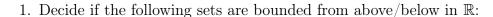
Selected exercises 03



(a)
$$S = \{1, 2, 3\}$$

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 (c) $S = \{0\} \cup \{x : x > 0\}$ (e) $S = \{x^3 : x \in \mathbb{Z}\}$ (b) $S = \{x : x \ge 5\}$ (d) $S = \{x^2 : x < -2\}$ (f) $S = \{x^3 - x : x \ge 1\}$

(e)
$$S = \{x^3 : x \in \mathbb{Z}\}$$

(b)
$$S = \{x : x > 5\}$$

(d)
$$S = \{x^2 : x < -2\}$$

(f)
$$S = \{x^3 - x : x \ge 1\}$$

2. Compute the supremum and infimum (if they exist) in the set S and prove by defini-

(a)
$$S = [0, 2), S \subset \mathbb{R}$$

(b)
$$S = (-1, \sqrt{2}), S \subset \mathbb{Q}$$

3. Find sets S and T with the following properties:

(a)
$$\sup S = \inf T$$
, $S \cap T = \emptyset$

(b)
$$\inf S = \sup T, S \subseteq T$$

(c)
$$\inf T = \min S$$
, T has no minimal element.

4. Let $S \subseteq \mathbb{R}^{\geq 0}$ be a bounded set. Prove that the set T is bounded as well.

(a)
$$T = \{5s : s \in S\}$$

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 (b) $T = \{-s^2 : s \in S\}$ (c) $T = \{s^2 - s : s \in S\}$

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$$T = \{s^2 - s : s \in S\}$$

5. Let A and B be non-empty bounded subsets of \mathbb{R} , and let A+B be the set of all sums a+b where $a \in A$ and $b \in B$. Analogously, define A-B. Prove the following equalities, or give a counterexample.

(a)
$$\sup (A+B) = \sup A + \sup B$$

 (c) $\inf (A+B) = \inf A + \inf B$

(c)
$$\inf (A + B) = \inf A + \inf B$$

(b)
$$\sup (A - B) = \sup A - \sup B$$
 (d) $\inf (A - B) = \inf A - \sup B$

(d)
$$\inf (A - B) = \inf A - \sup B$$

6. (a) Given the complex numbers a = 2 + 3i and b = -1 + i, compute |a - 5b|.

- (b) Give a complex number with modulus (a.k.a. magnitude or absolute value) 2, and that is not a real number.
- 7. Give first few members of the following sequences. For each sequence determine if it is bounded, increasing/decreasing, convergent and calculate the limit (if it exists):

(a)
$$a_n = \frac{3}{3n+7}$$

(d)
$$a_n = \frac{n^2 + r}{r-1}$$

(g)
$$a_n = \frac{2n^3 + 6n - 3}{7n - 3n^3 + 2}$$

(a)
$$a_n = \frac{3}{3n+7}$$
 (d) $a_n = \frac{n^2+n}{n-1}$
(b) $a_n = \sqrt{n^4 + n^2} - n^2$ (e) $a_n = \cos(n\pi)$
(c) $a_n = \frac{n+(-1)^n}{n-(-1)^n}$ (f) $a_n = (-3)^n$

(e)
$$a_n = \cos(n\pi)$$

(c)
$$a_n = \frac{n+(-1)^n}{n-(-1)^n}$$

$$(f) a_n = (-3)^n$$