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A) i) $\lim_{n\to\infty} \frac{1}{n^5} = 0$

$\forall \varepsilon > 0 \ \exists N \text{ s.t. } n > N \Rightarrow |s_n - s| < \varepsilon$

$n^5 > N^5$

Then we take $\varepsilon = \frac{1}{N^5}$ $\quad 0 < \frac{1}{n^5} < \frac{1}{N^5} \Rightarrow \left|\frac{1}{n^5} - 0\right| = \frac{1}{n^5} < \varepsilon$

Since $\frac{1}{N^5}$ is positive

ii) $\lim_{n\to\infty} \frac{1}{\sqrt[5]{n}} = 0$

$\forall \varepsilon > 0 \ \exists N \text{ s.t. } n > N \Rightarrow |s_n - s| < \varepsilon$

$\left|\frac{1}{\sqrt[5]{n}} - 0\right| = \frac{1}{\sqrt[3]{n}} < \varepsilon \quad 1 < \varepsilon \sqrt[5]{n}$

$\sqrt[3]{n} > \frac{1}{\varepsilon}$

then we take $N = \frac{1}{\varepsilon^5}$ s.t. $n > N$ $\quad n > \frac{1}{\varepsilon^5} \Rightarrow \frac{1}{\sqrt[5]{n}} < \varepsilon = \frac{1}{\sqrt[5]{n}}$

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B i) $\lim_{n\to\infty} (4+n)/2n = 1/2$

$\forall \varepsilon > 0, \left| \frac{4+n}{2n} - \frac{1}{2} \right| < \varepsilon$

$\left| \frac{4+n-n}{2n} \right| < \varepsilon \quad \left| \frac{4}{2n} \right| < \varepsilon \quad \frac{2}{n} < \varepsilon$

$n > \frac{2}{\varepsilon}$

for $N = \frac{2}{\varepsilon}$ if $n > N$ we have that

$\Downarrow$

$|s_n - s| = \left| \frac{2}{n} \right| < \varepsilon$

ii) $\lim_{n\to\infty} 2/n + 3/(n+1) = 0$

$\forall \varepsilon > 0 \quad |s_n - s| = \left| \frac{\frac{2}{n} + 3}{n+1} - 0 \right|$

$=$

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1c] (s_n) (t_n) $s_n = t_n$ ~~excep~~ except some values of n

Let s_n and t_n be the last different elements.

then we have $\forall \varepsilon > 0$, $n > N_{s_m} \Rightarrow |s_m - s| < \varepsilon$

and $n > N_{t_m} \Rightarrow |t_m - s| < \varepsilon$

we take $N = \max(N_{s_m}, N_{t_m})$ s.t.

$$n > N \begin{cases} \Rightarrow |s_m - s| < \varepsilon \\ \Rightarrow |t_m - s| < \varepsilon \end{cases}$$

So now we have $N_0 = N + 1$ for which $\begin{cases} n > N_0 \Rightarrow |s_m - s| < \varepsilon \\ n > N_0 \Rightarrow |t_n - s| < \varepsilon \end{cases}$

But now we know that $s_n = t_n$ so:

$n > N_0 \Rightarrow |t_n - s| < \varepsilon$

for $N_0 = N + 1$

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1) i) $(s_n), (t_n)$ bounded sequences s.t. s_n and t_n are bounded

$$|s_n + t_n| \le |s_n| + |t_n|$$

$$-(|s_n| + |t_n|) \le |s_n + t_n| \le |s_n| + |t_n|$$

so, every element of $(s_n + t_n)$, even the biggest one, will have a bigger number in $|s_n| + |t_n|$ (which are numbers from a bounded set)

And that also goes for it's least element that always has a smaller or equal $-(|s_n| + |t_n|)$.

ii) S_n has a range which is a bounded set containing all the possible values we can get from S_n.

$\alpha = \mathbb{R}$ let's say that the set is bounded by x and y.

$$S = (x, y) \quad x \le S_n \le y$$

$$T = \{\alpha s_n : s_n \in S\}$$

$$x \le S_n \quad s_n \le y$$

$\alpha x \le \alpha S_n \quad \alpha S_n \le \alpha y$ meaning that αx is a lower bound and αy is an upper bound of T which now is a bounded set containing all elements of $(\alpha \cdot S_n)$

Thus $(\alpha \cdot S_n)$ is bounded.

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[E] i) $\lim_{n\to\infty} s_n = s$

$a \in \mathbb{R}$

prove $s \geq a$

assume $s < a$

$\varepsilon > 0$

s_m is the biggest element that $s_m < a$

$n > m. \Rightarrow s_n > s_m$

$|s_n - s| < \varepsilon \quad s - \varepsilon < s < s + \varepsilon$

$s_n > s - \varepsilon$

$s_n < s + \varepsilon$

$s_m < a$

(*) $s_n \geq a$

Since $\varepsilon > 0$ and $a \geq s$ we can take ε to be as small as $a - s$

than we have that $s_n < s + a - s = a$

but (*) states otherwise and that makes s_m not the biggest element less than a //.

ii) $a \in \mathbb{R}$ (s_n)

example: Let (s_n) be the sequence $\frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$

$\forall \varepsilon > 0 \quad n \geq N \Rightarrow \left|\frac{1}{n} - 0\right| < \varepsilon \qquad \frac{1}{n} < \varepsilon \qquad 1 < \varepsilon n$

$n > \frac{1}{\varepsilon}$

$N = \frac{1}{\varepsilon}$ will suffice,

so we know the $\lim_{n\to\infty} \frac{1}{n} = 0$

And also since $n \in \mathbb{N}^+$ we have that $\frac{1}{n}$ will always be positive so $s_n > 0$ //.