

Algebra I

2. KOLOKVIJ

– 18. JANUAR 2022 –

Čas pisanja: 135 minut. Maksimalno število točk: 100. Dovoljena je uporaba pisala in kalkulatorja. Pišite razločno in utemeljite vsak odgovor. Srečno!

- Kdaj je matrica simetrična? Zapišite definicijo transponirane matrike in naštejte vsaj tri lastnosti transponiranja. Dokažite naslednjo izjavo: Za vsako kvadratno matriko A je $A + A^T$ simetrična matrika. (6 točk)
 - Zapišite definicijo $n \times n$ determinante (z uporabo permutacij) in naštejte vsaj štiri primere uporabe determinant. (6 točk)
 - Zapišite in dokažite Cramerjevo pravilo za reševanje sistema linearnih enačb. (8 točk)
- Za katere vrednosti $\beta \in \mathbb{R}$ bo imel naslednji sistem linearnih enačb neskončno mnogo rešitev? Za vse dobljene vrednosti tudi poiščite rešitve.

$$\begin{aligned}x + y + z &= 1 \\2x + y + 4z &= \beta \\4x + y + 10z &= \beta^2\end{aligned}$$

(20 točk)

- Za katere vrednosti $a \in \mathbb{R}$ bo determinanta matrike

$$A = \begin{bmatrix} 1 & a & 3 & 2 \\ 2 & 2 & -2 & 1 \\ 3 & 3 & -5 & 1 \\ 4 & 4 & -7 & 5 \end{bmatrix}$$

enaka 30?

(20 točk)

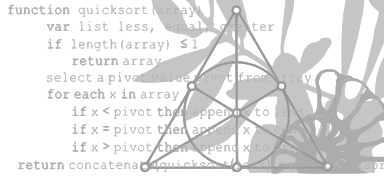
- Za matrike $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ in $C = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$, rešite matrično enačbo $(A + 3I)(X - I) = B$. (10 točk)

- Naj bo $E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in \mathbb{R}^{2 \times 2}$. Pokažite da za matriko $A \in \mathbb{R}^{2 \times 2}$ velja

$$\det(A) = 1 \Leftrightarrow A^T E A = E.$$

(10 točk)

- Zapišite matriko $A = \begin{bmatrix} 2 & -2 & 1 \\ 6 & -1 & 5 \\ 3 & 7 & 4 \end{bmatrix}$ v obliki $A = LU$, kje je L spodnje trikotna matrika ki ima na glavni diagonali vse elemente enake 1, in U zgornje trikotna matrika. (20 točk)



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MIDTERM 2
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Time: 135 minutes. Maximum number of points: 100. You are allowed to use a pen and a calculator. Write clearly, and justify all your answers. Good luck!

- When is a matrix symmetric? Give the definition of a transpose of a matrix and write at least three properties of transposing. Prove the next statement: For every square matrix A the matrix $A + A^T$ is symmetric as well. (6 points)
 - State the definition of a $n \times n$ determinant (using permutations) and give at least four examples of use of the determinant. (6 points)
 - Write down and prove Cramer's rule for solving systems of linear equations. (8 points)
- For which values of $\beta \in \mathbb{R}$ will the next system have infinitely many solutions? For each value that you got compute also the solutions.

$$\begin{aligned}x + y + z &= 1 \\2x + y + 4z &= \beta \\4x + y + 10z &= \beta^2\end{aligned}$$

(20 points)

- For which values $a \in \mathbb{R}$ will the determinant of the matrix

$$A = \begin{bmatrix} 1 & a & 3 & 2 \\ 2 & 2 & -2 & 1 \\ 3 & 3 & -5 & 1 \\ 4 & 4 & -7 & 5 \end{bmatrix}$$

equal 30?

(20 points)

- For matrices $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$, solve the matrix equation $(A + 3I)(X - I) = B$. (10 points)

- Let $E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in \mathbb{R}^{2 \times 2}$. Show that for matrix $A \in \mathbb{R}^{2 \times 2}$ it holds that

$$\det(A) = 1 \Leftrightarrow A^T E A = E.$$

(10 points)

- Write the matrix $A = \begin{bmatrix} 2 & -2 & 1 \\ 6 & -1 & 5 \\ 3 & 7 & 4 \end{bmatrix}$ in the form $A = LU$, where L is a lower triangular matrix with all coefficient on the main diagonal equal to 1, and U is an upper triangular matrix. (20 points)