

## Univerza na Primorskem UP FAMNIT Študijsko leto 2021/2022

## Algebra I 1. KOLOKVIJ – 23. NOVEMBER 2021 –

Čas pisanja: 135 minut. Maksimalno število točk: 100. Dovoljena je uporaba pisala in kalkulatorja. Pišite razločno in utemeljite vsak odgovor. Srečno!

- (a) Zapišite definicijo skalarnega produkta in naštejte vsaj 3 njegove lastnosti. Zapišite in dokažite karakterizacijo pravokotnosti med dvema vektorjema s pomočjo skalarnega produkta.
   (6 točk)
  - (b) Zapišite definicijo mešanega produkta in njegov geometrijski pomen. (6 točk)
  - (c) Dokažite naslednjo trditev: Razdalja d med nevzporednima premicama  $\ell_1=\vec{r}_1+\lambda_1\vec{v}_1$  in  $\ell_2=\vec{r}_2+\lambda_2\vec{v}_2$  ( $\lambda_1,\lambda_2\in\mathbb{R}$ ) se izračuna kot

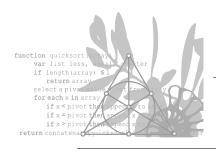
$$d = \frac{|\langle \vec{v}_1 \times \vec{v}_2, \vec{r}_2 - \vec{r}_1 \rangle|}{|\vec{v}_1 \times \vec{v}_2|}.$$

(8 točk)

- 2. Točke A(3,2,t), B(3,-3,1) in C(5,t,2) predstavljajo oglišča paralelograma ABCD.
  - (a) Za t = 1 izračunajte koordinate točke D. (6 točk)
  - (b) Za katere vrednosti  $t \in \mathbb{R}$  bo  $|\overrightarrow{AC}| = \sqrt{22}$ ? (7 točk)
  - (c) Za katere vrednosti  $t \in \mathbb{R}$  bo točka A oddaljena od izhodišča za 7 enot? (7 točk)
- 3. Dane imamo vektorje  $\vec{a}=(8-t,3,-1-t)$ ,  $\vec{b}=(7,1,0)$  in  $\vec{c}=(7,7,0)$ . Poiščite vse vrednosti  $t\in\mathbb{R}$ , za katere bo  $\angle(\vec{a},\vec{b})=\angle(\vec{a},\vec{c})=\varphi$  in določite ta kot.

Namig: Kot med dvema vektorjema dobimo s pomočjo enačbe  $\cos \varphi = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| \cdot |\vec{v}_2|}$ . (20 točk)

- 4. Zapišite enačbo premice  $\ell$ , ki vsebuje točko A(1,2,1), je vzporedna z ravnino x-y+z=4 in seka premico  $p=(0,0,2)+\lambda(1,2,0)$ . Izračunajte tudi presečišče med premicama  $\ell$  in p. (20 točk)
- 5. Premica p je podana s presečiščem ravni 3x-2y+z+3=0 in 4x-3y+4z+1=0. Poiščite vrednosti  $\alpha,\beta\in\mathbb{R}$  za katere je premica p pravokotna na ravnino  $\alpha x+8x+\beta z=-2$ . (20 točk)



## University of Primorska UP FAMNIT Academic year 2021/2022

## Algebra I MIDTERM 1 - NOVEMBER 23, 2021 -

Time: 135 minutes. Maximum number of points: 100. You are allowed to use a pen and a calculator. Write clearly, and justify all your answers. Good luck!

- (a) Give the definition of the scalar (dot) product and state at least 3 of its properties.
   Write and prove the characterisation of orthogonality between two vectors in terms of the scalar product.
   (6 points)
  - (b) Give the definition of the box product and state its geometrical meaning. (6 points)
  - (c) Prove the next claim: The distance d between two non-parallel lines  $\ell_1 = \vec{r}_1 + \lambda_1 \vec{v}_1$  and  $\ell_2 = \vec{r}_2 + \lambda_2 \vec{v}_2$  ( $\lambda_1, \lambda_2 \in \mathbb{R}$ ) is computed as

$$d = \frac{|\langle \vec{v}_1 \times \vec{v}_2, \vec{r}_2 - \vec{r}_1 \rangle|}{|\vec{v}_1 \times \vec{v}_2|}.$$

(8 points)

- 2. Points A(3,2,t), B(3,-3,1) and C(5,t,2) are corners of the parallelogram ABCD.
  - (a) For t = 1 find the coordinates of D.

(6 points)

(b) For which values  $t \in \mathbb{R}$  will  $|\overrightarrow{AC}| = \sqrt{22}$ ?

(7 points)

(c) For which values  $t \in \mathbb{R}$  will A be at distance 7 from the origin?

(7 points)

3. We are given vectors  $\vec{a}=(8-t,3,-1-t)$ ,  $\vec{b}=(7,1,0)$  and  $\vec{c}=(7,7,0)$ . Find all values  $t\in\mathbb{R}$ , for which  $\angle(\vec{a},\vec{b})=\angle(\vec{a},\vec{c})=\varphi$  and compute that angle.

Hint: The angle between two vectors can be obtained from the equation  $\cos \varphi = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| \cdot |\vec{v}_2|}$ . (20 points)

- 4. Find the line  $\ell$  that contains point A(1,2,1), is parallel to the plane x-y+z=4 and intersects the line  $p=(0,0,2)+\lambda(1,2,0)$ . Compute also the intersection between lines  $\ell$  and p.
- 5. The line p is given by the intersection of planes 3x 2y + z + 3 = 0 and 4x 3y + 4z + 1 = 0. Find the values  $\alpha, \beta \in \mathbb{R}$  for which the line p is orthogonal to the plane  $\alpha x + 8x + \beta z = -2$ . (20 points)