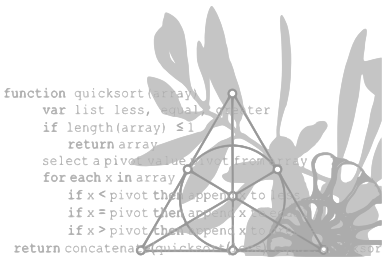


Univerza na Primorskem
UP FAMNIT
Študijsko leto 2020/2021

Algebra I

Pisni izpit

– 9. julij 2021 –

Čas pisanja: 90 *minut. Maksimalno število točk:* 50. *Dovoljena je uporaba pisala in enega ročno napisanega lista formata A*4 *z definicijami in formulami (brez rešenih primerov). Pišite razločno in utemeljite vsak odgovor. Srečno!*

1. Dani sta točka $T(1,3,4)$ in premica p z enačbo $x = \frac{y}{-2} = \frac{z+1}{2}$.

   (a) Poiščite koordinate pravoketne projekcije točke T na premico p (tj. poiščite koordinate točke na premici p, ki je najbližja točki T). *(8 točk)*

   (b) Izračunajte ploščino paralelograma, ki ga razpenjata krajevni vektor točke T in smerni vektor premice p. *(8 točk)*

2. Poiščite vse točke na premici p podani z enačbo $(1,0,0) + \lambda(1,1,1)$, ki so enako oddaljene od ravnin $\Sigma : x + y - z = -1$ in $\Pi : x - y + z = 5$. *(9 točk)*

3. Dani sta matriki $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 1 & 0 & -1 \end{bmatrix}$ in $B = \begin{bmatrix} 4 & 3 & -1 \\ 5 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

   Rešite matrično enačbo $AX - B^T = X + AB$. *(9 točk)*

4. Dan je naslednji sistem linearnih enačb:

$$\begin{array}{rcrcrcl} 3x & + & 4y & + & 4z & = & 2 \\ 3x & + & 2y & + & 3z & = & 3 \\ 4x & + & 5y & + & tz & = & 4 \end{array}$$

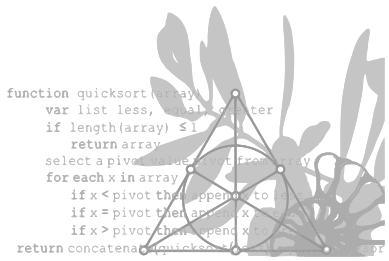
   (a) Za katere vrednosti $t \in \mathbb{R}$ bo sistem protisloven? *(4 točke)*

   (b) Za $t = 7$ poiščite rešitev sistema. *(4 točke)*

5. Izračunajte determinanto:

$$\begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{vmatrix}$$

   (8 točk)



University of Primorska
UP FAMNIT
Academic year 2020/2021

Algebra I

Written exam

– July 9, 2021 –

Time: 90 minutes. Maximum number of points: 50. You are allowed to use a pen and one A4 hand-written piece of paper with definitions and formulas (and with no solved exercises). Write clearly, and justify all your answers. Good luck!

1. We are given the point $T(1,3,4)$ and the line p with equation $x = \frac{y}{-2} = \frac{z+1}{2}$.

   (a) Find the coordinates of the projection of point T on line p (i.e. find the coordinates of the point on line p that is closest to T). *(8 points)*

   (b) Determine the area of the parallelogram, determined by the vector $\overrightarrow{OT}$ and the direction vector of the line p. *(8 points)*

2. Find all the points on the line p given by $(1,0,0) + \lambda(1,1,1)$ that are equidistant from planes $\Sigma : x + y - z = -1$ and $\Pi : x - y + z = 5$. *(9 points)*

3. We are given matrices $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 1 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 & -1 \\ 5 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

   Find matrix X from the equation $AX - B^T = X + AB$. *(9 points)*

4. Consider the following system of linear equations:

$$\begin{aligned} 3x &+ 4y &+ 4z &= 2 \\ 3x &+ 2y &+ 3z &= 3 \\ 4x &+ 5y &+ tz &= 4 \end{aligned}$$

   (a) For which values $t \in \mathbb{R}$ will the system be inconsistent? *(4 points)*

   (b) Fot $t = 7$ find the solution of the system. *(4 points)*

5. Compute the determinant:

$$\begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{vmatrix}$$

   (8 points)