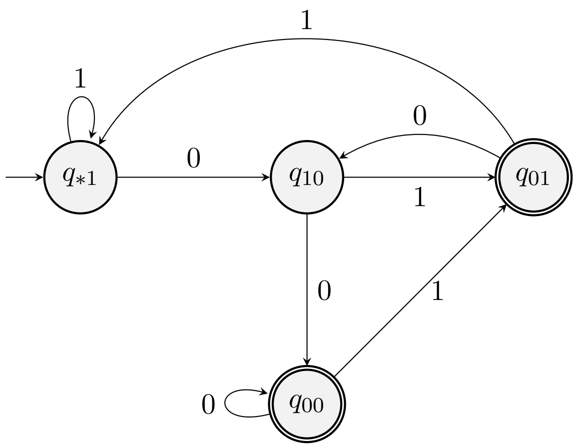
NONDETERMINISTIC FINITE AUTOMATA (NFA)

A **nondeterministic finite automaton** (NFA) can be defined as $M = (Q, \Sigma, \delta, q_0, F)$, where:

- Q is the finite set of states
- Σ : input alphabet (finite set of symbols)
- $q_0 \in Q$: starting state
- $F \subseteq Q$: set of accepting states
- $\delta : Q \times \Sigma \to \mathcal{P}(Q)$, where $\mathcal{P}(Q)$ is the power set of Q

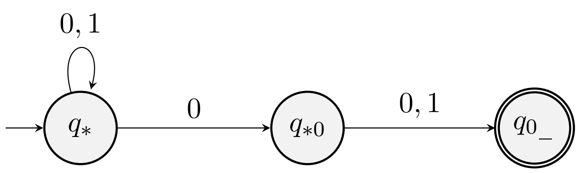
Exercise 1

Let language $L = \{$strings where the second letter from the back is 0$\}$ and alphabet $\Sigma = \{0, 1\}$. Create an NFA and DFA that accepts L.



	0	1
$\to q_{*1}$	q_{10}	q_{*1}
q_{10}	q_{00}	q_{01}
$*q_{01}$	q_{10}	q_{*1}
$*q_{00}$	q_{00}	q_{01}

DFA



	0	1
$\to q_*$	$\{q_*, q_{*0}\}$	$\{q_*\}$
q_{*0}	$\{q_{0\_}\}$	$\{q_{0\_}\}$
$*q_{0\_}$	$\emptyset$	$\emptyset$

NFA

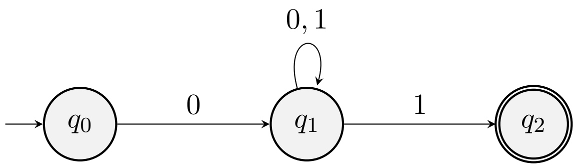
Exercise 2

Let language $L = \{$strings where the third letter from the back is 1$\}$ and alphabet $\Sigma = \{0, 1\}$. Create an NFA and DFA that accepts L.

Exercise 3

Create an NFA for each of the following languages over alphabet $\Sigma = \{0, 1\}$:

(a) $L_1 = \{0(0|1)^*1\}$



	0	1
$\rightarrow q_0$	$\{q_1\}$	$\emptyset$
q_1	$\{q_1\}$	$\{q_1, q_2\}$
$*q_2$	$\emptyset$	$\emptyset$

NFA

(b) $L_2 = \{(0|1)^*(00|11)\}$

(c) $L_3 = \{(0|1)^*1(0|1)\}$. *Is this familiar from Exercise 1?*

(d) $L_4 = \{(0|1)^*1(0|1)^k\}$. *How would you construct this if you knew the value of k?*

(e) $L_5 = \{w | w_{|1|} = 1\}$.

Exercise 4

Create NFA-s for each of the languages in Exercise 10-13 from last week (DFA).

(a) $L_{10} = \{b^n a b^m | n, m \geq 0\}$

(b) $L_{11} = \{b^n a b^m | n, m \geq 1\}$

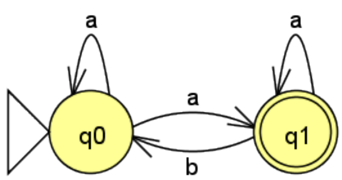
(c) $L_{12} = \{a^{2n} b^{2m+1} | n, m \geq 0\}$.

(d) $L_{13} = \{a^n b a^m | n, m \geq 0, (n + m)\%2 = 0\}$.

Exercise 5

Transform the following NFA into DFA.

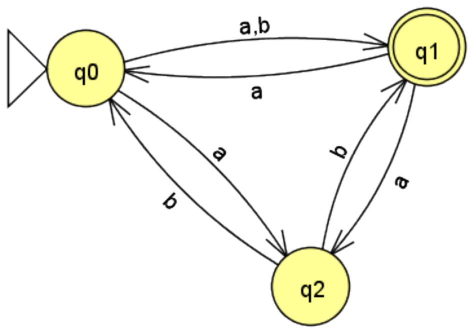
(a)



The states and transitions of the DFA version of 5.a are given in the following table:

	a	b
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\emptyset$
$*\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\emptyset$	$\emptyset$	$\emptyset$

(b)

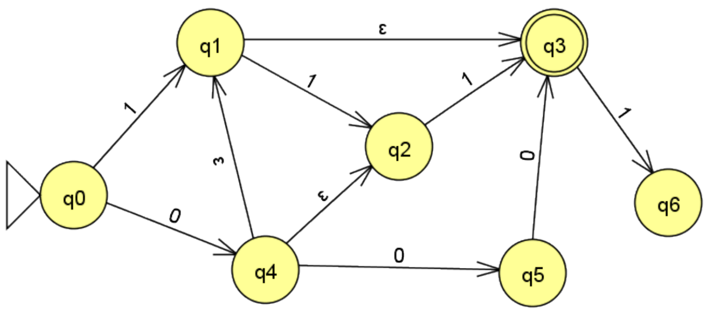


The states and transitions of the DFA version of 5.b are given in the following table:

	a	b
$\rightarrow \{q_0\}$	$\{q_1, q_2\}$	$\{q_1\}$
$*\{q_1\}$	$\{q_0, q_2\}$	$\emptyset$
$*\{q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1\}$
$*\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1\}$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$
$\emptyset$	$\emptyset$	$\emptyset$

Exercise 6

Transform the following ε-NFA into an NFA.



The transition table of the ε-NFA of exercise 6 is given below:

	0	1	ε
$\rightarrow q_0$	$\{q_4\}$	$\{q_1\}$	$\emptyset$
q_1	$\emptyset$	$\{q_2\}$	$\{q_3\}$
q_2	$\emptyset$	$\{q_3\}$	$\emptyset$
$*q_3$	$\emptyset$	$\{q_6\}$	$\emptyset$
q_4	$\{q_5\}$	$\emptyset$	$\{q_1, q_2, q_3\}$
q_5	$\{q_3\}$	$\emptyset$	$\emptyset$
q_6	$\emptyset$	$\emptyset$	$\emptyset$

The states that have a closure with size greater than 1 are:
$CL(q_1) = \{q_1, q_3\}$
$CL(q_4) = \{q_1, q_2, q_3, q_4\}$
These will be used for the transformation. The states and transitions of the transformed NFA are given by the transition table below. Notice that the transition values changed for non-ε symbols where we had states with a closure size > 1, or where the removed states from the ε column had a transition on either 0 or 1. Also notice the new accepting states q_1 and q_4, because they have an accepting state in their closure.

	0	1
$\rightarrow q_0$	$\{q_1, q_2, q_3, q_4\}$	$\{q_1, q_3\}$
$*q_1$	$\emptyset$	$\{q_2, q_6\}$
q_2	$\emptyset$	$\{q_3\}$
$*q_3$	$\emptyset$	$\emptyset$
$*q_4$	$\{q_5\}$	$\{q_2, q_3, q_6\}$
q_5	$\{q_3\}$	$\emptyset$
q_6	$\emptyset$	$\emptyset$

Exercise 7

Create an ε-NFA that accepts the following language $L = \{a^n | n\%2 = 0$ or $n\%3 = 0\}$, $\Sigma = \{a\}$. Transform it into an NFA.