



Algebra I

2. KOLOKVIJ

– 12. JANUAR 2018 –

Čas pisanja: 90 minut. Maksimalno število točk: 50. Dovoljena je uporaba pisala in enega ročno napisanega lista formata A4 z definicijami in formulami (brez rešenih primerov). Pišite razločno in utemeljite vsak odgovor. Srečno!

1. Dani sta matriki $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & -1 & 2 \\ -1 & 3 & 0 \end{bmatrix}$ in $B = \begin{bmatrix} 2 & -3 & 0 \\ -1 & 2 & 3 \end{bmatrix}$.

Rešite matrično enačbo $A^2X - B^T = 3X$. (12 točk)

2. Za katere vrednosti $c \in \mathbb{R}$ je naslednji sistem rešljiv?

$$\begin{aligned} x + 2y + 3z + 5w &= 1 \\ 2x + 2y - z + w &= 2 \\ x + y + w &= 3 \\ y + 2z + 3w &= c \end{aligned}$$

Poiščite rešitev. (13 točk)

3. Pokažite, da je determinanta $n \times n$ matrike oblike

$$A_n = \begin{bmatrix} 3 & 2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 3 & 2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 3 \end{bmatrix}$$

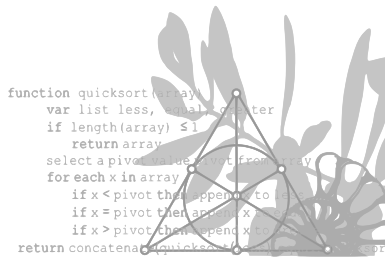
enaka $2^{n+1} - 1$. (12 točk)

(Namig: Izračunajte determinante za $n \in \{1, 2, 3, 4, \dots\}$).

4. Določite rang matrike

$$A = \begin{bmatrix} 1 & a+1 & -1 \\ 2 & a+3 & a^2-3 \\ 1 & 2 & -1 \end{bmatrix}$$

v odvisnosti od števila $a \in \mathbb{R}$. (13 točk)



Algebra I
MIDTERM 2
– JANUARY 12, 2018 –

Time: 90 minutes. Maximum number of points: 50. You are allowed to use a pen and one A4 hand-written piece of paper with definitions and formulas (and with no solved exercises). Write clearly, and justify all your answers. Good luck!

1. We are given the matrices $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & -1 & 2 \\ -1 & 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 & 0 \\ -1 & 2 & 3 \end{bmatrix}$.

Solve the matrix equation $A^2X - B^T = 3X$. (12 points)

2. For which values of $c \in \mathbb{R}$ is the following system solvable?

$$\begin{aligned} x + 2y + 3z + 5w &= 1 \\ 2x + 2y - z + w &= 2 \\ x + y + w &= 3 \\ y + 2z + 3w &= c \end{aligned}$$

Find its solution. (13 points)

3. Show that the determinant of the $n \times n$ matrix of the form

$$A_n = \begin{bmatrix} 3 & 2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 3 & 2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 3 \end{bmatrix}$$

equals $2^{n+1} - 1$. (12 points)

(Hint: Compute the determinant for $n \in \{1, 2, 3, 4, \dots\}$).

4. Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & a+1 & -1 \\ 2 & a+3 & a^2-3 \\ 1 & 2 & -1 \end{bmatrix}$$

depending on the value of $a \in \mathbb{R}$. (13 points)