Algebra I - Matrični račun: 12. vaje

Ponavljanje - priprava za 2. kolokvij

1. Dani sta matriki $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & x \\ 1 & x & 6 \end{bmatrix}$ in $B = \begin{bmatrix} -1 & -2 & 1 \\ 0 & -1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$.

   Določite $x \in \mathbb{R}$ tako, da bo $\det(AB) = 6$.

2. Za matriki $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 1 & 0 & -1 \end{bmatrix}$ in $B = \begin{bmatrix} 4 & 3 & -1 \\ 5 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ rešite matrično enačbo

$$AX - B^T = X + AB.$$

3. Dan je sistem enačb:

$$\begin{array}{rcrcrcl} x & + & 2y & - & 3z & = & 8 \\ -2x & + & 2y & + & z & = & 3 \\ 3x & - & 2y & + & \alpha z & = & 9 \end{array}$$

   (a) Določite tak parameter $\alpha \in \mathbb{R}$, da bo sistem protisloven.

   (b) Za $\alpha = 8$ rešite sistem s pomočjo matrik.

4. Za poljuben $n \in \mathbb{N}$ izračunajte

$$det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \dots \cdot \begin{bmatrix} n & n-1 \\ n+1 & n \end{bmatrix}\right).$$

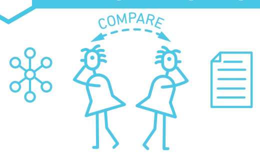
LEARN TO STUDY USING…

Dual Coding

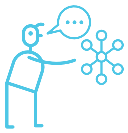
COMBINE WORDS AND VISUALS

LEARNINGSCIENTISTS.ORG

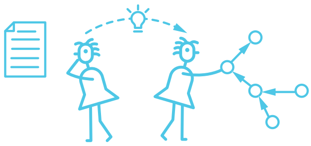
HOW TO DO IT



Look at your class materials and find visuals. Look over the visuals and compare to the words.



Look at visuals, and explain in your own words what they mean.

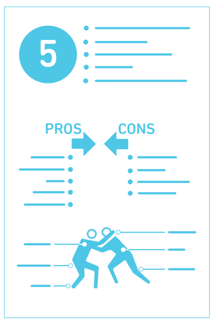


Take information that you are trying to learn, and draw visuals to go along with it.

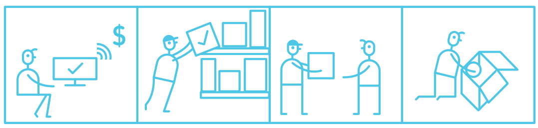
HOLD ON!

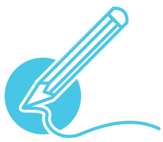
Try to come up with different ways to represent the information visually, for example an infographic, a timeline, a cartoon strip, or a diagram of parts that work together.

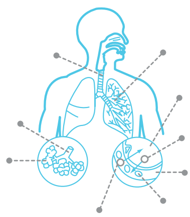
INFOGRAPHIC



CARTOON STRIP

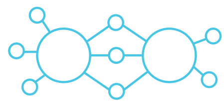






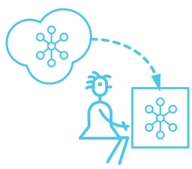
DIAGRAM

TIMELINE



GRAPHIC ORGANIZER

Work your way up to drawing what you know from memory.



RESEARCH

Read more about dual coding as a study strategy

Mayer, R. E., & Anderson, R. B. (1992). The instructive animation: Helping students build connections between words and pictures in multimedia learning. *Journal of Educational Psychology*, *4*, 444-452.

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Navodila.

1. Dani sta matriki $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & x \\ 1 & x & 6 \end{bmatrix}$ in $B = \begin{bmatrix} -1 & -2 & 1 \\ 0 & -1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$.

   Določite $x \in \mathbb{R}$ tako, da bo $\det(AB) = 6$.

① $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & x \\ 1 & x & 6 \end{bmatrix}$ $B = \begin{bmatrix} -1 & -2 & 1 \\ 0 & -1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

I. možnost: zmnožimo matriki in izračunamo determinanto

II. upoštevamo: $\det(AB) = \det A \cdot \det B$

(II) $\det A = 0 + x + x - 2 - 0 - 6 = 2x - 8$

$\det B = 1 - 8 + 0 + 2 + 2 + 0 = -3$

$\det A \cdot \det B = (2x-8) \cdot (-3) = -6x + 24 = 6$

$-x + 4 = 1$

$\boxed{x = 3}$

2. Za matriki $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 1 & 0 & -1 \end{bmatrix}$ in $B = \begin{bmatrix} 4 & 3 & -1 \\ 5 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ rešite matrično enačbo

$$AX - B^T = X + AB.$$

② $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 1 & 0 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 & -1 \\ 5 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$AX - B^T = X + AB$

$AX - X = AB + B^T$

$(A - I)X = AB + B^T \quad / (A-I)^{-1}$ z leve!

$\underbrace{(A-I)^{-1}(A-I)}_{I}X = (A-I)^{-1}(AB+B^T)$

$X = (A-I)^{-1}(AB+B^T)$

$C = A - I = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 1 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 5 \\ 1 & 0 & -2 \end{bmatrix}$

$\det(A - I) = 0 + 10 + 0 - 8 + 0 + 0 = 2$

$C_{11} = \begin{vmatrix} 2 & 5 \\ 0 & -2 \end{vmatrix} = -4$ $C_{21} = -\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = 4$ $C_{31} = \begin{vmatrix} 2 & 4 \\ 2 & 5 \end{vmatrix} = 2$

$C_{12} = -\begin{vmatrix} 0 & 5 \\ 1 & -2 \end{vmatrix} = 5$ $C_{22} = \begin{vmatrix} 0 & 4 \\ 1 & -2 \end{vmatrix} = -4$ $C_{32} = -\begin{vmatrix} 0 & 4 \\ 0 & -2 \end{vmatrix} = 0$

$C_{13} = \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} = -2$ $C_{23} = -\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} = 2$ $C_{33} = \begin{vmatrix} 0 & 2 \\ 0 & 2 \end{vmatrix} = 0$

$(A-I)^{-1} = \frac{1}{\underset{2}{\det(A-I)}} \begin{bmatrix} -4 & 4 & 2 \\ 5 & -4 & 0 \\ -2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 2 & 1 \\ 5/2 & -2 & 0 \\ -1 & 1 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 18 & 7 & 3 \\ 20 & 6 & 5 \\ 3 & 3 & -2 \end{bmatrix}$ $B^T = \begin{bmatrix} 4 & 5 & 1 \\ 3 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$AB + B^T = \begin{bmatrix} 22 & 12 & 4 \\ 23 & 8 & 5 \\ 2 & 3 & -1 \end{bmatrix}$

$X = (A-I)^{-1}(AB+B^T) = \begin{bmatrix} -2 & 2 & 1 \\ 5/2 & -2 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 22 & 12 & 4 \\ 23 & 8 & 5 \\ 2 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 4 & -5 & 1 \\ 9 & 14 & 0 \\ 1 & -4 & 1 \end{bmatrix}$

3. Dan je sistem enačb:

$$\begin{array}{rcrcrcr} x & + & 2y & - & 3z & = & 8 \\ -2x & + & 2y & + & z & = & 3 \\ 3x & - & 2y & + & \alpha z & = & 9 \end{array}$$

(a) Določite tak parameter $\alpha \in \mathbb{R}$, da bo sistem protisloven.

(b) Za $\alpha = 8$ rešite sistem s pomočjo matrik.

③ $x+2y-3z=8$
$-2x+2y+z=3$
$3x-2y+\alpha z=9$

a) določi sistem, da bo sistem protisloven
I možnost: Gauss
II možnost: determinanta $(=0)$

I $A|b = \left[\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ -2 & 2 & 1 & 3 \\ 3 & -2 & \alpha & 9 \end{array}\right] \begin{matrix} \sim \\ v_2+2v_1 \\ v_3-3v_1 \end{matrix} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ 0 & 6 & -5 & 19 \\ 0 & -8 & \alpha+9 & -15 \end{array}\right] 3v_3+4v_2 \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ 0 & 6 & -5 & 19 \\ 0 & 0 & 3\alpha+7 & 31 \end{array}\right]$

$3\alpha+7=0 \Rightarrow \boxed{\alpha=-\frac{7}{3}}$

II $\begin{vmatrix} 1 & 2 & -3 \\ -2 & 2 & 1 \\ 3 & -2 & \alpha \end{vmatrix} = 2\alpha+6-12+18+2+4\alpha =$
$= 6\alpha+14$

$6\alpha+14=0$
$\alpha = -\frac{14}{6} = -\frac{7}{3}$

! Preveri, da je v tem primeru sistem protisloven (ker ima lahko neskončno mnogo rešitev)

b) za $\alpha=8$ reši sistem → Gauss v a primeru

$A|b = \left[\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ -2 & 2 & 1 & 3 \\ 3 & -2 & 8 & 19 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ 0 & 6 & -5 & 19 \\ 0 & 0 & 31 & 31 \end{array}\right] /:31 \sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ 0 & 6 & -5 & 19 \\ 0 & 0 & 1 & 1 \end{array}\right] v_2+5v_3 \sim$

$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & 8 \\ 0 & 6 & 0 & 24 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{matrix} v_1-2v_2+3v_3 \\ \sim /:6 \end{matrix} \sim \begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right] \end{array}$

$\boxed{\begin{array}{l} x=3 \\ y=4 \\ z=1 \end{array}}$

$8-8+3=3$

4. Za poljuben $n \in \mathbb{N}$ izračunajte $det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \dots \cdot \begin{bmatrix} n & n-1 \\ n+1 & n \end{bmatrix}\right)$.

④ $det\Big(\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}}_{A_1} \cdot \underbrace{\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}}_{A_2} \cdot \ldots \cdot \underbrace{\begin{bmatrix} n & n-1 \\ n+1 & n \end{bmatrix}}_{A_n}\Big)$

$n=1$: $det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\right) = 1 = det\, A_1$

$n=2$: $det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\right) = det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\right) \cdot det\left(\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\right) = 1 \cdot 1 = 1$

$n=3$: $det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix}\right) = det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\right) \cdot det\left(\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}\right) = 1 \cdot 1 = 1$

Vemo: $det\,(AB) = det\,A \cdot det\,B$

$det\,(A_1 \cdot A_2 \cdots A_n) = det\,A_1 \cdot det\,(A_2 \cdot A_3 \cdot \ldots \cdot A_n) = \ldots = det\,A_1 \cdot det\,A_2 \cdot \ldots \cdot det\,A_n$

$\forall n \in \mathbb{N}$ $A_n = \begin{bmatrix} n & n-1 \\ n+1 & n \end{bmatrix}$ $det\,A_n = n^2 - (n-1)(n+1) = \cancel{n^2} - \cancel{n^2} + 1 = 1$ vsi členi so enaki 1

$det\,A_n = 1 \quad \forall n \in \mathbb{N}$

$\Rightarrow det\,(A_1 \cdot A_2 \cdot \ldots \cdot A_n) = 1 \quad \forall n \in \mathbb{N}$