Univerza na Primorskem
UP FAMNIT
Študijsko leto 2021/2022

Algebra I

1. kolokvij

– 23. november 2021 –

Čas pisanja: 135 *minut. Maksimalno število točk:* 100. *Dovoljena je uporaba pisala in kalkulatorja. Pišite razločno in utemeljite vsak odgovor. Srečno!*

1. (a) Zapišite definicijo skalarnega produkta in naštejte vsaj 3 njegove lastnosti. Zapišite in dokažite karakterizacijo pravokotnosti med dvema vektorjema s pomočjo skalarnega produkta. *(6 točk)*

   (b) Zapišite definicijo mešanega produkta in njegov geometrijski pomen. *(6 točk)*

   (c) Dokažite naslednjo trditev: Razdalja d med nevzporednima premicama $\ell_1 = \vec{r}_1 + \lambda_1 \vec{v}_1$ in $\ell_2 = \vec{r}_2 + \lambda_2 \vec{v}_2$ $(\lambda_1, \lambda_2 \in \mathbb{R})$ se izračuna kot

   $$d = \frac{|\langle \vec{v}_1 \times \vec{v}_2, \vec{r}_2 - \vec{r}_1 \rangle|}{|\vec{v}_1 \times \vec{v}_2|}.$$

   (8 točk)

2. Točke $A(3,2,t), B(3,-3,1)$ in $C(5,t,2)$ predstavljajo oglišča paralelograma $ABCD$.

   (a) Za $t = 1$ izračunajte koordinate točke D. *(6 točk)*

   (b) Za katere vrednosti $t \in \mathbb{R}$ bo $|\overrightarrow{AC}| = \sqrt{22}$? *(7 točk)*

   (c) Za katere vrednosti $t \in \mathbb{R}$ bo točka A oddaljena od izhodišča za 7 enot? *(7 točk)*

3. Dane imamo vektorje $\vec{a} = (8-t, 3, -1-t)$, $\vec{b} = (7,1,0)$ in $\vec{c} = (7,7,0)$. Poiščite vse vrednosti $t \in \mathbb{R}$, za katere bo $\measuredangle(\vec{a}, \vec{b}) = \measuredangle(\vec{a}, \vec{c}) = \varphi$ in določite ta kot.

   Namig: Kot med dvema vektorjema dobimo s pomočjo enačbe $\cos\varphi = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| \cdot |\vec{v}_2|}$. *(20 točk)*

4. Zapišite enačbo premice ℓ, ki vsebuje točko $A(1,2,1)$, je vzporedna z ravnino $x - y + z = 4$ in seka premico $p = (0,0,2) + \lambda(1,2,0)$. Izračunajte tudi presečišče med premicama ℓ in p. *(20 točk)*

5. Premica p je podana s presečiščem ravni $3x - 2y + z + 3 = 0$ in $4x - 3y + 4z + 1 = 0$. Poiščite vrednosti $\alpha, \beta \in \mathbb{R}$ za katere je premica p pravokotna na ravnino $\alpha x + 8x + \beta z = -2$. *(20 točk)*

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Academic year 2021/2022

Algebra I
Midterm 1
– November 23, 2021 –

Time: 135 *minutes. Maximum number of points:* 100*. You are allowed to use a pen and a calculator. Write clearly, and justify all your answers. Good luck!*

1. (a) Give the definition of the scalar (dot) product and state at least 3 of its properties. Write and prove the characterisation of orthogonality between two vectors in terms of the scalar product. *(6 points)*

   (b) Give the definition of the box product and state its geometrical meaning. *(6 points)*

   (c) Prove the next claim: The distance d between two non-parallel lines $\ell_1 = \vec{r}_1 + \lambda_1 \vec{v}_1$ and $\ell_2 = \vec{r}_2 + \lambda_2 \vec{v}_2$ ($\lambda_1, \lambda_2 \in \mathbb{R}$) is computed as
$$d = \frac{|\langle \vec{v}_1 \times \vec{v}_2, \vec{r}_2 - \vec{r}_1 \rangle|}{|\vec{v}_1 \times \vec{v}_2|}.$$
   (8 points)

2. Points $A(3,2,t), B(3,-3,1)$ and $C(5,t,2)$ are corners of the parallelogram $ABCD$.

   (a) For $t = 1$ find the coordinates of D. *(6 points)*

   (b) For which values $t \in \mathbb{R}$ will $|\overrightarrow{AC}| = \sqrt{22}$? *(7 points)*

   (c) For which values $t \in \mathbb{R}$ will A be at distance 7 from the origin? *(7 points)*

3. We are given vectors $\vec{a} = (8-t, 3, -1-t)$, $\vec{b} = (7,1,0)$ and $\vec{c} = (7,7,0)$. Find all values $t \in \mathbb{R}$, for which $\measuredangle(\vec{a}, \vec{b}) = \measuredangle(\vec{a}, \vec{c}) = \varphi$ and compute that angle.

   Hint: The angle between two vectors can be obtained from the equation $\cos\varphi = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| \cdot |\vec{v}_2|}$. *(20 points)*

4. Find the line ℓ that contains point $A(1,2,1)$, is parallel to the plane $x - y + z = 4$ and intersects the line $p = (0,0,2) + \lambda(1,2,0)$. Compute also the intersection between lines ℓ and p. *(20 points)*

5. The line p is given by the intersection of planes $3x - 2y + z + 3 = 0$ and $4x - 3y + 4z + 1 = 0$. Find the values $\alpha, \beta \in \mathbb{R}$ for which the line p is orthogonal to the plane $\alpha x + 8x + \beta z = -2$. *(20 points)*