

A)

i) ~~$x = \sqrt{10}$~~ $x = \sqrt{10}$

$$x = A \setminus B = \{x \in \mathbb{R}^{\geq 0} : x^2 < 10\} \mid \{x \in \mathbb{R}^{\geq 0} : x^2 \geq 10\}$$

ii) ~~$x = \sqrt[3]{2}$~~ $x = \sqrt[3]{2}$

$$x = A \setminus B = \{x \in \mathbb{R}^{\geq 0} : x^3 < 2\} \mid \{x \in \mathbb{R}^{\geq 0} : x^3 \geq 2\}$$

iii) ~~$x = \sqrt[3]{2} + \sqrt{3}$~~ $x = \sqrt[3]{2} + \sqrt{3}$

 $x = A \setminus B = \sqrt[3]{2}$ - we already have this in (ii)

$$y = \sqrt{3}$$

$$y = A \setminus B = \{y \in \mathbb{R}^{\geq 0} : y^2 < 3\} \mid \{y \in \mathbb{R}^{\geq 0} : y^2 \geq 3\}$$

$$\sqrt[3]{2} + \sqrt{3} = x + y = A \setminus B = \{x + y \in \mathbb{R}^{\geq 0} : x^3 < 2 \in A_x, y^2 < 3 \in A_y, x, y \in \mathbb{R}^{\geq 0}\}$$

$$B := \mathbb{R}^{\geq 0} \setminus A$$

B.

1) $A \cup B = \mathbb{Q}$

2) $a \in A$ $a = x + y$ $b < a$
 $b < x + y$

$$x_1 < x \text{ so } x_1 \in A$$

$$\text{but } b = x_1 + y \text{ so } b \in A$$

$$b < x = y \text{ so } y_1 < y \text{ so } y_1 \in A$$

$$b = x + y_1 \text{ so } b \in A$$

3) B is a non empty set as $z \in B$, $w \in B$, $z, w \in B$
A (so A has no greatest element since A_1, A_2 do not have a greatest element)

$$(if x = y \in A, x \cdot y \in A, \text{ for any } x > x \text{ in } A)$$

Homework 2

Marko Taleski 89201031

$F = \{0, 1\}$ is a field with 2 operations $(+, \cdot)$
only if the following is true:

I) $\langle F \setminus \{0\}, \cdot \rangle$ is a group with identity 1

- (i) - is ~~not~~ closed under \cdot

- (ii) - \cdot is associative

- (iii) - has identity 1 $(1 \cdot 1 = 1 \cdot 1 = 1)$

- (iv) - has inverse $(1 \cdot 1 = 1 \cdot 1 = 1)$

II) $\langle F, + \rangle$ is a group with identity 0

- (i) - is closed under $+$

- (ii) - $+$ is associative

- (iii) - has identity 0 $(0 + 0 = 0 + 0 = 0)$

- (iv) - has inverse $(0 + 0 = 0 + 0 = 0)$

III) $+, \cdot$ are commutative $0 + 1 = 1 + 0$ $1 \cdot 1 = 1 \cdot 1$

IV)

$+, \cdot$ are distributive $0 \cdot (0 + 1) = (0 \cdot 0) + (0 \cdot 1)$