Analysis I
UP - FAMNIT
Fall 2022 October 12/13, 2022

Selected exercises 02

1. Give the Dedekind cuts in $\mathbb{R}^{\geq 0}$ corresponding to the following. Your definition should not refer to the elements themselves.

   (a) $\sqrt{7}$ (c) $2+\sqrt[3]{5}$ (e) $\sqrt{2}+1$ (g) $3-\sqrt{2}$
   (b) $\sqrt[3]{11}$ (d) $\sqrt[4]{8}$ (f) $4-\sqrt{7}$ (h) $3-\sqrt{3}$

2. Prove that the following are not rational numbers

   (a) $\sqrt{7}$ (b) $\sqrt{3}$ (c) $\sqrt{2}+\sqrt{17}$ (d) $\sqrt{3}+\sqrt{13}$

3. Prove that the square root of a positive integer is either an integer or an irrational.

4. Decide if following sets are bounded from above/below.

   (a) $S=\{1,2,3\}$ (c) $S=\{0\}\cup\{x \mid x>0\}$ (e) $S=\{x^3 \mid x\in\mathbb{Z}\}$
   (b) $S=\{x \mid x\geq 5\}$ (d) $S=\{x^2 \mid x<-2\}$ (f) $S=\{x^2-x \mid x\geq 1\}$

5. Let $F=\{a-b\sqrt{5} \mid a,b\in\mathbb{Q}\}$. Prove that F is a field. (*Hint*: use that $\mathbb{R}$ is a field.)

6. Let $n\in\mathbb{N}$, $n>0$. Prove that $\sqrt{n+\sqrt{n}}$ is irrational.

7. Prove that the set of remainders modulo 5 is a field.