

$$\textcircled{D.} \text{ i) } \lim_{n \rightarrow \infty} \frac{2n^2+2}{5n^3-3n+1} = \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{2n^2+2}{5n^2-3-1/n} = \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{\frac{2n^2}{n^2} + \frac{2}{n^2}}{\frac{5n^2}{n^2} - \frac{3}{n^2} - \frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{2 + 2/n^2}{5 - 3/n^2 - 1/n^3} \quad (\text{By Mol})$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{2 + 2/n^2}{5 - 3/n^2 - 1/n^3} = \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{2}{5} = 0 \cdot \frac{2}{5} = 0 \quad (\text{By Mol})$$

$$\text{ii) } \lim_{n \rightarrow \infty} \frac{\sqrt{4n^3+4n}}{2n-2} = \lim_{n \rightarrow \infty} \frac{2\sqrt{n^3+1}}{2(n-1)} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n^3}{n^2} + \frac{1}{n^2}}}{\frac{n}{n} - \frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n + \frac{1}{n^2}}}{1 - \frac{1}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{n + \frac{1}{n^2}}}{1 - \frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{1} = \lim_{n \rightarrow \infty} \sqrt{n} = +\infty$$

$$\text{iii) } \lim_{n \rightarrow \infty} \frac{3n^4+2n^2}{3n^3+2n^2-5n} = \lim_{n \rightarrow \infty} n \cdot \frac{3n^3+2n}{3n^3+2n^2-5n} = \lim_{n \rightarrow \infty} n \cdot \frac{\frac{3n^3}{n^3} + \frac{2n}{n^3}}{\frac{3n^3}{n^3} + \frac{2n^2}{n^3} - \frac{5n}{n^3}} = \lim_{n \rightarrow \infty} n \cdot \frac{3 + 2/n^2}{3 + 2/n - 5/n^2} \quad (\text{By Mol})$$

$$= \lim_{n \rightarrow \infty} n \cdot \frac{3 + 2/n^2}{3 + 2/n - 5/n^2} = \lim_{n \rightarrow \infty} n \cdot \frac{3}{3} = \lim_{n \rightarrow \infty} n = +\infty$$

$\textcircled{E.}$ If $\lim_{n \rightarrow \infty} S_{2n} = 2$ and $\lim_{n \rightarrow \infty} S_{2n+1} = 2$, then show that $\lim_{n \rightarrow \infty} S_n = 2$.

S_{2n} - even terms of S_n $S_{2n} \cup S_{2n+1} = S_n$

S_{2n+1} - odd terms of S_n

We have:

$$S_{2n} \text{ index } = 0 \quad 2 \quad 4 \quad 6 \quad 8$$

$$S_n \text{ index } = 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$S_{2n+1} \text{ index } = 1 \quad 3 \quad 5 \quad 7 \quad 9$$

$$\lim_{n \rightarrow \infty} \inf S_{2n} \leq \lim_{n \rightarrow \infty} \inf S_n \leq \lim_{n \rightarrow \infty} \inf S_{2n+1} = 2$$

$$\lim_{n \rightarrow \infty} \sup S_{2n} \leq \lim_{n \rightarrow \infty} \sup S_n \leq \lim_{n \rightarrow \infty} \sup S_{2n+1} = 2$$

$$\lim_{n \rightarrow \infty} \inf S_n = \lim_{n \rightarrow \infty} \sup S_n = 2$$

$$\lim_{n \rightarrow \infty} S_n = 2$$