Equations for
https://zstoebs.github.io/notes/lcp

Zach Stoebner

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1   LCPs

Linear program:

$$\min_x cx$$
$$\text{s.t. } Ax \geq b$$
$$x \geq 0$$

Complementarity:

$$x, y \geq 0$$
$$\text{s.t. } x_i y_i = 0 \text{ for all i}$$
$$\text{Shorthand: } x \geq 0 \perp y \geq 0$$

LCP:

$$\text{Find w, z where:}$$
$$w = Mz + q$$
$$w \geq 0 \perp z \geq 0$$

KKT:
General form a program:

$$\min_x f(x)$$
$$\text{s.t. } g(x) \geq 0$$
$$x \geq 0$$

$$\nabla_x f(x) - (\nabla_x g(x))^T \lambda_x = 0$$
$$g(x) \geq 0 \perp \lambda_x \geq 0$$

$$c - A^T \lambda_x = 0$$
$$Ax - b \geq 0 \perp \lambda_x \geq 0$$

$$
\begin{aligned}
L(x, \lambda_x) &= \begin{bmatrix} c - A^T \lambda_x \\ Ax - b \end{bmatrix} \\
&= \begin{bmatrix} 0 & -A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda_x \end{bmatrix} + \begin{bmatrix} c \\ b \end{bmatrix} \\
&= Mz + q \\
&= w
\end{aligned}
$$
$$w - Mz = q$$

Lemke:

w	z	
I	-M	q

$$Ax = b \rightarrow Bx_B + Dx_D = b$$