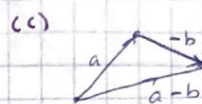
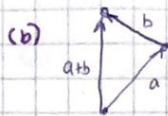
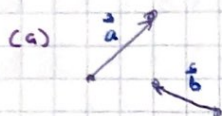


Quiz 1

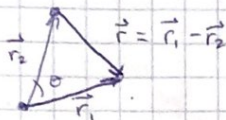
- ① For given vectors \vec{a} and \vec{b} we have



TRUE

easy to verify

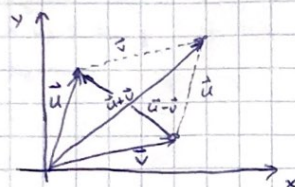
- ② For given vectors \vec{r}_1 and \vec{r}_2 we have



by the arrows direction, it is easiest to consider $\vec{r}_2 + \vec{r}_1 = \vec{r}_2 + (\vec{r}_1 - \vec{r}_2) = \vec{r}_1$
This is true from the picture!

TRUE

- ③ For given vectors \vec{u} and \vec{v} we have



$\vec{u} + \vec{v}$ is easy to verify.

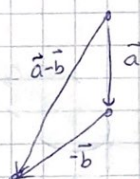
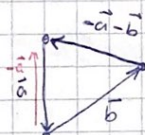
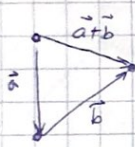
For $\vec{u} - \vec{v}$ use same method as above.

$$\vec{v} + (\vec{u} - \vec{v}) = \vec{u}$$

This is true!

TRUE

- ④ For given vectors \vec{a} and \vec{b} we have



TRUE

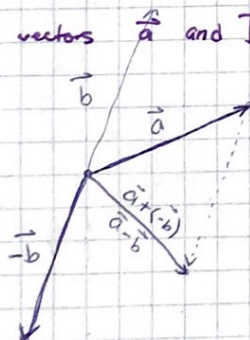
only the middle picture is difficult to verify.

Just as before, follow the "path" of arrows

$$\vec{b} + (-\vec{a} - \vec{b}) = -\vec{a}$$

Which agrees with the photo!

- ⑤ For given vectors \vec{a} and \vec{b} we have



easy to verify

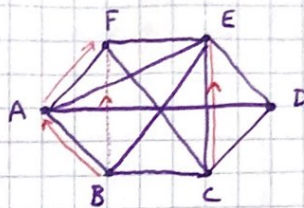
TRUE

⑥ For a given regular hexagon all three statements are true:

(a) $\vec{EA} = \vec{EF} + \vec{FA}$

(b) $\vec{BE} = -\vec{CB} - \vec{DC} - \vec{ED}$

(c) $\vec{CE} = \vec{BA} + \vec{AF}$



(a) easy

(b) $-\vec{CB} - \vec{DC} - \vec{ED} = \vec{BC} + \vec{CD} + \vec{DE} = \vec{BE}$

easy

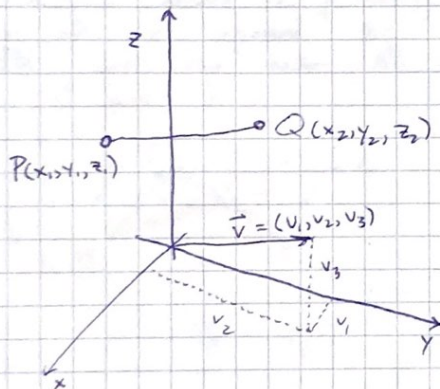
(c) almost the same vector ...

T

Same length & parallel !!

⑦ If $\vec{v} = (v_1, v_2, v_3)$ is represented by the directed line segment \vec{PQ} , where the initial point is $P(x_1, y_1, z_1)$ and the terminal point is $Q(x_2, y_2, z_2)$ then $x_1 + v_1 = x_2$, $y_1 + v_2 = y_2$, and $z_1 + v_3 = z_2$.

Thus, $v_1 = x_2 - x_1$, $v_2 = y_2 - y_1$, and $v_3 = z_2 - z_1$ are the components of \vec{PQ} .



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Exercises for beginners

- ① Give a formula for the distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$

$$\vec{P_1P_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$|\vec{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- ② Compute the distance between $P_1(2, 1, 5)$ and $P_2(-2, 3, 0)$.

$$\begin{aligned} |\vec{P_1P_2}| &= \sqrt{(-2-2)^2 + (3-1)^2 + (0-5)^2} \\ &= \sqrt{(16) + (4) + (25)} = \sqrt{45} = \boxed{3\sqrt{5}} \end{aligned}$$

- ③ Let $\square ABCD$ denote a parallelogram and let S be the intersection of line segment AC and BD . If $\vec{a} = \vec{AS}$ and $\vec{b} = \vec{SB}$ write the vector \vec{AD} as a linear combination of vectors \vec{a} and \vec{b} .



"Easy" way.... start with \vec{a} and observe the "want" vector is needed... This is just $-\vec{b}$!

$$\boxed{\vec{AD} = \vec{a} + (-\vec{b})}$$

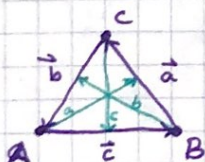
"Formal" way: $\vec{AB} = \vec{a} + \vec{b} \Rightarrow \vec{BA} = -\vec{a} - \vec{b}$

$$\vec{AC} = \vec{a} + \vec{a} = 2\vec{a}$$

$$\vec{AC} + \vec{BA} = \vec{AD}$$

$$2\vec{a} + (-\vec{a} - \vec{b}) = \boxed{\vec{a} + (-\vec{b}) = \vec{AD}}$$

- ④ In the given triangle $\triangle ABC$, vectors \vec{a} , \vec{b} , and \vec{c} are sides BC , CA , and AB , respectively. All three medians write as linear combinations of vectors \vec{a} , \vec{b} , and \vec{c} .



$$a = \vec{c} + \frac{1}{2}\vec{a}$$

$$b = \vec{a} + \frac{1}{2}\vec{b}$$

$$c = \vec{b} + \frac{1}{2}\vec{c}$$

- ⑤ If \vec{v} is a three-dimensional vector equal to the vector with initial point at the origin and terminal point (v_1, v_2, v_3) , then the component form of \vec{v} is $\vec{v} = (v_1, v_2, v_3)$. Find the component form and length of the vectors with the given endpoints.

(i) $A(-3, 4, 1)$ and $B(-5, 2, 2)$

$$\begin{aligned} \vec{AB} &= (-5 - (-3), 2 - 4, 2 - 1) \\ &= (-2, -2, 1) \\ |\vec{AB}| &= \sqrt{(-2)^2 + (-2)^2 + (1)^2} \\ &= \sqrt{4 + 4 + 1} \\ &= 3 \end{aligned}$$

(ii) $A(-3, 6, -5)$ and $B(1, 2, 7)$

$$\begin{aligned} \vec{AB} &= (1 - (-3), 2 - 6, 7 - (-5)) \\ &= (4, -4, 12) \\ |\vec{AB}| &= \sqrt{16 + 16 + 144} = \sqrt{176} \end{aligned}$$

(iii) $A(0, 4, 3)$ and $B(1, 2, 5)$

$$\begin{aligned} \vec{AB} &= (1 - 0, 2 - 4, 5 - 3) \\ &= (1, -2, 2) \\ |\vec{AB}| &= 3 \end{aligned}$$

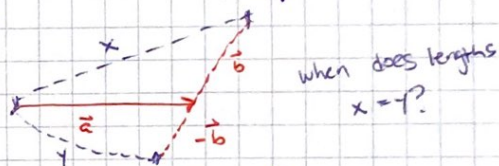
(iv) $A(-7, -8, 1)$ and $B(10, 8, 1)$

$$\begin{aligned} \vec{AB} &= (10 - (-7), 8 - (-8), 1 - 1) \\ &= (17, 16, 0) \\ |\vec{AB}| &= \sqrt{17^2 + 16^2 + 0^2} = \sqrt{545} \end{aligned}$$

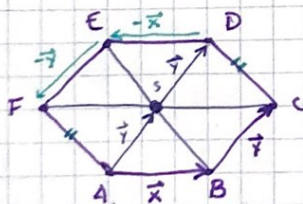
Standard exercises

- ① What can we say about the angle between vectors \vec{a} and \vec{b} in the plane if we know that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$?

Right angle!



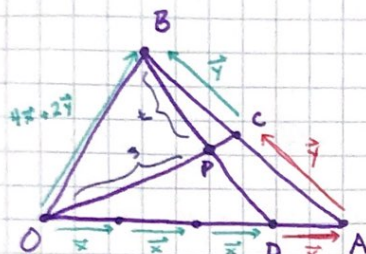
- ② Let the points A, B, C, D, E, and F denote the successive vertices of a regular hexagon and let $\vec{x} = \vec{AB}$ and $\vec{y} = \vec{BC}$. Express vectors \vec{AC} , \vec{AD} , \vec{BE} , \vec{AE} , \vec{BF} , and \vec{DF} in terms (as a linear combination) of vectors \vec{x} and \vec{y} .



$$\begin{aligned} \vec{AC} &= \vec{x} + \vec{y} \\ \vec{AD} &= \vec{x} + \vec{y} + (-\vec{x}) + \vec{y} = 2\vec{y} \\ \vec{BE} &= 2(-\vec{x} + \vec{y}) \\ \vec{AE} &= 2\vec{y} - \vec{x} \\ \vec{BF} &= -2\vec{x} + \vec{y} \\ \vec{DF} &= -\vec{x} - \vec{y} \end{aligned}$$

Problems from Exam

- ① In the given triangle $\triangle OAB$, C is the midpoint of the line segment AB and let D lie on the line segment OA such that $|OD|:|DA| = 3:1$. Denote by P the intersection point between line segments OC and BD . Determine the ratios $|OP|:|PC|$ and $|BP|:|PD|$.



Notice that $\exists s \in \mathbb{R}$ s.t. $s \cdot \vec{OC} = \vec{OP}$
and $t \in \mathbb{R}$ s.t. $t \cdot \vec{DB} = \vec{BP}$

$$\vec{OC} = 4\vec{x} + \vec{y}$$

$$\vec{DB} = \vec{x} + 2\vec{y}$$

$$|OP|:|PC| = \frac{4}{7}:\frac{3}{7} =$$

$$|OP|:|PC| = 4:3$$

$$|BP|:|PD| = \frac{4}{7}:\frac{3}{7}$$

$$|BP|:|PD| = 4:3$$

$$4\vec{x} + 2\vec{y} = s \cdot \vec{OC} + t \cdot \vec{DB}$$

$$= s \cdot (4\vec{x} + \vec{y}) + t \cdot (\vec{x} + 2\vec{y})$$

$$4\vec{x} = 4s\vec{x} + t\vec{x} \quad \wedge \quad 2\vec{y} = s\vec{y} + 2t\vec{y}$$

$$4 = 4s + t$$

$$2 = s + 2t$$

$$4s = 4 - t \Rightarrow s = 1 - \frac{t}{4} \quad s = 1 - \frac{4/7}{4}$$

$$2 = 1 - \frac{t}{4} + 2t$$

$$1 = \frac{7t}{4} \Rightarrow$$

$$t = \frac{4}{7}$$

$$s = \frac{6}{7}$$