





A. Let He set of all points (x,ylin R² s.t. x² +xy+3y²=3. Show that H is closed subset of R² (considered with the Eaclidean Metric). Is H bounded?

H= { (x,y) | x2 + xy + 3y 2=3}

Now we let (x_n, y_n) be $-7(x_iy)$ and $(y_n, x_n) \in H$ s.t. $x_n^2 + x_n y_n + 3y_n^2 = 3$. Now we fore if as a limit: $\lim_{n \to \infty} (x_n^2 + y_n y_n + 3y_n^2) = 3 = > x^2 + xy + 3y^2 = 3 = > (x_iy) \in H$

To check i) it's 60 and ed: (asing math (ogiclisty) $x^2 + xy + 3y^2 = 3$ = $7\frac{7}{2}x^2 + \frac{7}{2}(x+y)^2 + \frac{5}{2}y^2 = 3/2$ $x^2 + (x+y)^2 + 5y^2 = 6$ $x^2 \le 6$ and $5y^2 \le 6$

=7 lyl <V and lyles

which means (x,y) E[-V6, V6]x.[-V6] x.[-V6] -> thise

from which we conclude that AEM.

[X1, y1] -> (x,y) (=> x1-7 x N yn-7 y)

Now we prove:

We prove in opposite direction E(E): Let $x_n - 7x \wedge y_n - 7y_n$ so we have: $f(x_1 - x_1) + f(x_1 - x_1) = f(x_1 - x_1) + f(x_1 - x_1) = f(x_1 - x_1) + f(x_1 - x_1) +$

>7 lim X1 = X







B. Given a metric space M with metric d, verily that any E-ball is an open set.

- Let point a & M.

From the E-Gall det, we have

BE(a)={xeM|d(x, a) < E}, or Br(a).

From taxing Ex as follows: 2 E* = F = ily. a)

and KEBs Cyl we get d(x,y)<E*

From A-inequality, we get.

d(x, a) = d(x, y) + d(y, a) < E-d(y, a), which is equal to E* d(x, a) = d(x, y) + d(y, a) < E*+d(y, a), - 11 - to E or 2(x, a) < E

From the above we get that $B_{\epsilon}^{(d)}$ a is open ball we also have that $B_{\epsilon}^{(d)}(y) \subseteq B_{\epsilon}^{(d)}(a)$

With this, from the open set det, we have that De (a) is open set, which means that -any E-ball is an open set.





Prove in an Er. The Dicture on p73 of the notes may be helpful.

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there is I such that IZO. And, we apply the ded. for open sets, where we choose ICE, from which we get da (a161cd =7 dz (a161c & @)

prove that these metrices definned the same open set A

then Bo (a, d) c Bo (a, E)

If A is an open set in do, then Bo NEzo,

Bola, d) c A = 7 Bola, Ele A

Which means A is open set.

The now, the dela, bled, a, bl, then Be la, Ele B, la, Sl.

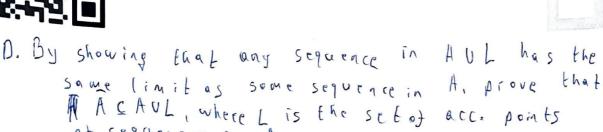
If A is an open set in de, then Be la, Ele A, de on Ezo,

Bela, Ele A => B, la, Sle A.

Which means A is open set







By knowing thet Lis the set of acc. points of a seq. in A, we have that

AUL contains all the points ar.

of sequences in A.

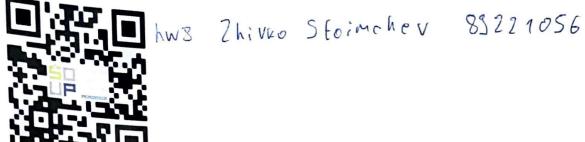
Bernase II .. I centains the limit of seq. ion A.

Because HUL centains all limit soints of A, we conclude that AVL is closed,

By letting a EMA (for our example), we have a EA, which is the same as a EA \A.

Since aEA is trivial, we work with a EA lA, and we get that a & A and a & A.

And because A doesn't contain limit points, we conclude that a is a limit of point of A, where a EL So, from a EA and a EL, we conclude that A LAUL.







E. Show that both R and R2 may be covered by countably many open balls

We have Q= U{Vn}

In = (Vn-1, Vn +1), where Vn EQ

Uln=R s.t. => ln SR n Ulngh

(going backwards)

(=) x=R 5.E. = 1 (x-\frac{1}{3}, x+\frac{1}{3} \n Q \dagger o, where \(\tau \) is the cross-section!

which means that 3 VX FQ and VX E(X-1/3, X+1/3) where => XE(TX-1, VX fil for some VX.

Because of Ux - x/ < = 2 c1, we have that RCUI,

 $Q^2 = \bigcup \{(x_1)_A\}$

In = B(Vn, ya)

=> 1 = { (x, y) | V(x-x)2 + (y-y6)2 < 1} Uln=R2

(going backwards) (= | We now let (xx) Eh2, and get B(x,y)(== {x, y) | d(x, y), (x,y) | == } B(x,y)(3/n (2)

from which we can conclude lastry math logic that:]: [Z, . Z] + B and V(x-Z)2+y-Z2/2 < 3 < 7

to have at last d ((x,y), (zn, Zz)) < 1 which means that the

(x,y) = 1