# TCS1 - 1st midterm

UP FAMNIT

22.11.2022

Name: _____

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| Exercise | Points | Total |
|---|---|---|
| **1** | | 10 |
| **2** | | 10 |
| **3** | | 10 |
| **4** | | 10 |
| **Total** | | 40 |

**Instructions:**

1. You have **90 minutes** to complete the examination. As a courtesy to your classmates, we ask that you not leave during the last fifteen minutes.
2. When you finish please staple your pieces of papers that you would like to submit. The stapler will be provided. All pieces of the submitted paper must contain your name and student ID.
3. Please do not shuffle your solutions. The solution of each exercise should be uninterrupted on the same page, or at least on adjacent pages. Justify all your answers.
4. You may use one (1) double-sided A4 pages with notes that you have prepared. You may not use any other resources, including lecture notes, books, or other students.
5. Please sign the Honor Code statement below.

## Good luck!

In recognition of and in the spirit of the University of Primorska Honor Code, I certify that I will neither give nor receive unpermitted aid on this examination.

Signature: _____

**1.** Let $A$, $B$, $C$, and $D$ be atomic propositions. We are given a proposition

$$\mathcal{I} = A \wedge ((\neg A \Rightarrow B) \Rightarrow \neg(\neg A \wedge \neg B)) \wedge C.$$

(a) Write the truth table of $\mathcal{I}$.

(b) Write the **negation** of $\mathcal{I}$ in conjuncive normal form.

(c) Write $\mathcal{I}$ by using connectives $(\Rightarrow, \neg)$ only[1].

(d) Draw the switching circuit equivalent to $\mathcal{I}$.

**2.** Show the correctness of the following implications. If the statement is correct, prove it without using a truth table. If the statement is false, find a counterexample.

(a) $(A \Rightarrow B) \wedge (A \Rightarrow C) \wedge A \Rightarrow \neg A \wedge B \wedge C$

(b) $\neg A \wedge (A \Leftrightarrow B) \Rightarrow \neg B$

(c) $(A \Rightarrow B) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \Rightarrow C))$

**3.** On an island of knights and servants, it is November 22nd, i.e. *the dancing day*. This means people stand in circles, holding hands, dancing. It is a time of joy, everybody is happy, and distinguishing knights from servants is particularly difficult[2].

(a) Four individuals, namely Barack, Donald, George, and Joe (in cyclic order), are holding hands in a circle. They all seem to be saying:

*"I am in contact with precisely one knight!"*

Is this possible? If so, find at least one arrangement of knights/servants, regarding the mentioned four individuals. Can you find a solution where Donald is servant? Find all solutions!

(b) On the other side of the village, another circle formed. They are named (in cyclic order) Bill, Elon, Jeff, Sam, Satoshi, and Warren, and seem to be having a great time too! Holding hands, they all chant the very same sentence:

*"I am in contact with precisely one knight!"*

Is this possible? If so, find at least one arrangement of knights/servants, regarding the mentioned four individuals. Can you find a solution where Sam is servant? Find all solutions!

$(\cdot)$ Suppose $n$ individuals is dancing in a circle, holding hands, and chanting:

*"I am in contact with precisely one knight!"*

Determine the number of legal solutions, for arbitrary value of $n$. Justify your answer.[3]

**4.** Prove:

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

[1] **Hint:** simplify $\mathcal{I}$ first!

[2] It should be noted that each of individuals is touching precisely two other individuals, that is, no genetic mutations or disabilities are considered. Circles are of length at least three.

[3] Solving this part gives an extra bonus of up to 5 points.