TOR I – Discrete Structures I

– Problem Set 1 –

1. The following two propositions are given:
   A: "It is cold outside."
   B: "It is raining outside.".
   Write the following compound propositions in natural language: (a) $\neg A$, (b) $A \wedge B$, (c) $A \vee B$, (d) $B \vee \neg A$.

2. The following two propositions are given:
   A: "Janez is rich."
   B "Janez is happy."
   Write the following propositions symbolically:

   (a) If Janez is rich, then he is unhappy.

   (b) Janez is neither happy nor rich.

   (c) Janez is happy only if he is poor.

   (d) Janez is poor if and only if he is unhappy.

3. Find truth tables for the examples from the previous task.

4. Solve the following problems about knights and servants (knights are always telling the truth, while servants always lie):

   (i) Artur: It is not true that Cene is a servant.
   Bine: Cene is a knight or I am a knight.
   Cene: Bine is a servant.
   For each of them determine whether they are knights or servants!

   (ii) Artur: Cene is a servant or Bine is a servant.
   Bine: Cene is a knight and Artur is a knight.
   For each of them determine whether they are knights or servants!

   Rešitev. Označimo: A – Arthur je vitez, B – Bine je vitez, C – Cene je vitez.
   (i) Velja naslednja sestavljena izjava

   $$(A \Leftrightarrow C) \wedge (B \Leftrightarrow C \vee B) \wedge (C \Leftrightarrow \neg B).$$

   S pomočjo pravilnostne tabele vidimo, da je izjava pravilna le za nabor $A = 1$ in $B = C = 0$.
   (ii) Velja naslednja sestavljena izjava

   $$(A \Leftrightarrow \neg C \vee \neg B) \wedge (B \Leftrightarrow C \wedge A).$$

S pomočjo pravilnostne tabele vidimo, da je izjava pravilna le za nabor $A = 1$ in $B = C = 0$.

5. Using basic connectives $\neg$ and $\wedge$ express the following compound propositions:

   (i) $A \vee B$

   (ii) $A \Rightarrow B$

   (iii) $A \Leftrightarrow B$

6. Verify the validity of the following equivalences:

   (i) $A \wedge (B \vee C) \Leftrightarrow \neg(A \wedge B) \Rightarrow (A \wedge C)$

   (ii) $\neg A \wedge (A \Rightarrow B) \Leftrightarrow A \Rightarrow \neg A \wedge B$

   (iii) $A \vee B \vee C \Leftrightarrow \neg(A \vee B) \Rightarrow C$

   (iv) $(A \Rightarrow B) \wedge (B \Rightarrow A) \Leftrightarrow (A \wedge B) \vee (\neg A \wedge \neg B)$