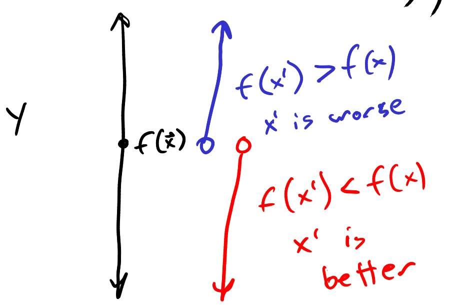
Multi-Objective Optimization

Example: maximize speed and fuel efficiency and safety

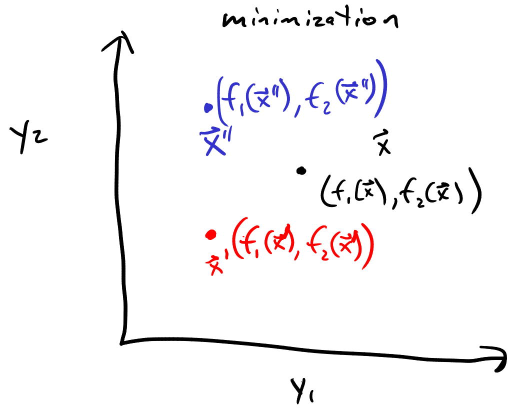
$f_1(\vec{x})$ $\quad$ $f_2(\vec{x})$ $\quad$ $f_3(\vec{x})$

Y_1 $\quad$ Y_2 $\quad$ Y_3

single objective (minimizing)



minimization

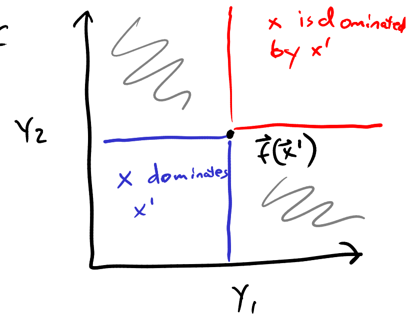


Dominance

Design point $\vec{x}$ is said to dominate $\vec{x}'$ if

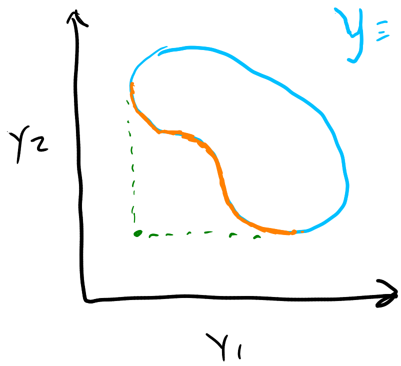
$$f_i(\vec{x}) \le f_i(\vec{x}') \quad \forall i$$

and $f_j(\vec{x}) < f_j(\vec{x}')$ for at least 1 j



Pareto Optimality

Design point $\vec{x}$ is Pareto optimal if there is no $\vec{x}'$ in the feasible set that dominates $\vec{x}$.



$\mathcal{Y} \equiv$ image of $\mathcal{X}$ through $\vec{f}$

Pareto Frontier: set of all Pareto-optimal design points.

Utopia Point

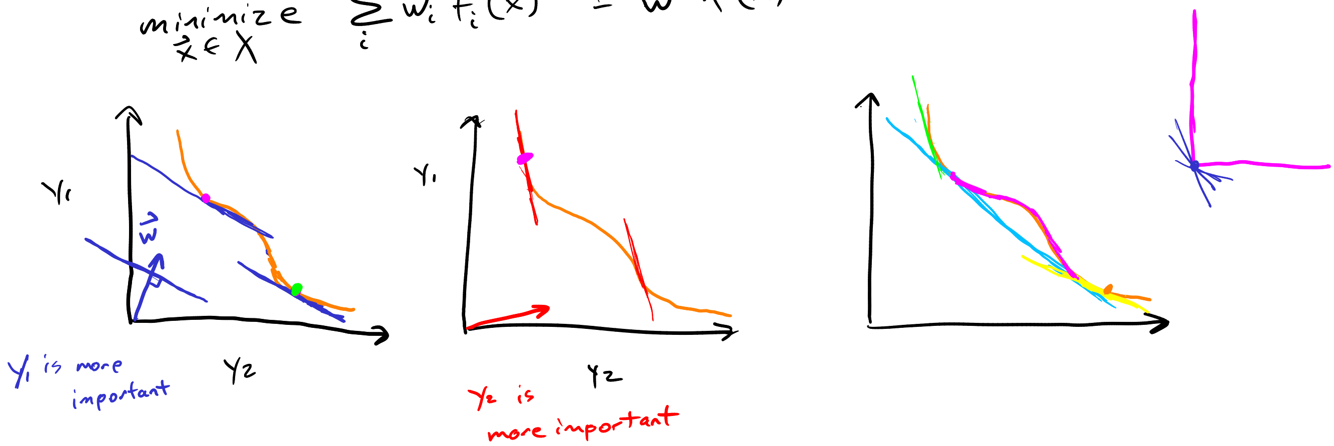
$\vec{y}^{\,utopia}$ is the point in the criterion-space consisting of the component-wise optimum

$$y_i^{utopia} \equiv \underset{\vec{x} \in \mathcal{X}}{\text{minimize}}\, f_i(\vec{x})$$

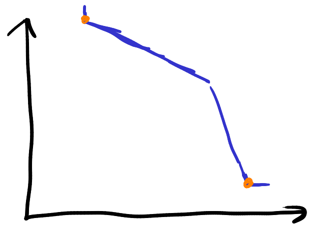
How to find design points on Pareto frontier

Method 1: weighting

$$\underset{\vec{x}\in X}{\text{minimize}} \sum_i w_i f_i(\vec{x}) = \vec{w}^T \vec{f}(\vec{x})$$



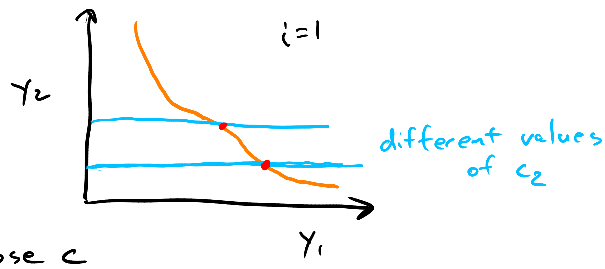
Cannot find points not on the convex hull of the Pareto front



Method 2: Constraints

$$\underset{x\in X}{\text{minimize}}\ f_i(\vec{x})$$

subject to $f_j(\vec{x}) \le c_j \quad \forall j \neq i$



downside: sometimes difficult to chose c

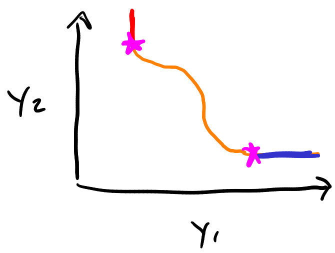
Method 3: Lexicographic Method

$$\underset{\vec{x}\in X}{\text{minimize}}\ f_1(\vec{x}) \rightarrow y_1^*$$

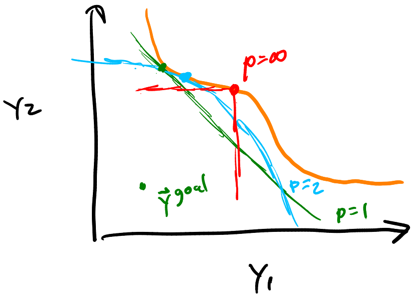
$$\underset{\vec{x}\in X}{\text{minimize}}\ f_2(\vec{x}) \quad \text{s.t.} \quad f_1(\vec{x}) \le y_1^*$$

y_2^*

$$\underset{x\in X}{\text{minimize}}\ f_3(\vec{x}) \quad \text{s.t.} \quad f_1(\vec{x}) \le y_1^*, \quad f_2(\vec{x}) \le y_2^*$$



Method 4: Goal Programming



$$\underset{x \in X}{\text{minimize}} \; \|f(\vec{x}) - \vec{Y}^{goal}\|_p$$

Method 5 weighted min-max method

$$\text{minimize} \; \max_i \left(w_i \left(f(\vec{x}) - y_i^{goal} \right) \right)$$

Can identify all points on Pareto front by sweeping through weights

equivalent to

$$\underset{\vec{x}, s}{\text{minimize}} \; s$$

$$\text{s.t.} \; \vec{w} \odot \left(\vec{f}(\vec{x}) - \vec{y}^{goal} \right) - s\mathbb{1} \leq 0$$