Analysis I
UP - FAMNIT
Fall 2022 October 05/06, 2022

Selected exercises 01

1. Prove by induction that the sum of all positive natural numbers until n is $\frac{n(n+1)}{2}$.
2. Prove that for all positive natural numbers n
$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$
3. Prove that $3 + 11 + \cdots + (8n - 5) = 4n^2 - n$, for all positive integers n.
4. Prove the following statements for every positive natural number n:
   (a) $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$.
   (b) $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.
5. Prove that for any $n \in \mathbb{N}$, the integer $11^n - 4^n$ is a natural number multiple of 7.
6. Prove that $4^n + 15n - 1$ is divisible by 9 for all natural numbers n.
7. Prove associativity for addition in $\mathbb{Z}$ and $\mathbb{Q}$.
8. Prove that the set of all even integers is a group for addition.
9. Prove the Bernoulli inequality: for every natural number n and for every number $a > -1$ it holds that:
$$(1 + a)^n \geq 1 + na.$$
10. Prove that $n^{n+1} > (n + 1)^n$ for all natural numbers $n \geq 3$.