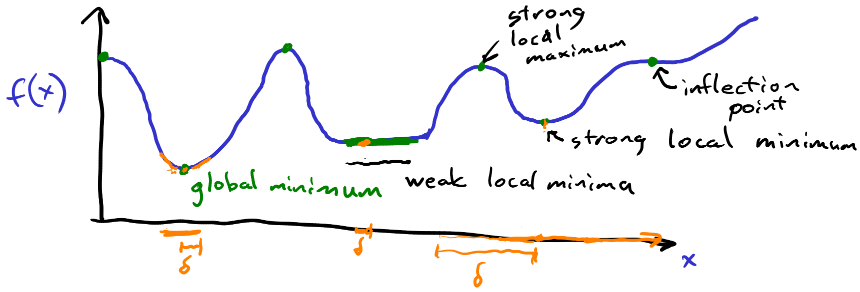
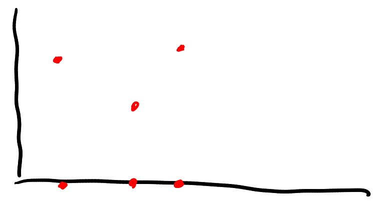
1-Dimensional Optimization



critical points

$f'(x) = 0$



hard to determine

global minimizer

x^* s.t. $f(x^*) \le f(x) \quad \forall_x \in X$

local minimizer

x^* s.t. $\exists\, \delta > 0$ s.t. $f(x^*) \le f(x) \quad \forall_x$ s.t. $|x - x^*| < \delta$

strong local minimizer

x^* s.t. $\exists\, \delta > 0$ s.t. $f(x^*) < f(x) \quad \forall_x$ s.t. $|x - x^*| < \delta$

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continuous

$f'(x) = 0$
$f''(x) > 0$ $\Rightarrow$ strong local minimum

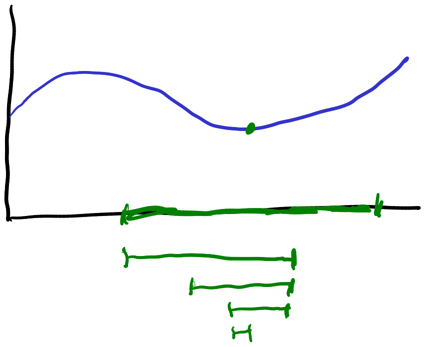
$f'(x) = 0$ FONC
$f''(x) \ge 0$ SONC

for a local minimum

not sufficient for local minimum (inflection point)

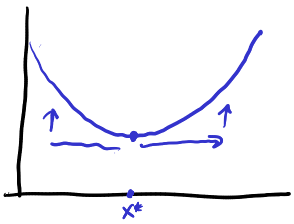
Bracketing

1. Identify interval containing local minimum
2. Shrink interval

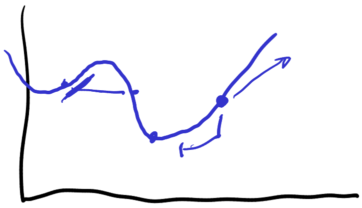


Property: Unimodality

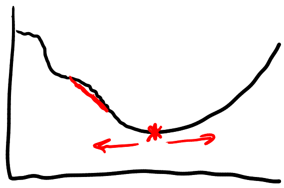
A function f is unimodal if $\exists$ unique optimizer x^* such that f is monotonically decreasing for $x \le x^*$ and monotonically increasing for $x \ge x^*$



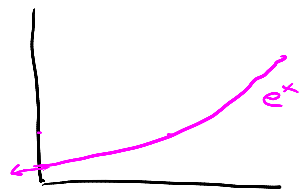
unimodal and convex



not unimodal and not convex



unimodal not convex



not unimodal convex

f unimodal $\longrightarrow$ bracketing finds a global minimum

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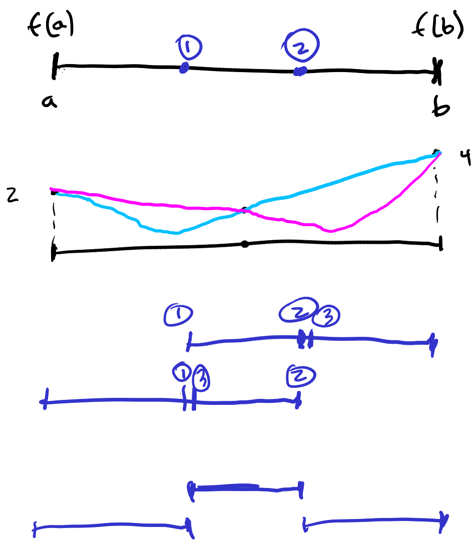
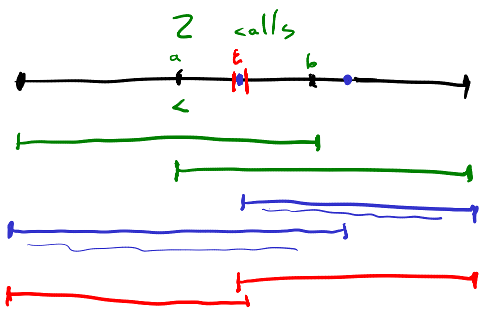
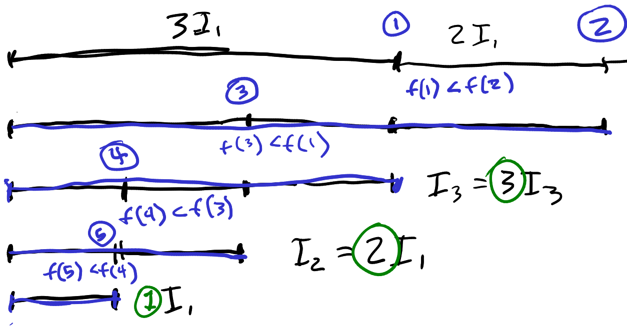
Break:



unimodal function
have 3 function calls
how do you use the function calls to shrink maximally





$I_5 = 8 I_1$ $\quad 8 = 5 + 3$

$I_4 = 3I_1 + 2I_1 = 5 I_1$ $\quad 5 = 3 + 2$

$I_3 = 3 I_3$ $\quad 3 = 2 + 1$

$I_2 = 2 I_1$

$1 I_1$