(A) $1\cdot 2 + 2\cdot 3 + \dots n(n+1) = \frac{n(n+1)(n+2)}{3}$ $n \in \mathbb{N}$

base case:
$n=0$

$0 \cdot 1 = \frac{0(0+1)(0+2)}{3}$

$0 = 0$

$n=1$

$1 \cdot 2 = \frac{1(1+1)(1+2)}{3}$

$2 = \frac{2 \cdot 3}{3}$

$2 = 2$

induction step

n

$1\cdot 2 + 2\cdot 3 + \dots n(n+1) = \frac{n(n+1)(n+2)}{3}$ ⇒ Suppose it's true

$n+1$

$1\cdot 2 + 2\cdot 3 + \dots n(n+1) + (n+1)(n+2) = \frac{(n+1)(n+2)(n+3)}{3}$

$\frac{n(n+1)(n+2)}{3} + (n+1)(n+2) = \frac{(n+1)(n+2)(n+3)}{3}$

$\frac{n(n+1)(n+2) + 3(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$

$\frac{(n+1)(n+2)(n+3)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$

(B) $P_n = n^3 + n + 1$ is even $n \in \mathbb{N}$

i) for every $n \in \mathbb{N}$ we have that $P_n \Rightarrow P_{n+1}$

ii) P_n is always 1

$n=0$

$n^3 + n + 1$

$0 + 0 + 1 = 1$

$n=1$

$1+1+1=3$

$n=2$

$8+2+1=11$

$n=3$

$27+3+1=31$

$2n$ $2n+1$

$(2n)^3 + 2n + 1 = 2n^3 + 2n + 1 = 2n(n^2+1) + 1$

$n+1$

$(n+1)^3 + n + 1$

$n^3 + 3n^2 + 3n + 1$

u - user
g - group
o - other people/outside world

ctrl + C → terminate current command
ctrl + D → close shell
ctrl + L → clear screen / clear terminal
ctrl + A - beginning
ctrl + E - end
alt F → go forward
alt b → go back

d → directory
- → file
rwx rwx rwx → owner, group, everyone
r → read
w → write
x → execute

most ← → least

$\neg, \vee, \wedge, \Rightarrow, \Leftrightarrow$

$A \vee B \sim B \vee A$

$A \wedge B \sim B \wedge A$

$(A \wedge B) \wedge C \sim A \wedge (B \wedge C)$

$(A \vee B) \vee C \sim A \vee (B \vee C)$

$A \vee (B \wedge C) \sim A \vee B \wedge B \vee C$

$A \wedge (B \vee C) \sim (A \wedge B) \vee (B \wedge C)$

$A \wedge A \sim A$, $A \vee A \sim A$, $A \wedge \neg A \to \bot$, $A \vee \neg A \to \top$

$\neg(A \wedge B) \equiv (\neg A \vee \neg B)$

$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

$A \Leftrightarrow B \sim \neg A \Leftrightarrow \neg B$

$A \Leftrightarrow B \sim (\neg A \vee B) \wedge (A \vee \neg B)$

$A \Leftrightarrow B \sim (A \wedge B) \vee (\neg A \wedge \neg B)$

$A \Leftrightarrow B \sim A \Rightarrow B \wedge B \Rightarrow A$

$\neg(A \Leftrightarrow B) \sim \neg A \Leftrightarrow B$

$A \Rightarrow B \sim \neg B \Rightarrow \neg A$

$A \Rightarrow B \sim \neg A \vee B$

$A \wedge (A \Rightarrow B) \Rightarrow B$

$\neg B \wedge (A \Rightarrow B) \Rightarrow \neg A$

$\neg A \wedge (A \vee B) \Rightarrow B$

$A \wedge B \Rightarrow A$

$A \Rightarrow A \vee B$

$A \wedge \neg A \Rightarrow B$

$(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$

$(A \Rightarrow B) \Rightarrow ((C \Rightarrow A) \Rightarrow (C \Rightarrow B))$

$(A \Rightarrow B) \Rightarrow (A \wedge C \Rightarrow B \wedge C)$

$A \Rightarrow B \Rightarrow (A \wedge C \Rightarrow B \wedge C)$

$A \Rightarrow B \Rightarrow ((A \Leftrightarrow C) \Rightarrow (B \Leftrightarrow C))$

$A \Rightarrow B \Rightarrow (A \vee C \Rightarrow B \vee C)$

$(A \Leftrightarrow B) \Rightarrow (A \Rightarrow B)$

$(A \Leftrightarrow B) \Rightarrow (B \Rightarrow A)$

$A \wedge (A \Leftrightarrow B) \Rightarrow B$

$\neg A \wedge (A \Leftrightarrow B) \Rightarrow \neg B$

$B \Rightarrow (A \Leftrightarrow A \wedge B)$

$\neg B \Rightarrow (A \Leftrightarrow A \vee B)$

$A \Rightarrow (B \wedge \neg B) \Rightarrow \neg A$