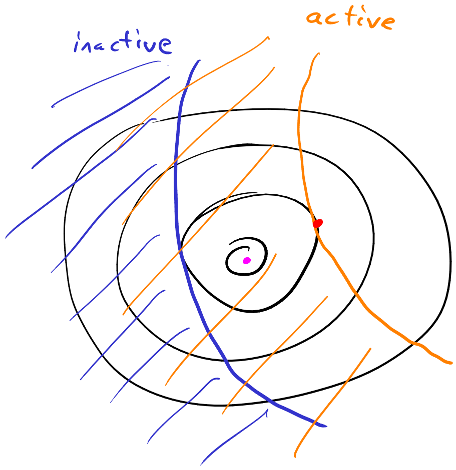
# Constraints

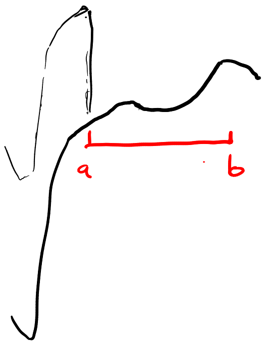
$$\underset{\vec{x}}{\text{minimize}} \quad f(\vec{x})$$
$$\text{subject to} \quad \vec{x} \in \mathcal{X}$$




$$\text{minimize} \quad f(\vec{x})$$
$$\text{subject to} \quad \vec{h}(\vec{x}) = 0 \qquad \text{equality}$$
$$\vec{g}(\vec{x}) \le 0 \qquad \text{inequality}$$

$g_i(\vec{x})$ is <u>active</u> if $g_i(\vec{x}^*) = 0$

## General strategy: Convert constrained to unconstrained




$$\min f(x)$$
$$\text{s.t.} \quad x \in [a, b]$$

how to convert to unconstrained?

$$\min f(x) + \begin{cases} 9001 & \text{if } x \notin [a, b] \\ 0 & \text{o.w.} \end{cases}$$

## Penalty

increase objective where constraints violated

(1) $$\underset{\vec{x}}{\text{minimize}} \; f(\vec{x}) + \rho\, p(\vec{x})$$

$$p_{\text{count}}(\vec{x}) = \sum_i (g_i(\vec{x}) > 0)$$
$$p_{\text{quad}}(\vec{x}) = \sum_i \max(g_i(\vec{x}), 0)^2$$

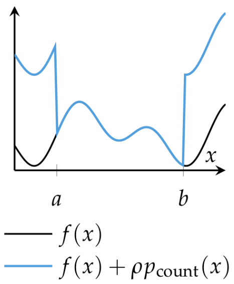



Figure 10.8. The original and count-penalized objective functions for minimizing $f$ subject to $x \in [a, b]$.

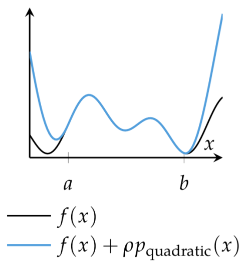



Figure 10.9. Using a quadratic penalty function for minimizing $f$ subject to $x \in [a, b]$.

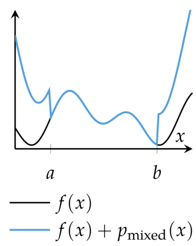



Figure 10.10. Using both a quadratic and discrete penalty function for minimizing $f$ subject to $x \in [a, b]$.

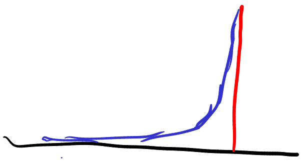
raise $\rho$ until solution feasible

```
loop
    solve (1)
    if solution not feasible
        ρ ← 2ρ
```

## Barrier (Interior Point)

increase objective near constraint

minimize $f(x) + \frac{1}{\rho} p(\vec{x})$



continuous

non-negative

approaches infinity at constraint

inverse

$$-\sum_i \frac{1}{g_i(\vec{x})}$$

log

$$-\sum_i \begin{cases} \log(g_i(\vec{x})) & \text{if } g_i(\vec{x}) \geq -1 \\ 0 & \text{ow} \end{cases}$$

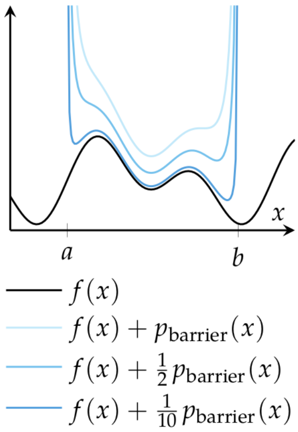



Figure 10.12. Applying the interior point method with an inverse barrier for minimizing $f$ subject to $x \in [a, b]$.

## Lagrange Multiplier

all possible optima → unconstrained critical pts

$$\underset{x}{\text{minimize}} \; \underset{\mu \geq 0}{\text{maximum}} \; \mathcal{L}(\vec{x}, \vec{\mu})$$

$$\mathcal{L}(\vec{x}, \vec{\mu}) = f(\vec{x}) + \vec{\mu}^T \vec{g}(\vec{x})$$