Large-scale nonlinear programming

IPOPT, short for "**I**nterior **P**oint **OPT**imizer, pronounced I-P-Opt", is a software library for large scale nonlinear optimization of continuous systems.

It is written in C++ (after migrating from Fortran and C) and is released under the EPL (formerly CPL). IPOPT implements a primal-dual interior point method, and uses line searches based on Filter methods (Fletcher and Leyffer).

fmincon Algorithms

`fmincon` has five algorithm options:

- `'interior-point'` (default)
- `'trust-region-reflective'`
- `'sqp'`
- `'sqp-legacy'`
- `'active-set'`

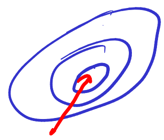
Sequential Quadratic Programming (SQP)

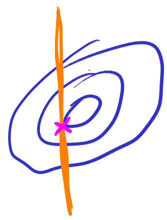
(notation from EDO)

Key Ideas:

- Like Newton's method, but with constraints
- An optimization problem with
  - Quadratic objective
  - Linear Equality Constraints

  can be solved with one matrix inversion





- Inequality constraints in the active set behave just-like equality constraints

General NLP

minimize $f(x)$

s.t. $h(x) = 0$

$\cancel{g(x) \le 0}$

$$\mathcal{L}(x,\lambda) = f(x) + \lambda^T h(x)$$

$H_{\mathcal{L}}$ Hessian of $\mathcal{L}$ w.r.t. x

J_h of h

KKT: $\nabla_x \mathcal{L} = 0$, $h(x) = 0$

Equality-Constrained QP

minimize $x^TQx + q^Tx$

s.t. $Ax + b = 0$

$\Longleftrightarrow$

Locally, near some x

minimize $\frac{1}{2}p^T H_{\mathcal{L}} p + \nabla_x \mathcal{L}^T p$

s.t. $J_h p + h = 0$

$$\mathcal{L}(x,\lambda) = \tfrac{1}{2}x^TQx + q^Tx + \lambda^T(Ax+b)$$

$$\text{KKT:}\begin{cases} \nabla_x \mathcal{L}(x,\lambda) = Qx + q + A^T\lambda = 0 \\ Ax + b = 0 \end{cases}$$

$$\begin{bmatrix} Q & A^T \\ A & 0 \end{bmatrix}\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} -q \\ -b \end{bmatrix}$$

solve this to get x^*, λ^*

$$\begin{bmatrix} H_{\mathcal{L}} & J_h^T \\ J_h & 0 \end{bmatrix}\begin{bmatrix} p_x \\ p_\lambda \end{bmatrix} = \begin{bmatrix} -\nabla_x \mathcal{L} \\ -h \end{bmatrix} \qquad (1)$$

SQP with equality constraints

loop

calculate $H_{\mathcal{L}}, J_h$ at x_k, λ_k

solve ① to find p_x, p_λ

$x_{k+1} \leftarrow x_k + \alpha p_x$

$\lambda_{k+1} \leftarrow \lambda_k + \alpha_k p_\lambda$

chosen through line search

Inequality Constraints

Key: identifying active set

Local approximation of NLP Lagrangian

$$\underset{s}{\text{minimize}} \quad \frac{1}{2} s^T H_{\mathcal{L}} s + \nabla_x \mathcal{L}^T s$$
$$\text{s.t.} \quad J_h s + h = 0$$
$$J_g s + g \le 0$$

$$\underset{x}{\text{minimize}} \quad \frac{1}{2} x^T Q x + q^T x$$
$$\text{s.t.} \quad Ax + b = 0$$
$$Cx + d \le 0$$

W_k = working set approximation of active set

$C_w x + d_w = 0$

active set rows

$C_n x + d_n < 0$

inactive set rows

Start assuming that we have x_k that satisfies

$$A x_k + b = 0$$
$$C_w x_k + d_w = 0$$
$$C_n x_k + d_n < 0$$

$$\underset{p}{\text{minimize}} \quad \frac{1}{2}(x_k + p)^T Q (x_k + p) + q^T (x_k + p)$$
$$\text{s.t.} \quad A(x_k + p) + b = 0$$
$$C_w (x_k + p) + d_w = 0$$