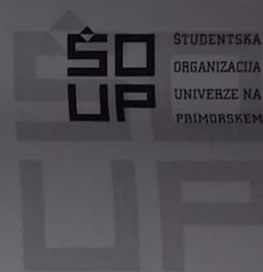




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[A] i)

$$\lim_{n \rightarrow \infty} \frac{1}{n^5} = 0$$

$$\forall \varepsilon > 0 \quad \exists N \text{ s.t. } n > N \Rightarrow |s_n - s| < \varepsilon$$
$$n^5 > N^5$$

Then we take $\varepsilon = \frac{1}{N^5}$ $0 < \frac{1}{n^5} < \frac{1}{N^5} \Rightarrow \left| \frac{1}{n^5} - 0 \right| = \frac{1}{n^5} < \varepsilon$

Since $\frac{1}{N^5}$ is positive

ii) $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n}} = 0$

$$\forall \varepsilon > 0 \quad \exists N \text{ s.t. } n > N \Rightarrow |s_n - s| < \varepsilon$$

$$\left| \frac{1}{\sqrt[5]{n}} - 0 \right| = \frac{1}{\sqrt[5]{n}} < \varepsilon \quad 1 < \varepsilon \sqrt[5]{n}$$
$$\sqrt[5]{n} > \frac{1}{\varepsilon}$$

then we take $N = \frac{1}{\varepsilon^5}$ s.t. $n > N \quad n > \frac{1}{\varepsilon^5} \Rightarrow \frac{1}{\sqrt[5]{n}} < \varepsilon = \frac{1}{\sqrt[5]{N}}$



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B) i) $\lim_{n \rightarrow \infty} (4+n)/2n = 1/2$

$$\forall \epsilon > 0, \left| \frac{4+n}{2n} - \frac{1}{2} \right| < \epsilon$$

$$\left| \frac{4+n - n}{2n} \right| < \epsilon \quad \left| \frac{4}{2n} \right| < \epsilon \quad \frac{2}{n} < \epsilon$$
$$n > \frac{2}{\epsilon}$$

for $N = \frac{2}{\epsilon}$ if $n > N$ we have that

$$\Downarrow$$
$$|s_n - s| = \left| \frac{2}{n} \right| < \epsilon$$

ii) $\lim_{n \rightarrow \infty} 2/n + 3/(n+1) = 0$

$$\forall \epsilon > 0 \quad |s_n - s| = \left| \frac{2}{n} + \frac{3}{n+1} - 0 \right|$$



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Proof:

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PRIMORSKEM

[C] $\{s_n\} \{t_n\}$ $s_n = t_n$ ~~except~~ except some values of n

Let s_n and t_n be the last different elements.

then we have $\forall \epsilon > 0; n > N_{s_m} \Rightarrow |s_m - s| < \epsilon$

and $n > N_{t_m} \Rightarrow |t_m - s| < \epsilon$

we take $N = \max(N_{s_m}, N_{t_m})$ s.t.

$$\left. \begin{array}{l} n > N \\ n > N \end{array} \right\} \begin{array}{l} \Rightarrow |s_m - s| < \epsilon \\ \Rightarrow |t_m - s| < \epsilon \end{array}$$

so now we have $N_0 = N + 1$ for which $\left. \begin{array}{l} n > N_0 \Rightarrow |s_m - s| < \epsilon \\ n > N_0 \Rightarrow |t_n - s| < \epsilon \end{array} \right\}$

But now we know that $s_n = t_n$ so:

$$n > N_0 \Rightarrow |t_n - s| < \epsilon$$

for $N_0 = N + 1$



KUP;

1D i) $(S_n), (t_n)$ Bounded sequences s.t. S_n and t_n are Bounded

$$|S_n + t_n| \leq |S_n| + |t_n|$$

$$-(|S_n| + |t_n|) \leq S_n + t_n \leq |S_n| + |t_n|$$

so, every element of $(S_n + t_n)$, even the biggest one, will have a bigger number in $|S_n| + |t_n|$ (which are numbers from a Bounded set)

And that also goes for it's least element that always has a smaller or equal $-(|S_n| + |t_n|)$.

ii) S_n has a range which is a Bounded set containing all the possible values we can get from S_n .

$X = \mathbb{R}$ Let's say that the set is Bounded by x and y .

$$S = (x, y) \quad x \leq S_n \leq y$$

$$T = \{x S_n : S_n \in S\}$$

$$x \leq S_n \quad S_n \leq y$$

$$x x \leq x S_n \quad x S_n \leq x y \quad \text{meaning that } x x \text{ is a}$$

lower bound and $x y$ is an upper bound of T which now is a Bounded set containing all elements of $(x \cdot S_n)$

Thus $(x \cdot S_n)$ is Bounded.



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[E] i) $\lim_{n \rightarrow \infty} S_n = s$

$a \in \mathbb{R}$

prove $s > a$

assume $s < a$

$\epsilon > 0$

S_m is the biggest element that $S_m < a$

$n > m \Rightarrow S_n > S_m$

$|S_n - s| < \epsilon \quad s - \epsilon < S_n < s + \epsilon$

$S_n > s - \epsilon$

$S_n < s + \epsilon$

$S_m < a$

(*) $S_n > a$

Since $\epsilon > 0$ and $a > s$ we can take ϵ to be as small as $a - s$

then we have ~~that~~ that $S_n < s + a - s = a$

But (*) states otherwise and that makes S_m not the biggest element less than a //

ii) $a = \mathbb{R} \quad (s_n)$

example: let (s_n) be the sequence $\frac{1}{n}$

$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

$\forall \epsilon > 0 \quad n > N \Rightarrow \left| \frac{1}{n} - 0 \right| < \epsilon \quad \frac{1}{n} < \epsilon \quad 1 < \epsilon n$

$n > \frac{1}{\epsilon}$

$N = \frac{1}{\epsilon}$ will suffice,

so we know the $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

And also since $n \in \mathbb{N}^+$ we have that $\frac{1}{n}$ will always be positive so $S_n > 0$ //