



General Remarks

- Since it has *not* been modified to account for constraints imposed at boundary and possibly interior nodes, the system property matrix **K** is singular at this stage of theanalysis.
- In order to specialize the problem, the *nodal* constraints must next be considered.

V.N. Kaliakin



General Remarks

- In finite element analyses, constraints on the nodal values of the primary dependent variables or their derivatives are required in order to account for, or approximate, the actual constraints imposed on the body being analyzed.
- It is only through such constraints that a problem can be uniquely posed.

V.N. Kaliakin



General Remarks

- Since the values of the primary dependent variables, or their derivatives, can be specified at any node in the mesh, the previous notion of "boundary conditions" must be generalized to *nodal* specifications.
- Along the total boundary Γ of a body, at nodes along element boundaries, or at any node within an element, two types of nodal specifications are typically possible, namely:



General Remarks

- Known values of the primary dependent variables are specified along the portion of Γ (or at an interior node) denoted by Γ_1 . These are referred to as essential or Dirichlet specifications.
- Known values of the gradients of the primary dependent variables are specified along the portion of Γ (or at an interior node) denoted by Γ_2 . These are referred to as *natural* or *Neumann* specifications.

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General Remarks

- With regard to a *specific* primary dependent variable at a specific node in a body, either an essential constraint or a natural constraint, but not both can be specified.
- Stated mathematically,

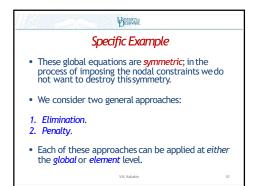
$$\Gamma_1 \cup \Gamma_2 = \Gamma$$
, $\Gamma_1 \cap \Gamma_2 = \emptyset$

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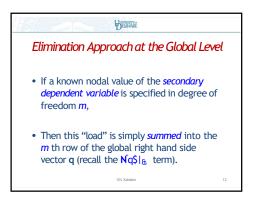


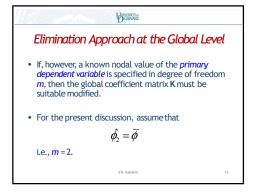
Specific Example

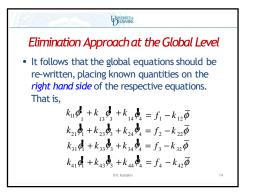
- To present the techniques used to specify nodal constraints, consider a hypothetical mesh with four degrees of freedom.
- The associated global equations are thus

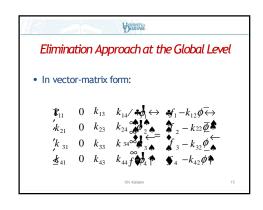


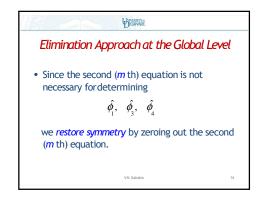


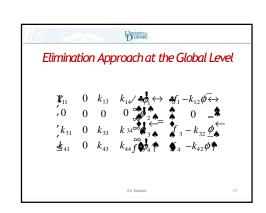


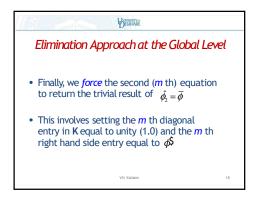


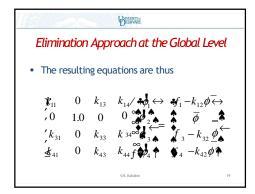


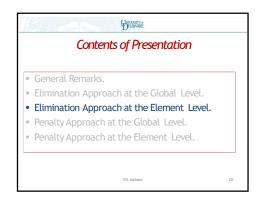


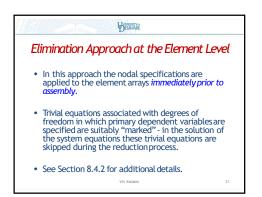


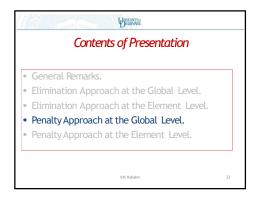


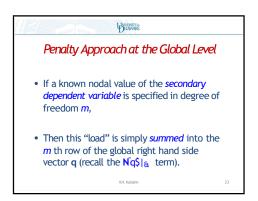


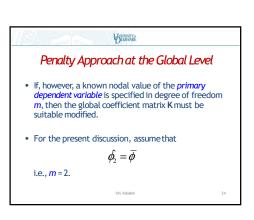


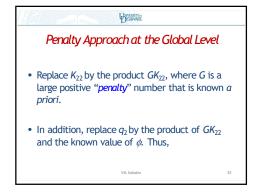


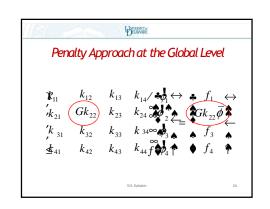


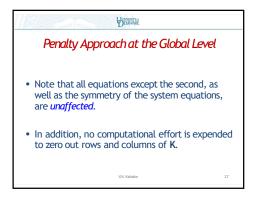


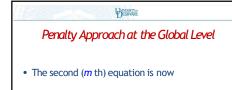












$$k_{21}\phi_1 + Gk_{22}\phi_2 + k_{23}\phi_3 + k_{24}\phi_4 = Gk_{22}\overline{\phi}$$

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• Since the magnitude of *G* has been chosen to be *large*, and since the elements of *K* are typically of the similar order, it follows that the term *GK*₂₂ will *dominate* the left hand side of the previous equation.

• This equation thus effectively reduces to

$$\phi_1 + G\phi_2 + \phi_3 + \phi_4 \approx G\phi$$

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Penalty Approach at the Global Level

• This equation thus effectively reduces to $\phi_1 + G\phi_2 + \phi_3 + \phi_4 \approx G\phi$ • Implying that $G\phi_2 \approx G\phi \implies \phi_2 \approx \overline{\phi}$



