343091e+08 1.002959e+09
## [21] 3.008868e+09 9.026426e+09 2.707940e+10 8.123806e+10 2.437147e+11
## [26] 7.311442e+11 2.193431e+12 6.580293e+12 1.974087e+13 5.922263e+13
## [31] 1.776679e+14 5.330037e+14 1.599011e+15 4.797033e+15 1.439110e+16
## [36] 4.317330e+16 1.295199e+17 3.885597e+17 1.165679e+18 3.497037e+18
## [41] 1.049111e+19 3.147333e+19 9.442000e+19 2.832600e+20 8.497800e+20
## [46] 2.549340e+21 7.648020e+21 2.294406e+22 6.883218e+22 2.064965e+23
##  [1] 1.000000e+00 3.000000e+00 1.500000e+01 6.100000e+01 1.730000e+02
##  [6] 5.300000e+02 1.581000e+03 4.836000e+03 1.436800e+04 4.329900e+04
## [11] 1.298060e+05 3.891220e+05 1.167610e+06 3.503026e+06 1.051351e+07
## [16] 3.154701e+07 9.464544e+07 2.839195e+08 8.517636e+08 2.555302e+09
## [21] 7.665961e+09 2.299786e+10 6.899314e+10 2.069786e+11 6.209352e+11
## [26] 1.862806e+12 5.588418e+12 1.676526e+13 5.029578e+13 1.508873e+14
## [31] 4.526619e+14 1.357986e+15 4.073957e+15 1.222187e+16 3.666562e+16
## [36] 1.099969e+17 3.299906e+17 9.899717e+17 2.969915e+18 8.909745e+18
## [41] 2.672923e+19 8.018770e+19 2.405631e+20 7.216893e+20 2.165068e+21
## [46] 6.495204e+21 1.948561e+22 5.845684e+22 1.753705e+23 5.261115e+23
##  [1] 1.000000e+00 3.000000e+00 1.300000e+01 4.600000e+01 1.320000e+02
##  [6] 4.070000e+02 1.225000e+03 3.685000e+03 1.099300e+04 3.321500e+04
## [11] 9.934400e+04 2.981030e+05 8.927700e+05 2.679048e+06 8.036237e+06
## [16] 2.410778e+07 7.230530e+07 2.169360e+08 6.507962e+08 1.952326e+09
## [21] 5.856904e+09 1.757072e+10 5.271246e+10 1.581370e+11 4.744122e+11
## [26] 1.423236e+12 4.269710e+12 1.280913e+13 3.842738e+13 1.152821e+14
## [31] 3.458464e+14 1.037539e+15 3.112618e+15 9.337853e+15 2.801356e+16
## [36] 8.404067e+16 2.521220e+17 7.563661e+17 2.269098e+18 6.807295e+18
## [41] 2.042188e+19 6.126565e+19 1.837970e+20 5.513909e+20 1.654173e+21
## [46] 4.962518e+21 1.488755e+22 4.466266e+22 1.339880e+23 4.019639e+23
##  [1] 1.000000e+00 5.000000e+00 9.000000e+00 1.800000e+01 5.400000e+01
##  [6] 1.750000e+02 4.960000e+02 1.562000e+03 4.740000e+03 1.417200e+04
## [11] 4.252700e+04 1.284410e+05 3.855920e+05 1.157128e+06 3.471386e+06
## [16] 1.041454e+07 3.123485e+07 9.370428e+07 2.811020e+08 8.432884e+08
## [21] 2.529866e+09 7.589564e+09 2.276850e+10 6.830507e+10 2.049151e+11
## [26] 6.147457e+11 1.844237e+12 5.532711e+12 1.659814e+13 4.979441e+13
## [31] 1.493832e+14 4.481497e+14 1.344449e+15 4.033347e+15 1.210004e+16
## [36] 3.630013e+16 1.089004e+17 3.267011e+17 9.801034e+17 2.940310e+18
## [41] 8.820930e+18 2.646279e+19 7.938837e+19 2.381651e+20 7.144954e+20
## [46] 2.143486e+21 6.430458e+21 1.929137e+22 5.787412e+22 1.736224e+23
##  [1] 1.000000e+00 2.000000e+00 7.000000e+00 1.700000e+01 5.100000e+01
##  [6] 1.450000e+02 4.210000e+02 1.294000e+03 3.953000e+03 1.180400e+04
## [11] 3.566600e+04 1.064290e+05 3.201300e+05 9.589950e+05 2.877538e+06
## [16] 8.638940e+06 2.591493e+07 7.774135e+07 2.332343e+08 6.996685e+08
## [21] 2.098965e+09 6.296838e+09 1.889057e+10 5.667218e+10 1.700161e+11
## [26] 5.100484e+11 1.530145e+12 4.590433e+12 1.377129e+13 4.131388e+13
## [31] 1.239417e+14 3.718249e+14 1.115475e+15 3.346424e+15 1.003927e+16
## [36] 3.011782e+16 9.035345e+16 2.710604e+17 8.131811e+17 2.439543e+18
## [41] 7.318630e+18 2.195589e+19 6.586767e+19 1.976030e+20 5.928090e+20
## [46] 1.778427e+21 5.335281e+21 1.600584e+22 4.801753e+22 1.440526e+23
##  [1] 1.000000e+00 2.000000e+00 9.000000e+00 2.900000e+01 7.100000e+01
##  [6] 1.940000e+02 5.910000e+02 1.815000e+03 5.479000e+03 1.627100e+04
## [11] 4.863600e+04 1.463260e+05 4.388790e+05 1.315746e+06 3.950426e+06
## [16] 1.185188e+07 3.555916e+07 1.066766e+08 3.200332e+08 9.600631e+08
```

```
## [21] 2.880151e+09 8.640446e+09 2.592118e+10 7.776329e+10 2.332904e+11
## [26] 6.998713e+11 2.099614e+12 6.298845e+12 1.889653e+13 5.668958e+13
## [31] 1.700687e+14 5.102062e+14 1.530619e+15 4.591856e+15 1.377557e+16
## [36] 4.132670e+16 1.239801e+17 3.719403e+17 1.115821e+18 3.347463e+18
## [41] 1.004239e+19 3.012717e+19 9.038150e+19 2.711445e+20 8.134335e+20
## [46] 2.440300e+21 7.320901e+21 2.196270e+22 6.588811e+22 1.976643e+23
##  [1] 1.000000e+00 2.000000e+00 4.000000e+00 1.800000e+01 5.500000e+01
##  [6] 1.540000e+02 4.360000e+02 1.277000e+03 3.925000e+03 1.185200e+04
## [11] 3.573200e+04 1.077940e+05 3.236230e+05 9.690780e+05 2.907401e+06
## [16] 8.727716e+06 2.617207e+07 7.850133e+07 2.354921e+08 7.064537e+08
## [21] 2.119411e+09 6.358243e+09 1.907479e+10 5.722457e+10 1.716739e+11
## [26] 5.150218e+11 1.545065e+12 4.635191e+12 1.390557e+13 4.171671e+13
## [31] 1.251501e+14 3.754504e+14 1.126351e+15 3.379054e+15 1.013716e+16
## [36] 3.041149e+16 9.123446e+16 2.737034e+17 8.211102e+17 2.463330e+18
## [41] 7.389991e+18 2.216997e+19 6.650992e+19 1.995298e+20 5.985893e+20
## [46] 1.795768e+21 5.387304e+21 1.616191e+22 4.848573e+22 1.454572e+23
##  [1] 1.000000e+00 5.000000e+00 1.200000e+01 2.100000e+01 6.000000e+01
##  [6] 1.600000e+02 4.580000e+02 1.381000e+03 4.065000e+03 1.226100e+04
## [11] 3.692100e+04 1.111960e+05 3.337540e+05 1.000157e+06 3.001961e+06
## [16] 9.002616e+06 2.699599e+07 8.097698e+07 2.429762e+08 7.289425e+08
## [21] 2.186804e+09 6.560699e+09 1.968215e+10 5.904645e+10 1.771398e+11
## [26] 5.314180e+11 1.594254e+12 4.782767e+12 1.434829e+13 4.304486e+13
## [31] 1.291346e+14 3.874037e+14 1.162211e+15 3.486633e+15 1.045990e+16
## [36] 3.137970e+16 9.413909e+16 2.824173e+17 8.472518e+17 2.541755e+18
## [41] 7.625266e+18 2.287580e+19 6.862740e+19 2.058822e+20 6.176466e+20
## [46] 1.852940e+21 5.558819e+21 1.667646e+22 5.002937e+22 1.500881e+23
##  [1] 1.000000e+00 2.000000e+00 9.000000e+00 2.500000e+01 5.900000e+01
##  [6] 1.690000e+02 5.190000e+02 1.539000e+03 4.483000e+03 1.370000e+04
## [11] 4.147300e+04 1.241370e+05 3.719900e+05 1.117218e+06 3.351972e+06
## [16] 1.005892e+07 3.017507e+07 9.052679e+07 2.715671e+08 8.147066e+08
## [21] 2.444145e+09 7.332438e+09 2.199715e+10 6.599189e+10 1.979761e+11
## [26] 5.939271e+11 1.781782e+12 5.345347e+12 1.603604e+13 4.810812e+13
## [31] 1.443244e+14 4.329731e+14 1.298919e+15 3.896758e+15 1.169027e+16
## [36] 3.507082e+16 1.052125e+17 3.156374e+17 9.469121e+17 2.840736e+18
## [41] 8.522209e+18 2.556663e+19 7.669988e+19 2.300996e+20 6.902989e+20
## [46] 2.070897e+21 6.212691e+21 1.863807e+22 5.591421e+22 1.677426e+23
##  [1] 1.000000e+00 2.000000e+00 3.000000e+00 4.000000e+00 1.500000e+01
##  [6] 5.900000e+01 1.660000e+02 5.210000e+02 1.523000e+03 4.519000e+03
## [11] 1.385500e+04 4.142700e+04 1.238060e+05 3.710510e+05 1.112822e+06
## [16] 3.338109e+06 1.001493e+07 3.005271e+07 9.015176e+07 2.704544e+08
## [21] 8.113416e+08 2.434036e+09 7.302130e+09 2.190627e+10 6.571902e+10
## [26] 1.971574e+11 5.914721e+11 1.774417e+12 5.323249e+12 1.596976e+13
## [31] 4.790928e+13 1.437279e+14 4.311836e+14 1.293551e+15 3.880652e+15
## [36] 1.164196e+16 3.492587e+16 1.047776e+17 3.143328e+17 9.429985e+17
## [41] 2.828996e+18 8.486987e+18 2.546096e+19 7.638288e+19 2.291486e+20
## [46] 6.874459e+20 2.062338e+21 6.187013e+21 1.856104e+22 5.568312e+22
```

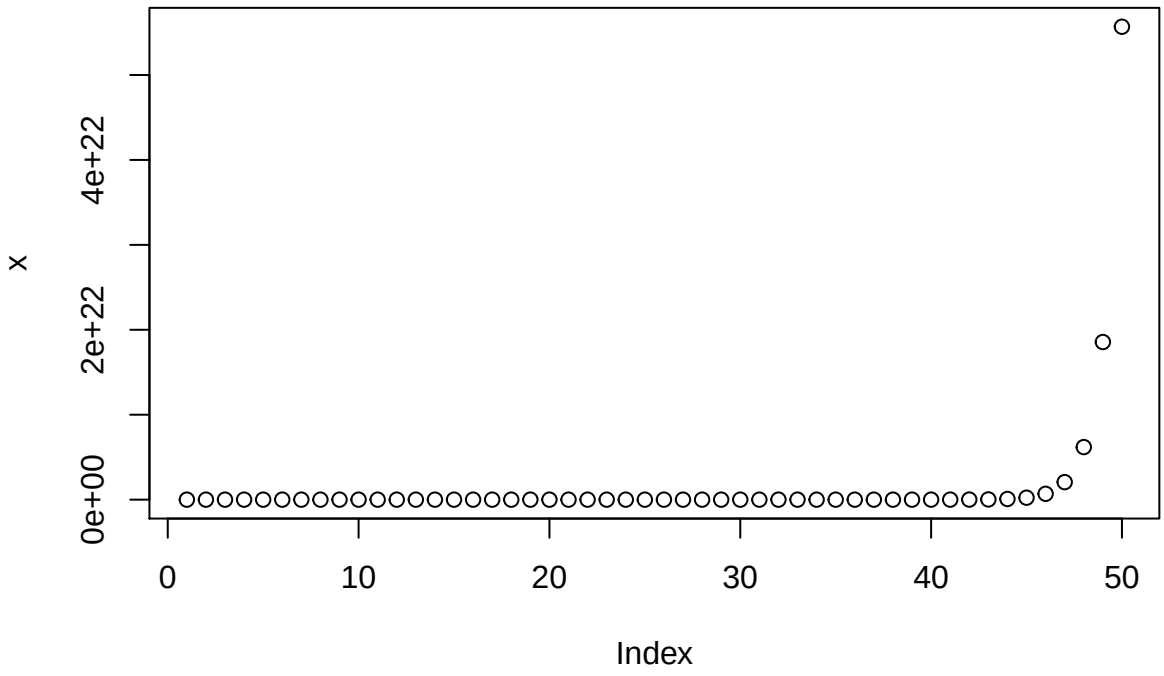
```
prob=b/k#la probabilité d'extinction de la population
print(paste0("la probabilité empirique d'extinction est:",prob))
```

```
## [1] "la probabilité empirique d'extinction est:0.02"
```

```
exp_ext = t/b
# espérance moyenne de l'extinction lorsqu'il y a extinction (conditionnellement à l'extinction)
print(paste0("l'espérance de temps d'extinction est: ",exp_ext, " dans le cas surcritique"))
```

```
## [1] "l'espérance de temps d'extinction est: 2 dans le cas surcritique"
```

```
plot(x)
```



On remarque que dans le cas surcritique, il existe des populations qui survivent et on observe ainsi que la probabilité empirique d'extinction est très petite dans le cas critique quand l'espérance est 3. Ce qui correspond bien au théorème de la référence. De plus, avec la variation du paramètre a, on peut ainsi dire que plus l'espérance grande, plus le nombre des populations qui ont survécu est important et plus la probabilité d'extinction de la population se rapproche de 0. Ensuite, on peut observer que les événements d'extinction correspondent le plus souvent à des extinctions qui arrivent très tôt dans la simulation. Ainsi, on voit que lorsqu'il y a extinction, le temps de la moyenne de l'extinction est très tôt (qui est égale à 2). Après cela, on trace la graphe de la dernière simulation (quand j=k), et on a une croissance d'exponentielle pour la génération en abscisse et les individus en ordonné. Et si on veut une croissance d'exponentielle brutale, on peut mettre l'espérance plus petite mais on remarque qu'il y a une risque d'extinction surout pour le paramètre qui est très petit et rappoche de 1.

Par exemple:

```
a=1.1
print("pour l'espérance 1.1,on trouve que la somme de ces 10 individus aléatoire")
```

```
## [1] "pour l'espérance 1.1,on trouve que la somme de ces 10 individus aléatoire"
```

```
print(paste0("qui suit la loi de Poisson est égale à " ,Sumpoisson(10)))
```

```
## [1] "qui suit la loi de Poisson est égale à 9"
```

```
k=30#nombre de simulation
n=50#nombre de génération
```

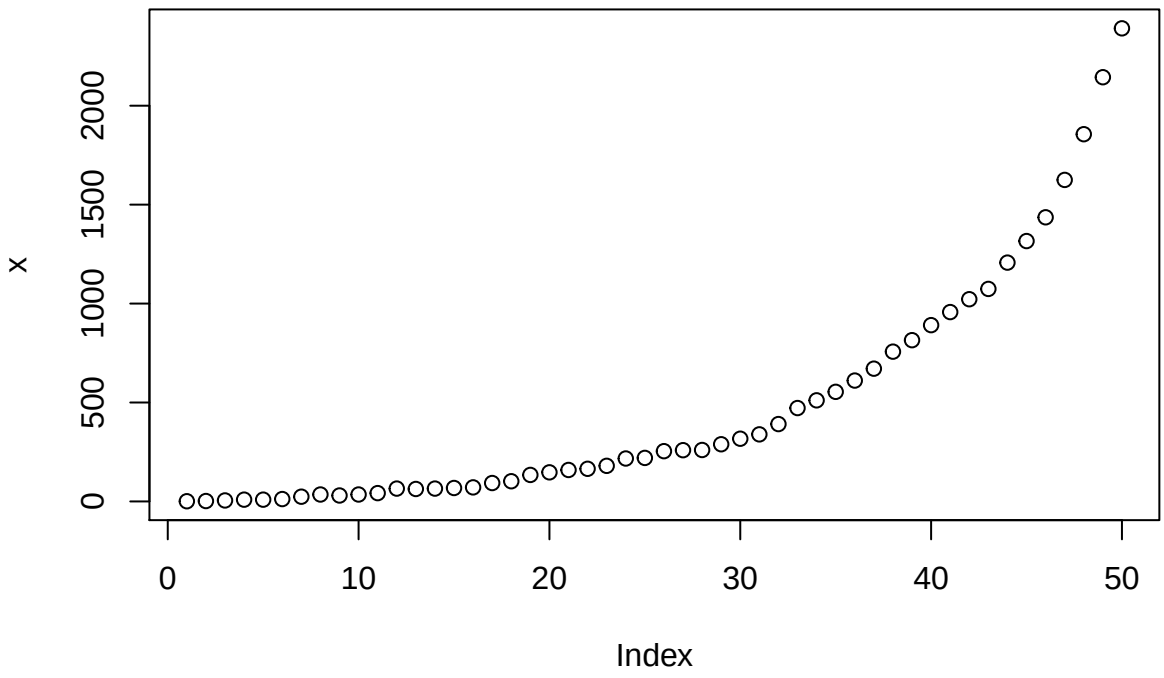
```
b=0#compteur pour calculer le nombre de d'extinction
t=0
for (j in 1:k){
x=1 # Le premier individu
  for (i in 2:n){
    x<-c(x,Sumpoisson(x[length(x)])) # ajouter un individu qui suit une loi de Poisson à la fin de x
    if (x[length(x)]==0){ #si le dernier individu de x n'a pas la génération
      b=b+1
      t=t+i
      break
    }
}
#print(x)
}
prob=b/k#la probabilité d'extinction de la population
print(paste0("la probabilité empirique d'extinction est:",prob))
```

```
## [1] "la probabilité empirique d'extinction est:0.7"
```

```
exp_ext = t/b
# espérance moyenne de l'extinction lorsqu'il y a extinction (conditionnellement à l'extinction)
print(paste0("l'espérance de temps d'extinction est: ",exp_ext, " dans le cas surcritique"))
```

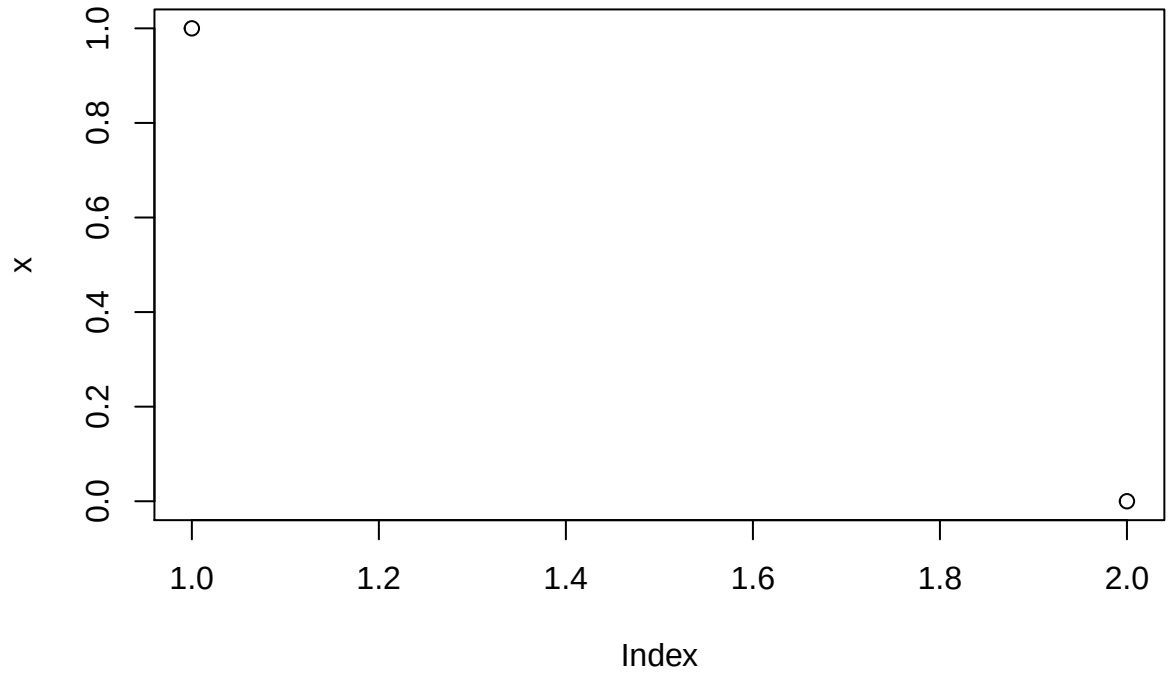
```
## [1] "l'espérance de temps d'extinction est: 4.19047619047619 dans le cas surcritique"
```

```
plot(x)
```




```
k=30#nombre de simulation
n=50#nombre de génération


b=0#compteur pour calculer le nombre de d'extinction
t=0
for (j in 1:k){
x=1 # Le premier individu
  for (i in 2:n){
    x<-c(x,Sumpoisson(x[length(x)])) # ajouter un individu qui suit une loi de Poisson à la fin de x
    if (x[length(x)]==0){ #si le dernier individu de x n'a pas la génération
      b=b+1
      t=t+i
      break
    }
}
#print(x)
}
plot(x)
```



On observe qu'il y a une extinction et une croissance exponentielle plus brutale pour le parametrè 1.1.

Avec immigration:

```
k=30 #nombre de simulation
n=30  #nombre de génération

b=0 #compteur pour calculer le nombre de d'extinction
t=0 #temps
for (j in 1:k){
x=1 # Le premier individu
  for (i in 2:n){
    x<-c(x,Sumpoisson(x[length(x)]))
    if (x[length(x)]==0){#si le dernier individu de x n'a pas la génération
      x<-c(x,sample(1:10,1))#On ajoute un nombre des individus d'immigration entre 1 et 10
    }
    else{
      x<-c(x,sample(1:10,1))
    }
  }
print(x)
}
```

```
##  [1]  1  1  3  5  3  3  9  8  6  6  4  7  1  3  1  1  4  6  5  2  1  0  7 13  7
## [26]  9  9 11  4  5  1  1  1  1  9  8  2  1  4  4  9  6  6  3  2  1 10 10  2  4
## [51]  3  5  4  3  5  7  4  6  5
##  [1]  1  2  8  9 10  6  1  1  2  2  4  2  8  8 10  7  7  5  3  6  7  9  5  5  8
## [26] 12  3  5  3  3  4  1  2  3  4  3  7  4  7  8  1  2 10 11  9 12  6  8  4  4
```

```
## [51]  8 10  5  6  3  2  4  7  4
##  [1]  1  1  4  6  4  4  2  1  3  8  9  8  6  8 10 15  6  9 10 12  3  3  9 15  1
## [26]  1 10 11  2  9  8 12  8 10  3  1  5  5  8 12  6  7 10  7  6  5  1  1  1  0
## [51]  8  7  3  2  7 11  3  2  8
##  [1]  1  1  2  0  6  9 10 11  9 12  1  0  2  1 10  9  5  9  1  3  5  5  4  5  1
## [26]  0 10  9  9  9  2  1  6 10  6  8  2  1  5  9  8  9  2  4  4  5  3  4  5  5
## [51]  7  9  7 11  6  7  4  2  1
##  [1]  1  0  9  9  6  9  9 11  5  6  3  3  7  8  7  6  4  4 10  8  4  4  4  5  7
## [26]  6  7  7  4  4  3  5  3  5  8  9  3  2  7 10  9  9  1  0  6  5  3  1  1  1
## [51] 10  8  5  7 10 14  4  4  4
##  [1]  1  2  4  5  7  9  4  4  2  0  4  2 10  8  7  9  8 15  8 12  2  3  6  6  3
## [26]  4  8 12  9 15  2  3  1  2  8  7  1  1 10 14  3  3  6  8  2  1  7  9  6  8
## [51]  7  7  5  5  2  3  6  5  3
##  [1]  1  1  3  0  8 10  5  7  2  1  7 13  5  5  9  8  7  3  7  6  5 10  7  8  7
## [26]  8 10 13  2  2  1  3  2  5  5  3 10  9  1  2  7  9  1  1  2  7  3  2  3  2
## [51] 10 15  3  3  3  6  2  2  5
##  [1]  1  3  8 11  9 10  4  4  5  1  8  7  2  2  2  2  7  4  9 11  4  3  3  5  3
## [26]  3  2  2  4  8  3  2  1  0  5  7  4  5 10 10  4  2  8  8  1  1  2  4  3  5
## [51]  3  4  6  2  7  7  7  8 10
##  [1]  1  1  1  0  2  3  7 16  4  4  1  0 10 15  4  3  7  5  2  2  4  6  2  3 10
## [26] 10 10  7  7 13  3  5  2  2  4  2  3  1  9  7  2  2  9 12  5  4  3  4  6  3
## [51]  8  9 10 10  5 12  9 16  4
##  [1]  1  0  2  1  4  8  6  7  7  9  5  6  6  8  4  3 10 15  3  5  1  1  1  0  2
## [26]  1  3  2  3  3 10  7  1  4  5  7  1  4  7  8  2  2  2  0  4  3  1  1  7 10
## [51]  2  1  2  0  2  7 10 17  3
##  [1]  1  0  4  6  9  9  8  4  3  3  7  6  5  7  6  6  6  6  9 13  2  3  7 12  7
## [26]  8  5  4  2  2  1  0  9 10  9  8  5  0  2  4  4  5  7  9  6  4  8  6  6  6
## [51]  9 12  5  8  2  0  2  2  5
##  [1]  1  0  3  5  4  6  6  4  9  7  4  2  5  5  9 13  6  6  5  3  7  7  7  9  3
## [26]  5  8 10  7  6  4  4  6  6 10 13  8  5  9 11  6  7  8  7  7  3  2  0  2  2
## [51]  1  1  8  5  3  2  2  1  9
##  [1]  1  1  9 12  9  6  6  5  7 10  8  7  8 11  6  5 10 11  1  1  3  5  9  9  7
## [26]  5  3  2  4  4  5  5  5  7  6  7  8  5  7  5  6  9  7  6  1  1  4  5  4  6
## [51]  2  3  8  7  7 12  3  6 10
##  [1]  1  0  1  1  6  6 10 15  4  2  6  5  6  5 10  9  5  5  3  3  1  2  7  5  5
## [26]  5  4  2  7  5  1  2  2  3  4  5  2  5  6  5  3  4  5  6  5 11  9 10  8 13
## [51]  1  0  9 12  5  7  6  6  1
##  [1]  1  1 10 14  6  7  3  4  3  1  4  6  1  2  9  7  1  0  9  7  4  5  3  3  7
## [26] 10  7  6  6  9  2  3  7  6  5  7  8  3  4  7  2  3  5  4  2  3  8  8  2  1
## [51]  8  3  7  8  6  7  1  4  5
##  [1]  1  5  3  2  3  2 10 12 10 11  7  7 10 11  6  6  2  1  3  4  4  4  5  4  3
## [26]  5  1  2  1  2  7  7 10 10 10 16  7 11  3  3  7  8  6  6  6  7  7  5  8  8
## [51]  6 10  7  7  3  5  7  2  5
##  [1]  1  1  7  8  5  3  2  3  5  3  2  4  8  7  3  2  1  1  2  6  6  4  6  6  6
## [26]  5  2  3 10 10  6  8  3  6  8 10  2  2  4  6 10 11  7  5  3  7  9  7  4  5
## [51]  1  0  4  7  9  6  7 11  1
##  [1]  1  0  8 10  2  4  8  9  4  2  8 10  3  9  7  7  5  9  6 11  6  8  2  0  9
## [26]  7  9  5  9  7  6 10  1  0  2  1  8  6  2  5 10 13  6  5  8 12  6  9  1  0
## [51] 10  8  1  1  6 10  7 12  5
##  [1]  1  0  7 12  1  2  4  7  6  4 10 17 10 10 10 10  2  2  7  3  4  2 10 11  5
## [26]  3  2  4  8  8  9  9 10 12  2  2 10  8  2  1  6  6  6  6  1  1  5  3 10 10
## [51]  8 14  9 11  1  0  2  1  1
##  [1]  1  2  6  5 10 15  3  4  7  4 10 17  2  2 10 10  7  9  1  2  2  2  7 13  9
## [26] 11  3  2  5  9  2  3  3  2  1  1  2  0 10  8  3  5  8 11  4  7  2  4  7  7
```

```
## [51]  3  4  5  7  7  8  3  5  1
##  [1]  1  0  5  5  5  7  3  2  6  6 10 11  3  5  2  3  2  0  1  0  1  0  4  3 10
## [26] 16  8 11  3  2  2  1  1  1  3  0  8 11 10  7  9 13  9  8  5  6  1  1  7  4
## [51] 10  9  4  0  5  3 10 11  4
##  [1]  1  0  4  5  5  6  4  5  7  8  9 17  5  7 10  7  7 11  2  2 10 11 10  9  4
## [26]  2  3  1  7  8  9  5  7  5  1  1  6  7  6  9  1  2  7 11  3  0  4  5 10 14
## [51]  3  7  8 11 10 11 10 12  4
##  [1]  1  1  3  5  9 11  5  2 10 17  6  4 10 11  3  0 10 16  2  4  7  8  4  8  3
## [26]  8  4  5  6  6  2  0  8  8  7  6 10 13 10  9 10 12 10 10  8  5  5  5  9 11
## [51]  2  1  9  7  9  7  1  0  7
##  [1]  1  4  1  0  3  4 10 12  5  3  2  2  9 11  9 13  4  3  7  4  7  5  1  1  9
## [26] 10  9 10  3  3  7  6  6  4  5  6  9  7  1  3  6  6 10  8  5  5  7  6  8  5
## [51]  5  7  7 15 10 12  9  4 10
##  [1]  1  2  8 11  2  3  8  7  4  5  7  8  9  9  6 12  4  4  6  7  4  3  8  5 10
## [26] 15  6  3  9 12  5  6  2  2 10 12  9 11  9 11  1  1  7  4  2  1  4  7  9  8
## [51]  4  5  3  4  8  8  3  4  5
##  [1]  1  1  8  7  9  5  4  2 10 11  4  4  7  9 10  9  5  0  1  1  3  1  8  9  5
## [26]  5  8  7  4  2  7  6  4  4  5  8  3  5  6  6  6  9  2  2  7 12  7  9  5  6
## [51]  6  7 10 13  3  4  6  4  2
##  [1]  1  0  4  6  5  6  2  0  9  9  1  0  2  3  3  3  9  7  8  8 10 11  2  5  2
## [26]  4  5  6  6  8  6  5  9 12  8  9  7  1  1  1  8  9  2  3  1  0  6  8  6  3
## [51]  8  8 10 10 10 14  2  1  5
##  [1]  1  1  5  5  5  6  9  9  2  1  2  1  2  1  9 13  4  2  3  3  2  1  8  8  3
## [26]  3  8  9  9 15  1  3  2  0  7  8 10 12  8  9  7  9  5  5  7  7  7  6  8 13
## [51]  4  2  3  3  1  3  3  2  5
##  [1]  1  1  4  2  7  4  6  9  8  6 10 10  9  5  9  6  5  8  8  7  3  5  9 10  7
## [26] 12  3  4  5  7  2  0  1  1  5 10  8  9  8 15  5  1  8  6  8  7  1  1  3  3
## [51]  3  3  3  1  3  5  3  2  3
##  [1]  1  2  7  4  1  1  2  2  4  3  2  1  1  1  1  1  5  8  7  6  1  2  8 10  4
## [26]  5 10  7  1  0  5  9  6  3  2  3  7  6  4  4  6  7  7 10  7  1  2  2  5  4
## [51]  1  1  3  3 10 11 10 10  2
```

Remarque: On a chosi un nombre aléatoire entre 1 et 10 pour le nombre des individus d'immigration puisque si c'est un nombre très grand, la population va être explosée très vite. Et on observe que la population ne va jamais s'éteindre puisqu'il y a des individus d'immigration. Ainsi, 0 n'est plus un état absorbant.