Homework 7 Marko Taleski 89201031

A) It is a metric if the following statements are true

For $x, y, z \in M$:

- $d(x,y) \geq 0$; $d(x,y) = 0 <=> x = y$
- $d(x,y) = d(y,x)$
- $d(x,z) \leq d(x,y) + d(y,z)$

Now we prove for $d((x_1, x_2, x_3), (y_1, y_2, y_3)) =$

$= |x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3|$ in $\mathbb{R}^3$

* $x = y$ $\quad d(x,y) = 0 <=> x = y$

$$\begin{cases} x_1 = y_1 \\ x_2 = y_2 \\ x_3 = y_3 \end{cases} \qquad \begin{matrix} x_1 - y_1 = 0 \\ x_2 - y_2 = 0 \\ x_3 - y_3 = 0 \end{matrix}$$

$d(x,y) \geq$ only if $x \neq y$ $\quad d(x,y) > 0$

$|x_1 - y_1| > 0$

$|x_2 - y_2| > 0$

$|x_3 - y_3| > 0$ $\blacksquare$

This holds because we are using distances

* $d(x,y) = d(y,x)$

$|x_1 - y_1| = |y_1 - x_1|$

$|x_2 - y_2| = |y_2 - x_2|$

$|x_3 - y_3| = |y_3 - x_3|$ $\blacksquare$

This holds because we are using absolute value

* $d((x_1, x_2, x_3), (y_1, y_2, y_3)) = |x_1 - y - z_1 + z_1| + |x_2 - y_2 - z_2 + z_2| + |x_3 - y_3 - z_3 + z_3|$

$= |x_1 - z_1| + |z_1 - y| + |x_2 - z_2| + |z_2 - y_2| + |x_3 - z_3| + |z_3 - y_3|$

$\leq d(x,z) + d(z,y) + d(x,z) + d(z,y) + d(x,z) + d(z,y) =$

$= d(x,z) + d(z,y)$ $\blacksquare$

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i) $\sum_{n=1}^{\infty} \frac{2^n+3^n}{5^n} = \sum_{n=1}^{\infty} \frac{2^n}{5^n} + \sum_{n=1}^{\infty} \frac{3^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n + \sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n =$

$= \frac{1}{1-\frac{2}{5}} + \frac{1}{1-\frac{3}{5}} = \frac{1}{\frac{5-2}{5}} + \frac{1}{\frac{5-3}{5}} = \frac{1}{\frac{3}{5}} + \frac{1}{\frac{2}{5}} =$

$= \frac{5}{3} + \frac{5}{2} = \frac{10+15}{6} = \frac{25}{6}$ convergent

ii) $\sum_{n=1}^{\infty} \sin^n(3)$

$\sum_{n=1}^{\infty} \sin^n(3)$ converges to $\frac{1}{1-\sin(3)}$ because of the rule

$\sum_{n=1}^{\infty} r^n = \frac{1}{1-r}$ if $|r|<1$ and $\sin 3 = 0{,}14112$

iii) $\sum_{n=1}^{\infty} \frac{1}{n+1}$ it is divergent because $\frac{1}{n}$ diverges to infinity when we add 1 to n it doesn't change the fact that it diverges