Homework 4

(A) $\lim_{n\to\infty} S_n = 3$ $\quad \lim_{n\to\infty} t_n = \infty$ $\quad \lim_{n\to\infty} S_n \cdot t_n = \infty$

want to show: $S_n \cdot t_n > M$

$[n > N] \Rightarrow |S_n - 3|$ [redacted] $< \varepsilon$ $\qquad [n \Rightarrow N] \Rightarrow [t_n > M_0]$

$3 - S_n < \varepsilon$ $\qquad M_0 = \dfrac{M}{3-\varepsilon}$

$3 - \varepsilon < S_n$

$\forall M > 0, \exists N$ s.t. $[n \Rightarrow N] \Rightarrow \left[S_n \cdot t_n > \cancel{(3-\varepsilon)} \cdot \dfrac{M}{\cancel{(3-\varepsilon)}} = M\right]$

5/5 (B) if $\lim_{n\to\infty} S_n = \infty$, then $\lim_{n\to\infty} \sqrt{S_n} = \infty$

want to show: $n > N \Rightarrow \sqrt{S_n} > M$

$\forall M > 0\ \exists N$ such that $[n > N] \Rightarrow [S_n > M]$

We take an $M_0 = M^2$

and for that have $n > N \Rightarrow S_n > M_0 \Leftrightarrow S_n > M^2 \Leftrightarrow \sqrt{S_n} > M$

$\forall M > 0, \exists N$ such that $[n > N] \Rightarrow [S_n > M]$

5/5 (C) i) $\displaystyle \lim_{n\to\infty} \frac{3n^2+1}{2n^2+2n-6} \overset{\div n^2}{=} \lim_{n\to\infty} \frac{\frac{3n^2}{n^2} + \frac{1}{n^2}}{\frac{2n^2}{n^2} + \frac{2n}{n^2} - \frac{6}{n^2}} = \lim_{n\to\infty} \frac{3 + \frac{1}{n^2} \to 0}{2 + \frac{2}{n} \to 0 - \frac{6}{n^2} \to 0} = \frac{3}{2}$ (By AoL)

ii) $\displaystyle \lim_{n\to\infty} \frac{\sqrt{n^2+4n}}{2n+3} \overset{\div n}{=} \lim_{n\to\infty} \frac{\sqrt{\frac{n^2}{n^2} + \frac{4n}{n^2}}}{\frac{2n}{n} + \frac{3}{n}} = \lim_{n\to\infty} \frac{\sqrt{1 + 4/n \to 0}}{2 + \frac{3}{n} \to 0} = \frac{\sqrt{1}}{2} = \frac{1}{2}$ (By AoL)

iii) $\displaystyle \lim_{n\to\infty} \frac{n^3+2}{3n^3+2n^2+5n} \overset{\div n^3}{}$

$\displaystyle = \lim_{n\to\infty} \frac{\frac{n^3}{n^3} + \frac{2}{n^3}}{\frac{3n^3}{n^3} + \frac{2n^2}{n^3} + \frac{5n}{n^{3}}} = \lim_{n\to\infty} \frac{1 + 2/n^3 \to 0}{3 + 2/n \to 0 + 5/n^2 \to 0} = \frac{1}{3}$ By AoL

D. i) $\lim\limits_{n\to\infty} \dfrac{2n^2+2}{5n^3-3n-1} = \lim\limits_{n\to\infty} \dfrac{1}{n} \cdot \dfrac{2n^2+2}{5n^2-3-1/n} \stackrel{\cdot / : n^2}{=} \lim\limits_{n\to\infty} \dfrac{1}{n} \cdot \dfrac{\frac{2n^2}{n^2}+\frac{2}{n^2}}{\frac{5n^2}{n^2}-\frac{3}{n^2}-\frac{1}{n^3}} =$

(By AoL) By AoL

$= \lim\limits_{n\to\infty} \dfrac{1}{n} \cdot \dfrac{2+\cancel{2/n^2}^{\to 0}}{5-\cancel{3/n^2}_{\to 0}-\cancel{1/n^3}_{\to 0}} = \lim\limits_{n\to\infty} \dfrac{1}{n} \cdot \dfrac{2}{5} = 0 \cdot \dfrac{2}{5} = 0$

ii) $\lim\limits_{n\to\infty} \dfrac{\sqrt{4n^3+4n}}{2n-2} = \lim\limits_{n\to\infty} \dfrac{\cancel{2}\sqrt{n^3+n}}{\cancel{2}(n-1)} \stackrel{\cdot / : n}{=} \lim\limits_{n\to\infty} \dfrac{\sqrt{\frac{n^3}{n^2}+\frac{n}{n^2}}}{\frac{n}{n}-\frac{1}{n}} =$

$= \lim\limits_{n\to\infty} \dfrac{\sqrt{n+\cancel{\frac{1}{n}}^{\to 0}}}{1-\cancel{\frac{1}{n}}_{\to 0}} = \lim\limits_{n\to\infty} \dfrac{\sqrt{n}}{1} = \lim\limits_{n\to\infty} \sqrt{n} = +\infty$

iii) $\lim\limits_{n\to\infty} \dfrac{3n^4+2n^2}{3n^3+2n^2-5n} = \lim\limits_{n\to\infty} n \cdot \dfrac{3n^3+2n}{3n^3+2n^2-5n} \stackrel{\cdot / : n^3}{=} \lim\limits_{n\to\infty} \dfrac{\frac{3n^3}{n^3}+\frac{2n}{n^3}}{\frac{3n^3}{n^3}+\frac{2n^2}{n^3}-\frac{5n}{n^3}} \cdot n =$

By AL

$= \lim\limits_{n\to\infty} n \cdot \dfrac{3+\cancel{2/n^2}^{\to 0}}{3+\cancel{2/n}_{\to 0}-\cancel{5/n^2}^{\to 0}} = \lim\limits_{n\to\infty} n \cdot \dfrac{3}{3} = \lim\limits_{n\to\infty} n = +\infty$

E. If $\lim\limits_{n\to\infty} S_{2n} = 2$ and $\lim\limits_{n\to\infty} S_{2n+1} = 2$, then show that $\lim\limits_{n\to\infty} S_n = 2$.

4/5

S_{2n} - even terms of S_n $\quad S_{2n} \cup S_{2n+1} = S_n$

S_{2n+1} - odd terms of S_n

$S_{2n} \le S_n \le S_{2n+1}$

doesn't work tho

We have:

S_{2n} index	=	0	2	4	6	8
S_n index	=	0	1	2	3	4
S_{2n+1} index	=	1	3	5	7	9

have to

the rel. between index and value is not clear

$\liminf\limits_{n\to\infty} S_{2n} \le \liminf\limits_{n\to\infty} S_n \le \liminf\limits_{n\to\infty} S_{2n+1} = 2$

$\limsup\limits_{n\to\infty} S_{2n} \le \limsup\limits_{n\to\infty} S_n \le \limsup\limits_{n\to\infty} S_{2n+1} = 2$

$\liminf\limits_{n\to\infty} S_n = \limsup\limits_{n\to\infty} S_n = 2$

$\lim\limits_{n\to\infty} S_n = 2$