Element Interpolation Functions

Pertinent reading: *Chapter 9* in textbook

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Contents of Presentation

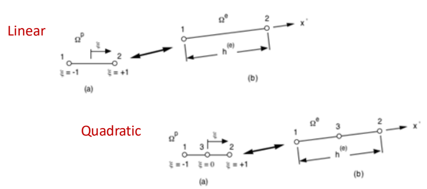
- General Approach.
- General Requirements for Element Interpolation Functions.
- Approach Involving Matrix Inversion.

General Approach

- Develop the element interpolation functions in a "parent" domain $\wedge^p$.
- Uniquely map $\wedge^p$ to the actual element domain ($\wedge^e$) for each element.
- *Chapter 10* in the textbook gives additional details pertaining to element mapping.

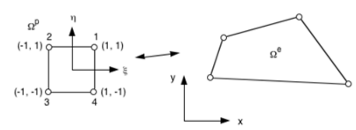
General Approach

Example: mapping of a one-dimensional line elements



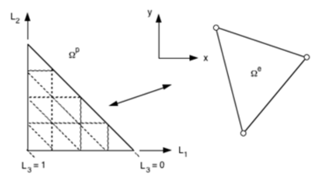
General Approach

Example: mapping of a 4-node quadrilateral element (see Example 9.1 in textbook)



General Approach

Example: mapping of a 3-node triangular element



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General Requirements for Element Interpolation Functions

- All element interpolation functions must satisfy the *fundamental* ("Kronecker delta") *requirement* for all interpolation functions; viz.,

$$N_i\left(\xi_j,\eta_j,\zeta_j\right)=\delta_{ij}$$

General Requirements for Element Interpolation Functions

- Satisfy (hopefully) the *compatibility criterion.*
- Satisfy the *completeness criterion.*

General Requirements for Element Interpolation Functions

- In addition, as part of the *completeness* criterion, the element interpolation functions must satisfy the following relation:

$$\sum_{i=1}^{N_{dof}} N_i = 1$$

where N_{dof} = number of element *degrees of freedom* associated with *a given primary dependent variable*.

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Approach Involving Matrix Inversion

- Recall the approach used in Chapter 7 to derive element interpolation functions.
- The *interpolating polynomial* was written in the following general form:

"basis" functions

$$\hat{\phi}^{(e)} = \alpha_1 + \alpha_2\xi + \alpha_3\eta + \alpha_4\zeta + \text{K} = \chi\alpha$$

"generalized" coordinates

Approach Involving Matrix Inversion

- The polynomial is next evaluated at each of the nodes in the element.
- The number of nodes corresponds to the number of (unknown) generalized coordinates appearing in the polynomial.
- This gives the following general result:

$$\hat{\boldsymbol{\varphi}}_{\mathbf{n}}^{(\mathbf{e})} = \mathbf{A}\boldsymbol{\alpha}$$

Approach Involving Matrix Inversion

- Inverting $\mathbf{A}$ gives the following expression:

$$\boldsymbol{\alpha} = \mathbf{A}^{-1}\hat{\boldsymbol{\varphi}}_{\mathbf{n}}^{(\mathbf{e})}$$

- Then, upon substitution

$$\hat{\phi}^{(e)} = \chi\boldsymbol{\alpha} = \chi\mathbf{A}^{-1}\hat{\boldsymbol{\varphi}}_{\mathbf{n}}^{(\mathbf{e})} = \mathbf{N}\hat{\boldsymbol{\varphi}}_{\mathbf{n}}^{(\mathbf{e})}$$

Approach Involving Matrix Inversion

Remarks

- For some types of elements $\mathbf{A}^{-1}$ may not exist for all orientations of the element in the global coordinate system.
- For large values of N_{dof} the analytic determination of $\mathbf{A}^{-1}$ may require a substantial computational effort. This effort is, however, lessened by the availability of software capable of carrying out symbolic arithmetic operations.

Approach Involving Matrix Inversion

Remarks

- In developing element interpolation functions using generalized coordinates, it is not always an easy task to satisfy *spatial isotropy* (see *Section 7.5.2* in textbook).

Approach Involving Matrix Inversion

- It is thus desirable to develop a procedure by which the interpolation functions can be written down *directly*, thus avoiding the potential pitfalls and excessive computational effort associated with the aforementioned approach.
- Such a direct approach is particularly useful when *higher-order* elements are required.

Approach Involving Matrix Inversion

- *Higher-order* elements maintain the interelement continuity of lower order elements, but employ a higher-order approximation (e.g., more terms in the polynomial).
- Relatively small numbers of higher-order elements are typically capable of more accurate representations than the linear ones discussed in the previous chapters.

Approach Involving Matrix Inversion

- As will be discussed in *Chapter 10*, the boundary edges and surfaces of such elements can also be curved, thus allowing for more accurate representations of element domains.

Approach Involving Matrix Inversion

- As compared to the basic linear elements, higher-order elements are, however, *more expensive* to formulate; as a result, the cost-effectiveness of various elements represents an area of on-going dispute.

- Since the optimal choice of element type is very often problem-dependent, it follows that no single element is exclusively preferred.