



A. Show that any seq. of positive real numbers either has a subsequence that converges, or else a subseq. that diverges to N.

- Let an be a seq. of positive real numbers. Need to prove: al an has a sabseq. that converge 6) by has a subseq. that diverge to A.

Proof of al Suppose that an is bounded. Then there exists Teal number Mzo s.t. an = M for all nEN. In that case, the seq- on is bounded of positive real numbers. B With this, by Bolzano-Weierstrass theorem, an has a convergent subseq. ank. This concludes that the subsequilland converges. Cin this casel

BW Thm: every bounded seq. has a convergent subsequence - (From Tutorials)

Proof of 6) Suppose that an is unbounded. Then] subseq. and of an s.t. ank-zw as K-zw. This concludes that the subseq. (and) diverges to w. (in this case).



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B. for each of the following series, determine whether the series converges or diverges. (Sam from 1 to 20) for those that converge, find the value that they

i)
$$\sum (4^n + 2^n)/3^n = \frac{4^n + 2^n}{3^n}$$
By applying ratio test, we have

By applying ratio test, we have
$$\frac{4^{n+1}}{2n} = \frac{4^{n+1}}{2^n} + \frac{2^{n+1}}{2^n} = \frac{1}{1} + \frac{1}{1} = 2 > 1 = 7 \text{ diverges}$$

The seq. $sin^{n}(3)$ $Z r^{n} = \frac{1}{1+c}$ if |c| < 1 and sin < 3 > 0, 14

Since $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$ also converges, by direct comparison test we have that $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$ also converges, to 1.

iv) $\sum (\sqrt{n+2} + \sqrt{n})$ = $\frac{1}{\sqrt{n+2} + \sqrt{n}} = \frac{1}{\sqrt{n+2} + \sqrt{n}} = \frac{2}{\sqrt{n+2} + \sqrt{n$







C. for each of the following series, determine whether the series converges, diverges to ±00, or diverges, not to 100. Don't try to find what the series converge to

i) $\sum sin^2(3n+2)/(2n^2-3)$, $a_n = sin^2(3n+2)/(2n^2-3)$

we define $6n = \frac{1}{n^2}$, and it converges, so we have

 $\lim_{n\to\infty}\frac{q_n}{q_n}=\lim_{n\to\infty}\frac{3\ln^2\frac{3n+2}{2n^2-3}}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n^2+12n+4}{2n^2+3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n^2+12n+4}{2n^2+3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2+3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2+3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2+3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2+3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}=\lim_{n\to\infty}\frac{n^2\left(\frac{3n+2}{2n^2-3}\right)^2}{\frac{1}{n^2}\left(\frac{3n+2}{2n^2-3}\right)^2}$

that means that Zan and Zon have the same nature, and Zan is convergent.

ii) \(\(1/(n-2\)\), \(a_n = 1/(n-2\)\)\) \(\) \(\) \(\)_n = \(\)_n \(\) and \(\) \(\)_n is divergent &n = 1

=7 Lim an = Cim n-211 = 00 =1 E IR =7 this means that

Zan and Zen have some

and this shows that

=7 Za, is divergent

iii) \(\(\n + 1 \) / (n > + 2) \\ \quad \alpha_n = n + 1 / (n > + 2) \\ \alpha \tau \) \\ \delta \quad \(\lambda_n = n \) \| \lambda_n = n \] \| \lambda_n = n \) \| \lambda_n = n \] \

= Z6n is convergent (because 2>2>1), and we have

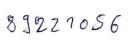
=7 $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^3+n^2}{n^3+n^2} = 1 \in \mathbb{R}^+$ =7 this means

that Zan and Elen have same

nature, and this shows that

=7 Zan converges







O. Determine whether each of the following series are a scolately convergent, conditionally envergent, ar livergent.

$$\begin{aligned} |\tilde{a}| & \geq \frac{(-3)^n}{(3n^2+6^n)} = \frac{3^n}{3n^2+6^n} = \frac{3^n}{3n^2+$$

? New we use route test (from Tutorials), and get = 2 Since by is convergent, |a, | absolutely converges by comparison test.

Find the since on is convergent, that
$$\frac{1}{\sqrt{2n+1}} = \frac{1}{\sqrt{2n+1}}$$

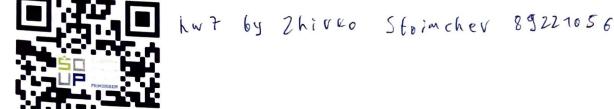
with absolute value, we have $\frac{1}{\sqrt{2n+1}} \sim \frac{1}{\sqrt{2n+1}}$

we new compate $\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{1}{\sqrt{2n+1}} \sim \lim_{n \to \infty} \frac{1}{\sqrt{2n+1}} = \lim_{n \to \infty} \frac{1}{\sqrt{2n+1}} =$

now ove can see that $\sum_{n=10}^{100} \sqrt{2n} + \sum_{n=10}^{100} \sqrt{2n} = \sum_{n=10}^{100} \sqrt{2n} =$

 $a_n = \frac{1}{\sqrt{2n+1}}$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\sqrt{2n+2}} = 0$, and $a_{n+1} = \frac{1}{\sqrt{2n+3}} < a_n = \frac{1}{\sqrt{2n+2}}$

a trow there we conclude that Z(-11" the is divergent







E. Show that if $a_n = 5n^3 - 14n^2 + 7n - 3$, then $a_n = \Phi(n^3)$ $\exists d > 0$ and $\exists \beta > 0$, $\exists N$ s.t. $\forall n > N$ $d n^3 \le 5n^3 - 14n^2 + 7n - 3 \le \beta n^3$ $1 \cdot n^3 \le 5n^3 - 14n^2 + 7n - 3 \le 5n^2 + 14n^3 + 7n^3 + 3n^2 = 23n^3$ that is equivalent with $0 \le 4n^3 - 14n^2 + 7n - 8$ from what we get $14n^2 - 4n + 3 \le 4n^3$

New are show that $\exists N=10$ and $\forall n>N$, it holds. $\exists d=1$ and $\exists \beta=2$? $\exists N=10 \ \ N>N$ $d-\beta_n \leq q_n \leq \beta \cdot b_n$