Homework 2 Marko Taleski 89201031

A)

i) $x=\sqrt{10}$

$x = A|B = \{x \in \mathbb{R}^{\geq 0} : x^2 \leq 10\} \mid \{x \in \mathbb{R}^{\geq 0} : x^2 \geq 10\}$

ii) $x=\sqrt[3]{2}$

$X = A|B = \{x \in \mathbb{R}^{\geq 0} : x^3 < 2\} \mid \{x \in \mathbb{R}^{\geq 0} : x^3 \geq 2\}$

iii) $x = \sqrt[3]{2} + \sqrt{3}$

$x = A_x|B_x = \sqrt[3]{2}$ - we already have this in (ii)

$y = \sqrt{3}$

$y = A_y|B_y = \{y \in \mathbb{R}^{\geq 0} : y^2 < 3\} \mid \{y \in \mathbb{R}^{\geq 0} : y^2 \geq 3\}$

$\sqrt[3]{2} + \sqrt{3} = x+y = A|B = \{x+y \in \mathbb{R}^{\geq 0} : x^3 < 2 \in A_x, y^2 < 3 \in A_y : x, y \in \mathbb{R}^{\geq 0}\}$

$B := \mathbb{R}^{\geq 0} \setminus A$

B.

1) $A \cup B = \mathbb{Q}$

2) $a \in A$ $\quad a = x+y \quad b < a$

$b < x+y$

$x_1 < x$ so $x_1 \in A_r$

but $b = x_1 - y$ so $b \in A$

$b < x = y$ $\quad y_1 < y$ so $y_1 \in A_s$

$b = x \cdot y_1$ so $b \in A$

3) B is a non empty set as $z \in B_r$, $w \in B_s$, $z, w \in B$

A (so A has no ~~gratest~~ gratest element since A_r, A_s do not have a ~~great~~ greatest element

(if $x = y \in A$, $x^a \cdot y \in A$, for any $x^a > x$ in A)

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Homework 2

$F=\{0,1\}$ is a field with 2 operations $(+,\cdot)$ only if the following is true:

I) $\langle F\setminus\{0\},\cdot\rangle$ is a group with identity 1

- (i) - is closed under $\cdot$
- (ii) - $\cdot$ is associative
- (iii) - has identity 1 $(1\cdot 1 = 1\cdot 1 = 1)$
- (iv) - has inverse $(1-1 = 1-1 = 0)$

II) $\langle F,+\rangle$ is a group with identity 0

- (i) - is closed under $+$
- (ii) - $+$ is associative
- (iii) - has identity 0 $(0+0 = 0+0 = 0)$
- (iv) - has inverse $(0-0 = 0-0 = 0)$

III) $+,\cdot$ are comutitive $0+1 = 1+0$ $1\cdot 0 = 0\cdot 1$

IV) $+,\cdot$ are distributive $0\cdot(0+1) = (0\cdot 0)+(0\cdot 1)$

Index of comments

1.1 Marko Taleski
A: 1/5
C: 4/5
D: 0/5 (not submitted)

1.2 Dedekind cuts are described using the set |Q (rational numbers) in order to construct the reals (|R). Hence, we cannot use |R to describe a Dedekind cut.

1.3 1/5

2.1 proof?

2.2 What is the inverse of 1?

2.3 There are more cases you should check here.

2.4 4-/5