Welcome to ASEN 6519

Optimization: Applications and Algorithms

Why Optimization?

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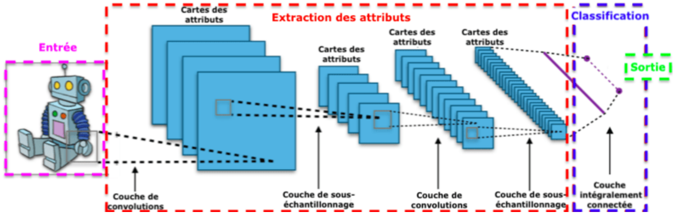

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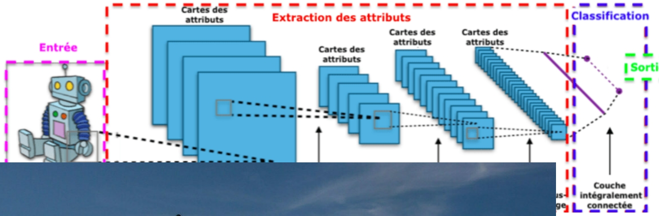

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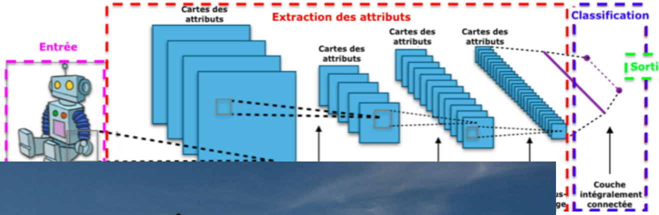

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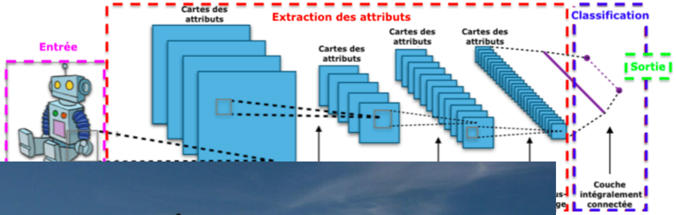

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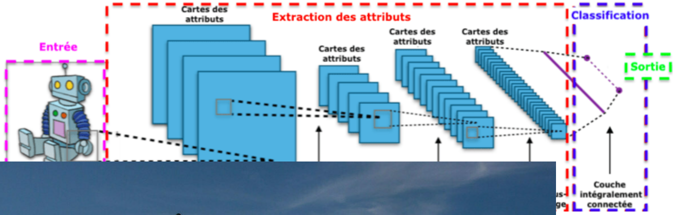
- Someday, you might design something that I use, and I want it to be optimized! ●



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https://imgur.com/gallery/musical-cars-sequel-to-15-point-turn-ADKFKpa

Optimization Problems and Solutions

Optimization Problem
Mathematical Program

objective

$$\text{minimize } f(\vec{x})$$

decision variables
"design point"

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

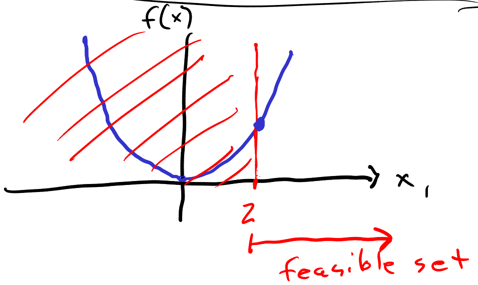
constraint

$$\text{subject to } \vec{x} \in X$$

feasible set

minimize $f(x)$ is equivalent to maximize $-f(x)$

$$\underset{x_1}{\text{minimize}}\ x_1^2$$
$$\text{subject to } x_1 \geq 2$$



$x^* = 2 \quad f(x^*) = 4$

Solution

if $X = \emptyset$ (empty) then problem is infeasible

$x_1 \leq -2$
$x_1 \geq 1$

A globally optimal solution is a design point $\vec{x}^* \in X$ where

$$f(\vec{x}^*) \leq f(\vec{x}) \quad \forall \vec{x} \in X$$

"solution"
"minimizer"

$f(x^*)$ is the value of the problem

$\underset{x}{\min} f(x)$ returns value

$\underset{x}{\operatorname{argmin}} f(x)$ returns x^*

Optimization in my career

1. Gradient descent for space domain awareness

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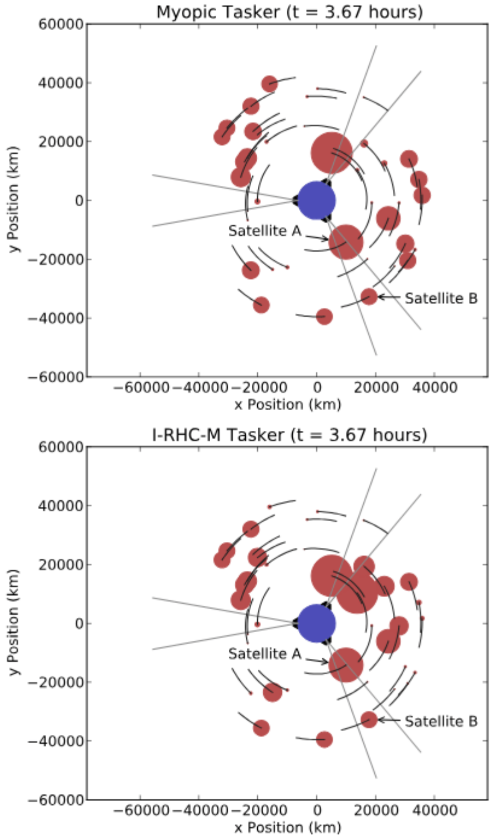

Fig. 3. Sensor network representation early in simulation 18.

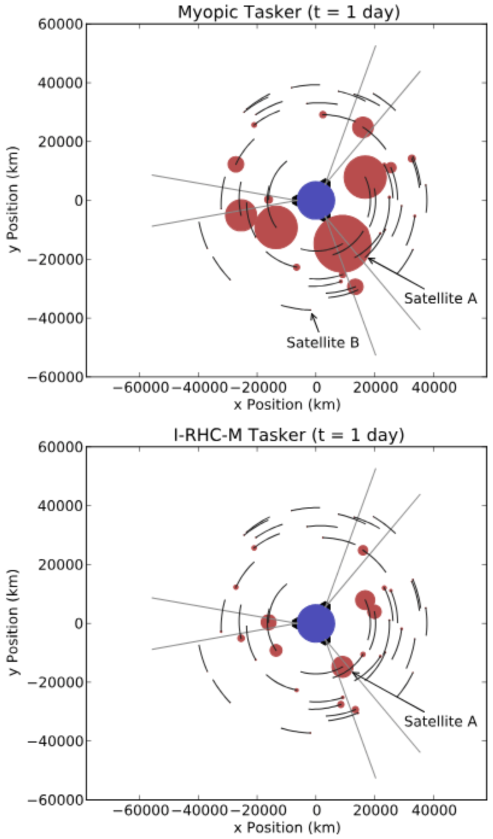

Fig. 4. Sensor network representation at end of simulation 18.

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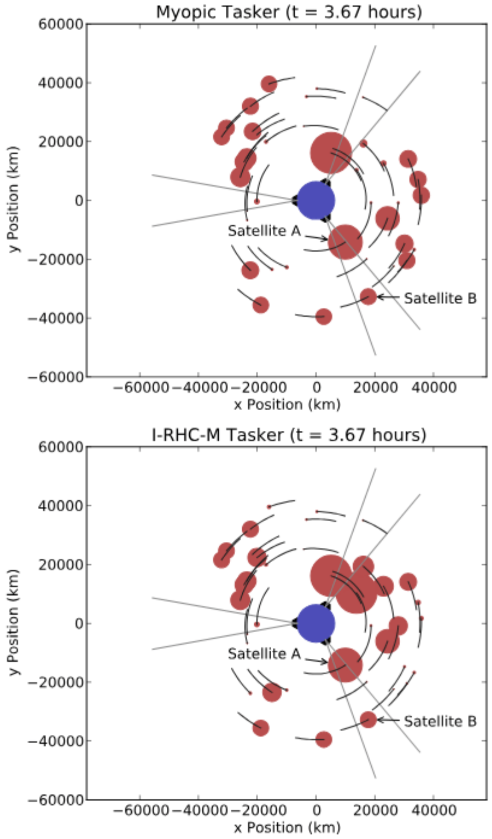

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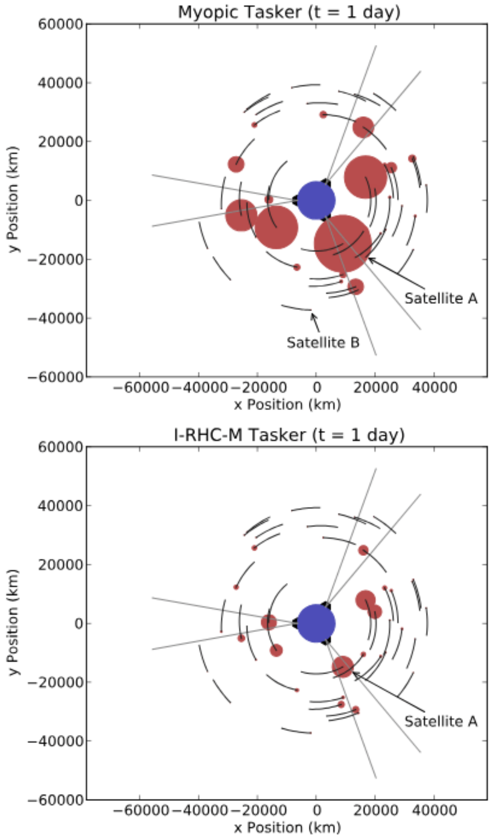

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$$\frac{\partial J_{\log}}{\partial \pi^{j}_{t,k}} = E\left[\sum_{i \in T^{j}(t)} \log(J(\chi_i, \bar{U})) \middle| u^{j}_{t} = k \right]$$

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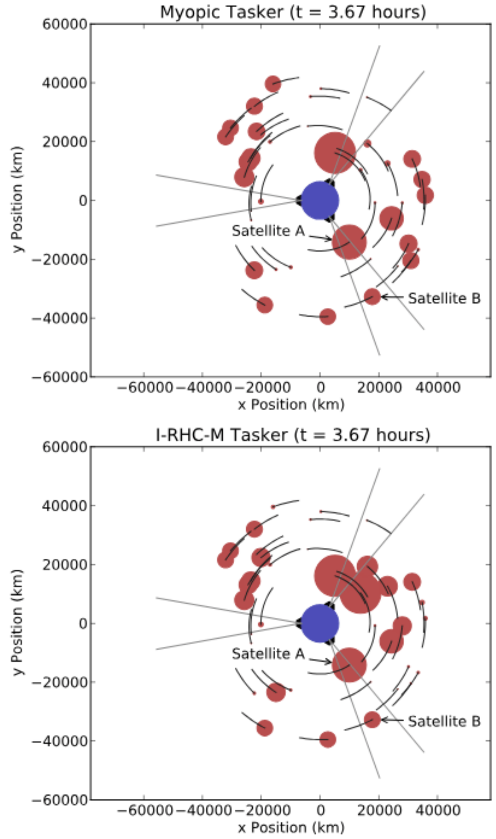

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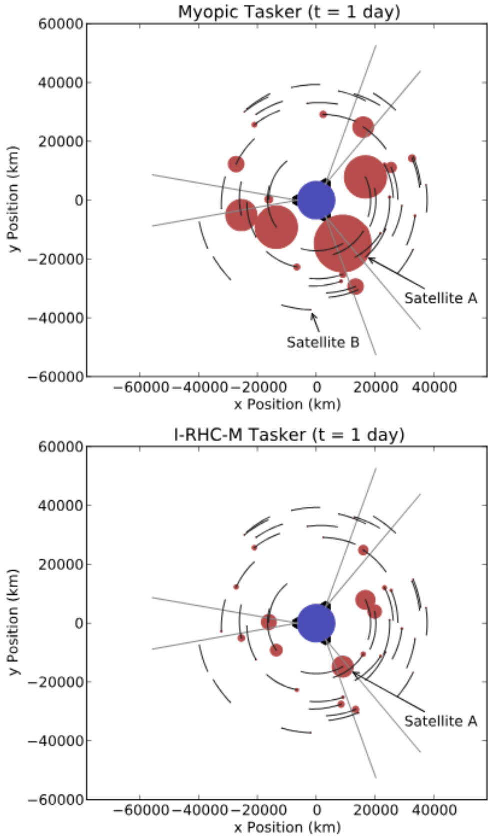

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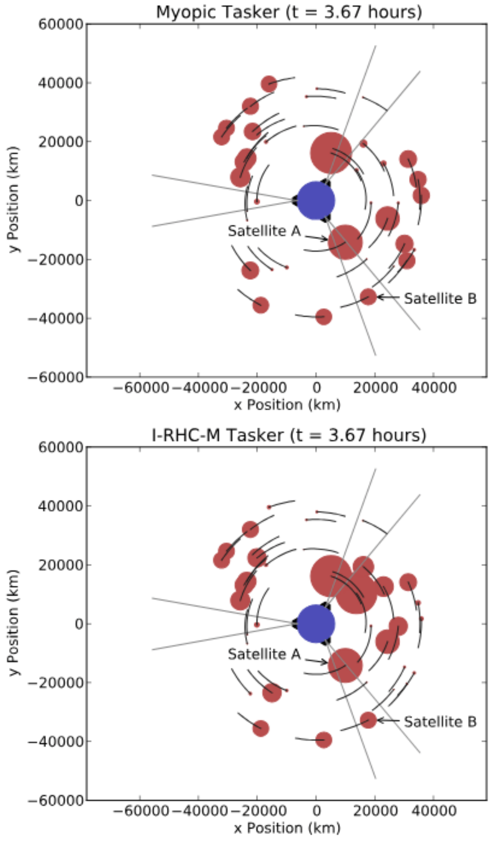

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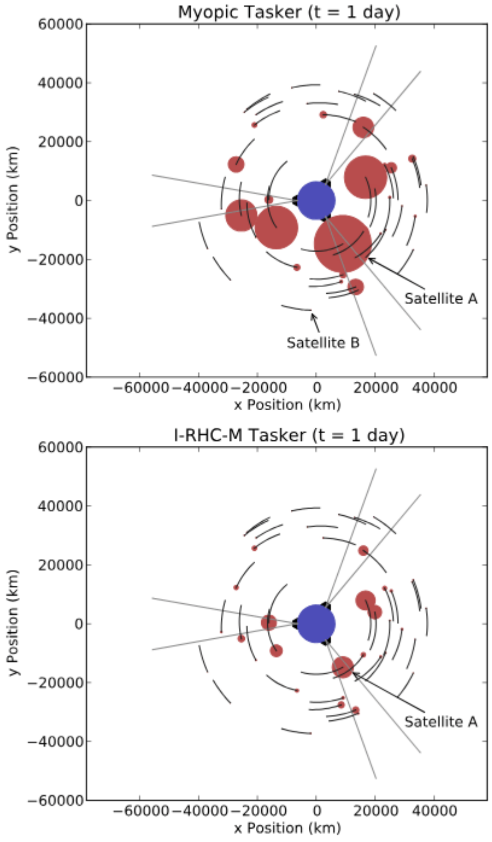

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$$\widehat{\frac{\partial J}{\partial \pi^{j}_{t,k}}} = \begin{cases} \frac{1}{\pi^{j}_{t,k}} \sum_{i \in T^{j}(t)} \log(J(\chi_i, \bar{U}, \omega)) & \text{if } u^{j}_{t} = k \\ 0 & \text{o.w.} \end{cases}$$

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2. Airborne collision avoidance

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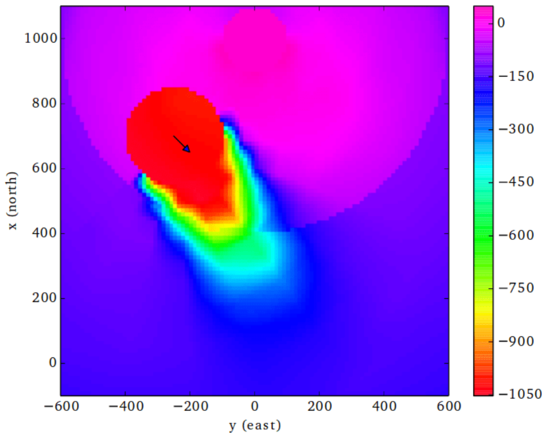

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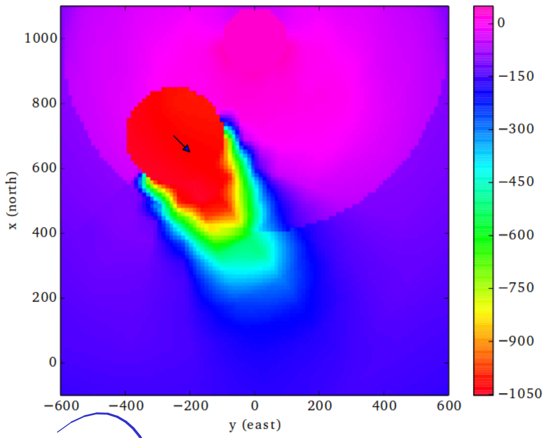

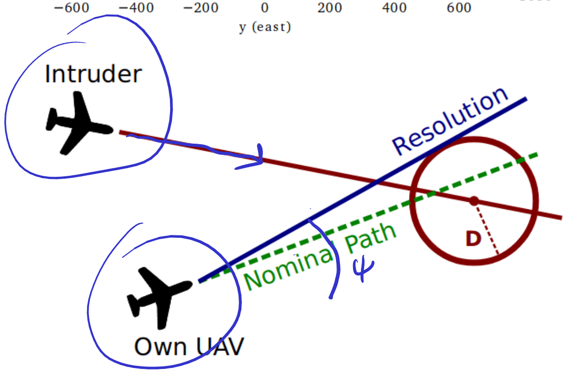

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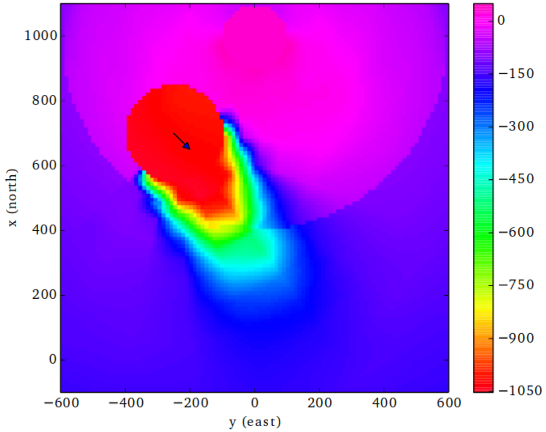

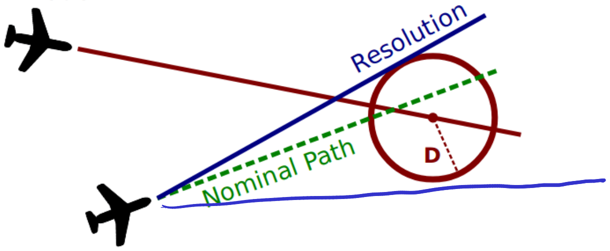

$$\underset{\psi}{\text{minimize}} \quad |\psi - \psi_{goal}|$$

$$\text{subject to} \quad d_{min}(\psi) \geq D$$

also an optimization problem

$$d_{min} \equiv \min_{\tau} d(\psi, \tau)$$

$$d(\psi, \tau) = \sqrt{\Delta x(\tau)^2 + \Delta y(\tau)^2}$$

$$\frac{d\, d(\psi,\tau)}{d\tau} = 0$$

$$\tau_{\min}\left(s, \psi^{(o)}_{\text{cand}}\right) = \max\left\{\frac{a+b}{c-2d}, 0\right\}$$

$$a := -v^{(i)} x^{(i)} \cos(\psi^{(i)}) - v^{(i)} y^{(i)} \sin(\psi^{(i)})$$

$$b := v^{(o)} x^{(i)} \cos(\psi^{(o)}_{\text{cand}}) + v^{(o)} y^{(i)} \sin(\psi^{(o)}_{\text{cand}})$$

$$c := v^{(o)\,2} + v^{(i)\,2} \cos^2(\psi^{(i)}) + v^{(i)\,2} \sin^2(\psi^{(i)})$$

$$d := v^{(o)} v^{(i)} (\cos(\psi^{(i)}) \cos(\psi^{(o)}_{\text{cand}}) + \sin(\psi^{(i)}) \sin(\psi^{(o)}_{\text{cand}})).$$

Optimization in my career

3. Assigning teams based on preferences

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Integer-Linear Program

$$\underset{x_{ij}}{\text{minimize}} \quad \sum_i \sum_j c_{ij} x_{ij} \quad (1)$$

Linear

$$\text{subject to} \quad x_{ij} \in \{0, 1\} \quad (2)$$

$$\sum_j x_{ij} = 1 \quad \forall i \quad (3)$$

$$\underline{s}_j \leq \sum_i s_i x_{ij} \leq \bar{s}_j \quad \forall j \quad (4)$$

$$\sum_i r_{ki} x_{ij} \geq \underline{r}_{kj} \quad \forall k, j \quad (5)$$

$$c_{ij} = 2^{\log_2(n)(p_{ij}-1)} s_i$$

Group: i ← not individuals

of friends

Project: j

Group size: s

Group skills: r

Group preference: p

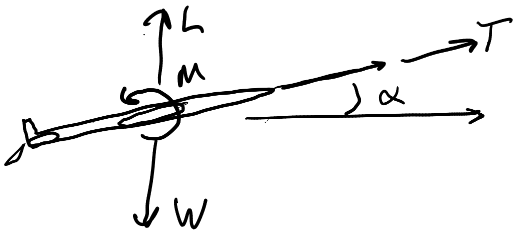
x_{ij}

$q_{ik} x_{ij} x_{kj}$

what didn't work: optimizing groups and preferences simultaneously

Optimization in my career

4. Finding trim points in aircraft dynamics



$\alpha, \delta_e, \delta_t \rightarrow L(\alpha, \delta_e)$, $M(\alpha, \delta_e)$

elevator throttle

$$C_{L_{trim}} = C_W = C_{L_\alpha}\alpha^* + C_{L_{\delta_e}}\delta_e^*$$

$$C_{M_{trim}} = 0 = C_{M_\alpha}\alpha^* + C_{M_{\delta_e}}\delta_e^*$$

Linear Equation

$$\underset{\alpha, \delta_e, \delta_t}{\text{minimize}} \quad |L + T\sin\alpha - W| + |M|$$

subject to $-10^\circ \leq \delta_e \leq 10^\circ$

$0 \leq \delta_t \leq 1$

Matlab : fmincon

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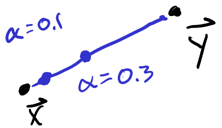
- Ease of implementation/use matters!
  - Can you either (1) put the problem into a form that an existing solver can use, or (2) code up the algorithm itself easily?
  - Even if an algorithm performs better, it will be neglected if it is more difficult to use.

A very important property: Convexity

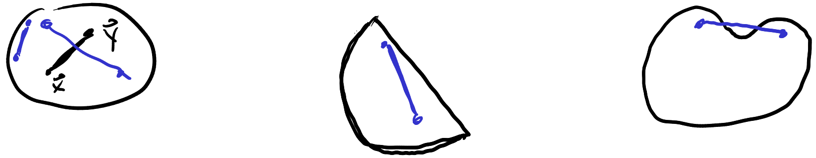
A convex combination of two points $\vec{x}$ and $\vec{y}$

$$\alpha\vec{x} + (1-\alpha)\vec{y} \quad \text{for } \alpha \in [0,1]$$



A set S is a convex set if and only if

$$\alpha\vec{x} + (1-\alpha)\vec{y} \in S \quad \forall \vec{x},\vec{y} \in S, \ \alpha \in [0,1]$$



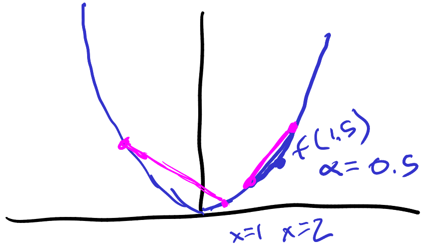
How many US states are convex?

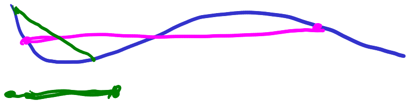
A very important property: Convexity

A function f is a convex function over convex set S iff

$$f(\alpha\vec{x}+(1-\alpha)\vec{y}) \le \alpha f(\vec{x}) + (1-\alpha) f(\vec{y}) \quad \forall \vec{x},\vec{y}\in S,\ \alpha\in[0,1]$$

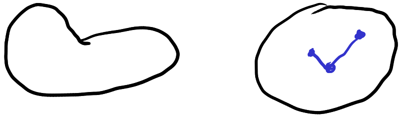
$f(x)=x^2$





An optimization problem is a convex optimization problem iff the objective function and feasible set are convex.

Convex optimization problems can be solved reliably in a (short) predictable amount of time.



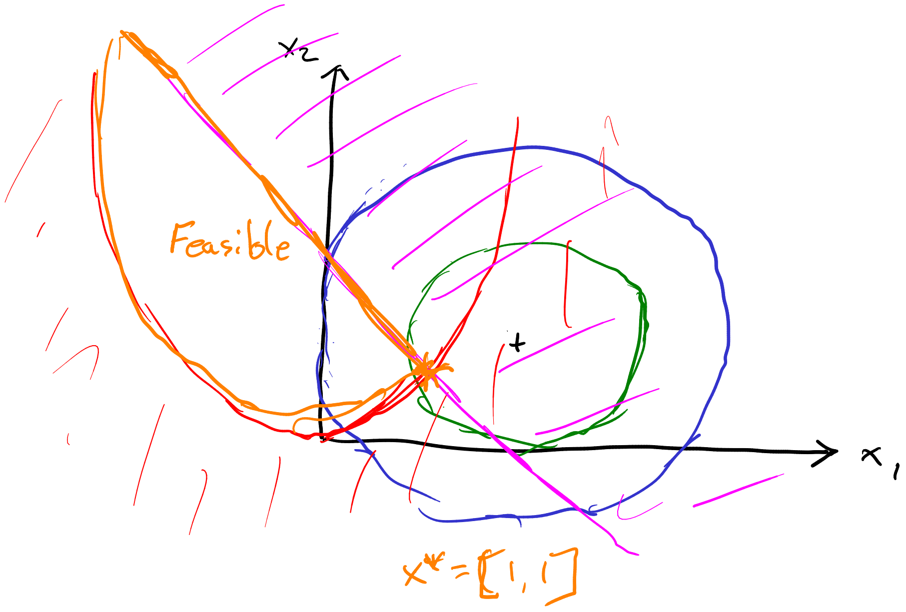
Simple Example

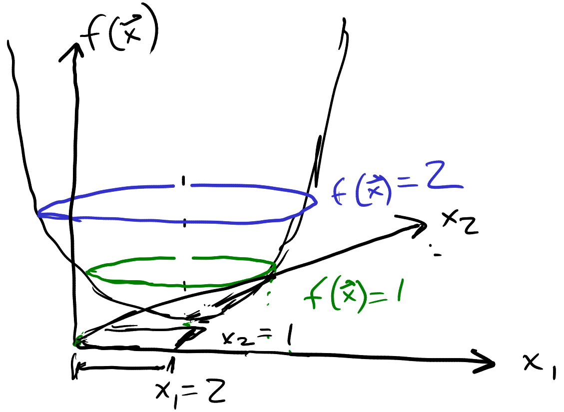
(N+W p. 3)

minimize $(x_1-2)^2 + (x_2-1)^2$

subject to $x_1^2 - x_2 \le 0 \rightarrow x_2 \ge x_1^2$

$x_1 + x_2 \le 2 \rightarrow x_2 \le 2 - x_1$





An Engineering Design Example: Problem Formulation

An Engineering Design Example: Graphical Solution

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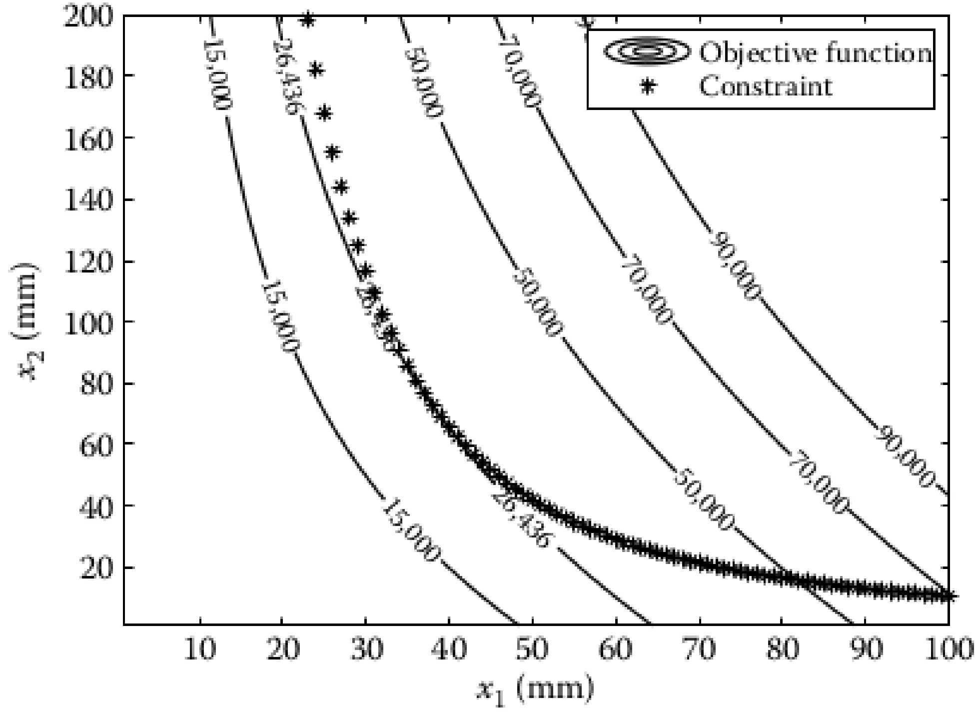
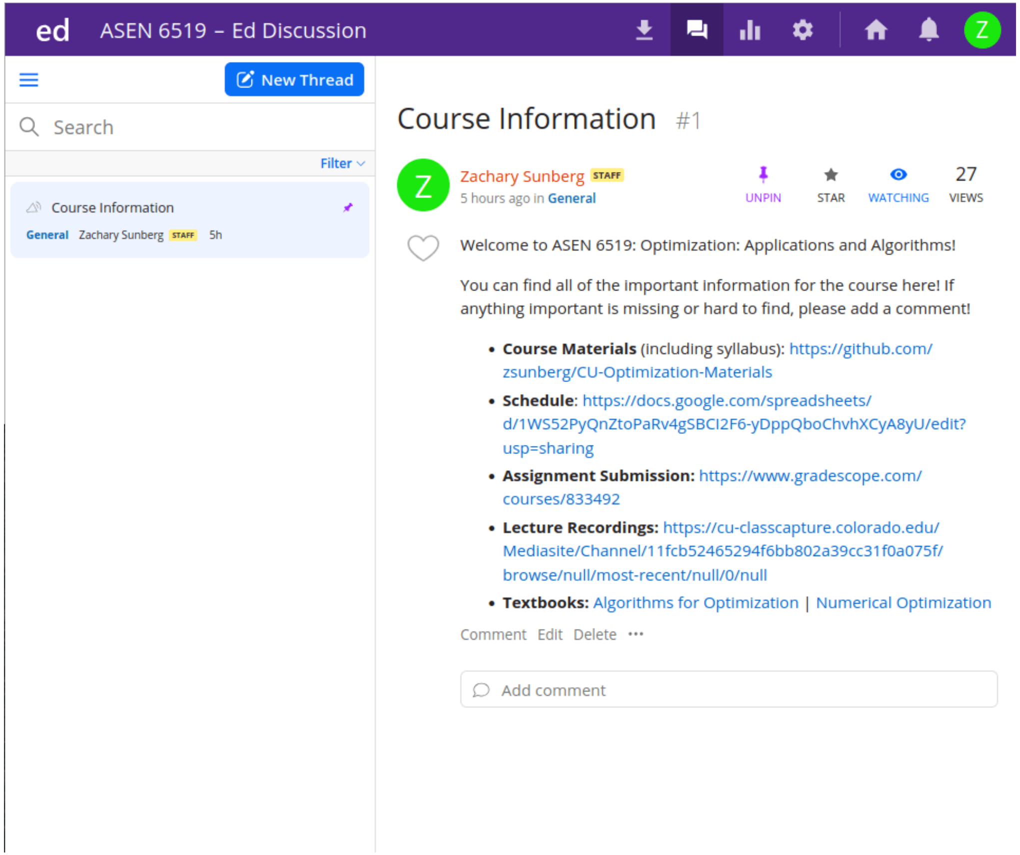


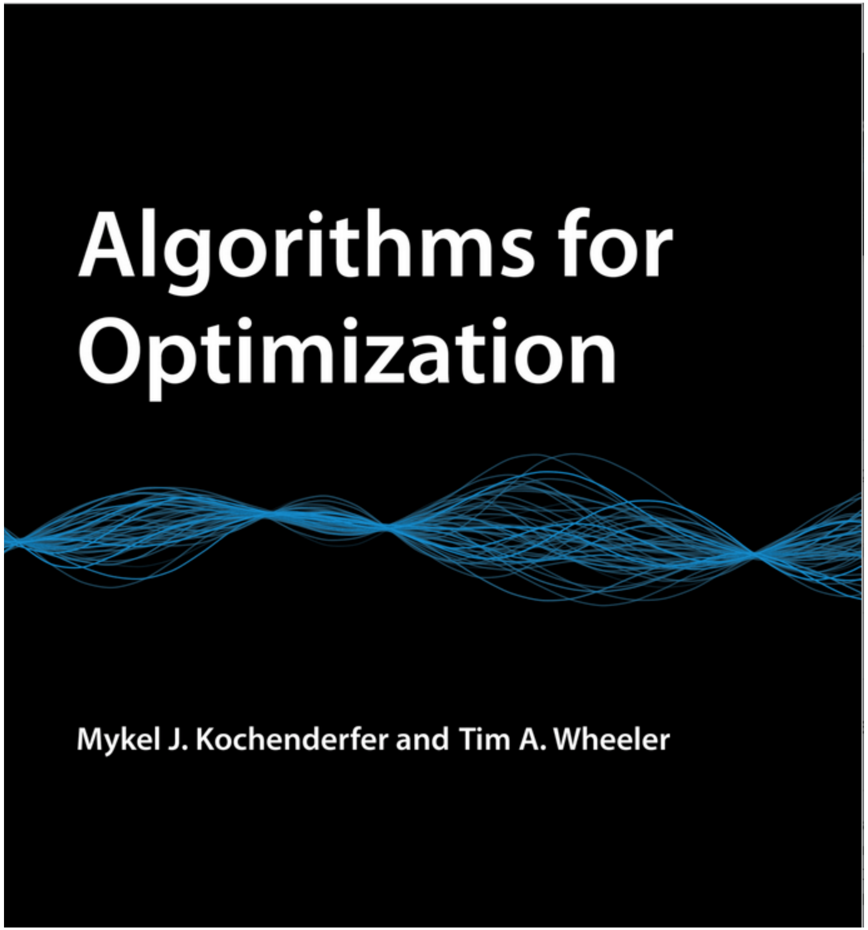
Course Hub: Edstem

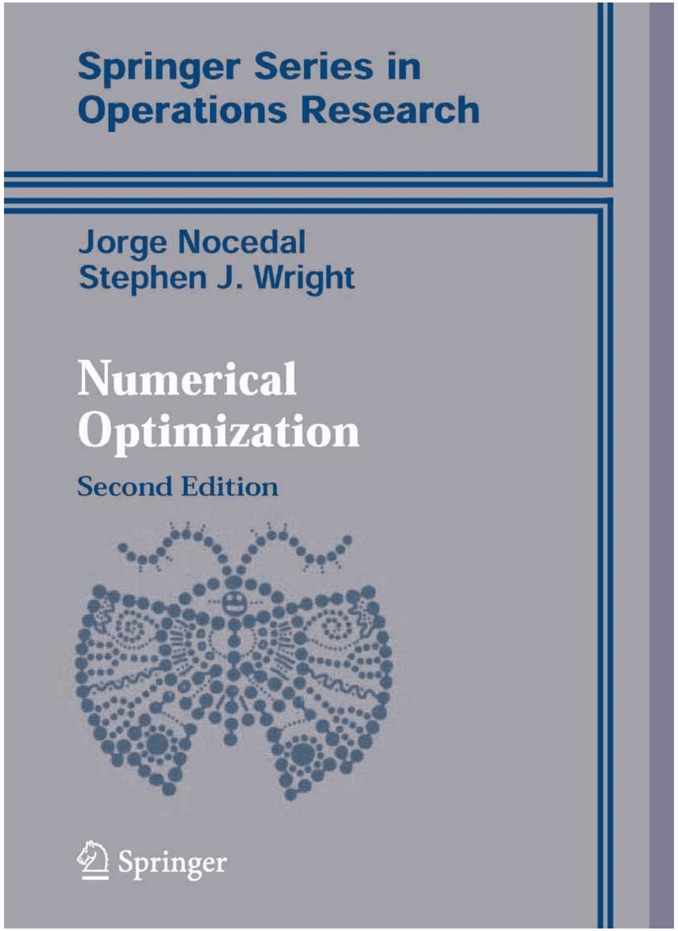


Syllabus:
https://github.com/zsunberg/CU-Optimization-Materials/blob/main/syllabus/syllabus.pdf

Join Ed:
https://edstem.org/us/join/txm78P

Books


Algorithms for Optimization
Mykel J. Kochenderfer and Tim A. Wheeler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