Scores	
1.	
2.	
3.	
4.	
5.	
Total:	

ANA-I Foundations of Analysis
1st Midterm Examination – 13 Nov, 2018

Name ________________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may have 1 handwritten A4-sized sheet of paper, but no other notes, books, or calculators.
80 total points.

1. (6 points each) Limit calculations. For each real-valued sequence, explain whether it is convergent, divergent to $\pm\infty$, or otherwise divergent (not to $\pm\infty$). If it is convergent, find its limit. If it is divergent, find its lim sup and lim inf. You may use any theorems we have proved in class or on homework.

   (a) $s_n = \dfrac{n^3 - 3n^2}{2n^2 + 3}$.

   (b) $s_n = \dfrac{\sqrt{n+1}}{\sqrt{n}+1}$.

   (c) $s_n = (-2)^n - \dfrac{1}{n}$.

   (d) $s_n = \dfrac{(-1)^n n - 2}{n \cdot \sqrt{4n+2}}$.

2. (6 points each) Examples. Justify briefly why each has the desired properties.

   (a) Give an example of a bounded monotonic sequence that converges to 2.

   (b) Give the Dedekind cut for $1 + \sqrt[3]{3}$. (Of course, your answer should not refer to $\sqrt[3]{3}$ or other irrational numbers.)

   (c) Using our construction of $\mathbb{Q} \supseteq \mathbb{Q}^{\geq 0} \supseteq \mathbb{N}$ via two applications of the method of order pairs, write the rational number $\frac{1}{2}$ as an ordered pair of ordered pairs.

3. (8 points) Let f_n be the nth Fibonacci number (so $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$). Find (with proof) an upper bound and a positive lower bound for the sequence of ratios f_{n+1}/f_n.

(-see reverse side-)

4. (12 points) Working directly from the definition (without using the Cauchy Completeness Theorem), show that if s_n is a real Cauchy sequence, then also $3s_n$ is Cauchy.

5. Given a sequence $(s_n)_{n\in\mathbb{N}}$, define a new sequence $t_n^{(s)}$ (depending on s) by the recursive formula $t_0^{(s)} = s_0$, and $t_n^{(s)} = \max\left\{t_{n-1}^{(s)}, s_n\right\}$.

   (a) (7 points) Show that for any given sequence (s_n), the new sequence $t_n^{(s)}$ is increasing.

   (b) (7 points) Show that if s_n is bounded, then $t_n^{(s)}$ is convergent.

   (c) (4 points) Given an example of a sequence s_n so that $t_n^{(s)}$ does not converge.