Scores	
1.	
2.	
3.	
4.	
5.	
Total:	

ANA-I Foundations of Analysis
1st Midterm Examination – 18 Nov, 2019

Name ________________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may have 1 handwritten A4-sized sheet of paper, but no other notes, books, or calculators.
72 total points.

1. (6 points each) Limit calculations. For each real-valued sequence, explain whether it is convergent, divergent to $\pm\infty$, or otherwise divergent (not to $\pm\infty$). If it is convergent, find its limit. If it is divergent, find its $\limsup$ and $\liminf$. You may use any theorems we have proved in class or on homework.

   (a) $s_n = \dfrac{2n-1}{n-\sqrt{2}}$

   (b) $s_n = \dfrac{(-1)^n \cdot 2n - 1}{4-5n}$

   (c) $s_n = \dfrac{(-1)^n \cdot 2\sqrt{n} + 12}{2n-3}$

   (d) $s_n = n - n^2$

2. (6 points each) Examples. Justify your answers briefly.

   (a) Give an example of a bounded sequence that diverges.

   (b) Using our construction of $\mathbb{Z}$ by the method of order pairs, explain why $2 - 3 = -1$.

   (c) Give the Dedekind cut for $\sqrt{3} + \sqrt{2}$. (Of course, your answer should not directly refer to irrational numbers such as $\sqrt{3}$ or $\sqrt{2}$.)

3. (12 points) Let (r_n) be a bounded sequence of real numbers from $[1/10, \infty)$ (that is, each entry r_n is $\geq 1/10$), and (s_n) be a sequence of real numbers with $\lim\limits_{n\to\infty} s_n = \infty$. Working directly from the definitions, show that also $\lim\limits_{n\to\infty} r_n \cdot s_n = \infty$.

4. (10 points) Consider the sequence recursively defined by the rule $s_n = 2s_{n-1}/n$ for $n \geq 1$, with the initial value of $s_0 = 1$. Show that s_n converges, and find its limit.

5. (8 points) Using any technique from this class that you like, show that if (s_n) is a Cauchy sequence of positive real numbers, then the sequence $\left(\sqrt{s_n}\right)$ is also Cauchy.