(B) $P_n = n^3 + n + 1 = 2k$ where $k \in \mathbb{N}$

i) Show that for every $n \in \mathbb{N}$, we have that $P_n \Rightarrow P_{n+1}$

$$(n+1)^3 + n + 1 + 1 = n^3 + 3n^2 + 3n + 1 + n + 2 =$$

$$= n^3 + 3n^2 + 4n + 3 = (n^3 + n + 1) + 3n^2 + 3n + 2 =$$

$$= (n^3 + n + 1) + \underline{3n(n+1) + 2}$$

$3n(n+1)$ will always be even, no matter of what number n is, because if its uneven when we add plus 1 to it $(n+1)$ it will become even, and if n is even when we multiply $3n$ with $(n+1)$ the result will always be even

ii) Prove that P_n is always false

Well we prove P_n to be true or false, by first proving P_0. Because in order for P_n to be true, P_0 must be always true

$n = 0$ $\quad 0^3 + 0 + 1 = 2k$

$0 + 0 + 1 = 2k$

$1 \neq 2k$

With P_0 being false, P_n will always be false

(C) In order to verify the distributive property for $\mathbb{Z}$, we start with distributivity formula $n \cdot (m + l) = nm + nl$. In integers on $\mathbb{Z}$, we mark (n, m) as $(n - m)$. So

$n: (x_1, y_1)$ or $(x_1 - y_1)$

$m: (x_2, y_2)$ or $(x_2 - y_2)$

$l: (x_3, y_3)$ or $(x_3 - y_3)$