Izpit

25. januar 2017

IME IN PRIIMEK: ______________________ VPISNA ŠT.: ☐☐☐☐☐☐☐☐

ŠTUDIJSKI PROGRAM: ______________________ LETNIK: ______________

1. (20 points) For the following logical statement draw the truth table, write down the canonical disjunctive and conjunctive forms and the logical circuit for this statement.

$$(A \vee B) \Rightarrow \neg(C \wedge B \Rightarrow A)$$

2.(15 points) Draw teh diagrams for the following categories

a. Category $\mathbb{A}$: Objects: A,B,C,D,E , Maps: 1_A, 1_B, 1_C, 1_D, 1_E $f : A \to B$, $g : B \to C$, $h : C \to D$, $i : D \to A$

b. Category $\mathbb{B}$: Objects: A,B,C,D,E, Maps: 1_A, 1_B, 1_C, 1_D, 1_E $f : A \to B$, $g : B \to A$, $h : C \to D$, $i : D \to E$

c. For the diagram below answer the following questions (note that the identities are not drawn). Write down the following images for the functor.

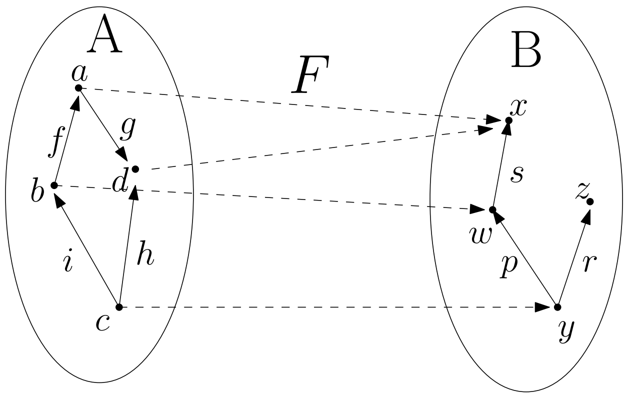
i. $F(a) =$

ii. $F(f) =$

iii. $F(g) =$

iv. $F(f \circ i) =$

v. $F(h) =$



3. (15 points) Prove $A \subseteq B \Rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$.

4. (15 points) Prove $f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$

5. (15 points) Prove $(A \Rightarrow B) \Rightarrow (A \vee C) \Rightarrow (B \vee C)$

6. (12 points) Determine whether the following statements are true or false

(a) If f is an injective function then $f^{-1}|_{\mathrm{im} f}$ is a function. YES NO

(b) If the consequence is a tautology then we can infer that the antecedent is a contradiction. YES NO

(c) If f is and injective function and g is an injective function then $g \circ f$ and $f \circ g$ are injective. YES NO

(d) $E = f^{-1}(f(E))$ YES NO

7. (12 points)Draw the following Venn diagrams

a. $A \cap B \cap C \neq \emptyset$

b. $(A \cup B) \subseteq C$

c. $A \cap B \neq \emptyset$, $A \subset C$, $B \subset C$ and mark $\bar{C} \backslash (A \cap B)$

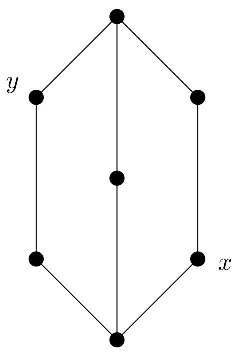
8. (15 points) Draw the Hasse diagram for the power set $\mathcal{P}(A)$, $A = \{1, 2, 3\}$ for teh following ordering relations

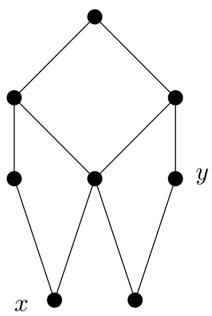
a. $xRy \Leftrightarrow x \subseteq y$

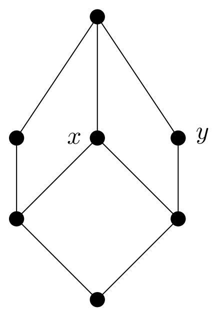
b. $xRy \Leftrightarrow y \backslash x = y$

c. $xRy \Leftrightarrow (|x| = |y|) \wedge \left(\sum_{i \in x} i \leq \sum_{j \in y} j\right)$. (Here $|\cdot|$ is the number of elements of a set and $\sum_{i \in x} i$ is the sum of the elements in set x. For example, if $x = A$, then $\sum_{i \in x} i = 1 + 2 + 3 = 6$)

9.(12 points) For the following Hasse diagrams mark $\inf(x, y)$, $\sup(x, y)$ if they exist and the immediate ancestors of x. Also mark the first element (initial element) if it exists and say whether the diagram represesnts a lattice (mreža)?







10.(12 points) How many possible functions are there $f : A \to B$ between the sets and also write how many bijections there are.

a. $A = \{0, 1, 2\}$, $B = \{0, 1, 2\}$

b. $A = \{0, 1, 2, 3\}$, $B = \{1, 2\}$

c. $A = \{0, 2\}$, $B = \{1, 2, 3\}$

11. (12 points) Write the negation of each statement with qualifiers:

a. $(\forall x)(\exists y)(x \in S \wedge y \in S \wedge x > y)$

b. $(\exists! x)(x \in S \wedge x \neq 2)$

c. $(\forall)(x \in S \wedge (x \leq 0 \Rightarrow x \leq -1))$

12. (9 points) Write down the definition of the intersection $\cap_{\lambda \in J} A_\lambda$ for the family if sets $\{A_\lambda; \lambda \in J\}$, where J is an arbitrary indexing set.