ASEN 6519: Optimization: Applications and Algorithms
Homework 2

September 18, 2024

1 Questions

Question 1. Analytically find the critical points of

$$f(\mathbf{x}) = x_1^4 + 3x_1^3 + 3x_2^2 - 6x_1x_2 - 2x_2$$

and classify them as local minima, maxima, or saddle points. Verify your answers by creating an appropriate contour plot.

Question 2. Implement two algorithms for finding the minimum of the Rosenbrock function from a starting point of [-1.2, 1]. One algorithm can be copied from the class notebooks or book. The other should be implemented from scratch (you can consult the book, but do not copy the code). Plot the path that both of the algorithms take as done in class and compare the number of iterations required to reach an approximate critical point (where $\|\nabla f(x)\| < 10^{-4}$).

Question 3. A weight of 30N is supported by three springs connected to the ceiling. The springs have spring constants $k_1 = 3$, $k_2 = 12$, and $k_3 = 94$ in N/m with unstretched lengths $l_1 = 1$ m, $l_2 = 1$ m, and $l_3 = 2$ m, and they are connected to the ceiling at points $p_1 = [0, 0, 0]$, $p_2 = [1, 0, 0]$, and $p_3 = [0, 0, 2]$. The y direction is up. The system will be in equilibrium when the potential energy of the system is minimized. The potential energy stored by a spring is $\frac{1}{2}k\Delta l^2$ where Δl is the length that the spring has stretched, and the potential energy of the weight is Wy, where y is the vertical location of the weight. Formulate this as an unconstrained optimization problem and solve it with one of the algorithms you implemented for the previous question to find the resting location of the weight.