Scores	
1.	
2.	
3.	
4.	
5.	
6.	
Total:	

ANA-I Foundations of Analysis
2nd Midterm Examination – 14 January 2022

Name ________________________________

General Instructions: Please answer the following, showing all your work and writing neatly. You may have 1 handwritten A4-sized sheet of paper, but no other notes, books, or calculators.
76 total points.

1. (6 points each) Limit calculations. For each real limit, explain whether it is convergent, divergent to $\pm\infty$, or otherwise divergent (not to $\pm\infty$). If it is convergent, find the limit. You may use any theorems we have proved in class or on homework.

   (a) $\lim\limits_{x\to 3} \dfrac{x^2-2x-3}{x+2}$

   (b) $\lim\limits_{x\to 3} \dfrac{x-4}{x^2-2x-3}$

   (c) $\lim\limits_{x\to 1+} \sqrt[3]{\dfrac{x^3-1}{x-1}}$
   Hint: $x^3-1=(x-1)(x^2+x+1)$.

2. (6 points each) Examples. Justify your answers briefly.

   (a) Give an example of a countable infinite, sequentially compact subset of $\mathbb{R}$.

   (b) Give an example of a real function that is increasing on the interval $(0,2)$, and decreasing on $(2,4)$.

   (c) Give examples of two series where the ratio test is inconclusive (gives no information), and where the first series converges and the second diverges.

3. (6 points each) For each series, determine whether it converges absolutely, converges conditionally, or diverges.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cos n}{3^n-2}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}+(-1)^n}{2\sqrt{n}-3n^3}$

(-see reverse side-)

4. (6 points) Explain how to reorder the terms of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ so that the resulting series sums to 2022.

5. (12 points) Working directly from definitions, prove that if f, g, and h are real functions so that $\lim_{x\to 2} f(x) = 3$, $\lim_{x\to 2} h(x) = 3$, and so that for all x on the interval $[1, 4]$ it holds that $f(x) \leq g(x) \leq h(x)$, then also $\lim_{x\to 2} g(x) = 3$. Your answer should involve the letter ϵ.

6. (10 points) Show that if $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$ is a sequence of "nested" nonempty sequentially compact subsets of $\mathbb{R}^2$, then $\bigcap_{n=1}^{\infty} A_n$ is nonempty.
   Hint: Construct a sequence with an accumulation point in the intersection.