Algebra I
Izpit
– 14. Junij 2022 –

Čas pisanja: 135 *minut. Maksimalno število točk:* 100. *Dovoljena je uporaba pisala in kalkulatorja. Pišite razločno in utemeljite vsak odgovor. Srečno!*

1. (a) Zapišite definicijo mešanega produkta, naštejte vsaj tri algebrske lastnosti mešanega produkta in eno dokažite. Kakšen je geometrijski pomen mešanega produkta? *(10 točk)*

   (b) Kdaj lahko sistem linearnih enačb $Ax = b$ rešujemo s pomočjo Cramerjevega pravila? Kaj lahko povemo o rešitvah sistema linearnih enačb, ki ga lahko s Cramerjevim pravilom rešimo? *(5 točk)*

   (c) Naštejte osnovne operacije nad vrsticami matrike in za vsako pojasnite, kako vpliva na vrednost determinante te matrike. *(5 točk)*

2. V previlnem šestkotniku $ABCDEF$ je točka G razpolovišče stranice EF, točka S pa presečišče premic AC in BG ter velja $|AS| : |SC| = 3 : 4$. Stranica šestkotnika meri 2 enoti. Označimo $\vec{a} = \overrightarrow{BA}$ in $\vec{b} = \overrightarrow{BC}$.

   (a) Izrazite vektor $\overrightarrow{SG}$ kot linearno kombinacijo vektorjev $\vec{a}$ in $\vec{b}$. *(10 točk)*

   (b) Izračunajte dolžino vektorja $\overrightarrow{AC}$. *(5 točk)*

   (c) Izračunajte kot φ med vektorjema $\overrightarrow{AC}$ in $\overrightarrow{AF}$. *(5 točk)*

3. Dani imamo točki $A(0, 2, 3)$ in $B(2, 7, 5)$.

   (a) Zapišite vektorsko obliko enačbe premice ℓ, ki vsebuje točki A in B. *(5 točk)*

   (b) Za katere vrednosti $x, z \in \mathbb{R}$ bo točka $P(x, 4, z)$ ležala na premici ℓ. *(5 točk)*

   (c) Poiščite presečišče med premico ℓ in ravnino Π, ki je podana z enačbo $-2x + 3y - z = 6$. *(5 točk)*

   (d) Izračunajte ploščino paralelograma, ki ga razpenjata krajevna vektorja točk A in B (tj. vektorja $\overrightarrow{0A}$ in $\overrightarrow{0B}$). *(5 točk)*
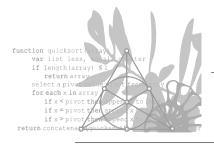
4. Izračunajte determinanto

$$\det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \ldots \cdot \begin{bmatrix} n & n-1 \\ n+1 & n \end{bmatrix}\right).$$

   za vsak $n \in \mathbb{N}$. *(20 točk)*

5. Za katere vrednosti $t \in \mathbb{R}$ bo sistem

$$tx - (2t)y + z = 3$$
$$-3x + 3y - 5z = -2$$
$$x - (t+1)y + z = 1$$

   protisloven? Poiščite rešitev sistema za $t = -1$. *(20 točk)*

Algebra I
Exam
– June 14, 2022 –

Time: 135 *minutes. Maximum number of points:* 100. *You are allowed to use a pen and a calculator. Write clearly, and justify all your answers. Good luck!*

1. (a) Write the definition of the box product, state at least three of its algebraic properties and prove one of them. What is the geometrical meaning of the box product?
(10 points)

   (b) In which case can we solve a system of linear equations $Ax = b$ using Cramer's rule? What can we say about the solution of a system of linear equations that can be solved using Cramer's rule?
(5 points)

   (c) List the basic row operations that we can use on a matrix and explain how each of them affects the value of the determinant of that matrix.
(5 points)

2. In a regular hexagon $ABCDEF$ point G is the midpoint of line segment EF, point S is the intersection of lines AC and BG, and $|AS| : |SC| = 3 : 4$. The side of the hexagon has length 2. Denote $\vec{a} = \overrightarrow{BA}$ and $\vec{b} = \overrightarrow{BC}$.

   (a) Write the vector $\overrightarrow{SG}$ as a linear combination of vectors $\vec{a}$ and $\vec{b}$. *(10 points)*

   (b) Compute the length of the vector $\overrightarrow{AC}$. *(5 points)*

   (c) Compute the angle φ between vectors $\overrightarrow{AC}$ and $\overrightarrow{AF}$. *(5 points)*

3. We are given points $A(0, 2, 3)$ and $B(2, 7, 5)$.

   (a) Find the equation (in vector form) of the line ℓ that contains points A and B. *(5 points)*

   (b) For which values $x, z \in \mathbb{R}$ the point $P(x, 4, z)$ lies on ℓ. *(5 points)*

   (c) Find the intersection of the line ℓ and the plane Π that is given by $-2x + 3y - z = 6$. *(5 points)*

   (d) Compute the area of the parallelogram, determined by vectors $\overrightarrow{0A}$ and $\overrightarrow{0B}$. *(5 points)*

4. Compute the determinant

$$\det\left(\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdots \cdots \begin{bmatrix} n & n-1 \\ n+1 & n \end{bmatrix}\right).$$

   for every $n \in \mathbb{N}$. *(20 points)*

5. For which values of $t \in \mathbb{R}$ will the system

$$tx - (2t)y + z = 3$$
$$-3x + 3y - 5z = -2$$
$$x - (t+1)y + z = 1$$

   be inconsistent? Find the solutions for $t = -1$. *(20 points)*