*Univerza na Primorskem*
*UP FAMNIT*
*Študijsko leto 2021/2022*

Algebra I

**2. KOLOKVIJ**

– 18. Januar 2022 –

*Čas pisanja:* 135 *minut. Maksimalno število točk:* 100. *Dovoljena je uporaba pisala in kalkulatorja. Pišite razločno in utemeljite vsak odgovor. Srečno!*

1. (a) Kdaj je matrka simetrična? Zapišite definicijo transponirane matrike in naštejte vsaj tri lastnosti transponiranja. Dokažite naslednjo izjavo: Za vsako kvadratno matriko $A$ je $A + A^T$ simetrična matrika. *(6 točk)*

   (b) Zapišite definicijo $n \times n$ determinante (z uporabo permutacij) in naštejte vsaj štiri primere uporabe determinant. *(6 točk)*

   (c) Zapišite in dokažite Cramerjevo pravilo za reševanje sistema linearnih enačb. *(8 točk)*

2. Za katere vrednosti $\beta \in \mathbb{R}$ bo imel naslednji sistem linearnih enačb neskončno mnogo rešitev? Za vse dobljene vrednosti tudi poiščite rešitve.

$$\begin{aligned} x + y + z &= 1 \\ 2x + y + 4z &= \beta \\ 4x + y + 10z &= \beta^2 \end{aligned}$$

*(20 točk)*

3. Za katere vrednosti $a \in \mathbb{R}$ bo determinanta matrike

$$A = \begin{bmatrix} 1 & a & 3 & 2 \\ 2 & 2 & -2 & 1 \\ 3 & 3 & -5 & 1 \\ 4 & 4 & -7 & 5 \end{bmatrix}$$

enaka 30? *(20 točk)*

4. (a) Za matrike $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ in $C = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$, rešite matrično enačbo $(A + 3I)(X - I) = B$. *(10 točk)*

   (b) Naj bo $E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in \mathbb{R}^{2\times 2}$. Pokažite da za matriko $A \in \mathbb{R}^{2\times 2}$ velja

$$det(A) = 1 \;\Leftrightarrow\; A^T E A = E.$$

*(10 točk)*

5. Zapišite matriko $A = \begin{bmatrix} 2 & -2 & 1 \\ 6 & -1 & 5 \\ 3 & 7 & 4 \end{bmatrix}$ v obliki $A = LU$, kje je $L$ spodnje trikotna matrika ki ima na glavni diagonali vse elemente enake 1, in $U$ zgornje trikotna matrika. *(20 točk)*

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# Algebra I
## MIDTERM 2
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*Time: 135 minutes. Maximum number of points: 100. You are allowed to use a pen and a calculator. Write clearly, and justify all your answers. Good luck!*

1. (a) When is a matrix symmetric? Give the definition of a transpose of a matrix and write at least three properties of transposing. Prove the next statement: For every square matrix $A$ the matrix $A + A^T$ is symmetric as well. *(6 points)*
   (b) State the definition of a $n \times n$ determinant (using permutations) and give at least four examples of use of the determinant. *(6 points)*
   (c) Write down and prove Cramer's rule for solving systems of linear equations. *(8 points)*

2. For which values of $\beta \in \mathbb{R}$ will the next system have infinitely many solutions? For each value that you got compute also the solutions.
$$\begin{aligned} x + y + z &= 1 \\ 2x + y + 4z &= \beta \\ 4x + y + 10z &= \beta^2 \end{aligned}$$
*(20 points)*

3. For which values $a \in \mathbb{R}$ will the determinant of the matrix
$$A = \begin{bmatrix} 1 & a & 3 & 2 \\ 2 & 2 & -2 & 1 \\ 3 & 3 & -5 & 1 \\ 4 & 4 & -7 & 5 \end{bmatrix}$$
equal 30? *(20 points)*

4. (a) For matrices $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$, solve the matrix equation $(A + 3I)(X - I) = B$. *(10 points)*
   (b) Let $E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in \mathbb{R}^{2\times 2}$. Show that for matrix $A \in \mathbb{R}^{2\times 2}$ it holds that
$$det(A) = 1 \iff A^T E A = E.$$
*(10 points)*

5. Write the matrix $A = \begin{bmatrix} 2 & -2 & 1 \\ 6 & -1 & 5 \\ 3 & 7 & 4 \end{bmatrix}$ in the form $A = LU$, where $L$ is a lower triangular matrix with all coefficient on the main diagonal equal to 1, and $U$ is an upper triangular matrix. *(20 points)*