Calculation of Secondary Dependent Variables

Pertinent reading: *Section 8.6* in textbook

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General Steps in Performing Finite Element Analyses

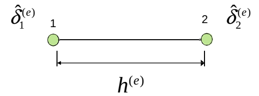
- Discretization of the solution domain.
- Assembly of *element* equations to form the *global* equations.
- Application of *nodal specifications*.
- Solution of the *global* equations.
- Calculation of the *secondary dependent variables* (e.g., “gradients”).
- Post-processing of the results.
- Interpretation of the results.

Calculation of Secondary Dependent Variables

- Typically the nodal values of the primary dependent variables are used to calculate values of the secondary dependent variables.
- This is a rather straightforward procedure (see Section 8.6 in the textbook).

Calculation of Secondary Dependent Variables

- Consider the specific case of a linear (two-node) element used in 1-d elastostatic analyses (see Example 7.7 in textbook).



Calculation of Secondary Dependent Variables

- The element interpolation functions for this element are

$$N_1 = \frac{1}{2}(1-\xi), \quad N_2 = \frac{1}{2}(1+\xi)$$

Calculation of Secondary Dependent Variables

The approximate *longitudinal displacement* (primary dependent variable) for a typical element is thus

$$\hat{\delta}^{(e)} = \sum_{m=1}^{2} N_m \hat{\delta}_m^{(e)} = \frac{1}{2}(1-\xi)\hat{\delta}_1^{(e)} + \frac{1}{2}(1+\xi)\hat{\delta}_2^{(e)}$$

Calculation of Secondary Dependent Variables

It follows that the approximate longitudinal (axial) *strain* for a typical element is thus

$$\hat{\varepsilon}^{(e)} = \frac{d\hat{\delta}^{(e)}}{dx} = \frac{d\hat{\delta}^{(e)}}{d\xi}\frac{d\xi}{dx} = \frac{d\hat{\delta}^{(e)}}{d\xi}\frac{2}{h^{(e)}}$$

(recall assumption of infinitesimal kinematics in Step 2 of Chapter 7).

Calculation of Secondary Dependent Variables

Substituting derivative of the interpolation functions gives

$$\hat{\varepsilon}^{(e)} = \frac{2}{h^{(e)}}\frac{d\hat{\delta}^{(e)}}{d\xi} = \frac{2}{h^{(e)}}\sum_{m=1}^{2}\frac{dN_m}{d\xi}\hat{\delta}_m^{(e)}$$

$$= \frac{2}{h^{(e)}}\left\{-\frac{1}{2}\hat{\delta}_1^{(e)} + \frac{1}{2}\hat{\delta}_2^{(e)}\right\} = \frac{1}{h^{(e)}}\left\{\hat{\delta}_2^{(e)} - \hat{\delta}_1^{(e)}\right\}$$

Calculation of Secondary Dependent Variables

The approximate element (axial) *stresses* follow directly; viz.,

$$\sigma^{(e)} = E^{(e)}\varepsilon^{(e)}$$

Calculation of Secondary Dependent Variables

Pseudo-code (for 1-d elements):

```
DO e=1,num_elements
  look up nodal coordinates associated w/element
  compute h^(e)
  look up material parameter E^(e)
  look up two nodal displacements associated w/element
  compute approximate strain & approximate stress
  possibly store & print results
END DO
```


Concluding Remarks

- The primary source of error associated with the calculation of secondary dependent variables is *cancellation error*.
- This is, however, minimized through the use of double precision floating point arithmetic for all calculations.