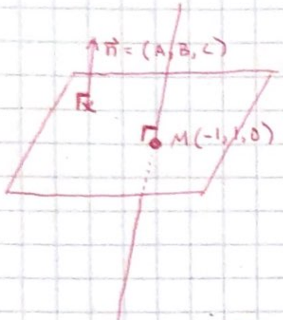
Lines & Planes

② Find the plane that is orthogonal to the line $\ell: \frac{x+1}{2} = \frac{y-1}{3} = z$ and contains the point where ℓ intersects the plane.



Pick some point from ℓ:

$M \in \ell \quad \boxed{M(-1,1,0)}$

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

$$A(x+1) + B(y-1) + Cz = 0$$

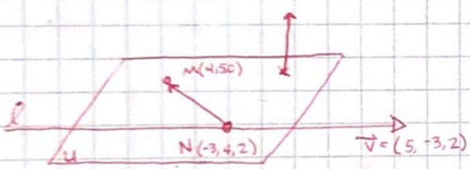
Need $\vec{n} = (A,B,C)$ ... use $\vec{v}$ (comes from ℓ)!

$$\boxed{\vec{v} = (2,3,1)}$$

Plane: $2(x+1) + 3(y-1) + z = 0$

$$\boxed{2x + 3y + z - 1 = 0}$$

① Find the plane which contains the point $M(4,5,0)$ and the line $\frac{x+3}{5} = \frac{y-4}{-3} = \frac{z-2}{2}$.



$\vec{v} = (5,-3,2)$, $\vec{NM} = (7,1,-2)$

$\vec{n} \perp \vec{v}$ & $\vec{n} \perp \vec{NM} \Rightarrow \vec{n} \parallel \vec{v} \times \vec{NM}$

$$\vec{v} \times \vec{NM} = \begin{vmatrix} i & j & k \\ 5 & -3 & 2 \\ 7 & 1 & -2 \end{vmatrix} = i\big((-3)(-2) - 2\big) - j\big(5(-2) - 2\cdot 7\big) + k\big(5 - (-3)7\big)$$

$$= (4, 24, 26)$$

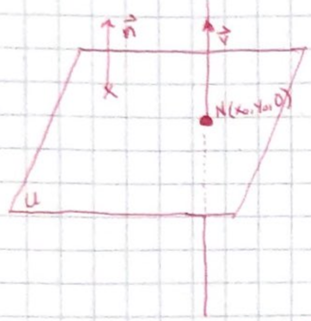
$u:\ A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

$4(x-4) + 24(y-5) + 26(z-0) = 0$

$4x - 16 + 24y - 120 + 26z = 0$

$$\boxed{4x + 24y + 26z - 136 = 0}$$

② Find the plane that is orthogonal to the line $\ell: \frac{x+1}{2} = \frac{y-1}{3} = z$ and contains the point where ℓ intersects the xy-plane.



$\vec{n} \parallel \vec{v} = (2,3,1)$:)

Need point that U contains.

Given: U contains the point where ℓ intersects xy-plane

$\Rightarrow$ U contains point N of line ℓ where $z=0$

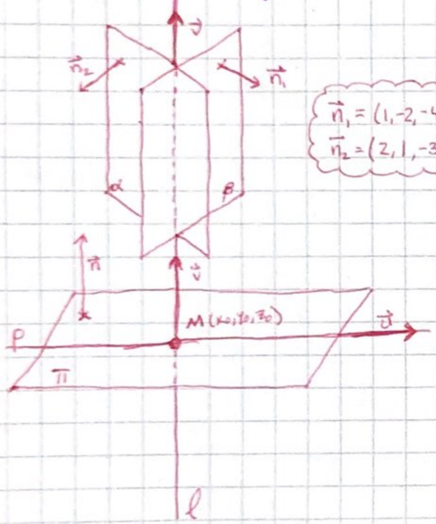
$\frac{x+1}{2} = \frac{y-1}{3} = 0 \Rightarrow x=-1,\ y=1$

$N(x_0,y_0,0) = \boxed{N(-1,1,0)}$

$U: 2(x+1) + 3(y-1) + 1(z-0) = 0$

$$\boxed{2x + 3y + z - 1 = 0}$$

③ Let $\Pi: x-4y+2z-7=0$ be a plane in space and let ℓ be the line given by the intersection of the planes $x-2y-4z=-3$ and $2x+y-3z+1=0$. Find the line p on the plane Π that is orthogonal to ℓ and contains the point given by the intersection between ℓ and Π.



Want: line p $\rightarrow$ Need: • vector $\vec{u}$, $\vec{u} \perp \vec{v}$ and $\vec{u} \parallel \Pi$

• point $M(x_0,y_0,z_0)$, $M \in \Pi$ and $M \in \ell$

$\vec{n}_1 = (1,-2,-4)$

$\vec{n}_2 = (2,1,-3)$

(1) To find vector $\vec{u}$, notice that $\vec{u} \perp \vec{v}$ and $\vec{u} \perp \vec{n}$

but $\vec{v} \parallel \vec{n}$ (can't use cross product), $\vec{n} = (1,-4,2)$ is given

⊛ $\vec{u} \cdot \vec{n} = \vec{0}$, $\vec{u} \cdot \vec{v} = \vec{0}$ ‼ Need to find $\vec{v}$ for system of equations

$\vec{v} \parallel \vec{n}_1 \times \vec{n}_2$, $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & -2 & -4 \\ 2 & 1 & -3 \end{vmatrix} = i\big((-2)(-3)+4\big) - j\big(-3+8\big) + k\big(1-(-2)(2)\big)$

$= \boxed{(10,-5,5) = \vec{v}}$

⊛ $u_1 - 4u_2 + 2u_3 = 0 \Rightarrow u_1 = 4u_2 - 2u_3$

$10u_1 - 5u_2 + 5u_3 = 0$

$35u_2 - 15u_3 = 0$

$35u_2 = 15u_3 \rightarrow$ Let $u_2 = 15,\ u_3 = 35$

$\boxed{\vec{u} = (-10, 15, 35)}$:)

$u_1 - 4u_2 + 2u_3 = u_1 - 60 + 70 = 0 \Rightarrow u_1 = -10$

(2) $\exists$ point $M(x_0,y_0,z_0) \in \alpha \cap \beta \in \Pi$ ‼

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 7 \\ 1 & -2 & -4 & -3 \\ 2 & 1 & -3 & -1 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 7 \\ 0 & 2 & -6 & -10 \\ 0 & 9 & -7 & -15 \end{array}\right] \xrightarrow{R_2/2} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 7 \\ 0 & 1 & -3 & -5 \\ 0 & 9 & -7 & -15 \end{array}\right] \xrightarrow[R_3-9R_2]{R_1+4R_2} \left[\begin{array}{ccc|c} 1 & 0 & -10 & -13 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 20 & 30 \end{array}\right]$$

$$\xrightarrow{R_3/20} \left[\begin{array}{ccc|c} 1 & 0 & -10 & -13 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 1 & 3/2 \end{array}\right]$$

$M(x_0,y_0,z_0) = \boxed{M(2, -\tfrac{1}{2}, \tfrac{3}{2})}$:)

$$\boxed{p: \frac{x-2}{-10} = \frac{y+\frac{1}{2}}{15} = \frac{z-\frac{3}{2}}{35}}$$

$\boxed{z = 3/2} \Rightarrow x - 10(3/2) = -13$

$x - 15 = -13 \Rightarrow \boxed{x=2}$

$y - 3(3/2) = -5$

$y - 9/2 = -\frac{10}{2} \Rightarrow \boxed{y = -1/2}$