Scores	
1.	
2.	
3.	
4.	
5.	
6.	
7.	
Total:	

ANA-I Foundations of Analysis
Final Examination B – 9 Feb 2021

Name ___________________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may have 1 handwritten A4-sized sheet of paper, but no other notes, books, or calculators.
110 total points.

1. (6 points each) Calculate the following limits, or explain why they diverge. You may use any theorems we have proved in class or on homework.
   (a) $\lim\limits_{n\to\infty} \dfrac{\sqrt{2n^2-n+3}}{3n+4}$
   (b) $\lim\limits_{n\to\infty} \sqrt[3]{\dfrac{n^2+1}{n^3-1}}$
   (c) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x^2-5}$
   (d) $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-3x+2}$
   (e) $\lim\limits_{x\to 0} \dfrac{|x|}{x}$.

2. Series
   (a) (6 points) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^2+5n+2}{(1+\frac{1}{10})^n}$ converges absolutely, converges conditionally, or diverges.
   (b) (7 points) Let $f(x) = \displaystyle\sum_{n=0}^{\infty} x^n$. For what values of x does the expression converge? For these values of x, write $f(x)$ in the form of an elementary function.

3. (6 points each) Examples. Justify your answers briefly.
   (a) There is a function $f : \mathbb{R} \to \mathbb{R}$ so that $f(-1) = -1$ and $f(1) = 1$, but so that f is never 0. What else can you say about f?
   (b) Explain briefly why $\left\{x \in \mathbb{Q}^{\geq 0} : x < 5\right\} \mid \left\{x \in \mathbb{Q}^{\geq 0} : x \geq 5\right\}$ is a Dedekind cut. What positive real number does it represent?
   (c) Give an example of a sequence a_n whose image set $\{a_n : n \in \mathbb{N}\}$ is not compact.
   (d) Give an example of a subset of $\mathbb{R}^2$ that is neither open nor closed.

4. (10 points) Let a_n be recursively defined by $a_0 = 2$, $a_{n+1} = \sqrt{5a_n}$ for $n \geq 0$. Show that the sequence converges, and find $\lim\limits_{n\to\infty} a_n$.

5. (11 points) How many terms are needed to estimate $\sum_{n=0}^{\infty} \frac{10+(-1)^n \cdot n}{5^n}$ to within 0.1? Justify your answer!

6. (12 points) Show directly from definition that if a_n, b_n are sequences so that $\lim_{n\to\infty} a_n = 2$ and $\lim_{n\to\infty} b_n = 4$, then $\lim_{n\to\infty} a_n \cdot b_n = 8$.

7. (10 points) Prove that if f is a continuous function from $[2,5]$ to $[2,5]$, then f has a fixed point.