1-Dimensional Optimization

$f(x)$

strong local maximum

inflection point

strong local minimum

global minimum weak local minima

δ δ

x
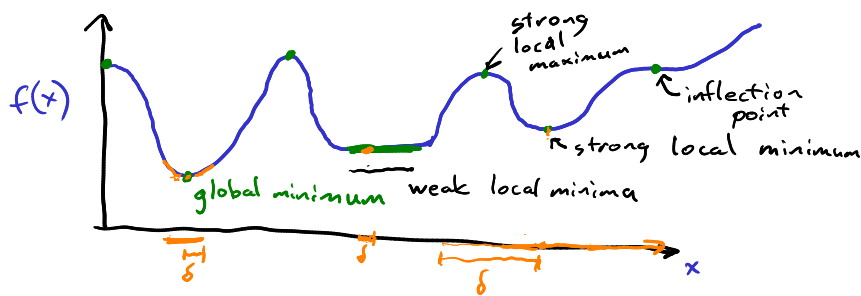
critical points

$f'(x) = 0$

hard to determine

global minimizer

x^* s.t. $f(x^*) \leq f(x)$ $\forall_x \in X$

local minimizer

x^* s.t. $\exists \delta > 0$ s.t. $f(x^*) \leq f(x)$ $\forall_x$ s.t. $|x - x^*| < \delta$

strong local minimizer

x^* s.t. $\exists \delta > 0$ s.t. $f(x^*) \leq f(x)$ $\forall_x$ s.t. $|x - x^*| < \delta$

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continuous

$f'(x) = 0$
$f''(x) > 0$ $\Rightarrow$ strong local minimum
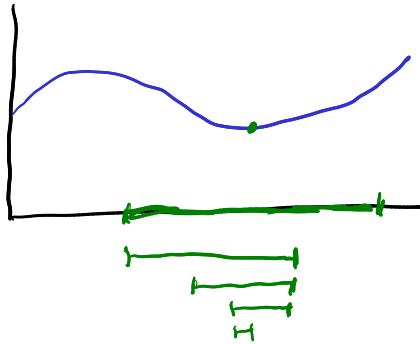
$f'(x) = 0$ FONC for a local minimum
$f''(x) \geq 0$ SONC

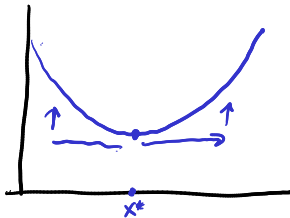not sufficient for local minimum (inflection point)

Bracketing

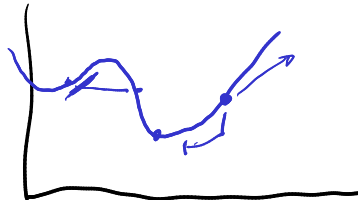1. Identify interval containing local minimum
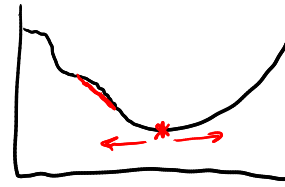2. Shrink interval

Property: Unimodality

A function f is <u>unimodal</u> if $\exists$ unique optimizer x^* such that f is <u>monotonically decreasing</u> for $x \leq x^*$ and monotonically increasing for $x \geq x^*$
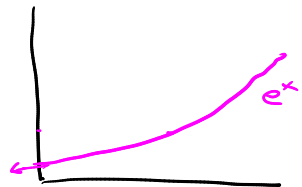
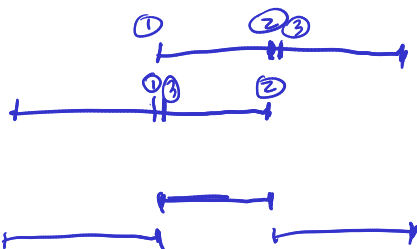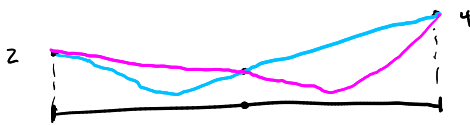

unimodal and convex
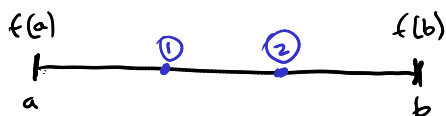
not unimodal and not convex

unimodal not convex

not unimodal convex

f unimodal $\longrightarrow$ bracketing finds a global minimum

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Break:

$f(a)$ ① ② $f(b)$

a b

Unimodal function have 3 function calls
how do you use the function calls
to shrink maximally

2 calls

$3I_1$ ① $2I_1$ ② $3I_1$ $\qquad I_5 = ⑧I_1$ $8=5+3$

$f(1) < f(2)$

③ $I_4 = 3I_1 + 2I_1 = ⑤I_1$ $5 = 3+2$

$f(3) < f(1)$

④ $I_3 = ③I_3$ $3 = 2+1$

$f(4) < f(3)$

⑤ $I_2 = ②I_1$

$f(5) < f(4)$

$①I_1$