CIEG 401/601

Introduction to the Finite Element Method

Homework #5
Due no later than *5:00 p.m.*
on
Due on Wednesday, May 2, 2018.
in
Room 301[1] or 360F P. S. DuPont Hall

• Problem 11

Please solve Exercise 8.1 in the textbook.

• Problem 12

Please solve Exercise 8.3 in the textbook.

[1] CIEG departmental office

• Problem 13

Draw the simplest mesh with which the problem shown in Figure 1 can be modeled for purposes of an elastostatic finite element analysis. Carefully indicate node and element numbers. Assume that the three materials are characterized by elastic moduli E_1, E_2, and E_3, respectively. In addition, assume that the cross-sectional areas of the three materials are A_1, A_2, and A_3, respectively.

Assuming that an analysis of this problem was actually performed, how could an analyst check its validity?

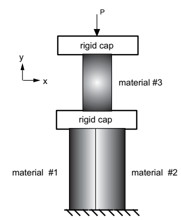

Figure 1: "Series-Parallel" Arrangement of Bars

NOTE: Do not forget that only *vertically* oriented one-dimensional elements shall be used.

NOTE: The interface between materials 1 and 2 is assumed to be *smooth*. As a result, no frictional forces are developed along this interface.

NOTE: There is no need to actually solve this problem.

• Problem 14

The velocity of fully developed *laminar*, *viscous* flow of an *incompressible* fluid through a circular pipe is given by [?]

$$v_x = -\frac{1}{4\mu}\frac{dP}{dx}\left[1 - \left(\frac{2r}{d}\right)^2\right]$$

where dP/dx is the pressure gradient, r is the radial coordinate, d is the pipe diameter [units of L^2], and μ is the absolute (or dynamic) viscosity [units of FtL^{-2}].

The volume of flow Q [units of $\mathrm{L}^3\mathrm{t}^{-1}$] is obtained by integrating v_x over the pipe cross-section, giving

$$Q = -\frac{\pi d^4}{128\mu}\frac{dP}{dx}$$

where the negative sign indicates that the flow is in the direction of negative pressure gradient.

Following the five-step procedure discussed in Chapter 7, the equations for a linear (2-node) element associated with this physical problem (Figure 2) are found to be

$$\frac{1}{R^{(e)}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \hat{P}_1^{(e)} \\ \hat{P}_2^{(e)} \end{Bmatrix} = \begin{Bmatrix} Q_1^{(e)} \\ Q_2^{(e)} \end{Bmatrix}$$

where

$$R^{(e)} = \frac{128\mu^{(e)}h^{(e)}}{\pi\left[d^{(e)}\right]^4}$$

is called the *pipe resistance*. Here $\mu^{(e)}$ is the element absolute (or dynamic) viscosity, $h^{(e)}$ is the element length, and $d^{(e)}$ is the element diameter.

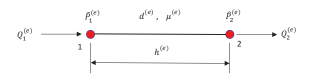

Figure 2: Typical linear element for pipe flow.

Consider the network of circular pipes shown in Figure 3. The length and diameter of each pipe is shown in the figure.

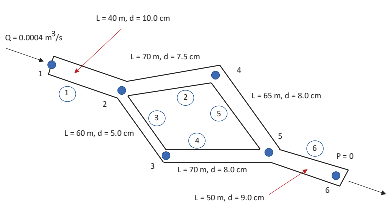

Figure 3: Pipe flow example (after Reddy [?]).

Please do the following:

a) Using a *minimal* number of elements, create a mesh for the problem.

b) Assuming that for each element $\mu^{(e)} = 8.90 \times 10^{-4}$ Pa-s, form the equations for each element.

c) Assemble the equations to form the global arrays.

d) Using the elimination approach at the global level, apply the nodal specifications to the global equations.

e) Solve the resulting equations for the unknown values of the primary dependent variables (i.e., the nodal pressures).

• Problem 15

Consider the beam shown in Figure 4. The material is characterized by an elastic modulus of $E = 10 \times 10^3$ kips/in^2, where 1 kip = 1000 pounds. The moment of inertia for the cross-section is $I = 320$ in^4.

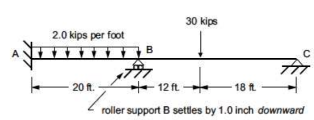

Figure 4: Beam subjected to uniformly distributed load and a point load with settling support.

The boundary conditions at point A are

$$w = \bar{w} = 0, \quad \text{and} \quad \frac{dw}{dx} = \bar{\theta} = 0$$

At points B and C, the boundary conditions are

$$w = \bar{w} = 0, \quad \text{and} \quad EI\frac{d^2w}{dx^2} = \bar{M} = 0$$

Using the equations for the Bernoulli-Euler beam element developed in lecture and provided on the hand-out, please do the following:

a) Using a *minimal* number of elements, create a mesh for the problem.

b) Form the equations for each element.

c) Assemble the equations to form the global arrays.

d) Using the elimination approach at the global level, apply the nodal specifications to the global equations.

e) Solve the resulting equations for the unknown values of the primary dependent variables (i.e., the nodal values of the transverse displacement and the rotation).

NOTE: there is no need to re-derive any of the element equations given in lecture or in the associated hand-out.