Exercise session 3

Exercise 21. Find the minimum, maximum, infimum, supremum for the following sets (if possible):

a) $A = \{-2, -3\} \cup [0, 2]$,

b) $A = [5, 9)$,

c) $A = [5, 9] \cup [12, 13]$,

d) $A = [0, 5] \cap (3, 6)$,

e) $A = \{x \in \mathbb{R} \mid x \geq \sqrt{3}\}$,

f) $A = \{x \in \mathbb{Q} \mid x^2 \leq 2\}$,

g) $A = \{x \in \mathbb{N} \mid x > \sqrt{5}\}$,

h) $A = \{1 + \frac{1}{n} \mid n \in \mathbb{N}^+\}$,

i) $A = \{\frac{1}{n^2} \mid n \in \mathbb{N}, n \geq 5\}$,

Exercise 22. Compute sup and inf (if it exists) in S and prove it by definition.

a) $S = [0, 2), S \subseteq \mathbb{R}$

b) $S = (-1, \sqrt{2}), S \subseteq \mathbb{Q}$

Exercise 23. Find sets S and T with the following properties:

a) $\sup S = \inf T$, $S \cap T = \emptyset$,

b) $\inf S = \sup S$, $S \subseteq T$,

c) $\inf T = \min S$, T has no minimal element.

Exercise 24. Let A and B be non-empty bounded subsets of $\mathbb{R}$, and let $A + B$ be the set of all sums $a + b$ where $a \in A$ and $b \in B$. Prove the following equalities, or give a counterexample:

a) $\sup(A + B) = \sup A + \sup B$,

b) $\sup(A - B) = \sup A - \sup B$,

c) $\inf(A + B) = \inf A + \inf B$,

d) $\inf(A - B) = \inf A - \inf B$.

Exercise 25. Given $a = 2 + 3i$ and $b = -1 + i$, compute $|a - 5b|$.

Exercise 26. Prove directly from the definition of limit:

a) $\lim_{n\to\infty} \frac{1}{3n^4} = 0$

b) $\lim_{n\to\infty} \frac{1}{\sqrt{n^2 + 1}} = 0$

c) $\lim_{n\to\infty} \frac{3n + 1}{2n - 1} = \frac{3}{2}$,

d) $\lim_{n\to\infty} \frac{1}{n} + \frac{\sin n}{n + 1} = 0$