

# Text as Data

## Meeting 5: Supervised Machine Learning

Petro Tolochko

# Machine Learning

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  - Focus on prediction

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  - Text regression (scaling)

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→ *Dictionary methods encode human theory; supervised ML discovers patterns from data.*

# Supervised ML Pipeline

- Create a labeled dataset
- Apply a function that maps features → outcome (ML step)
- Assess performance

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- **Goal:** learn a function  $f(X) \rightarrow Y$  that generalizes to unseen documents.

# Labeled Dataset

- How:
  - Human coders annotate parts of the corpus
  - Found data (e.g., self-reported profession in users' profile)
- Considerations:
  - Sampling should be representative for the corpus (e.g., Random, Stratified sample e.g., across time and source)
  - Quality of human coding matters (Assess the intercoder reliability)
  - Number of documents

## Labeled Dataset

- Number of documents
  - the higher the number of categories and the lower the reliability of the coders, the higher the number of documents (Barberá et al., 2021)
  - increase the sizes of manually coded validation dataset as large as possible (e.g., more than 1% of all data to be examined), assuming acceptable reliability (equal to or higher than .7) (Song et al., 2021)

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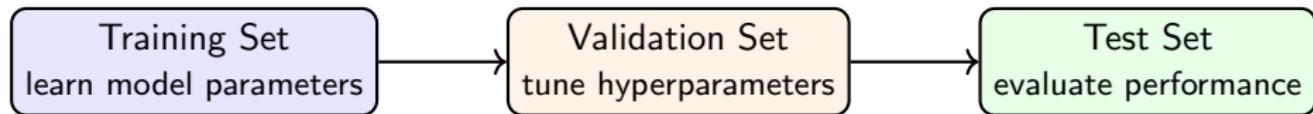
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- Test Data
  - The subset that is used to evaluate the final performance
  - Not used for learning

# Train–Validation–Test Split



Used during model development → Final evaluation only

*The validation set guides learning; the test set measures generalization.*

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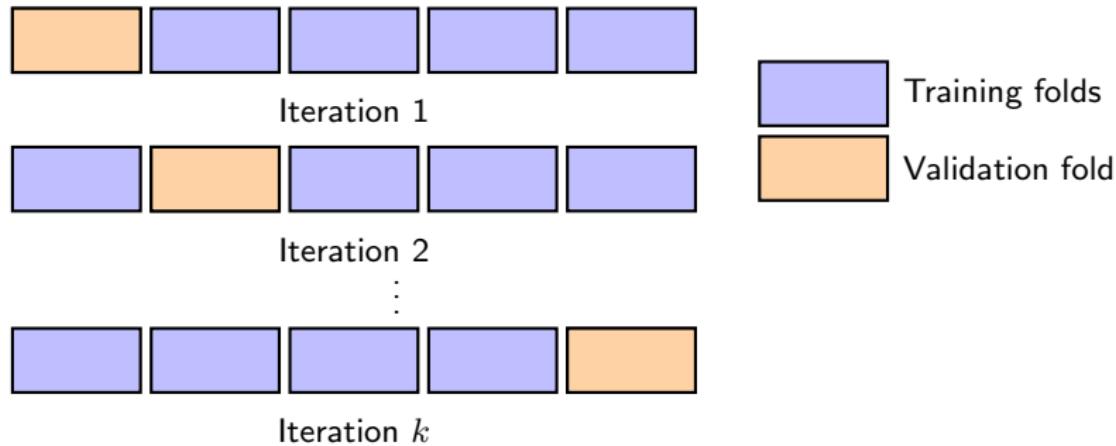
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- Repeat  $k$  times, each time with a different validation fold.
- Average performance across folds  $\Rightarrow$  robust estimate of generalization.

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*Each fold is used once for validation and  $k - 1$  times for training.*

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- Apply the model to *learn* which features in  $X$  (extracted from raw text) matter to recover  $Y$  (i.e., labels from the training data)

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# Questions?

# Naive Bayes

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*Bayesian idea: update prior beliefs with new evidence.*

## From Bayes' Rule to the Naive Bayes Classifier

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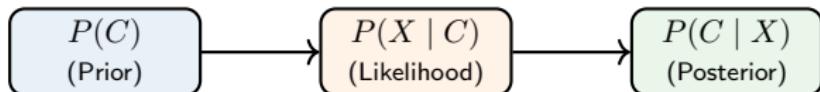
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→ *The classifier picks the class with the highest posterior probability.*

# How Priors and Likelihoods Combine



*Posterior = Prior × Likelihood (then normalized).*  
Naive Bayes picks the class with the highest posterior.

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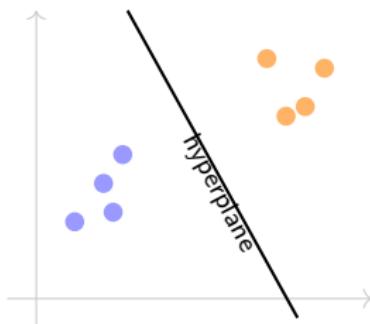
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- Interpretation: each class is a simple *language model* — choose the one most likely to have generated the document.

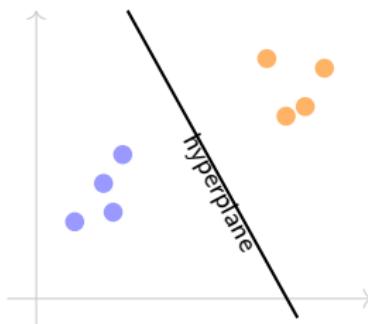
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# Support Vector Machines

# What is a Hyperplane?

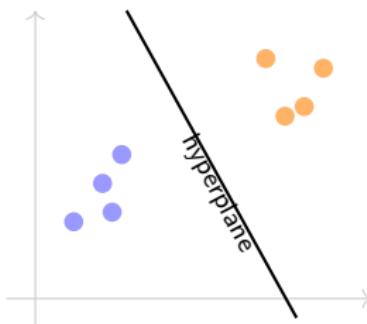


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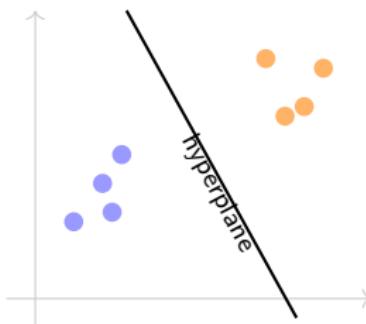
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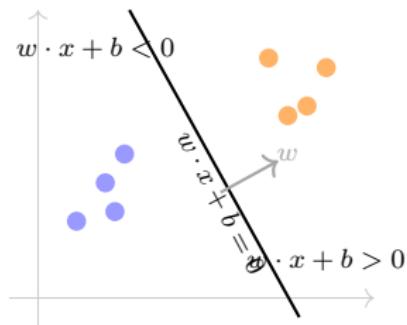


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- Used to separate points belonging to different classes.

# The Hyperplane Equation

$$\text{Hyperplane: } w \cdot x + b = 0$$

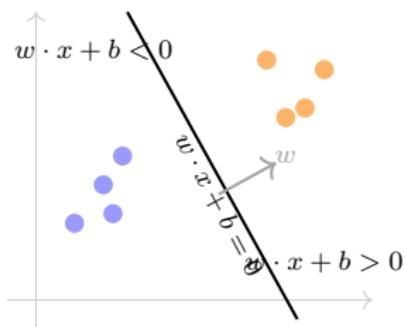
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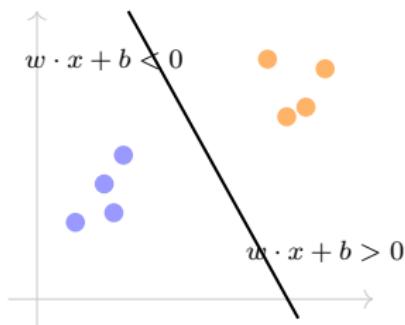


*The hyperplane is the boundary where  $w \cdot x + b = 0$ .*

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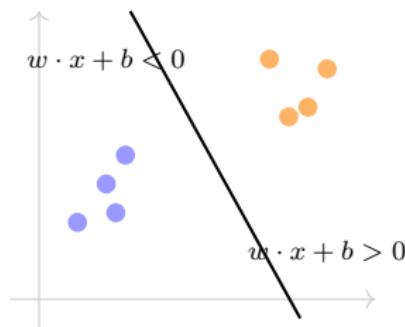
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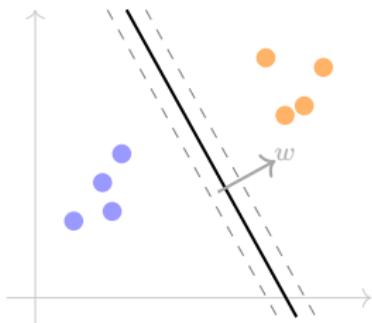


*The sign of  $(w \cdot x + b)$  determines on which side of the hyperplane a point lies.*

# Distance to the Hyperplane

Signed distance:  $d_i = \frac{w \cdot x_i + b}{\|w\|}$

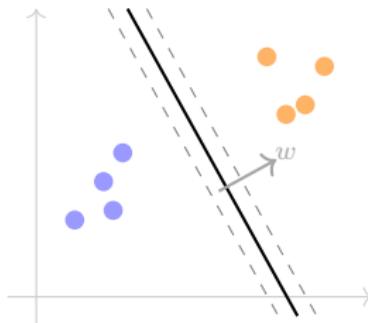
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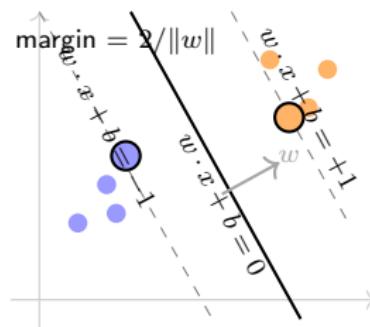
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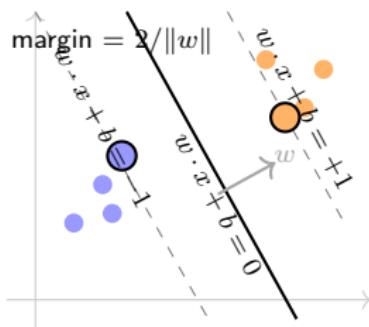


*SVM chooses the hyperplane that maximizes this margin.*

# Support Vectors and the Margin



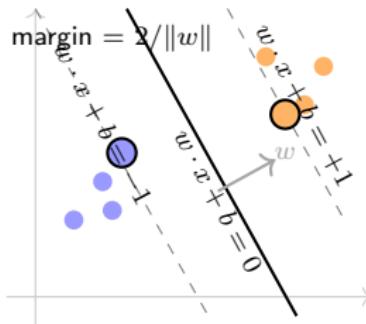
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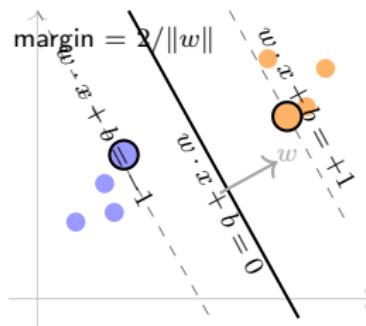


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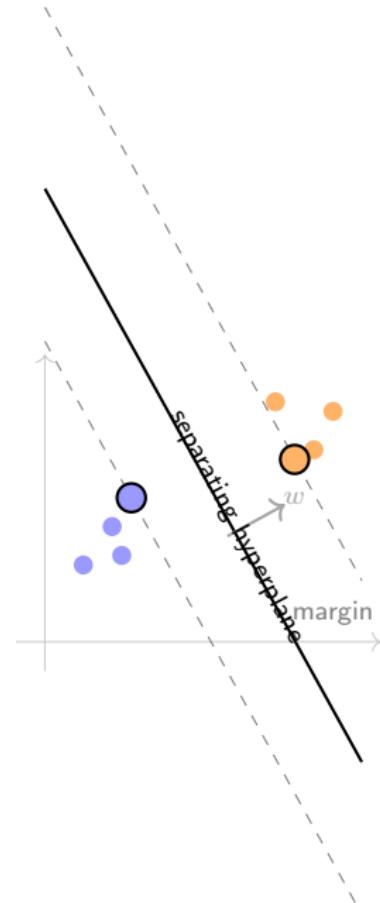
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- SVM maximizes the distance between these two margin planes.

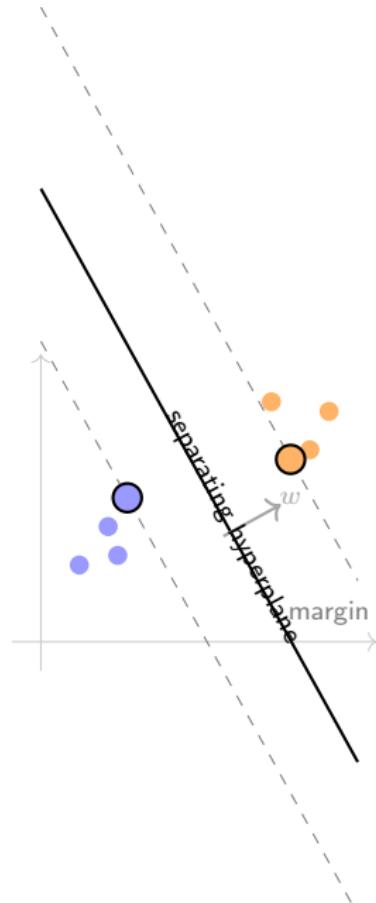
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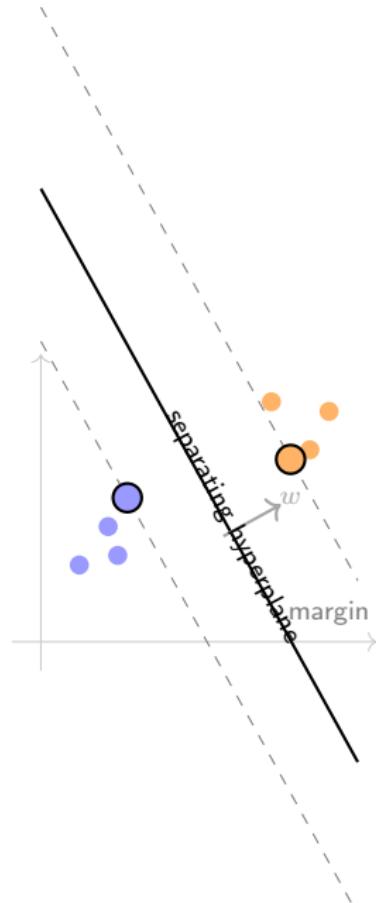
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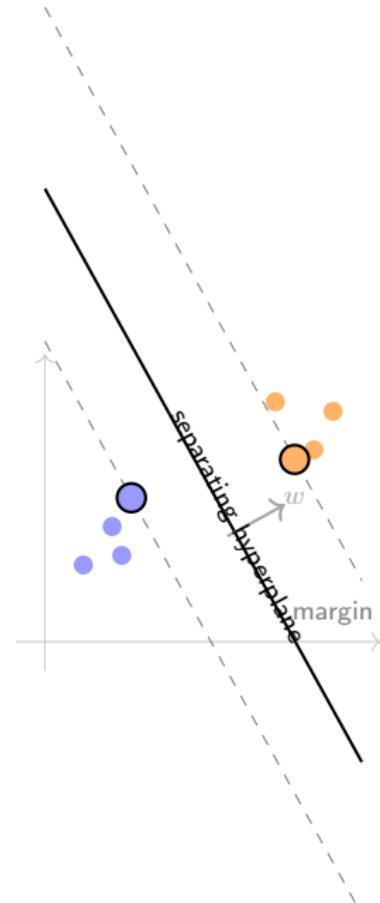
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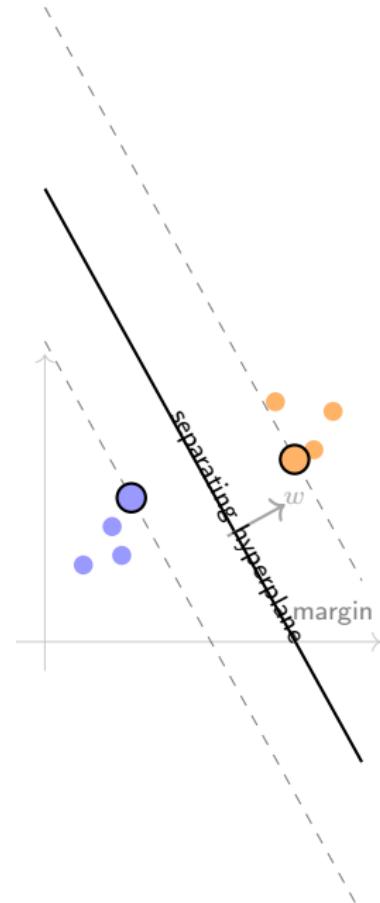


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- Works well for high-dimensional sparse data like text

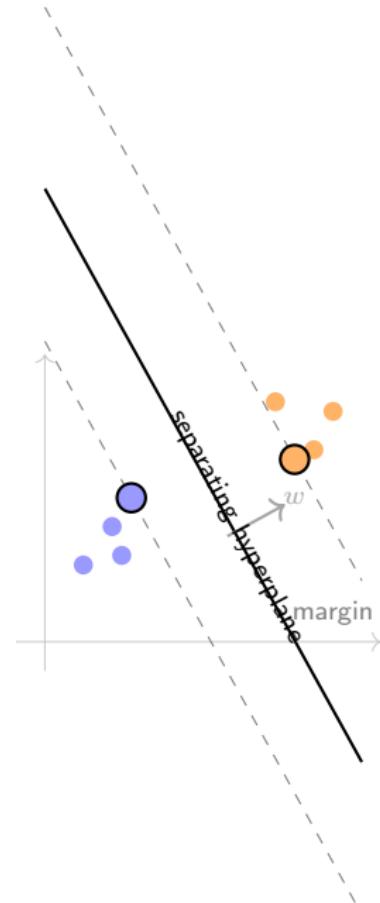


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- Represent each document as a feature vector (e.g., TF-IDF)
- SVM finds the hyperplane that best separates the classes
- **Goal:** maximize the margin between the closest points of different classes
- Decision rule:

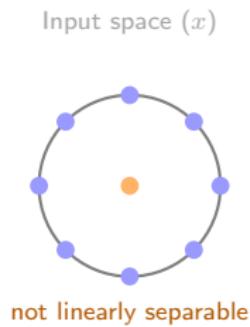
$$\hat{y} = \text{sign}(w \cdot x + b)$$

- Works well for high-dimensional sparse data like text



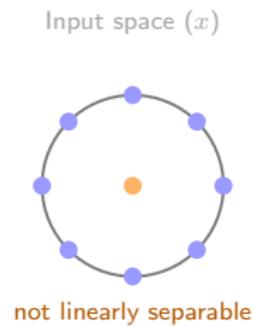
# When Linear Boundaries Fail

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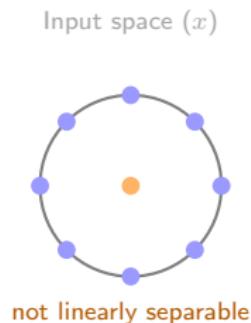
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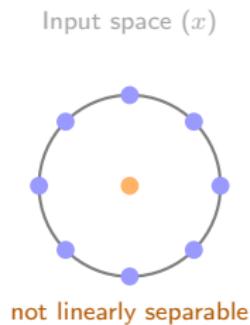
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*No linear hyperplane in 2D can separate these points.*



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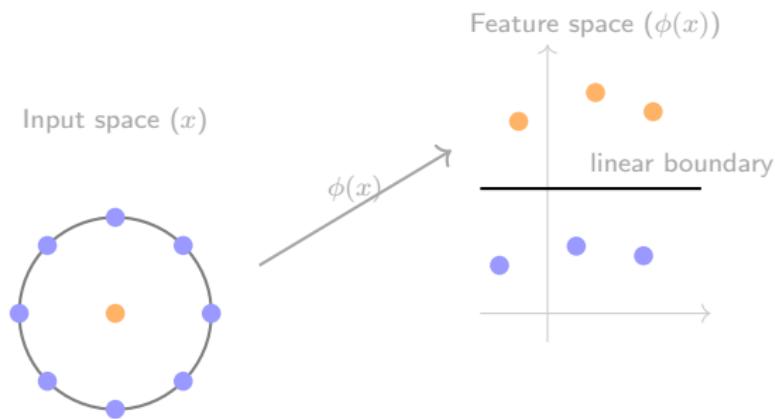
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- This is called the **kernel trick**, we get all the benefits of high-dimensional geometry without ever leaving the original feature space.

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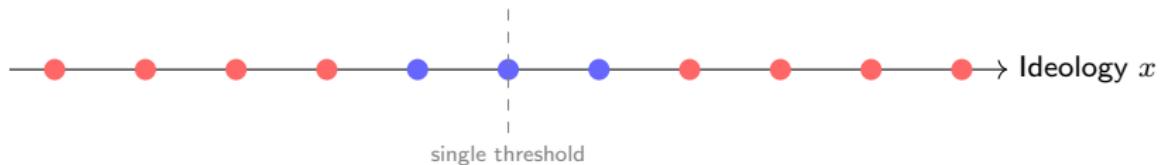


# Common Kernel Functions

Kernel	Formula	Intuition
Linear	$K(x_i, x_j) = x_i \cdot x_j$	Measures direct similarity (no transformation).
Polynomial	$K(x_i, x_j) = (x_i \cdot x_j + c)^d$	Captures feature interactions (e.g., word pairs).
RBF / Gaussian	$K(x_i, x_j) = e^{-\gamma \ x_i - x_j\ ^2}$	Distance-based similarity; very flexible and smooth.
Sigmoid	$K(x_i, x_j) = \tanh(\alpha x_i \cdot x_j + c)$	Behaves like a neural network activation.

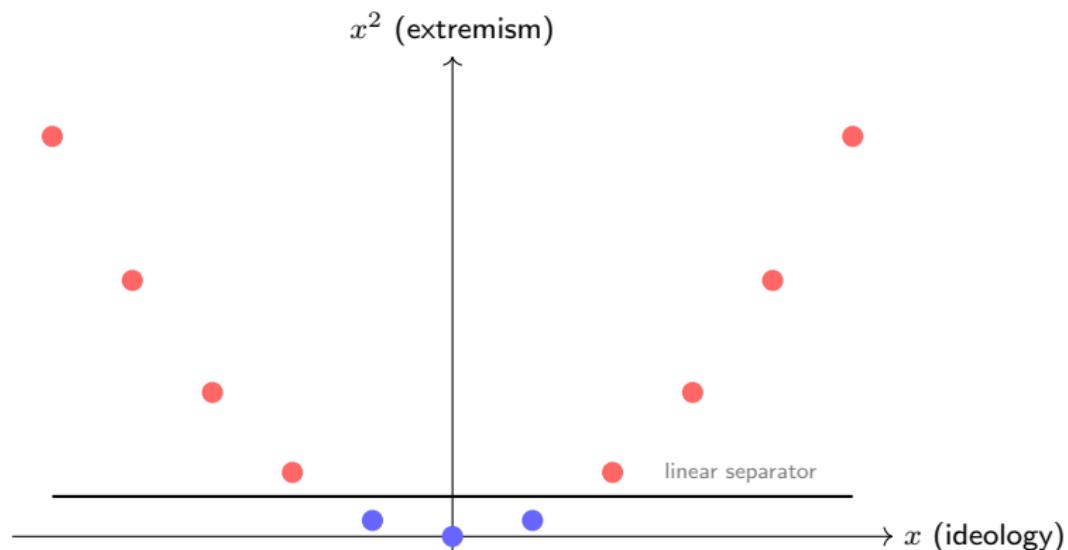
*Kernels measure similarity in an implicit feature space — they let SVMs handle nonlinear patterns without explicit transformation.*

# Ideology in 1D: Not Linearly Separable



*Extremes (red) lie on both sides of the center — one cut on a line can't separate them from moderates (blue).*

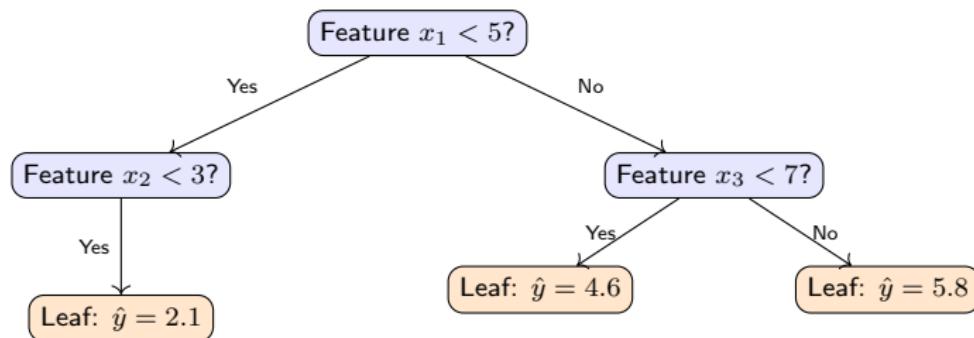
# Nonlinear Map $\phi(x) = (x, x^2)$ : Now Linearly Separable



After  $\phi(x)$ , a horizontal linear boundary separates extremes (red) from moderates (blue).

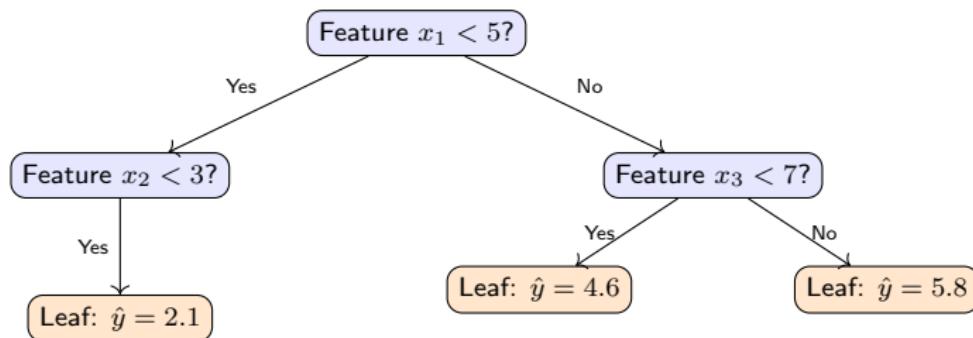
# Decision Trees

- Trees split the feature space into regions where target values are similar.



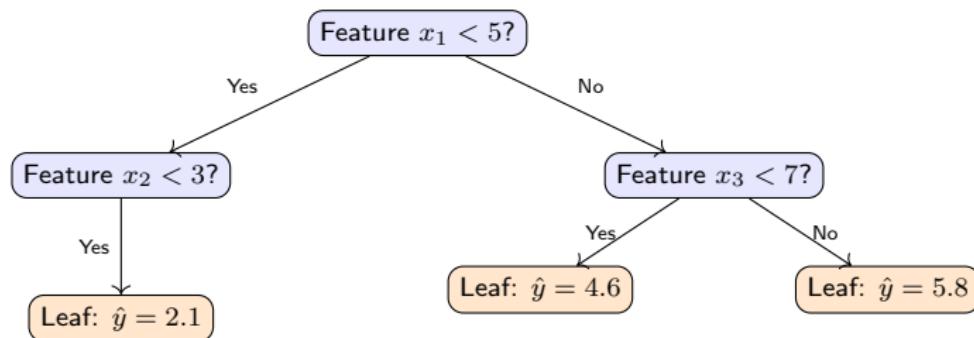
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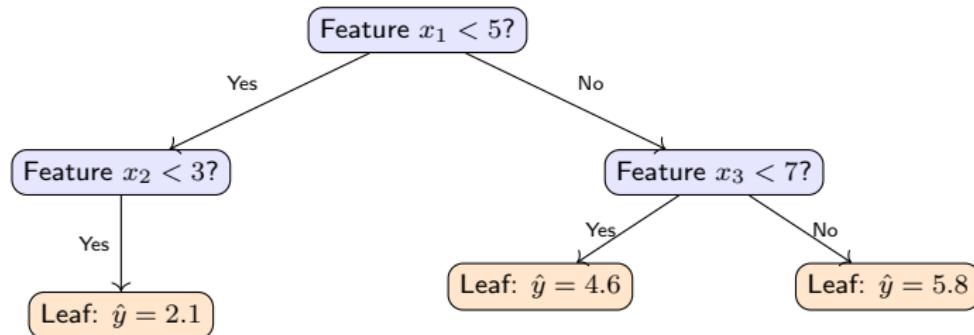
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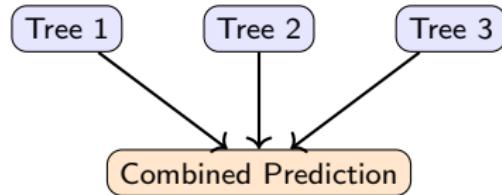
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- Each leaf predicts an outcome (class label or average value).
- The goal is to minimize a loss function over all splits (e.g., MSE or Gini impurity).



# Questions?

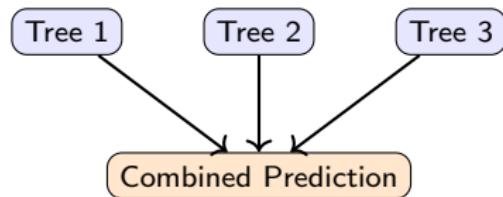
# From Trees to Ensembles

- A single tree is a weak learner — high variance and easy to overfit.



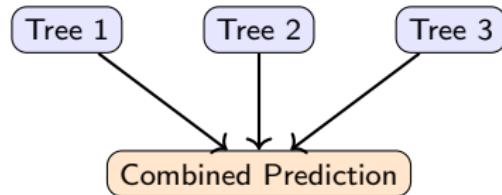
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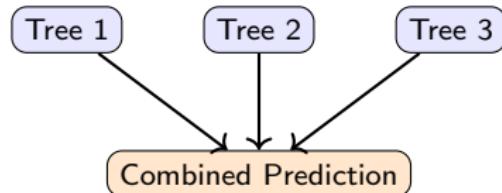
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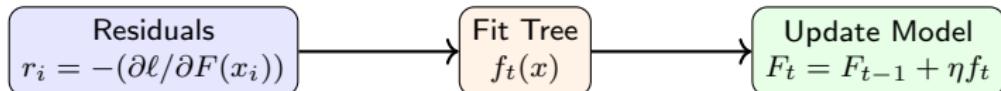
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- Ensemble prediction combines all trees' outputs for a stronger model.



# Gradient Boosting: Learning from Residuals

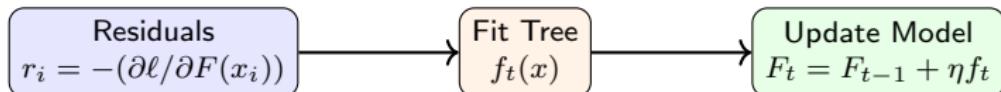
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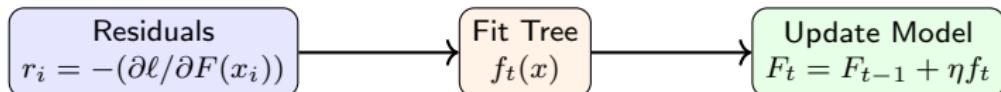


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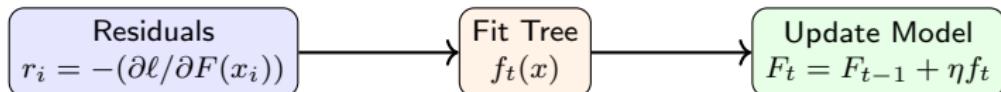
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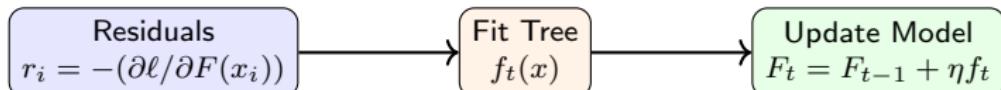
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