Week 2

① Solution of equation $5x = 7$ is $x = \frac{5}{7}$.

$\frac{5x}{5} = \frac{7}{5} \Rightarrow x = \frac{7}{5}$ $\boxed{\text{False}}$

② Equation $0 \cdot x = 7$ has infinitely many solutions.

Since $0 \cdot x = 0 \quad \forall x$ $\boxed{\text{False}}$

③ Equation $3 \cdot x = 0$ has infinitely many solutions.

$\frac{3 \cdot x}{3} = \frac{0}{3} \Rightarrow x = 0$ $\boxed{\text{False}}$

④ Let $x + y = 3$ is a given equation with 2 unknowns, x & y. This equation has infinitely many solutions.

Notice that $y = -x + 3$ is a linear equation.
Hence, $(x, y) = (t+3, t)$, $t \in \mathbb{R}$ is a general solution.
Since $\mathbb{R}$ is infinite, this is true! $\boxed{\text{True}}$

⑤ Equation $x + y + z = 7$ with 3 unknowns, $x, y,$ and z, is not possible to solve.

$4 + 2 + 1 = 7$
$1 + 1 + 5 = 7$
$\vdots$

Let $y = t \in \mathbb{R}$, $z = s \in \mathbb{R}$
then, $x = 7 - t - s$ $\boxed{\text{False}}$

$(x, y, z) = (7 - t - s, t, s)$
is a solution!

⑥ A set of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is linearly independent set if from the equation $\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \dots + \alpha_n \vec{v}_n = 0$ it follows that $\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$.

This is the definition of linear independence so $\boxed{\text{TRUE}}$

Ex: $\vec{v}_1 = (1,0,0)$
$\vec{v}_2 = (0,1,0)$
$\vec{v}_3 = (0,0,1)$

if $\alpha \vec{v}_1 + \beta \vec{v}_2 + \gamma \vec{v}_3 = (\alpha, \beta, \gamma) = (0,0,0)$
it must be that $\alpha = \beta = \gamma = 0$

⑦ If there exists some linear combination $\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \dots + \alpha_n \vec{v}_n = 0$ in which at least one of the integers α_i differs from 0, then the set of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is a linearly dependent set.

By definition, this is true. $\boxed{\text{TRUE}}$
Moreover, we can see that some $\alpha_i = \vec{0}$
and since $0 \cdot \alpha_j = \alpha_i$, $j \neq i$, it is not lin. ind.

⑧ A set of vectors that contains $\vec{0}$ is linearly dependent.

$\boxed{\text{TRUE}}$

⑨ A vector $\vec{v}$ of length 1 is called unit vector. $\boxed{\text{TRUE}}$

⑩ The unit vector $\vec{u}$ in the direction of the vector from $P(1,0,1)$ to $Q(3,2,0)$ is $\vec{u} = (2/3, 2/3, -1/3)$.

$$\vec{PQ} = \vec{v} = (2, 2, -1).$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$

$\boxed{\text{TRUE}}$

$$|\vec{v}| = \sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$\vec{u} = \frac{\vec{v}}{3} = (2/3, 2/3, -1/3)$$

⑪ Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ be vectors. The dot product of $\vec{u}$ and $\vec{v}$ is the number $u_1v_1 + u_2v_2 + u_3v_3$. The dot product is denoted as $\langle \vec{u}, \vec{v} \rangle$ or with $\vec{u} \cdot \vec{v}$.

$\boxed{\text{TRUE}}$

⑫ The given set of vectors are linearly independent: $\vec{u} = (-1,1,1)$, $\vec{v} = (1,2,3)$, and $\vec{w} = (0,1,8)$.

$$\alpha\vec{u} + \beta\vec{v} + \gamma\vec{w} = \vec{0}$$

$$\alpha(-1,1,1) + \beta(1,2,3) + \gamma(0,1,8) = (0,0,0)$$

$\boxed{\text{TRUE}}$

$$(-\alpha, \alpha, \alpha) + (\beta, 2\beta, 3\beta) + (0, \gamma, 8\gamma) = (0,0,0)$$

$$\begin{bmatrix} -\alpha & \beta & 0 \\ \alpha & 2\beta & \gamma \\ \alpha & 3\beta & 8\gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \alpha = \beta$$

$$\begin{aligned} -\alpha + \beta &= 0 \\ \alpha + 2\beta + \gamma &= 0 \\ \alpha + 3\beta + 8\gamma &= 0 \end{aligned}$$

$$\rightarrow \begin{aligned} 3\alpha + \gamma &= 0 \Rightarrow \gamma = -3\alpha \\ 4\alpha + 8\gamma &= 0 \end{aligned}$$

$$4\alpha + 8 \cdot {-3}\alpha = 0$$
$$4\alpha - 24\alpha = 0$$
$$-20\alpha = 0 \Rightarrow \boxed{\alpha = 0} \Rightarrow \boxed{\beta = \gamma = 0}$$

⑬ The given set of vectors are linearly independent: $\vec{u} = (-1, 1, 21)$, $\vec{v} = (1,2,3)$, and $\vec{w} = (0,1,8)$.

$$\alpha(-1,1,21) + \beta(1,2,3) + \gamma(0,1,8) = (0,0,0)$$

$\boxed{\text{FALSE}}$

$$\begin{aligned} -\alpha + \beta &= 0 \rightarrow \alpha = \beta \\ \alpha + 2\beta + \gamma &= 0 \rightarrow 3\alpha + \gamma = 0 \Rightarrow \gamma = -3\alpha \\ 21\alpha + 3\beta + 8\gamma &= 0 \rightarrow 24\alpha + 8(-3\alpha) = 0 \end{aligned}$$

$$24\alpha - 24\alpha = 0$$

(14) The standard unit vectors in $\mathbb{R}^3$ are $\vec{i} = (-1,0,0)$, $\vec{j} = (0,-1,0)$, and $\vec{k} = (0,0,1)$.

$\boxed{\text{FALSE}}$

(15) To find the unit vector $\vec{u}$ in the direction of the vector $\vec{PQ} = (2,2,1)$ we need to multiply $\vec{PQ}$ with $\frac{1}{2}$. As a solution we will get that $\vec{u} = (1,1,\frac{1}{2})$ is a unit vector.

$\boxed{\text{FALSE}}$

$$\vec{u} = \frac{\vec{PQ}}{|\vec{PQ}|} \qquad |\vec{PQ}| = \sqrt{2^2+2^2+1^2} = 3$$

$\vec{u} = \frac{1}{3}\vec{PQ}$ to be a unit vector

Moreover, for any unit vector, $\vec{w}$, $|\vec{w}| = 1$.

$$|\vec{u}| = \sqrt{1^2+1^2+(\tfrac{1}{2})^2} = \sqrt{\tfrac{5}{2}} \neq 1$$

(16) One of the algebraic properties of the dot product is that additivity holds: $\langle u+v, w\rangle = \langle u,w\rangle + \langle v,w\rangle$.

$\boxed{\text{TRUE}}$

$$\begin{aligned} LHS &= (u_1+v_1)\cdot w_1 + \cdots + (u_n+v_n)\cdot w_n \\ &= u_1\cdot w_1 + v_1\cdot w_1 + \cdots + u_n\cdot w_n + v_n\cdot w_n \\ &= u_1\cdot w_1 + \cdots + u_n\cdot w_n + v_1\cdot w_1 + \cdots + v_n\cdot w_n = RHS \end{aligned}$$

(17) One of the algebraic properties of the dot product is that homogeneity holds: $\langle \alpha u, v\rangle = \alpha\langle u,v\rangle$.

$\boxed{\text{TRUE}}$

$$\begin{aligned} LHS &= \alpha u_1\cdot v_1 + \cdots + \alpha u_n\cdot v_n \\ &= \alpha(u_1\cdot v_1 + \cdots + u_n\cdot v_n) = RHS \end{aligned}$$

(18) One of the algebraic properties of the dot product is that symmetricity holds: $\langle u,v\rangle = \langle v,u\rangle$

$\boxed{\text{TRUE}}$

$$\begin{aligned} LHS &= u_1\cdot v_1 + \cdots + u_n\cdot v_n \\ &= v_1\cdot u_1 + \cdots + v_n\cdot u_n = RHS \end{aligned}$$

Exercises for Beginners

① Solve the following systems of two linear equations with two unknowns

(i) $\begin{cases} 4x - 3y = -5 \\ 6x + 2y = 12 \end{cases}$

$$\left[\begin{array}{cc|c} 4 & -3 & -5 \\ 6 & 2 & 12 \end{array}\right] \xrightarrow[\frac{R_2}{2}]{\frac{R_1}{4}} \left[\begin{array}{cc|c} 1 & -3/4 & -5/4 \\ 3 & 1 & 6 \end{array}\right]$$

$$\xrightarrow{R_2 - 3 \cdot R_1} \left[\begin{array}{cc|c} 1 & -3/4 & -5/4 \\ 0 & 1 + \frac{9}{4} & 6 + 15/4 \end{array}\right] \xrightarrow[\text{up}]{\text{clean}} \left[\begin{array}{cc|c} 1 & -3/4 & -5/4 \\ 0 & 13/4 & 39/4 \end{array}\right]$$

$$\xrightarrow{R_2 \cdot \frac{4}{13}} \left[\begin{array}{cc|c} 1 & -3/4 & -5/4 \\ 0 & 1 & 3 \end{array}\right] \Rightarrow \boxed{y = 3} \Rightarrow x - 3/4 \cdot 3 = -5/4$$

$$x - \frac{9}{4} = -5/4 \Rightarrow \boxed{x = 1}$$

(ii) $\begin{cases} 4x - 3y = -5 \\ 8x - 6y = -10 \end{cases}$

$$\left[\begin{array}{cc|c} 4 & -3 & -5 \\ 8 & -6 & -10 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|c} 4 & -3 & -5 \\ 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow 4x - 3y = -5$ (infinitely many solutions)

(iii) $\begin{cases} x + y = 4 \\ 2x + 2y = 10 \end{cases}$

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 2 & 2 & 10 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 0 & 2 \end{array}\right] \lightning$$

$0 \neq 2$! No solution !

② Solve the following system of three linear equations with three unknowns.

(i) $\begin{cases} x - 3y + z = -1 \\ x + 3y - z = 5 \\ 2x + y - 2z = 0 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 1 & 3 & -1 & 5 \\ 2 & 1 & -2 & 0 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 0 & 6 & -2 & 6 \\ 0 & 7 & -4 & 2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 0 & 7 & -4 & 2 \\ 0 & 6 & -2 & 6 \end{array}\right]$$

$$\xrightarrow{R_2-R_3} \left[\begin{array}{ccc|c} 1 & -3 & 1 & -1 \\ 0 & 1 & -2 & -4 \\ 0 & 6 & -2 & 6 \end{array}\right] \xrightarrow[R_3-6R_2]{R_1+3R_2} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -13 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 10 & 30 \end{array}\right] \xrightarrow{\frac{R_3}{10}} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -13 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow[R_2+2R_3]{R_1+5R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow \boxed{\begin{array}{l} x=2 \\ y=2 \\ z=3 \end{array}}$$

(ii) $\begin{cases} x + 3y + z = 11 \\ x - 3y - z = -7 \\ 2x - 6y - 2z = -13 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 11 \\ 1 & -3 & -1 & -7 \\ 2 & -6 & -2 & -13 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 11 \\ 0 & -6 & -2 & -18 \\ 0 & -12 & -4 & -35 \end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 11 \\ 0 & -6 & -2 & -18 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\Rightarrow 0 = 1$ ↯ $\boxed{\text{No solution}}$

(iii) $\begin{cases} x + 3y + z = 11 \\ x - 3y - z = -7 \\ 2x - 6y - 2z = -14 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 11 \\ 1 & -3 & -1 & -7 \\ 2 & -6 & -2 & -14 \end{array}\right] \xrightarrow{R_3-2R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 11 \\ 1 & -3 & -1 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c} 2 & 0 & 0 & 4 \\ 1 & -3 & -1 & -7 \end{array}\right]$$

$\Rightarrow \boxed{x = 2}$

$R_2 \Rightarrow x - 3y - z = -7$
$2 - 3y - z = -7$
$-3y - z = -9$
$3y + z = 9$
$z = 9 - 3y$

$$\boxed{\begin{array}{l} x = 2 \\ y = s \\ z = 9 - 3s \end{array}}$$

Standard Exercises

① Let $\vec{a} = (1,2,1)$, $\vec{b} = (1,-1,2)$, and $\vec{c} = (5,-1,-3)$.
Compute $\vec{a}+\vec{b}$, $\vec{a}-2\vec{c}$, $(\vec{a}\cdot\vec{b})\cdot\vec{c}$ and $\vec{a}\cdot(\vec{b}\cdot\vec{c})$.

$$\vec{a}+\vec{b} = (1,2,1)+(1,-1,2) = (1+1,\ 2-1,\ 1+2) = \boxed{(2,1,3)}$$

$$\vec{a}-2\vec{c} = (1,2,1) - 2(5,-1,-3) = (1-10,\ 2+2,\ 1+6) = \boxed{(-9,4,7)}$$

$$(\vec{a}\cdot\vec{b})\cdot\vec{c} = (1\cdot 1 + 2\cdot(-1) + 1\cdot 2)\cdot\vec{c} = (1-2+2)\cdot\vec{c}$$

$$= 1\cdot(5,-1,-3) = \boxed{(5,-1,-3)}$$

$$\vec{a}\cdot(\vec{b}\cdot\vec{c}) = \vec{a}\cdot(1\cdot 5 + (-1)\cdot(-1) + 2\cdot(-3)) = \vec{a}\cdot(5+1-6)$$

$$= \vec{a}\cdot 0 = \boxed{\vec{0}}$$

② Can you express the vector $(3,-7,-3)$ as a linear combination of vectors $(1,0,-2)$, $(-4,3,8)$, and $(2,5,-4)$?

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right] \xrightarrow{R_3 + 2\cdot R_1} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right] \Rightarrow 0 = 3 \ \lightning$$

verified true ✓ { The vector $(3,-7,-3)$ can not be expressed as a linear combination of the given 3 vectors.

③ Are vectors $(8,2,-14)$, $(3,1,-5)$, and $(-2,0,4)$ linearly independent?

(★) $\alpha\cdot(8,2,-14) + \beta(3,1,-5) + \gamma(-2,0,4) = (0,0,0)$?

$$\begin{cases} 8\alpha + 3\beta - 2\gamma = 0 \\ 2\alpha + \beta = 0 \\ -14\alpha - 5\beta + 4\gamma = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 8 & 3 & -2 & 0 \\ 2 & 1 & 0 & 0 \\ -14 & -5 & 4 & 0 \end{array}\right] \xrightarrow[R_3 + 7R_2]{R_1 - 4R_2} \left[\begin{array}{ccc|c} 0 & -1 & -2 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right] \xrightarrow{R_3 + 2R_1} \left[\begin{array}{ccc|c} 0 & -1 & -2 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow \boxed{\alpha = \gamma}$ & $\boxed{\beta = -2\alpha}$

NOTE: $\alpha=\beta=\gamma=0$ is an option... Let's check if a nontrivial solution holds!

Pick $\alpha = 1$. $\Rightarrow \gamma = 1$, $\beta = -2$

Plug into (★):

$(8,2,-14) + (-2)(3,1,-5) + (-2,0,4)$

$= (8,2,-14) + (-6,-2,10) + (-2,0,4)$

$= (0,0,0)$ ☺

So, there exists a nontrivial solution for α, β, γ $\Rightarrow$ Vectors are not linearly independent ☺

④ For which values of x does the vector $(x,-3,-5)$ lie on the same plane as vectors $(1,0,-2)$ and $(-2,1,7)$?

$\vec{x} = (x,-3,-5)$, $\vec{a} = (1,0,-2)$, $\vec{b} = (-2,1,7)$

$\vec{x} = \alpha\vec{a} + \beta\vec{b}$

$(x,-3,-5) = (\alpha, 0, -2\alpha) + (-2\beta, \beta, 7\beta)$

$\Rightarrow \begin{cases} \beta = -3 \\ \alpha = -8 \end{cases}$ and $-5 = -2\alpha + 7\beta$ and $x = \alpha - 2\beta$

$-5 = -2\alpha + 7\cdot(-3)$

$-5 = -2\alpha - 21$

$16 = -2\alpha$

$x = -8 - 2(-3)$

$\boxed{x = -2}$

Problems from exam

① Show that vectors $\vec{a}$ and $\vec{b}$ are linearly independent if and only if vectors $\vec{a}-\vec{b}$ and $\vec{a}+\vec{b}$ are linearly independent.

($\Rightarrow$) Assume $\vec{a}$ and $\vec{b}$ are linearly independent.
Consider the equation

$$\alpha(\vec{a}+\vec{b}) + \beta(\vec{a}-\vec{b}) = 0.$$

Would like to show $\alpha = \beta = 0$.

$$\alpha(\vec{a}+\vec{b}) + \beta(\vec{a}-\vec{b}) = \alpha\vec{a} + \alpha\vec{b} + \beta\vec{a} - \beta\vec{b} = (\alpha+\beta)\vec{a} + (\alpha-\beta)\vec{b} = 0$$

$\{\vec{a}, \vec{b}\}$ linearly independent $\Rightarrow \alpha+\beta = \alpha-\beta = 0$

$\Rightarrow 2\alpha = 0$ and $2\beta = 0 \Rightarrow \alpha = 0, \beta = 0$

($\Leftarrow$) Assume that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are linearly independent.
Assume towards the contradiction that $\vec{a}$ and $\vec{b}$ are linearly dependent.
$\Rightarrow \exists\, \alpha$ s.t. $\vec{a} = \alpha\vec{b}$ (or $\vec{a} - \alpha\vec{b} = 0$)

Case 1: $\alpha = 1 \Rightarrow \vec{a} = \vec{b} \Rightarrow \vec{a} - \vec{b} = \vec{0}$ ↯

Case 2: $\alpha = -1 \Rightarrow \vec{a} = -\vec{b} \Rightarrow \vec{a} + \vec{b} = \vec{0}$ ↯

Case 3: $\alpha \neq \pm 1$. We know $\alpha_1(\vec{a}+\vec{b}) + \alpha_2(\vec{a}-\vec{b}) = \vec{0}$ iff $\alpha_1 = \alpha_2 = 0$.

Consider $\alpha = 2\alpha_1 - 1$ and $\alpha_2 = -(\alpha_1 - 1)$ $\rightarrow$ $\alpha_2 = 0 \Rightarrow \alpha_1 \neq 0$, $\alpha_1 = 0 \Rightarrow \alpha_2 \neq 0$

Then, $\alpha_1(\vec{a}+\vec{b}) + \alpha_2(\vec{a}-\vec{b}) = (\alpha_1+\alpha_2)\vec{a} + (\alpha_1 - \alpha_2)\vec{b}$

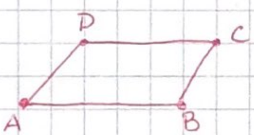
$= \vec{a} + \alpha\vec{b} = 0$ ↯

② In the given triangle $\triangle OAB$, C is the point on the line segment AB such that C divides line segment AB into the ratio 3:2 and let point D lie on the line segment OA such that D divides OA in the ratio 5:2. Denote by P the intersection point between line segments OC and BD. Find the ratio in which P divides OC and BD.

Similar to week 1 exercise !

③ Let $A(-3, 2, \lambda)$, $B(3, -3, 1)$, and $C(5, \lambda, 2)$ denote three vertices of parallelogram $\square ABCD$.

(a) Find coordinates of the vertex D.
(b) Calculate parameter λ so that $|\overrightarrow{AD}| = \sqrt{14}$
(c) Use part (b) and for the bigger value of λ, check is the set $\{\overrightarrow{AD}, \overrightarrow{BD}, \overrightarrow{AC}\}$ linearly independent. If the set is linearly dependent, write the vector $\overrightarrow{AC}$ as a linear combination of vectors $\overrightarrow{AD}$ and $\overrightarrow{BD}$.



(a) $\overrightarrow{BC} = (5-3,\ \lambda-(-3),\ 2-1) = (2, \lambda+3, 1)$

$D = A + \overrightarrow{BC} = (-3, 2, \lambda) + (2, \lambda+3, 1) = \boxed{(-1, \lambda+5, \lambda+1)}$

(b) $\overrightarrow{AD} = \overrightarrow{BC} = (2, \lambda+3, 1)$, $\sqrt{14} = |\overrightarrow{AD}| = \sqrt{(2)^2 + (\lambda+3)^2 + (1)^2}$

$14 = 4 + \lambda^2 + 6\lambda + 9 + 1$
$0 = \lambda^2 + 6\lambda$
$0 = \lambda(\lambda+6) \Rightarrow \boxed{\lambda = 0}$ or $\boxed{\lambda = -6}$

(c) $\vec{0} = \alpha \overrightarrow{AD} + \beta \overrightarrow{BD} + \gamma \overrightarrow{AC}$
$= \alpha(2,3,1) + \beta(-4,8,0) + \gamma(8,-2,2)$

$0 = 2\alpha - 4\beta + 8\gamma$
$0 = 3\alpha + 8\beta - 2\gamma$
$0 = 1\alpha + 2\gamma$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 3 & 8 & -2 & 0 \\ 2 & -4 & 8 & 0 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 8 & -8 & 0 \\ 0 & -4 & 4 & 0 \end{array}\right] \Rightarrow \boxed{\text{linearly dependent}}$$

$\overrightarrow{AC} = \alpha \overrightarrow{AD} + \beta \overrightarrow{BD}$
$(8, -2, 2) = (2\alpha, 3\alpha, \alpha) + (-4\beta, 8\beta, 0)$
$\boxed{\alpha = 2}$
$8 = 2\alpha - 4\beta$
$8 = 4 - 4\beta \Rightarrow \boxed{\beta = -1}$