

The distributive property would be:

$$(x_1 - y_1) [(x_2 - y_2) + (x_3 - y_3)] = (x_1 - y_1)(x_2 - y_2) + (x_1 - y_1)(x_3 - y_3)$$

In order to prove this formula we use this:

$$(a, b) + (c, d) = (a + c, b + d)$$

$$(a, b) \cdot (c, d) = (ac + bd, ad + bc)$$

So the proof would be

$$(x_1, y_1) \cdot (x_2, y_2 + x_3, y_3) = (x_1, y_1) \cdot (x_2 + x_3, y_2 + y_3) =$$

$$= (x_1 \cdot (x_2 + x_3) + y_1 \cdot (y_2 + y_3), x_1 \cdot (y_2 + y_3) + y_1 \cdot (x_2 + x_3)) =$$

$$= (x_1 x_2 + x_1 x_3 + y_1 y_2 + y_1 y_3, x_1 y_2 + x_1 y_3 + y_1 x_2 + y_1 x_3)$$

$$(x_1, y_1) \cdot (x_2, y_2) + (x_1, y_1) \cdot (x_3, y_3) =$$

$$= (x_1 x_2 + y_1 y_2, x_1 y_2 + y_1 x_2) + (x_1 x_3 + y_1 y_3, x_1 y_3 + y_1 x_3) =$$

$$= (x_1 x_2 + y_1 y_2 + x_1 x_3 + y_1 y_3, x_1 y_2 + y_1 x_2 + x_1 y_3 + y_1 x_3)$$

(D) In $\mathbb{Q}^{\geq 0}$ (n, m) is $\frac{n}{m}$, also in \mathbb{Q} (n, m) is $\frac{n}{m}$. This is derived from the lecture notes.

We order $\mathbb{Q}^{\geq 0}$ by $(n_1, m_1) < (n_2, m_2)$, this is only true if $n_1 m_2 < n_2 m_1$ is true

We were given the example $(1, 2), (0, 1)$ which is equal to $\frac{1}{2}$ or simplified $\frac{1}{2} - \frac{0}{1} = \frac{1}{2}$

So if we take the proposition from earlier

$(n_1, m_1) < (n_2, m_2)$ when $n_1 m_2 < n_2 m_1$

And we also count in that every n/m has a set of coordinates.