Automatic Flight Control Systems

- Autopilot
  - Controls a/c trajectory w/o requiring input from pilot
- Stability Augmentation System (SAS) ← adjusting stability derivatives C_{n_β}
  - Designed to improve dynamic stability
  - Relies on sensors, not on pilot inputs
- Control Augmentation System (CAS) ← adjusting control derivatives L_{δ_a}
  - Pilot input has two paths to control surfaces
    - Mechanical system
    - through the CAS electrical path
- Fly by Wire (FBW) ← choose your own derivatives
  - No mechanical link from pilot to actuators (a CAS with complete authority)

Autopilot: 1910s (A/C: Lockheed Vega (1930s))

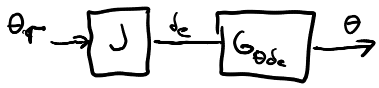
SAS (F-104) 1950s

CAS (F-15) Early '70s

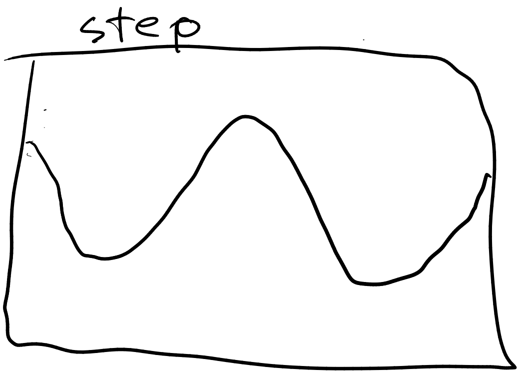
FBW (F-16) Late '70s

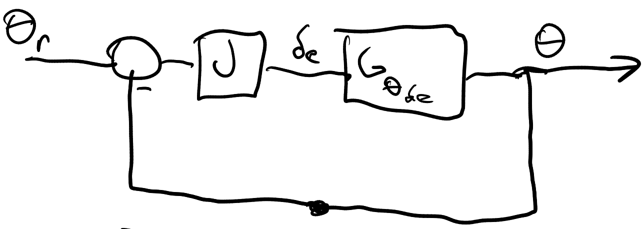
Control Design Task 1: Pitch Attitude Controller

Input: θ_r Goal: $\theta \to \theta_r$ quickly (minimal Phugoid)



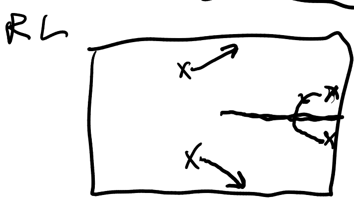
$J = 1$

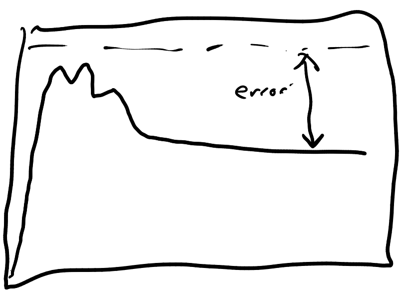




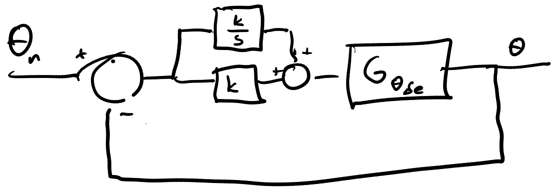
$J = K$

$K = -0.5$

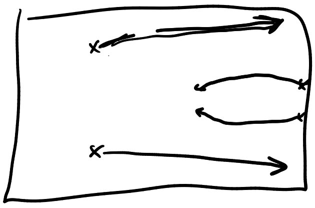




$J = K\left(1+\frac{1}{s}\right)$



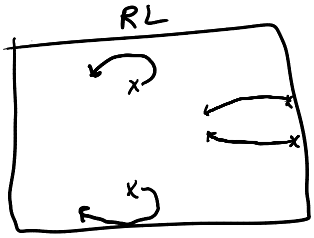
RL →



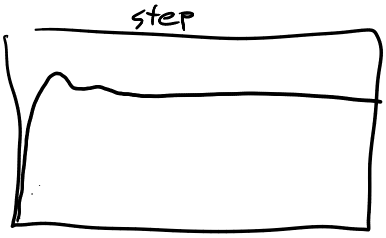
step

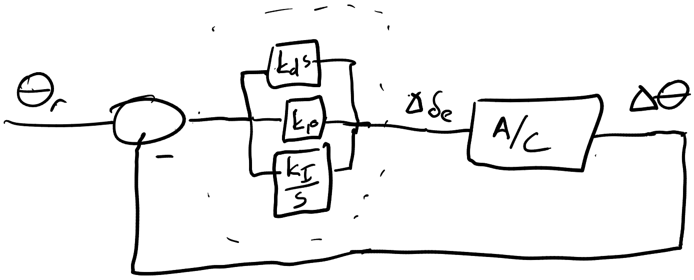
$J = K\left(1+\frac{1}{s}+s\right)$

RL



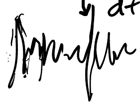
step

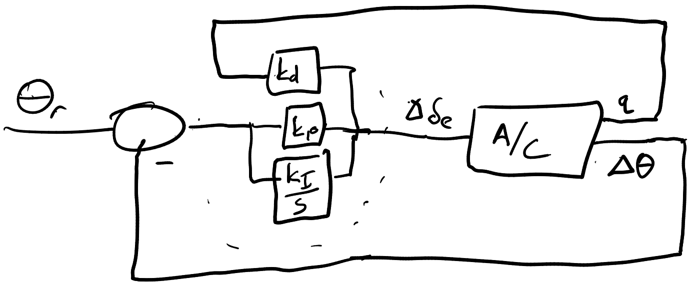




$\frac{d\Delta\theta}{dt}$ very large

$\Delta\theta$





$\Delta\delta_e = -k_p \Delta\theta$

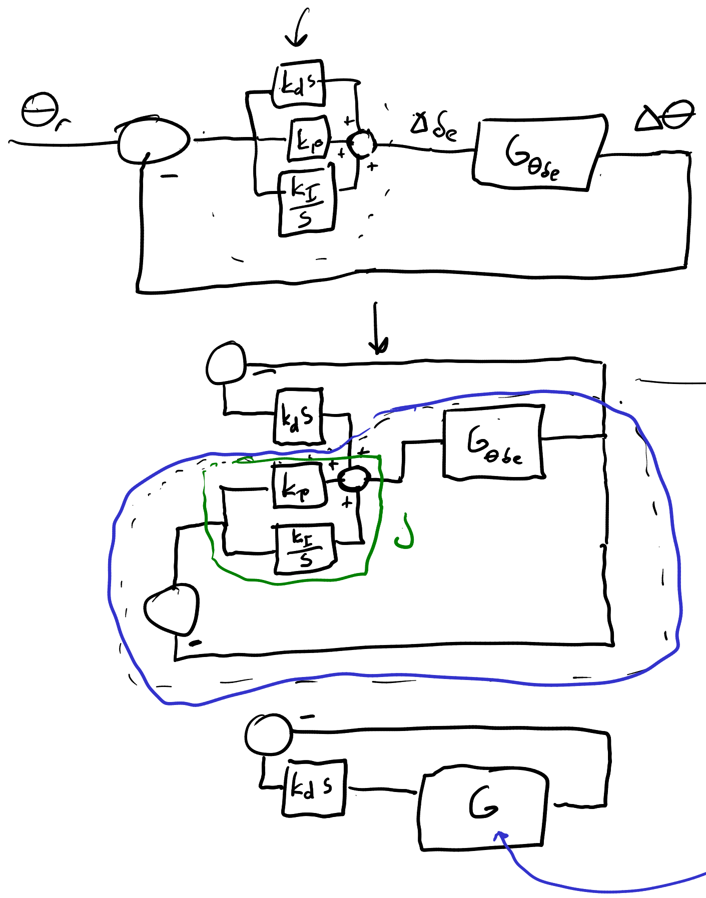
$\vec{u} = -K\vec{x}$

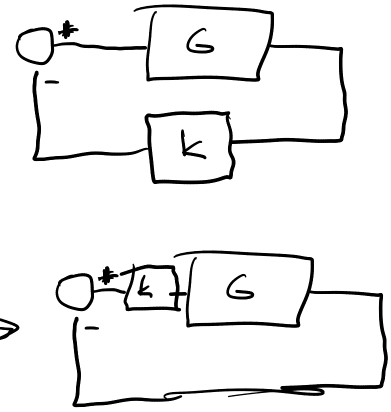
$$\begin{bmatrix}\delta_e\\ \delta_t\end{bmatrix} = -\begin{bmatrix}0&0&0&k_p\\0&0&0&0\end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta w\\ q\\ \Delta\theta\end{bmatrix}$$

$A-BK$

$$A_{lon} - \begin{bmatrix}-0.0002\\-17.85\\-1.158\\0\end{bmatrix}\begin{bmatrix}0&0&0&k_p\\0&0&0&0\end{bmatrix}$$

$$A_{lon} - \begin{bmatrix}0&0&0&-0.0002k_p\\0&0&0&-17.85k_p\\0&0&0&-1.158k_p\\0&0&0&0\end{bmatrix}$$





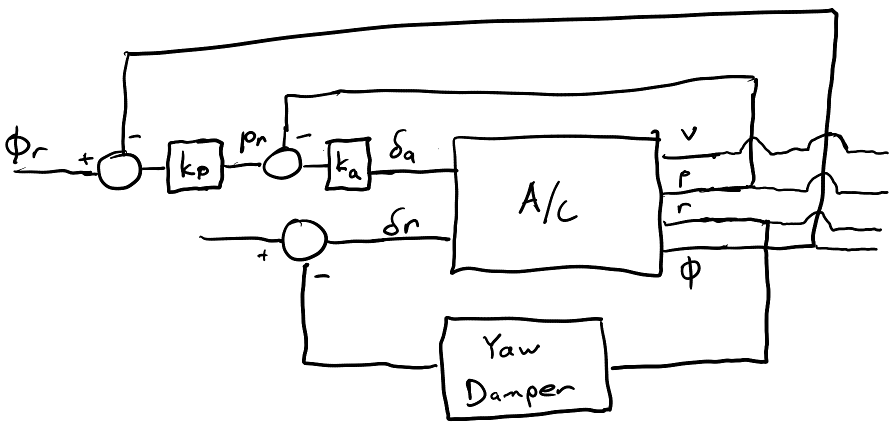
$$G = \frac{G_{\theta\delta_e}}{1 + J\,G_{\theta\delta_e}} = \frac{G_{\theta\delta_e}}{1+\left(k_p+\frac{k_I}{s}\right)G_{\theta\delta_e}}$$

Control Task 2: Roll Controller

Input: $\phi_r = \phi_c$ In Book

Goals: $\phi \rightarrow \phi_r$ quickly

Alleviate Dutch Roll



Part 1: Yaw Damper

$\delta_r = -k_r r$ from RL, choose $k_r = -1.9$

Problem: In steady right turn for $k_r = -1.9$, rudder will be positive (wrong way for coordinated turn)

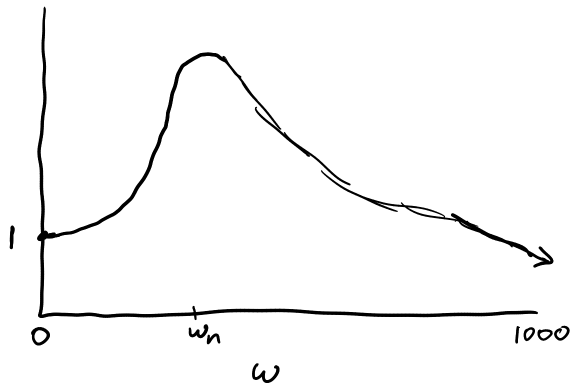
Want: $\delta_r \longrightarrow 0$ at low frequency

$\delta_r = -k_r r$ near dutch roll freq.

Review: Frequency Response

$|G(i\omega)|$

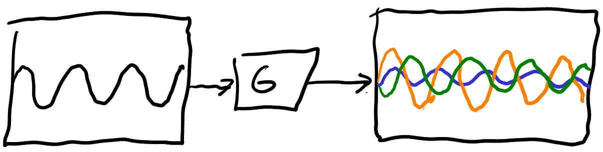
"Amplitude Response"



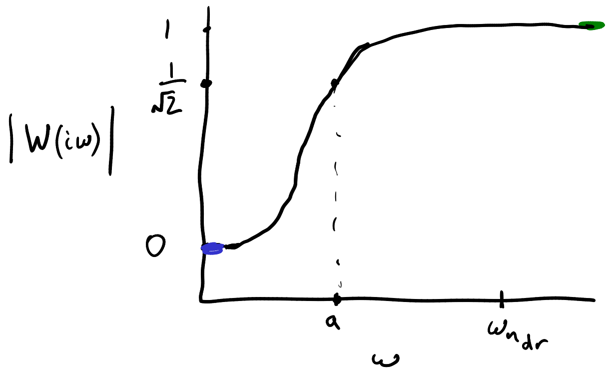
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$|G(0)| = 1$$

$$\lim_{\omega\to\infty} |G(i\omega)| = 0$$



"Washout"/high pass filter



$$W(s) = \frac{s}{s+a}$$

$$|W(0)| = 0$$

$$\lim_{\omega\to\infty} |W(\omega i)| = \lim_{\omega\to\infty} \left|\frac{\omega i}{\omega i}\right| = 1$$

$$|W(ai)| = \left|\frac{ai}{\underline{a+ai}}\right| = \left|\frac{a e^{\frac{\pi}{2}i}}{\sqrt{2}\, a e^{\frac{\pi}{4}i}}\right| = \frac{1}{\sqrt{2}}$$

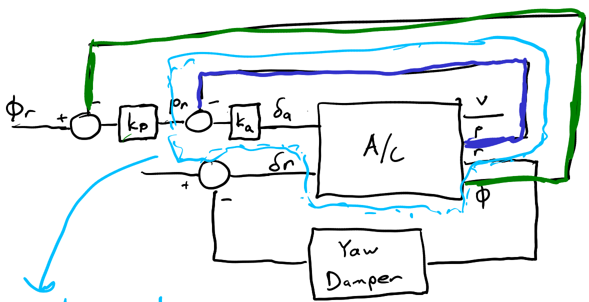
for 747 $\lambda_{dr} = -0.033 \pm 0.947i$

ω_d $\quad \omega_{n_{dr}} \approx 1$

choose $a = 0.1$

$$\boxed{\delta_r = -k_r \frac{s}{s+0.1}}$$

Part 2: Inner and Outer loop Aileron Controller



with $k_a = -1$
this looks like 0.25

$$\delta_a = (p_r - p)k_a$$

$$\dot{p} = \mathcal{L}_p p + \mathcal{L}_{\delta a}\delta_a$$

$-0.4342 = \frac{1}{\tau}$ $\qquad$ -0.1431

$$\dot{p} = \underbrace{(\mathcal{L}_p - \mathcal{L}_{\delta a}k_a)}_{=\frac{1}{\tau_{cl}}}p + \mathcal{L}_{\delta a}k_a p_r$$

from A_{lat}

$\lambda_r = -0.5625$

at steady state, $\dot{p} = 0$

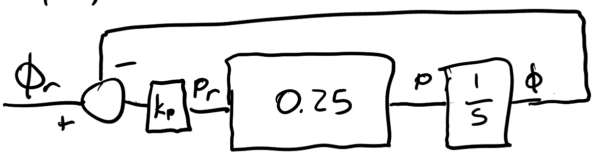
Choose $k_a = -1$

$$\hookrightarrow -\frac{\mathcal{L}_{\delta a}k_a}{\mathcal{L}_p - \mathcal{L}_{\delta a}k_a} = 0.25$$

$$p_\infty = -\underbrace{\frac{\mathcal{L}_{\delta a}k_a}{\mathcal{L}_p - \mathcal{L}_{\delta a}k_a}}_{\text{Not} = 1} p_r$$

Outer Loop

Assume: $\dot{\phi} = p$, roll rate dynamics (p, above) are much faster than ϕ response that we want

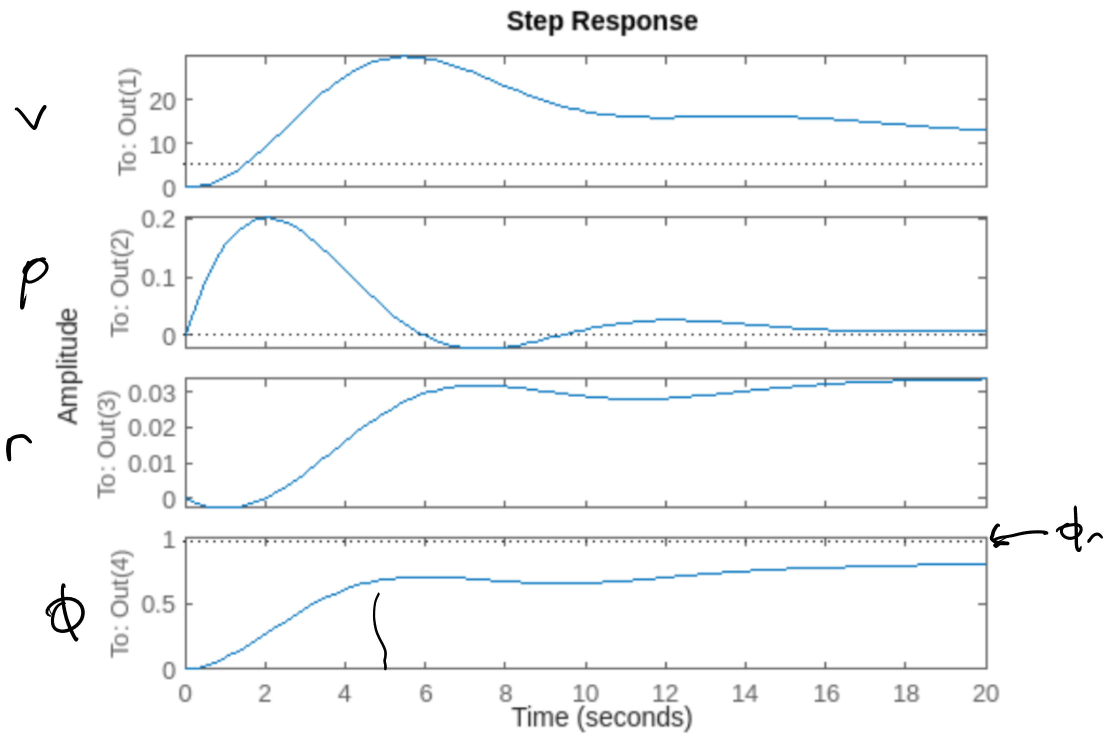


$$\dot{\phi} = 0.25\, k_p (\phi_r - \phi)$$

$$\dot{\phi} = -0.25\, k_p\, \phi + 0.25 k_p \phi_r$$

Based on Root Locus

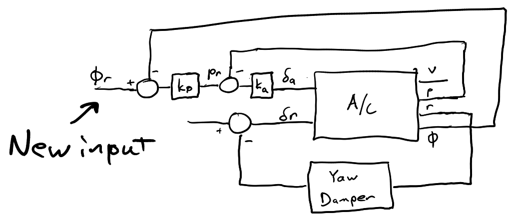
choose $k_p = 1.5$



State Space Gain Matrix for this Controller (without washout)

$$\vec{u} = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = -K\vec{x} = -\underbrace{\begin{bmatrix} 0 & k_a & 0 & k_a k_p \\ 0 & 0 & k_r & 0 \end{bmatrix}}_{K} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

$$\delta_a = -k_a p + k_a p_r$$
$$= -k_a p - k_a k_p \phi$$
$$\delta_r = -k_r r$$



$$A_{roll} = A_{lat} - B_{lat} K$$

$$\dot{\vec{x}} = A_{roll} \vec{x} + \underbrace{B_{lat} \begin{bmatrix} k_p k_a \\ 0 \end{bmatrix}}_{B_{roll}} \phi_c$$