ASEN 6519: Optimization: Applications and Algorithms
Homework 1

September 2, 2024

1 Questions

Question 1. Suppose we are given only three function evaluations to shrink a bracket of a unimodal function. In relative terms, how much tighter is the resulting interval when using Fibonacci search compared to the golden section search?

Question 2. Use dual numbers to find the derivative of the function $f(x) = x^2 + 2x + 1$ as a function of x and evaluate it at $x = -2$. Show all steps.

Question 3. Suppose minimization problem $P2$ is obtained by adding a new constraint to minimization problem $P1$. If the optimal value of $P1$ is v_1 and the optimal value of $P2$ is v_2, which of the following statements can be proven without any additional information?

(a) $v_1 = v_2$

(b) $v_1 < v_2$

(c) $v_1 > v_2$

(d) $v_1 \geq v_2$

(e) $v_1 \leq v_2$

If the statement is not provable, provide a counterexample. If the statement is provable, provide a short proof.

Question 4. For this question, we will focus on optimizing the function

$$f(x) = x^3 + x\,\cos(3\,x) + \frac{\sin\left(8\,x^2\right)}{2}.$$

on the interval $x \in [-1, 1]$.

(a) Is this function unimodal on $[-1, 1]$? Justify your answer.

(b) Is this function Lipschitz continuous on the interval $[-1, 1]$? If it is, find a Lipschitz constant for it.

(c) Find the critical points of $f(x)$ to a precision of 3 digits without evaluating the derivative of $f(x)$. Plot the critical points on the graph of $f(x)$.

(d) Use the Shubert-Piyavskii algorithm to find a lower bound for the function on the interval $[-1, 1]$. Plot the lower bound after 50 function evaluations. With this algorithm, the Lipschitz constant that you determined in part (b), and 50 function evaluations, is it possible to find the global minimum, x^*, with an error of less than 0.1?