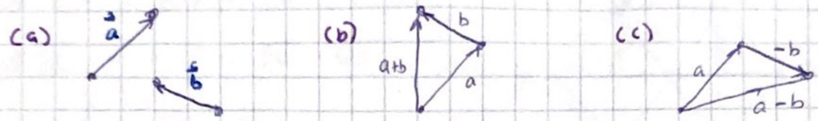
Quiz 1

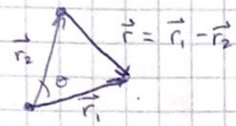
① For given vectors $\vec{a}$ and $\vec{b}$ we have



$\boxed{\text{TRUE}}$

easy to verify

② For given vectors $\vec{r}_1$ and $\vec{r}_2$ we have

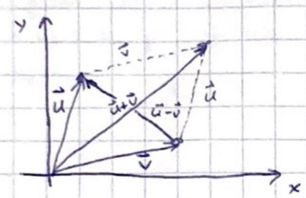


by the arrows direction, it is easiest to consider $\vec{r}_2 + \vec{r} = \vec{r}_2 + (\vec{r}_1 - \vec{r}_2) = \vec{r}_1$

This is true from the picture!

$\boxed{\text{TRUE}}$

③ For given vectors $\vec{u}$ and $\vec{v}$ we have



$\vec{u}+\vec{v}$ is easy to verify.

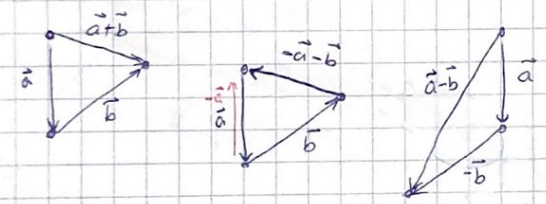
For $\vec{u}-\vec{v}$ use same method as above!

$\vec{v} + (\vec{u} - \vec{v}) = \vec{u}$

This is true!

$\boxed{\text{TRUE}}$

④ For given vectors $\vec{a}$ and $\vec{b}$ we have



$\boxed{\text{TRUE}}$

Only the middle picture is difficult to verify.

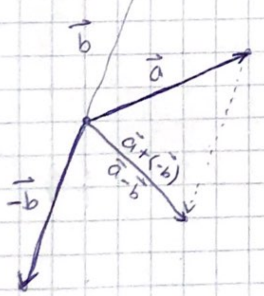
Just as before, follow the "path" of arrows

$b + (-\vec{a} - \vec{b}) = -\vec{a}$

Which agrees with the photo!

⑤ For given vectors $\vec{a}$ and $\vec{b}$ we have

$\boxed{\text{TRUE}}$



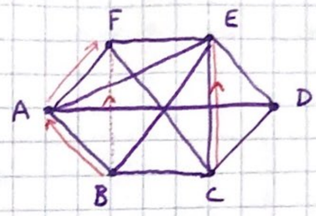
easy to verify

⑥ For a given regular hexagon all three statements are true:

(a) $\overrightarrow{EA} = \overrightarrow{EF} + \overrightarrow{FA}$

(b) $\overrightarrow{BE} = -\overrightarrow{CB} - \overrightarrow{DC} - \overrightarrow{ED}$

(c) $\overrightarrow{CE} = \overrightarrow{BA} + \overrightarrow{AF}$



(a) easy

(b) $-\overrightarrow{CB} - \overrightarrow{DC} - \overrightarrow{ED} = \underbrace{\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}}_{\text{easy}} = \overrightarrow{BE}$

(c) almost the same vector....

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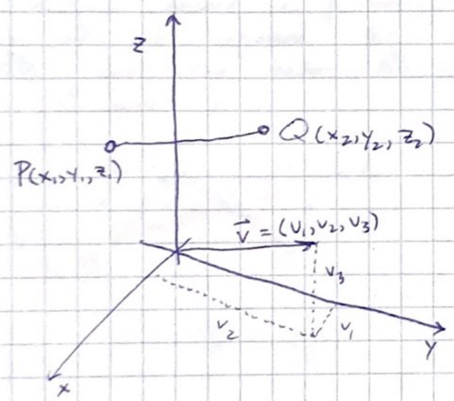
same length & parallel :)

⑦ If $\vec{v} = (v_1, v_2, v_3)$ is represented by the directed line segment $\overrightarrow{PQ}$, where the initial point is $P(x_1, y_1, z_1)$ and the terminal point is $Q(x_2, y_2, z_2)$ then $x_1 + v_1 = x_2$, $y_1 + v_2 = y_2$, and $z_1 + v_3 = z_2$.

Thus, $v_1 = x_2 - x_1$, $v_2 = y_2 - y_1$, and $v_3 = z_2 - z_1$ are the components of $\overrightarrow{PQ}$



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Exercises for beginners

① Give a formula for the distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$

$$\overrightarrow{P_1P_2} = (x_2 - x_1, \ y_2 - y_1, \ z_2 - z_1)$$

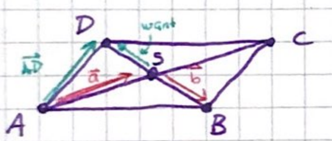
$$\boxed{|\overrightarrow{P_1P_2}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

② Compute the distance between $P_1(2,1,5)$ and $P_2(-2,3,0)$.

$$|\overrightarrow{P_1P_2}| = \sqrt{(-2-2)^2 + (3-1)^2 + (0-5)^2}$$

$$= \sqrt{(16) + (4) + (25)} = \sqrt{45} = \boxed{3\sqrt{5}}$$

③ Let $\square ABCD$ denote a parallelogram and let S be the intersection of line segment AC and BD. If $\vec{a} = \overrightarrow{AS}$ and $\vec{b} = \overrightarrow{SB}$ write the vector $\overrightarrow{AD}$ as a linear combination of vectors $\vec{a}$ and $\vec{b}$.



"Easy" way.... start with $\vec{a}$ and observe the "want" vector is needed... This is just $-\vec{b}$!

$$\boxed{\overrightarrow{AD} = \vec{a} + (-\vec{b})}$$

"Formal" way:

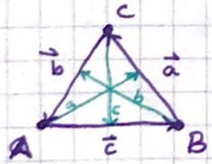
$$\overrightarrow{AB} = \vec{a} + \vec{b} \Rightarrow \overrightarrow{BA} = -\vec{a} - \vec{b}$$

$$\overrightarrow{AC} = \vec{a} + \vec{a} = 2\vec{a}$$

$$\overrightarrow{AC} + \overrightarrow{BA} = \overrightarrow{AD}$$

$$2\vec{a} + (-\vec{a} - \vec{b}) = \boxed{\vec{a} + (-\vec{b}) = \overrightarrow{AD}}$$

④ In the given triangle ΔABC, vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are sides BC, CA, and AB, respectively. All three medians write as linear combinations of vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$.



$a = \vec{c} + \frac{1}{2}\vec{a}$

$b = \vec{a} + \frac{1}{2}\vec{b}$

$c = \vec{b} + \frac{1}{2}\vec{c}$

⑤ If $\vec{v}$ is a three-dimensional vector equal to the vector with initial point at the origin and terminal point (v_1, v_2, v_3), then the component form of $\vec{v}$ is $\vec{v} = (v_1, v_2, v_3)$. Find the component form and length of the vectors with the given endpoints.

(i) $A(-3,4,1)$ and $B(-5,2,2)$

- $\vec{AB} = (-5-(-3),\ 2-4,\ 2-1) = \boxed{(-2,-2,1)}$
- $|\vec{AB}| = \sqrt{(-2)^2+(-2)^2+(1)^2} = \sqrt{4+4+1} = \boxed{3}$

(ii) $A(-3,6,-5)$ and $B(1,2,7)$

- $\vec{AB} = (1-(-3),\ 2-6,\ 7-(-5)) = \boxed{(4,-4,12)}$
- $|\vec{AB}| = \sqrt{16+16+144} = \boxed{\sqrt{176}}$

(iii) $A(0,4,3)$ and $B(1,2,5)$

- $\vec{AB} = (1-0,\ 2-4,\ 5-3) = \boxed{(1,-2,2)}$
- $|\vec{AB}| = \boxed{3}$

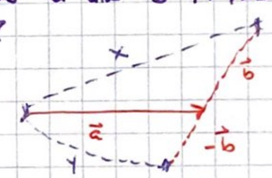
(iv) $A(-7,-8,1)$ and $B(10,8,1)$

- $\vec{AB} = (10-(-7),\ 8-(-8),\ 1-1) = \boxed{(17,16,0)}$
- $|\vec{AB}| = \sqrt{17^2+16^2+0^2} = \boxed{\sqrt{545}}$

Standard exercises

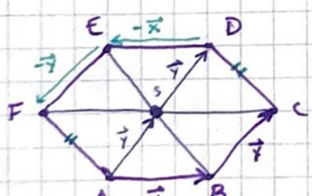
① What can we say about the angle between vectors $\vec{a}$ and $\vec{b}$ in the plane if we know that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$?

Right angle ?



when does lengths $x = y$?

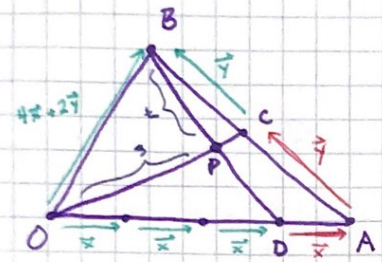
② Let the points A, B, C, D, E, and F denote the successive vertices of a regular hexagon and let $\vec{x} = \vec{AB}$ and $\vec{y} = \vec{BC}$. Express vectors $\vec{AC}$, $\vec{AD}$, $\vec{BE}$, $\vec{AE}$, $\vec{BF}$, and $\vec{DF}$ in terms (as a linear combination) of vectors $\vec{x}$ and $\vec{y}$.



- $\vec{AC} = \vec{x} + \vec{y}$
- $\vec{AD} = \vec{x} + \vec{y} + (-\vec{x}) + \vec{y} = 2\vec{y}$
- $\vec{BE} = 2(-\vec{x} + \vec{y})$
- $\vec{AE} = 2\vec{y} - \vec{x}$
- $\vec{BF} = -2\vec{x} + \vec{y}$
- $\vec{DF} = -\vec{x} - \vec{y}$

Problems from Exam

① In the given triangle ΔOAB, C is the midpoint of the line segment AB and let D lie on the line segment OA such that $|OD| : |DA| = 3:1$. Denote by P the intersection point between line segments OC and BD. Determine the ratio $|OP| : |PC|$ and $|BP| : |PD|$.



Notice that $\exists\, s \in \mathbb{R}$ s.t. $s \cdot \overrightarrow{OC} = \overrightarrow{OP}$
and $t \in \mathbb{R}$ s.t. $t \cdot \overrightarrow{BD} = \overrightarrow{BP}$

$\overrightarrow{OC} = 4\vec{x} + \vec{y}$
$\overrightarrow{DB} = \vec{x} + 2\vec{y}$

$$4\vec{x} + 2\vec{y} = s \cdot \overrightarrow{OC} + t \cdot \overrightarrow{DB}$$
$$= s \cdot (4\vec{x} + \vec{y}) + t \cdot (\vec{x} + 2\vec{y})$$

$$\underbrace{4\vec{x} = 4s\vec{x} + t\vec{x} \quad \& \quad 2\vec{y} = s\vec{y} + 2t\vec{y}}$$

$$4 = 4s + t$$
$$2 = s + 2t$$

$4s = 4 - t \Rightarrow s = 1 - \frac{t}{4}$ $\quad s = 1 - \frac{4/7}{4}$

$2 = 1 - \frac{t}{4} + 2t$
$1 = \frac{7t}{4} \Rightarrow \boxed{t = 4/7}$ $\quad \boxed{s = 6/7}$

$|OP| : |PC| = 6/7 : 1/7 \Leftrightarrow$

$\boxed{|OP| : |PC| = 6:1}$

$|BP| : |PD| = 4/7 : 3/7$

$\boxed{|BP| : |PD| = 4:3}$