Exercise session 1

Exercise 1. Prove that for all positive natural numbers n,

$$1 + 3 + 5 + (2n - 1) = n^2.$$

Exercise 2. Prove that $3 + 11 + \cdots + (8n - 5) = 4n^2 - n$, for all positive integers n.

Exercise 3. Prove that $1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$, for all positive integers n.

Exercise 4. Prove the following statements for every positive natural number n:

a) $1^2 + 2^2 + \ldots n^2 = \frac{1}{6}n(n + 1)(2n + 1)$,

b) $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.

Exercise 5. Prove that for any $n \in \mathbb{N}$, the integer $11^n - 4^n$ is a natural number multiple of 7.

Exercise 6. Prove that $4^n + 15n - 1$ is divisible by 9 for all natural numbers n.

Exercise 7. Prove the associativity for addition in $\mathbb{Z}$ and $\mathbb{Q}$.

Exercise 8. Prove that the set of all even integers is a group for addition.

Exercise 9. Prove Bernoulli inequality: for every natural number n and for every number $a \geq -1$ it holds that:

$$(1 + a)^n \geq 1 + na.$$

Exercise 10. Prove that $n^{n+1} > (n + 1)^n$ for all natural numbers $n \geq 3$.