ASEN 3728 Aircraft Dynamics
Written Homework 4

Due date listed on Gradescope.

Question 1. For this problem <u>assume the thrust is equal and opposite to the drag</u>. Consider an aircraft with the following parameters

m = 6.77 kg	c = 0.2 m	b = 3.3 m
S = 0.67 m^2	$C_{D_{\text{min}}} = 0.02$	K = 0.0224
$C_{L_\alpha} = 5.75$	$C_{L_q} = 10.14$	$C_{L_{\delta_e}} = 0.0079$
$C_{m_q} =$ -24.4	$C_{m_{\delta_e}} =$ -0.02	$C_{m_{\text{zero}}} = 0.12$
$C_{L_{\text{min}}} = 0$		

1. What is the drag coefficient of the aircraft when at trim with density $\rho = 1.10$ kg/m^3 and airspeed V_a = 21 m/s?
2. Consider an aircraft flying with with the same airspeed and density above. You wish to put the aircraft in trim with $\delta_e = 1.65°$ and $\alpha = 4.07°$. These values balance the forces but not the moment. You are allowed to change the location h of the center of gravity. If the neutral point $h_n = 0.75$, where must you locate the center of gravity of the aircraft for it to be in trim?

Question 2. Consider an aircraft cruising at an airspeed of 176 ft/s with the following longitudinal dynamics matrix for longitudinal perturbation state $\Delta\mathbf{x} = [\Delta u, \Delta w, \Delta q, \Delta\theta]^T$ and control input δ_e:

$$A_{\text{lon}} = \begin{bmatrix} -0.045 & 0.036 & 0 & -32.2 \\ -0.369 & -2.02 & 176 & 0 \\ 0.0019 & -0.0396 & -2.948 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad B_{\delta_e} = \begin{bmatrix} 0 \\ -28.2 \\ -11.9 \\ 0 \end{bmatrix}$$

where all units are English.

1. If the aircraft is operating near trim at $\Delta\mathbf{x} = [10\,\text{ft/s}, 0, 0, 0]^T$, what is the acceleration in the body x-direction, $\Delta\dot{u}$? Assume no control input.
2. Calculate the eigenvalues of A_{lon}. Indicate which eigenvalues correspond to the short period and phugoid modes of the aircraft. Briefly justify your answer.
3. Consider the control law $\delta_e = -k_1\,\Delta q - k_2\,\Delta\alpha$. Determine what the gain matrix K should be so that $A^{cl} = A_{\text{lon}} - B_{\delta_e}K$ would be the closed-loop dynamics matrix for the controlled system. The final answer should be K and should include entries with k_1, k_2, and possibly other quantities; you do not need to write out A^{cl}.
4. Calculate the eigenvalues of the closed-loop dynamics matrix A^{cl} using values of $k_1 = 0.1$ and $k_2 = 0.5$ in the control law from part 3. By what percentage do the natural frequencies of the short period and phugoid modes change between the open-loop and closed-loop dynamics using these gain values?

Question 3. Consider an aircraft with mass and geometric properties given in Table 1, flying at a trim condition described by Table 2. Assume the aircraft has a constant-power engine and the thrust is aligned with the body x-axis, i.e. the thrust IS NOT opposite the drag.

- Determine the lift and drag coefficients of the aircraft in trim. It may be helpful to draw a free body diagram of the aircraft.
- Calculate the dimensional stability derivatives Z_w, X_w, and M_w.

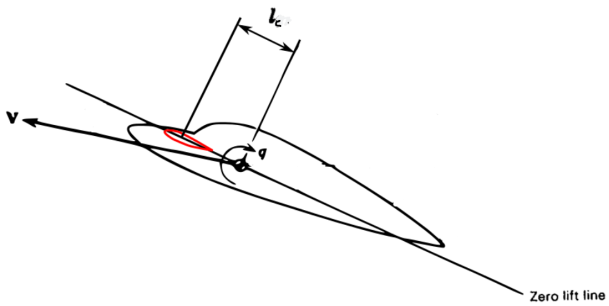
Table 1: Mass and Geometric Properties

Variable	Value		Variable	Value		Variable	Value
m	22,000 kg		S	87 m^2		c	3.9 m
b	22.3 m		a_{wb}	4.4/rad		static margin	.25
S_t	21.4 m^2		$\partial\epsilon/\partial\alpha$	0.3		a_t	3.67/rad
e	0.95						

Table 2: Trim condition

Variable	Value		Variable	Value		Variable	Value
ρ_0	1.225 kg/m^3		α_0	1.2°		u_0	125 m/s
γ_{a_0}	0		thrust T_0	14,000 N			

Question 4. Consider an aircraft with a canard, such that the aircraft has no tail, but has a control surface forward of the wings, as shown below. Assume the canard (shown in red) and the center of gravity are at the same vertical height relative to the zero lift line. You may also assume that $V = u_0$.



The pitching moment coefficient for the canard C_{m_c} is

$$C_{m_c} = \frac{l_c S_c}{\bar{c} S} C_{l_c} = V_{H_c} C_{l_c},$$

where S_c is the area of the canard, S is the area of the wing, l_c is the distance between the CG and the canard mean aerodynamic center, $\bar{c}$ is the length of the mean aerodynamic chord, and V_{H_c} is the canard volume ratio.

1. Use a diagram to determine $\Delta\alpha_c$, the change in the effective angle of attack for the canard, as a function of q.
2. Find $(C_{m_q})_{\text{canard}}$, the canard contribution to the pitching moment stability derivative.