

Automatic Flight Control Systems

- Autopilot
 - Controls a/c trajectory w/o requiring input from pilot
- Stability Augmentation System (SAS) ← adjusting stability derivatives
 - Designed to improve dynamic stability
 - Relies on sensors, not on pilot inputs
- Control Augmentation System (CAS) ← adjusting control derivatives
 - Pilot input has two paths to control surfaces
 - Mechanical system
 - through the CAS electrical path
- Fly by Wire (FBW) ← choose your own derivatives
 - No mechanical link from pilot to actuators
 - (a CAS with complete authority)



Autopilot: 1910s
(A/C: Lockheed Vega (1930s))



SAS (F-104) 1950s



CAS (F-15) Early '70s



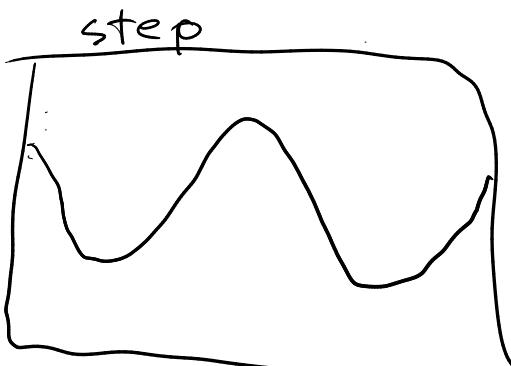
FBW (F-16) Late '70s

Control Design Task 1: Pitch Attitude Controller

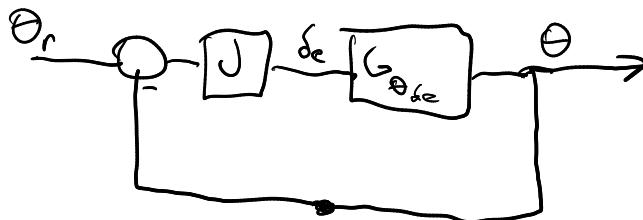
Input: θ_r Goal: $\theta \rightarrow \theta_r$ quickly (minimal Phugoid)



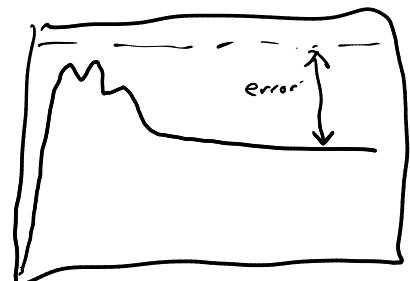
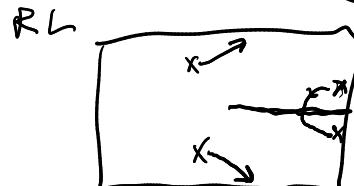
$J=1$



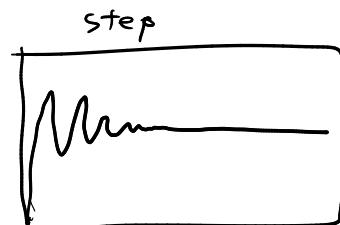
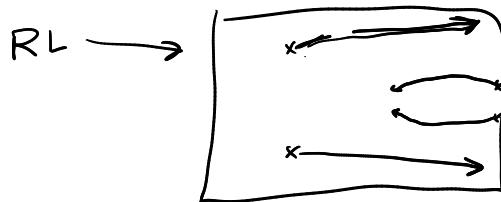
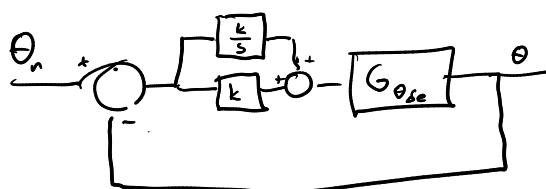
$J=K$



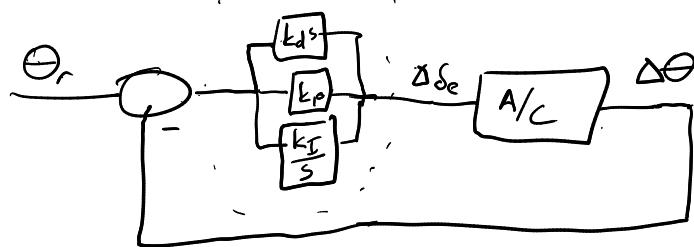
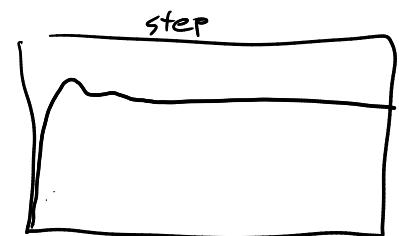
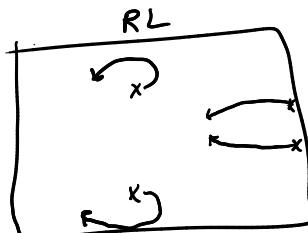
$K = -0.5$



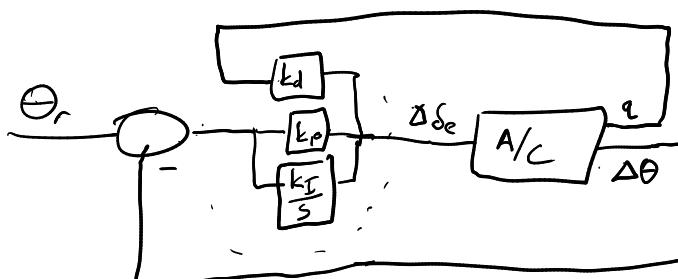
$$J = K \left(1 + \frac{1}{s}\right)$$



$$J = K \left(1 + \frac{1}{s} + s\right)$$



$\Delta \theta$ $\frac{d\Delta\theta}{dt}$ very large



$$\Delta \theta_e = -k_p \Delta \theta$$

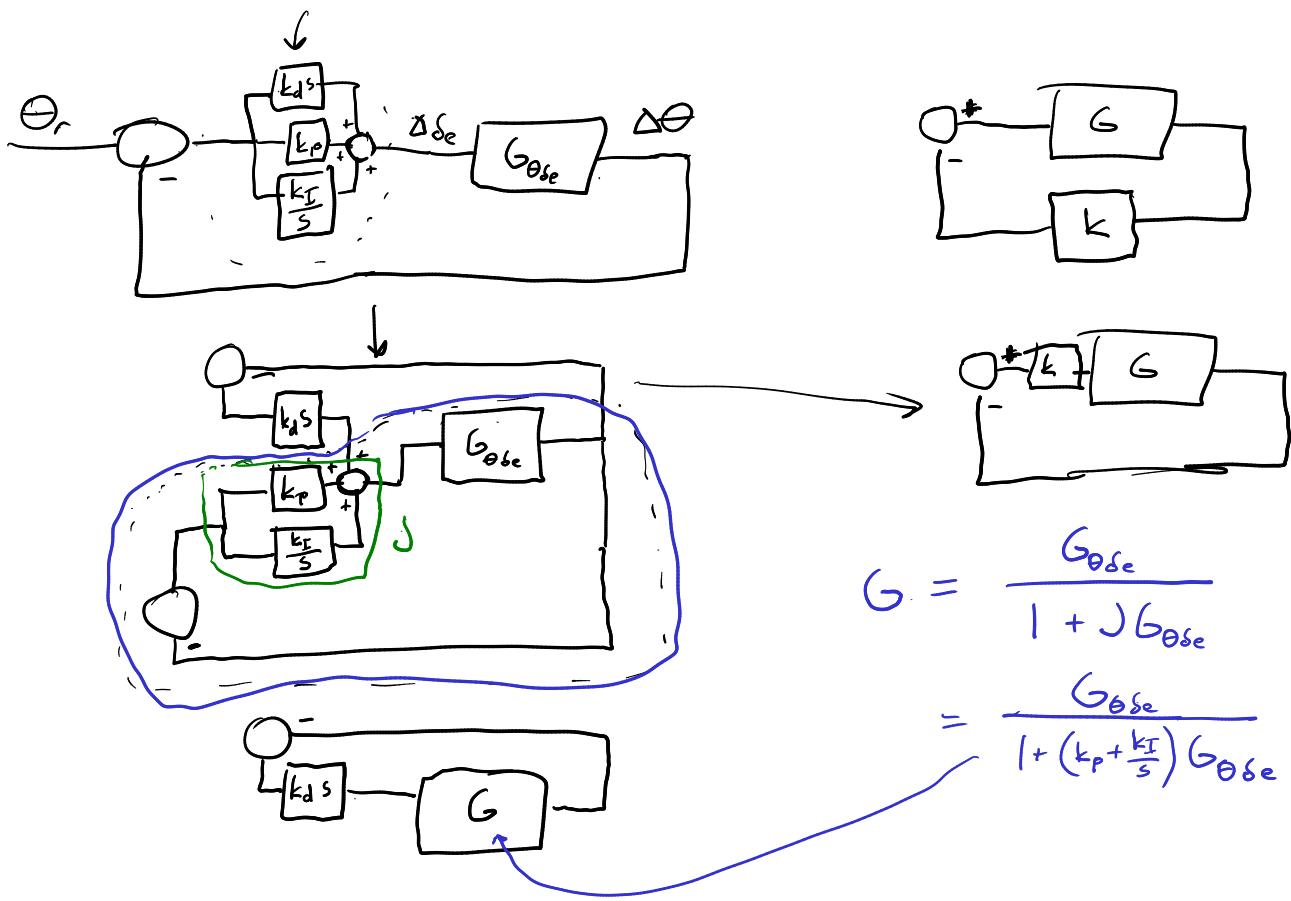
$$\bar{u} = -K \bar{x}$$

$$\begin{bmatrix} \dot{\theta}_e \\ \dot{\theta}_r \end{bmatrix} = - \begin{bmatrix} 0 & 0 & 0 & k_p \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{w} \\ q \\ \Delta \theta \end{bmatrix}$$

$A_{lat} - BK$?

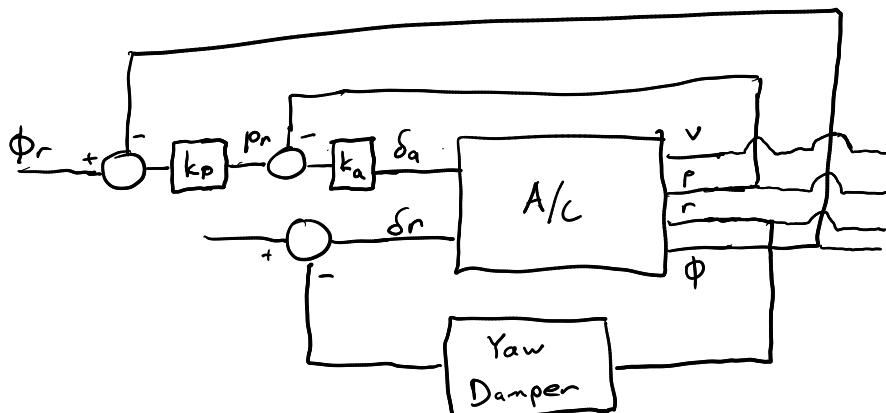
$$A_{lat} - \begin{bmatrix} 0 & 0 & 0 & k_p \\ -0.1431 & 0 & 0 & 0 \\ 0.0037 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{lat} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.14k_p \\ 0 & 0 & 0 & 0.0037k_p \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



Control Task 2: Roll Controller

Input: $\phi_r = \phi_c$ Goals: $\phi \rightarrow \phi_r$ quickly Alleviate Dutch Roll
 In Book



Part 1: Yaw Damper

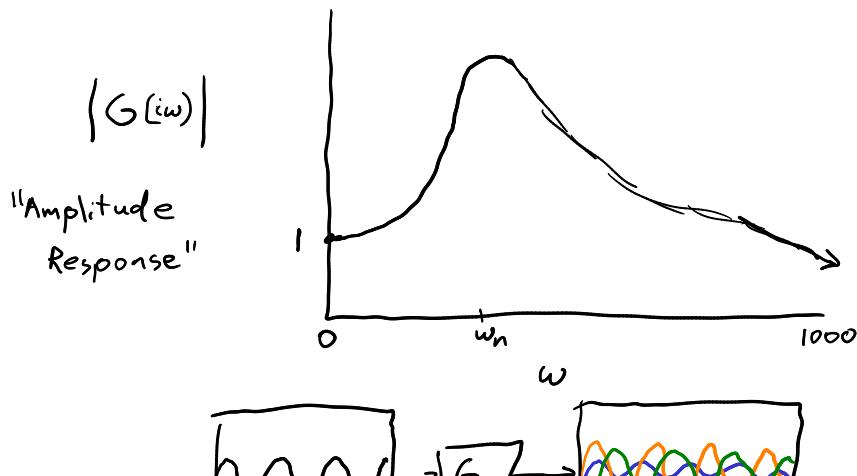
$$\delta_r = -k_r r \quad \text{from RL. choose } k_r = -1.9$$

Problem: In steady right turn for $k_r = -1.9$, rudder will be positive (wrong way for coordinated turn)

Want: $\delta_r \rightarrow 0$ at low frequency

$\delta_r = -k_r r$ near dutch roll freq.

Review: Frequency Response

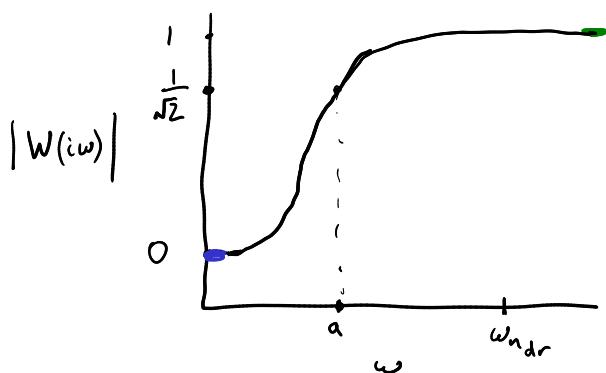


$$G(s) = \frac{\omega_n^2}{s^2 + 2s\omega_n s + \omega_n^2}$$

$$|G(0)| = 1$$

$$\lim_{\omega \rightarrow \infty} |G(i\omega)| = 0$$

"Washout" / high pass filter



$$W(s) = \frac{s}{s+a}$$

$$|W(0)| = 0$$

$$\lim_{\omega \rightarrow \infty} |W(\omega_i)| = \lim_{\omega \rightarrow \infty} \left| \frac{\omega_i}{\omega_i + a} \right| = 1$$

$$|W(a_i)| = \left| \frac{a_i}{a + a_i} \right| = \left| \frac{a e^{\frac{\pi i}{2}}}{\sqrt{2} a e^{\frac{\pi i}{2}}} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{a_i}{a} \sqrt{\frac{\pi i}{a}}$$

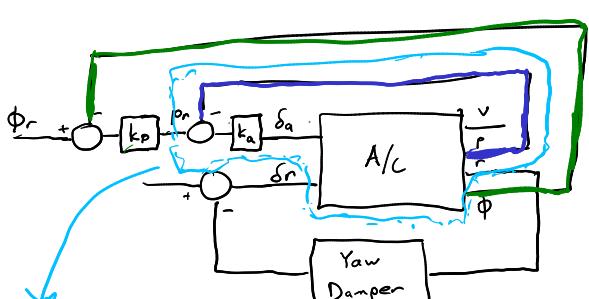
for 747 $\lambda_{dr} = -0.033 \pm 0.947i$

choose $a = 0.1$

$\omega_d \quad \omega_{n_{dr}} \approx 1$

$$\delta_r = -k_r \frac{s}{s+0.1}$$

Part 2: Inner and Outer loop Aileron Controller



$$\dot{\rho} = L_p \rho + L_{\delta_a} \delta_a$$

from A_{int}
 $\lambda_r = -0.5625$

$$\dot{\rho} = (L_p - L_{\delta_a} k_a) \rho + L_{\delta_a} k_a p_r$$

at steady state, $\dot{\rho} = 0$

choose $k_a = -1$

$$-\frac{L_{\delta_a} k_a}{L_p - L_{\delta_a} k_a} = 0.25$$

$$p_\infty = -\frac{L_{\delta_a} k_a}{\underbrace{L_p - L_{\delta_a} k_a}_{\text{Not } = 1}} p_r$$

Outer Loop

Assume: $\dot{\phi} = p$, roll dynamics are much faster than ϕ response that we want

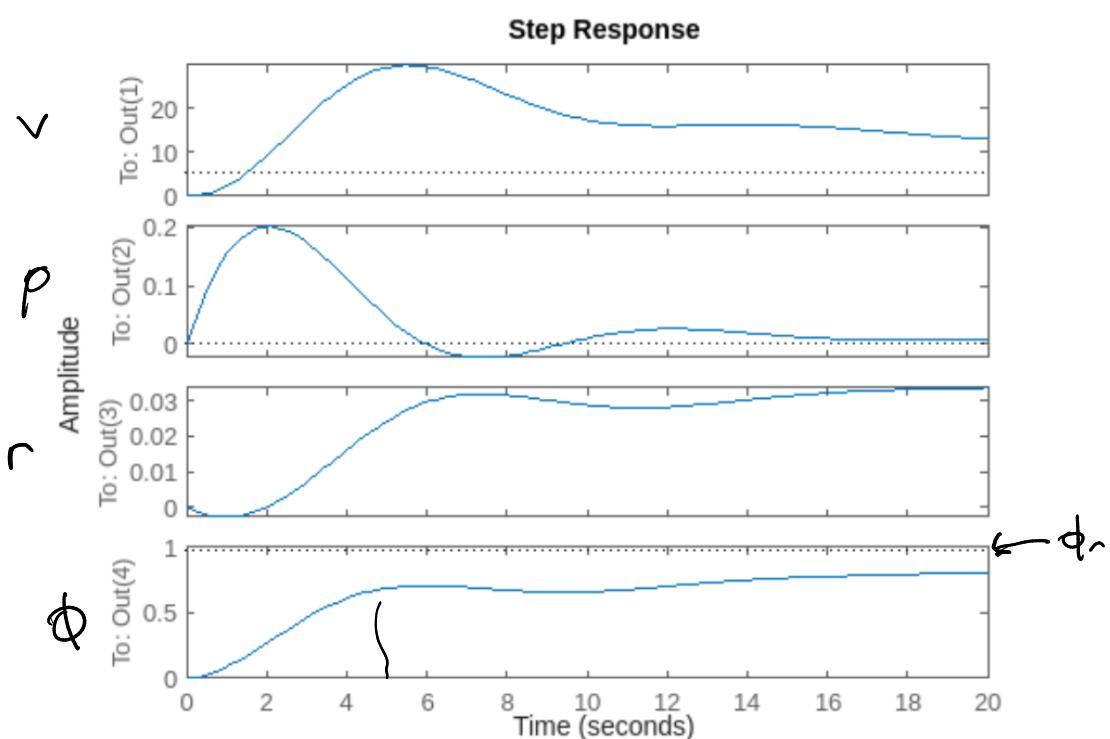


$$\dot{\phi} = 0.25 k_p (\phi_r - \phi)$$

$$\dot{\phi} = -0.25 k_p \phi + 0.25 k_p \phi_r$$

Based on Root Locus

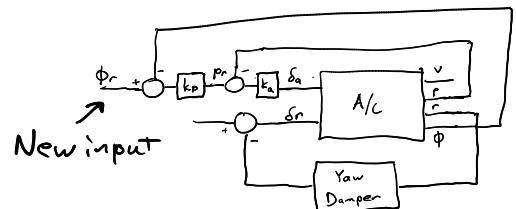
choose $k_p = 1.5$



State Space Gain Matrix for this Controller (without washout)

$$\dot{x} = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = -K \dot{x} = -\underbrace{\begin{bmatrix} 0 & k_a & 0 & k_a k_p \\ 0 & 0 & k_r & 0 \end{bmatrix}}_K \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

$$\begin{aligned} \delta_a &= -k_a p + k_a p_r \\ &= -k_a p - k_a k_p \phi \\ \delta_r &= -k_r r \end{aligned}$$



$$A_{roll} = A_{lat} - B_{lat} K$$

$$\dot{\vec{x}} = A_{roll} \vec{x} + B_{lat} \begin{bmatrix} k_p & k_a \\ 0 & 0 \end{bmatrix} \phi_c$$

