TCS1 - 2nd midterm

UP FAMNIT

19.1.2022

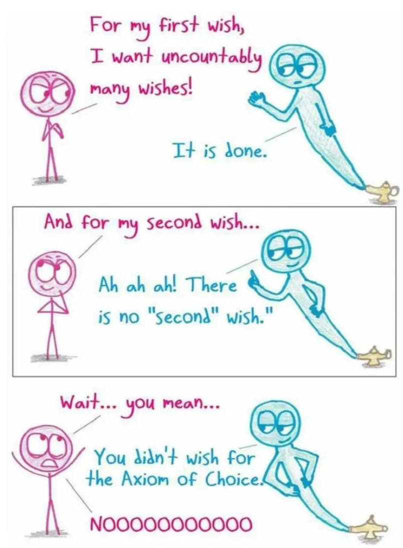
Name: ______________________________

Enrollment ID.: ______________________@student.upr.si

Exercise	Points
1	9
2	8
3	9
4	8
5	6
Total	40

Instructions:

1. You have **three 90 minutes** to complete the examination. As a courtesy to your classmates, we ask that you not leave during the last fifteen minutes.
2. When you finish please stample your pieces of papers that you would like to submit. The stampler will be provided. All pieces of the submitted paper must contain your name and student ID..
3. Please do not shuffle your solutions. The solution of each exercise should be uninterrupted, i.e. on adjacent pages.
4. You may use one (1) double-sided A4 pages with notes that you have prepared. You may not use any other resources, including lecture notes, books, or other students.
5. Please sign the Honor Code statement below.



In recognition of and in the spirit of the University of Primorska Honor Code, I certify that I will neither give nor receive unpermitted aid on this examination.

Signature: ______________________________

1. Let A be a non-empty funute set and let $S = \mathcal{P}(A)$. Define relation $R \subseteq S \times S$ by:

$$xRy \Leftrightarrow |x| = |y|,$$

where $|\cdot|$ denotes the set cardinality.

(i) Prove that R is an equivalence relation.

(ii) Find all equivalence classes for the case $A = \{1, 2, 3\}$.

2. Let A, B and C be sets, and let $g\colon A \to B$ and $f\colon B \to C$ be functions. Prove:

(i) If f, g are injective, then so is $f \circ g$.

(ii) If f, g are injective and at the same time $g \circ f$ is not, then $f \circ g$ is surjective.

3. On $S = \{3, 5, 7, 8, 9, 10, 12, 27, 30, 70\}$ we work with the divisibility relation R:

$$xRy \Leftrightarrow x \text{ divides } y.$$

(i) Draw the Hasse diagram of R.

(ii) Find all R-minimal elements.

(iii) Find all R-maksimal elements.

(iv) Find all non-empty subsets $U \subseteq S$, so that 3 is an R-lowerbound of U.

(v) Is R a lattice?

4. For the four pairs of sets below, determine whether or not they are equipollent.

i.)	$\{2, 5, 10\}$	$\{10, 5, 3\}$
iii.)	$\{2, 5, 10\}$	$\mathbb{R}$
iv.)	$\mathbb{Z}$	$\{2, 4, 6, \ldots\}$
ii.)	$[1, \infty)$	$[-1, 1)$

5. Let $G = (V, E)$ be a graph with vertex-set $V = \{1, 2, 3, 4, 5\}$ and edge-set

$$E = \{(1, 2), (3, 2), (4, 3), (1, 4), (2, 4), (1, 3)\}.$$

(a) Draw the graph.

Find

(b) maximal degree, i.e. $\Delta(G)$,

(c) minimal degree, i.e. $\delta(G)$,

(d) the size of biggest clique, i.e. $\omega(G)$,

(e) the size of biggest independent set, i.e. $\alpha(G)$, ter

(f) the minimal number of colours needed to color the graph, i.e. $\chi(G)$.