Scores	
1.	
2.	
3.	
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6.	
7.	
Total:	

ANA-I Foundations of Analysis
Final Examination A – 26 Jan, 2021

Name ____________________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may have 1 handwritten A4-sized sheet of paper, but no other notes, books, or calculators.
110 total points.

1. (6 points each) Calculate the following limits, or explain why they diverge. You may use any theorems we have proved in class or on homework.

   (a) $\lim_{n\to\infty} \dfrac{3n^2+n-1}{-2n^2+n-4}$

   (b) $\lim_{x\to 3-} \dfrac{x^2-9}{x+3}$

   (c) $\lim_{x\to -3} \dfrac{x^2-9}{x+3}$

   (d) $\lim_{x\to\infty} 2x+\sin 3x$

   (e) $\lim_{n\to\infty} \dfrac{3n+6ni}{|2+3i|\,n}$

2. (6 points each) Series

   (a) Find the exact value that $\sum_{n=1}^{\infty} \dfrac{3^n+1}{4^n}$ converges to, or else conclude the series diverges.

   (b) Determine whether $\sum_{n=1}^{\infty} \dfrac{(-1)^n n^{1/2}+3}{2n^2-1}$ converges absolutely, converges conditionally, or diverges.

3. (6 points each) Examples. Justify your answers briefly.

   (a) Give the Dedekind cut for $\sqrt[3]{5}$. Your answer should not refer directly to any irrational numbers.

   (b) Give an example of a sequence with exactly 3 accumulation points.

   (c) A function $\mathbb{R} \to \mathbb{R}$ that is strictly decreasing and continuous.

   (d) A sequentially compact subset of $\mathbb{R}^2$ that contains the point $(2,2)$.

4. (10 points) Let a_n be recursively defined by $a_0 = 15$, $a_{n+1} = \dfrac{1}{2}(a_n + \dfrac{9}{a_n})$ for $n \geq 0$. Show that the sequence converges, and find $\lim_{n\to\infty} a_n$.

5. (11 points) Using the method of bisection, estimate $\sqrt{10}$ to within 0.1.

6. (12 points) Show directly from definition that if a_n is a positive real sequence with $\lim_{n\to\infty} a_n = 0$ and $\lim_{x\to 0+} f(x) = L$,then $\lim_{n\to\infty} f(a_n) = L$.

7. (11 points) Show that if $S \times T$ is a closed subset of $\mathbb{R}^2$, then S and T are closed subsets of $\mathbb{R}$.