hw4 by Zhivko Stoimchev

p1

i) Using Arithmetic of Limits, find $\lim_{n\to\infty} 2n/(3n+5)$
ii) Working directly from the definition of limits, give a direct verification that your answer in (i) is correct (Your answer should involve the letter ε).

i) $\lim_{n\to\infty} \frac{2n}{3n+5} \Rightarrow \lim_{n\to\infty} \frac{2\cancel{n}}{\cancel{n}(3+\frac{5}{n})}$

Since, $2, 3, \frac{5}{n}$ are convergent, we can apply AoL.
and we get $\boxed{\frac{2}{3}}$ $\quad * \lim_{n\to\infty} \frac{2n}{3n+5} = \frac{2}{3} *$

ii) Discussion: For each $\varepsilon > 0$ we need to decide how big n must be to generate that

$$\left|\frac{2n}{3n+5} - \frac{2}{3}\right| < \varepsilon \quad \text{(aka find N)}$$

$$\Rightarrow \left|\frac{-10}{3(3n+5)}\right| < \varepsilon$$

Since $3(3n+5) > 0$, we can drop the absolute value and manipulate the expression.

$$\frac{10}{3(3n+5)} < \varepsilon \Rightarrow \frac{10}{3\varepsilon} < 3n+5$$

$$\Rightarrow \underbrace{\frac{10}{9\varepsilon} - \frac{5}{3}}_{\text{candidate for N}} < n$$

$\Rightarrow$ Since our steps are inversible, we can put $N = \frac{10}{9\varepsilon} - \frac{5}{3}$

Proof: Let $\varepsilon > 0$ and let $N = \frac{10}{9\varepsilon} - \frac{5}{3}$

Then we have $n > N$

$$\Rightarrow n > \frac{10}{9\varepsilon} - \frac{5}{3} \;/ * 3$$

$$\Rightarrow 3n > \frac{10}{3\varepsilon} - 5$$

hence $\left|\frac{2n}{3n+5} - \frac{2}{3}\right| < \varepsilon$

Notation:
i) is solved with help of lecture notes, and
ii) is solved with help of tutorial notes.

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B. Using the definitions of limit and/or infinite limit, show that $\lim_{n\to\infty} S_n = \infty$, then also $\lim_{n\to\infty} \sqrt[3]{S_n} = \infty$

From Lecture Notes

From the definition of infinite limits, for $\lim_{n\to\infty} S_n = \infty$, we have

$$\forall M > 0, \exists N \text{ s.t. } [n > N] \Rightarrow [S_n > M]$$ - diverges to ∞

From Tutorial Notes

For $\lim_{n\to\infty} \sqrt[3]{S_n}$ we must consider an arbitrary $M > 0$ and show that there is an N s.t.

$$n > N \Rightarrow \sqrt[3]{S_n} > M$$

- To see how big N must be, we solve for $\sqrt[3]{S_n} > M$ ~~and get~~ ~~S_n~~ and get $\boxed{S_n > M^3}$, thus $N = M^3$

- Let $M > 0$ and $N = M^3$

then

$$n > N \Rightarrow S_n > M^3$$

hence $\sqrt[3]{S_n} > M$ and also $S_n > M$, this shows that $\lim_{n\to\infty} \sqrt[3]{S_n} = \infty$ and also $\lim_{n\to\infty} S_n = \infty$.

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3.1

C. Find the following limits. You may use any theorem we have proved, such as Arithmetic of Limits. (Please do not use any theorem not discussed in class!) Indicate clearly what results you are using. The right answer without a justifiable reason will be given zero credit.

i) $\lim\limits_{n\to\infty} \frac{3n^2+1}{2n^2+2n-1} \Rightarrow \lim\limits_{n\to\infty} \frac{\cancel{n^2}(3+\frac{1}{n^2})}{\cancel{n^2}(2+\frac{2n}{n^2}-\frac{1}{n^2})} \Rightarrow \frac{3}{2}$

Since $3, \frac{1}{n^2}, 2, \frac{2n}{n^2}, -\frac{1}{n^2}$ are convergent, we get $\frac{3}{2}$ by using A.L.

ii) $\lim\limits_{n\to\infty} \frac{\sqrt{5n^2+2n}}{n+2} \Rightarrow \lim\limits_{n\to\infty} \frac{\sqrt{n^2(5+\frac{2}{n})}}{n(1+\frac{2}{n})} \Rightarrow \lim \frac{\cancel{n}\sqrt{5+\frac{2}{n}}}{\cancel{n}(1+\frac{2}{n})} \Rightarrow \frac{\sqrt{5}}{1}$

Since $1, \frac{2}{n}, \sqrt{5+\frac{2}{n}}$, are convergent, we get $\sqrt{5}$ by using A.L.

iii) $\lim \frac{-3n^3+4}{3n^3+2n^2-1n} \Rightarrow \lim \frac{\cancel{n^3}(-3+\frac{4}{n^3})}{\cancel{n^3}(3+\frac{2}{n}-\frac{1}{n^2})}$

Since $\pm 3, \frac{4}{n^3}, 3, \frac{2}{n}, -\frac{1}{n^2}$ are convergent, by A.L $\frac{-3}{3}$ and we get -1

Notation: solved from the examples in Lecture Notes

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D. —//— (same text as in C)

i) $\lim_{n\to\infty} \frac{n^2-4}{2n^3-3n+1} \Rightarrow \lim \Rightarrow \lim_{n\to\infty} \frac{\cancel{n^2}\left(1-\frac{4}{n^2}\right)}{n^{\cancel{3}}\left(2-\frac{3}{n^2}+\frac{1}{n^3}\right)} \Rightarrow \lim_{n\to\infty} \frac{1}{n} \cdot \frac{1-\cancel{\frac{4}{n^2}}}{2-\cancel{\frac{3}{n^2}}+\cancel{\frac{1}{n^3}}}$ in Limits

Since, $1, -\frac{4}{n^2}, 2, -\frac{3}{n^2}, \frac{1}{n^2}$ are convergent, by AoL and AoIL we know that $0 = \frac{1}{\infty}$, and $n=\infty$. So, we have

$\frac{1}{\infty} \cdot \frac{1}{2} = 0$

~~$\lim_{n\to\infty} \frac{\sqrt{3n^2+2n}}{n+2}$~~ ii) $\lim \frac{\sqrt{2n^3-1n}}{2n-3} \Rightarrow \lim_{n\to\infty} \frac{n}{n} \cdot \frac{\sqrt{2n-\frac{1}{n}}}{2-\frac{3}{n}}$

Since $2n, -\frac{1}{n}, 2, -\frac{3}{n}$ are convergent, by applying AoL and AoIL that $n=\infty$.

So we have $\frac{\sqrt{\infty}}{2} \Rightarrow \frac{\infty}{2} \Rightarrow \infty$

iii) $\lim_{n\to\infty} \frac{6n^2-2n^4}{12n^3+2n^2-17n} \Rightarrow \lim_{n\to\infty} \frac{n^4\left(\frac{6}{n^2}-2\right)}{n^3\left(12+\frac{2}{n}-\frac{17}{n^2}\right)} \Rightarrow \lim \frac{n\left(\cancel{\frac{6}{n^2}}-2\right)}{12+\cancel{\frac{2}{n}}-\cancel{\frac{17}{n^2}}}$

Since $\frac{6}{n^2}, -2, 12, \frac{2}{n}, -\frac{17}{n^2}$ are convergent, by applying AoL and AoIL $(n=\infty)$

we have $\frac{n(-2)}{12} \Rightarrow \frac{\infty(-2)}{12} \Rightarrow -\infty$

Notation: solved by the examples in Lecture Notes

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5.1

E.

i) Suppose that the S_n satisfies both $\lim_{n\to\infty} S_{2n} = 3$ and $\lim_{n\to\infty} S_{2n+1} = 3$. (That is, the sequence given by the even terms of S_n an that given by the odd terms of S_n both converge to 3.) Show that also $\lim_{n\to\infty} S_n = 3$

ii) Give an example of a sequence where the sequences given by the even and by the odd terms both converge, but where the entire sequence does not converge.

i We have $\lim_{n\to\infty} S_{2n} = 3$ and $\lim_{n\to\infty} S_{2n+1} = 3$. This means they 5.2

5.3 are bounded by 3 and we get:

$\Rightarrow m_1 \le S_{2n} \le M_1$ and $m_2 \le S_{2n+1} \le M_2$

Where M is $\max M_1, M_2$ and m is $\min m_1, m_2$.

This means that: $\Rightarrow m \le S_n \le M, \ \forall n \in \mathbb{N}$.

S_n must have limit points. 5.4

Let L be the set of limit points of S_n,

thus $L = 3$. 5.5 ($L = L_1 \cup L_2$, where L_1 is the set of limit points of the sequence of even terms, and L_2 is the set of limit points of the sequence of odd terms)

We know that bounded sequence with limit point is convergent, and her 5.6 the limit is 3, Therefore:

$$\boxed{\lim_{n\to\infty} S_n = 3}$$

ii Consider $S_n = \boxed{-1n}$, such that:

$S_{2n} = 1$ and $S_{2n+1} = -1$

Being constant sequence, both S_{2n} and S_{2n+1} are convergent, but S_n is not because it has two limits, 1 and -1, and for convergence the limit must be unique.

Index of comments

1.1 A: 5/5

1.2 Keep in mind that N should be an integer since it represents an index of a sequence.

3.1 C: 5/5

5.1 E: 3.5/5

5.2 You need to be a bit more rigorous in the proof, and explain what results you are using.

5.3 Convergent sequences must be bounded, but not by the limit!

5.4 Why? Explain what results you are using here.

5.5 L = {3}

5.6 Bounded sequence that has an accumulation point is not always convergent, with UNIQUE limit point it is.

6.1 the sequence -n diverges to minus infinity. The sequence you are referring to below is (-1)^n

6.2 It has no limit, but it has two accumulation points