Scores	
1.	
2.	
3.	
4.	
5.	
Total:	

ANA-I Foundations of Analysis
1st Midterm Examination – 19 November 2021

Name ______________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may have 1 handwritten A4-sized sheet of paper, but no other notes, books, or calculators.
76 total points.

1. (6 points each) Limit calculations. For each real-valued sequence, explain whether it is convergent, divergent to $\pm\infty$, or otherwise divergent (not to $\pm\infty$). If it is convergent, find its limit. If it is divergent, find its $\limsup$ and $\liminf$. You may use any theorems we have proved in class or on homework.
   (a) $s_n = \dfrac{3n+1}{4n-1}$
   (b) $s_n = \dfrac{(-1)^n + n}{2n-6}$
   (c) $s_n = \dfrac{(-1)^n n^2 - n}{3n-2}$
   (d) $s_n = \dfrac{2\sin n - \cos n}{n+2}$

2. (6 points each) Examples. Justify your answers briefly.
   (a) Give an example of a nonconstant complex sequence that converges to $5+2i$.
   (b) Let $A|B$ be the Dedekind cut for $\sqrt[3]{30}$. Which of the following numbers are in A? $1, \sqrt{2}, 2, 5/2, 7/2, 30$?
   (c) Explain why, if a_n is an increasing sequence of positive real numbers, then $1/a_n$ is a decreasing sequence of positive real numbers.
   (d) Give an example of a real sequence with accumulation points at $-\infty$, 0, 17, and ∞.

3. (6 points) Either find the limit of $z_n = \dfrac{n^2}{n - 2n^2 i}$, or else explain why the sequence diverges. Write your answer in the standard form $z = a + bi$.

4. (12 points) Working directly from definition, prove that if z_n and w_n are sequences of complex numbers with
$$\lim_{n\to\infty} z_n = 4+3i, \qquad \text{and} \qquad \lim_{n\to\infty} w_n = 4-3i,$$
then $\lim_{n\to\infty} z_n \cdot w_n = 25$. (You may use the fact that convergent complex sequences are bounded.)

5. (10 points) Let a_n be recursively defined by $a_0 = 1/2$, $a_{n+1} = \dfrac{a_n^2+1}{2}$ for $n \geq 0$. Show that a_n converges, and find its limit.