

# DATA REDUCTION

## 3-Point Bending

The diagram illustrates the relationship between two methods for calculating the Modulus of Elasticity ( $E_x$ ). On the left, a box labeled "Modulus" contains the equation  $E_x = \frac{\sigma_x}{\epsilon_x}$ . An arrow points from this box to the right, where another equation is shown:  $E_x = \frac{3PL}{2bh^2}$ . Below this equation, the text "Strain Gage Method" is written vertically. A vertical arrow points downwards from the "Modulus" box to the "Strain Gage Method" text, indicating that the two equations are equivalent.

$$E_x = \frac{3PL}{2bh^2}$$

Strain Gage Method

$$E_x = \frac{PL^3}{4bh^3\delta} (1 + S)$$

$$\text{Where } S = \frac{3h^2E_x}{2L^2G_{xz}}$$

$\delta$  = deflection of beam at  $L/2$

## ANALYSIS OF RESULTS

### Four-Point Bending

#### Assumptions

- Homogeneous
- $E_c = E_t$
- Small deflection

#### Euler Beam Solution

$$\sigma = \frac{Mc}{I} \longrightarrow \sigma = \frac{3PL}{4bh^2}$$

## DATA REDUCTION

### 4-Point Bending

$$\boxed{\text{Modulus}} \quad E_x = \frac{3 P L}{4 b h^2} \quad \text{Strain Gage Method}$$

Deflection Method

$$E_x = \frac{P L^3}{64 b h^3 \delta} (11 + 8 S)$$

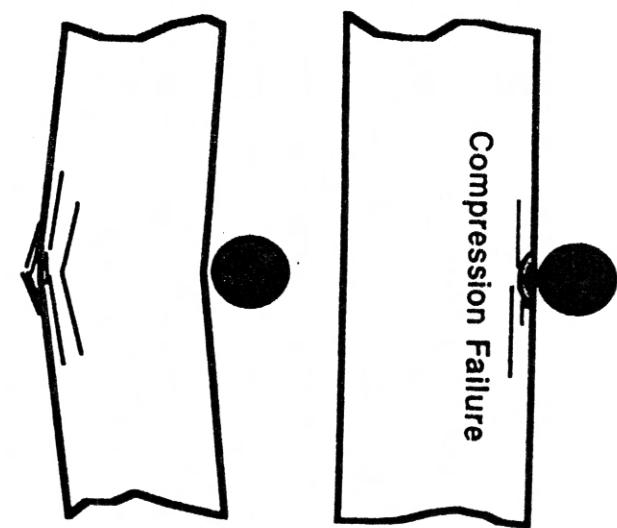
$$\text{Where } S = \frac{3 h^2 E_x}{2 L^2 G_{xz}}$$

$\delta$  = deflection of beam at  $L/2$

### CHARACTERISTICS OF SHEAR CORRECTION

- Neglecting  $S$  →  $E_{\text{apparent}} < E_{\text{tensile}}$
- Minimizing  $S$  → Decrease Influence of  $S$
- Increasing  $L/h$  → Decrease Influence of  $S$
- Decreasing  $G_{xz}$  → Increasing Influence of  $S$   
(for  $E_x$  constant)

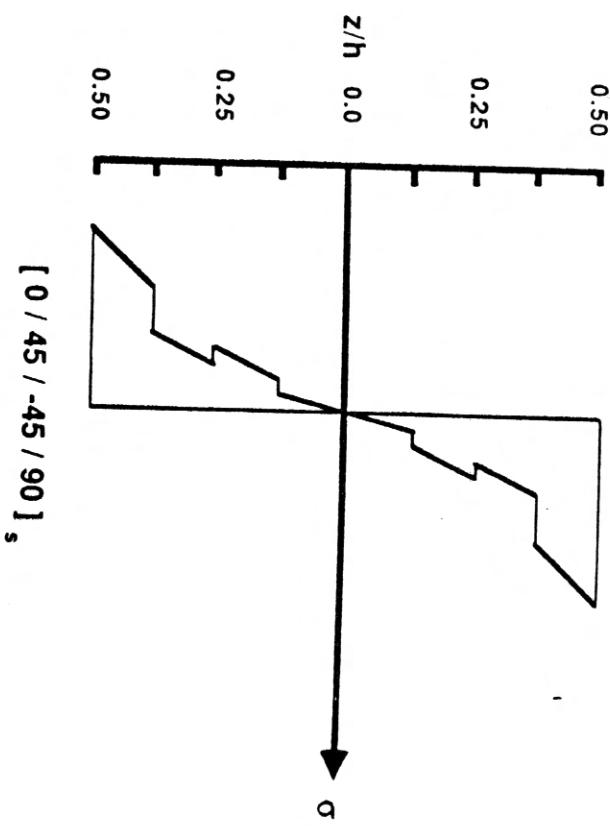
## FAILURE MODES



Tension Failure

Compression Failure

### Laminate Stresses



$[0 / 45 / -45 / 90]_s$

## COMPRESSION TESTS



### UNAXIAL COMPRESSION

#### 0° Coupon Properties

- $E_1^c$  – Compression Modulus in Fiber Direction
- $S_1^c$  – Compression Strength in Fiber Direction
- $\epsilon_1^c$  – Ultimate Compressive Strain in Fiber Direction

#### 90° Coupon Properties

- $E_2^c$  – Compression Modulus Transverse to Fibers
- $S_2^c$  – Compression Strength Transverse to Fibers
- $\epsilon_2^c$  – Ultimate Compressive Strain Transverse to Fibers

# **CRITICAL ASPECTS OF COMPRESSION TESTING**

- LOAD INTRODUCTION
  - Alignment
  - Damage Initiation
- TEST GEOMETRY
  - Uniform Stress
  - Fixity
  - Stability

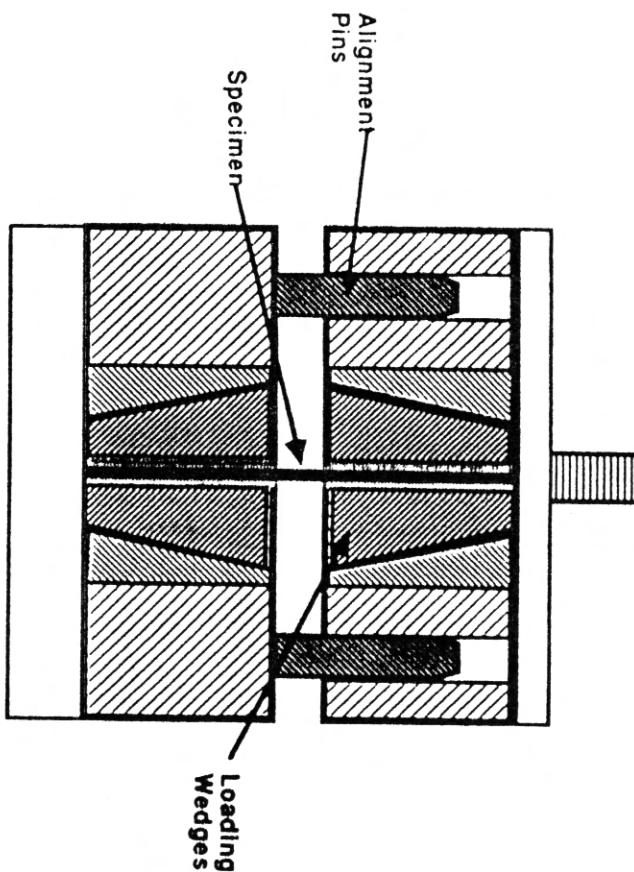
## **COMPRESSION TESTING**

### **LOADING THROUGH SHEAR**



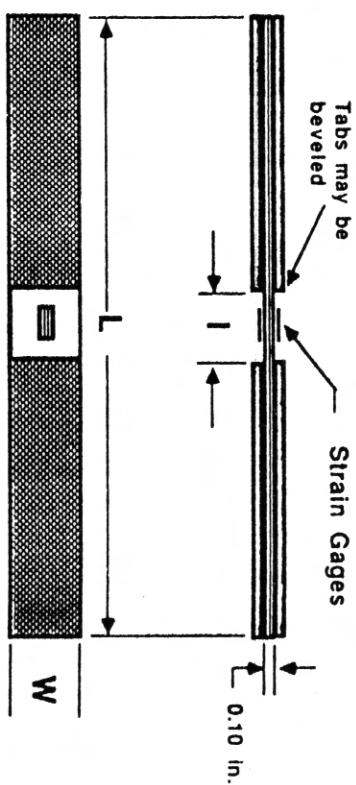
- | <b><u>Positive</u></b>    | <b><u>Negative</u></b>       |
|---------------------------|------------------------------|
| • Prevents loading damage | • Stress concentration       |
| • Complex fixturing       | • Potential tab bond failure |

# IITRI COMPRESSION FIXTURE



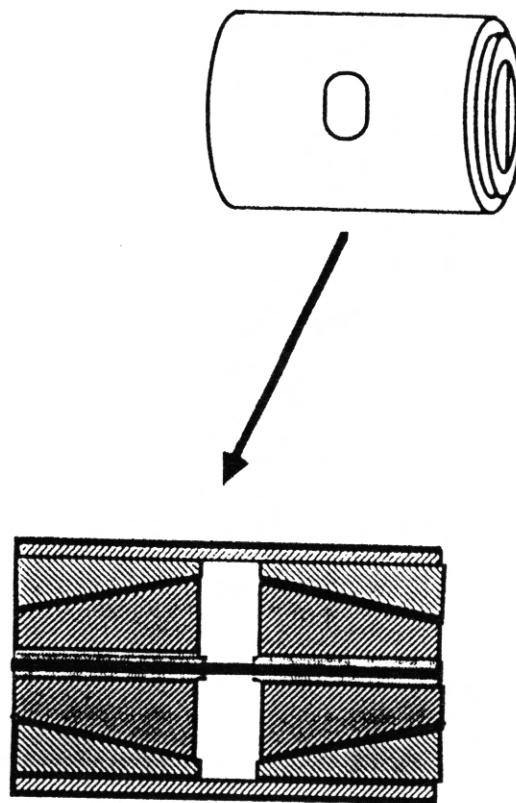
IITRI

## SPECIMEN GEOMETRY



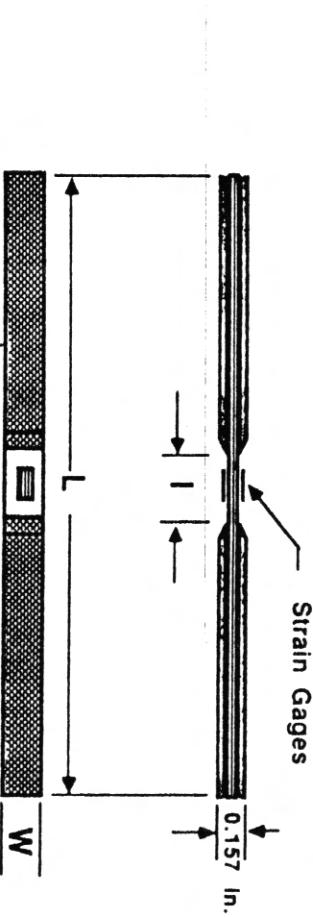
$W = 0.5$  in. ( 90° Specimen )       $I = 0.5$  in. ( nominal )  
 $W = 0.25$  in. ( 0° Specimen )       $L = 5$  in. ( nominal )

# CELANESE COMPRESSION FIXTURE



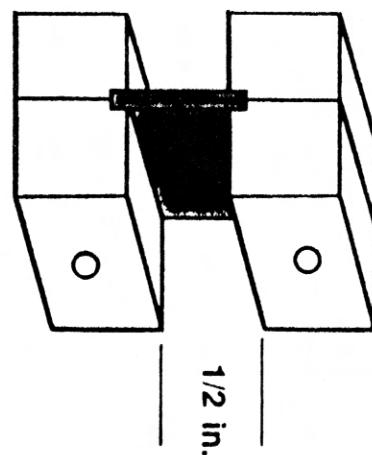
## CELANESE

### SPECIMEN GEOMETRY



$W = 0.25$  in. ( 0° Specimen )  
 $t = 0.10$  in.  
 $L = 5$  in. ( nominal )

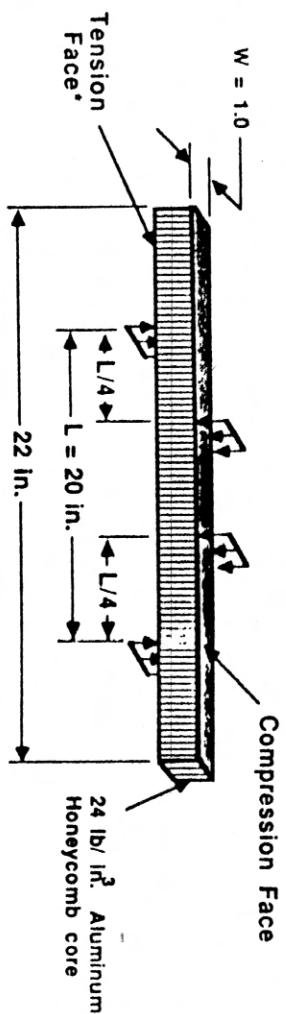
## RAE SPECIMEN



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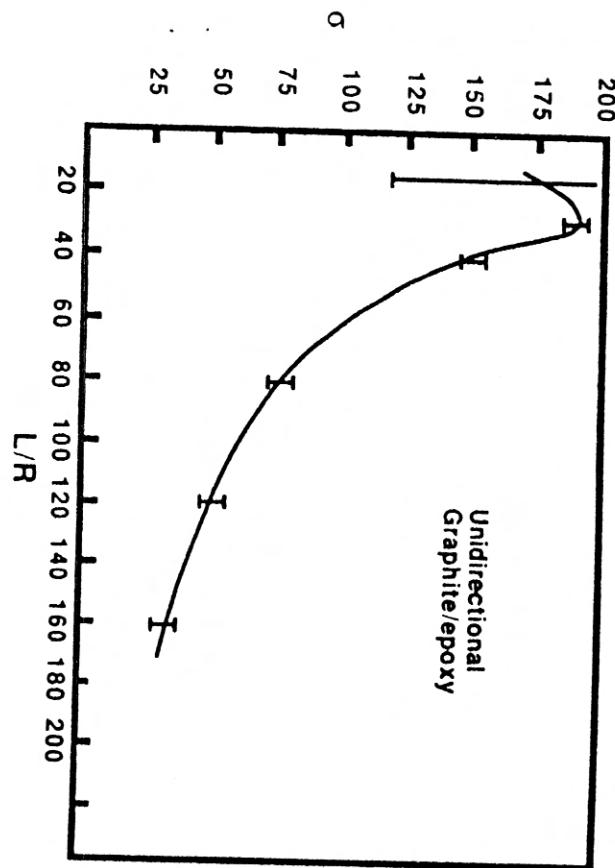
## SANDWICH BEAM METHOD

### Four-Point Bending

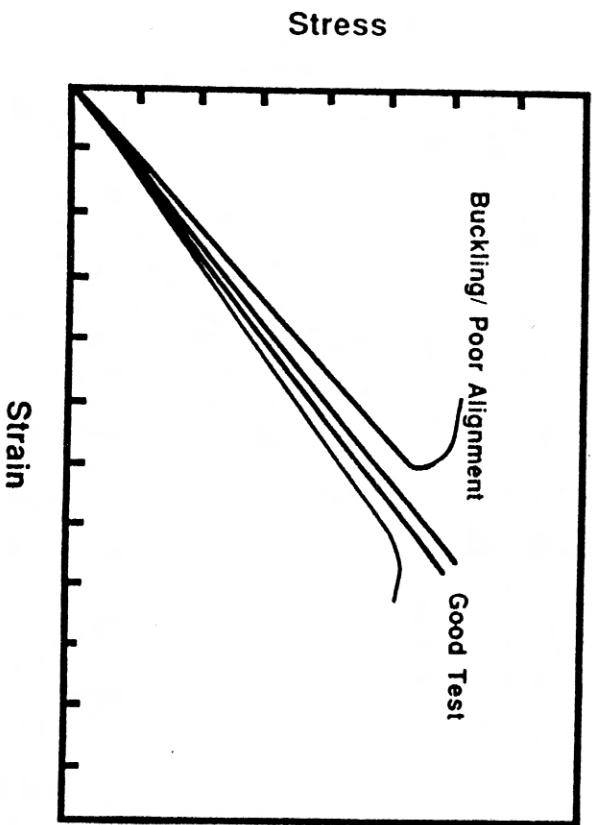


\*Tension face is designed to be stronger than the compression face.

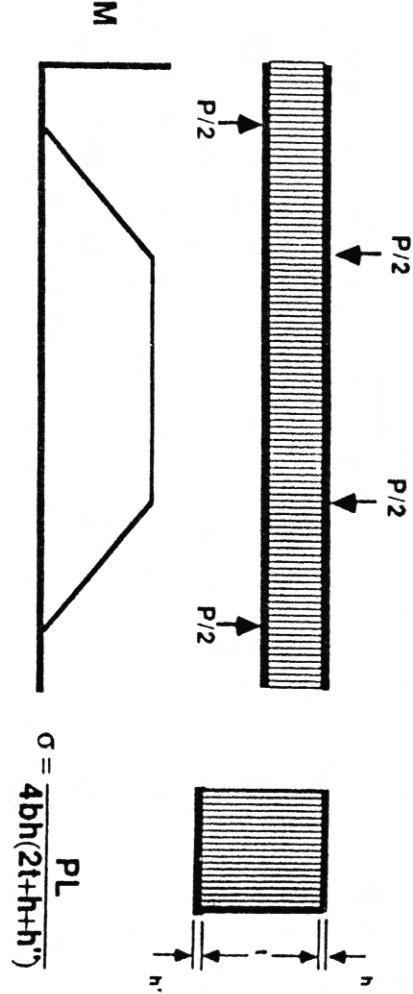
# ULTIMATE STRENGTH VERSUS ASPECT RATIO



## TYPICAL STRESS-STRAIN BEHAVIOR



# SANDWICH BEAM COMPRESSION

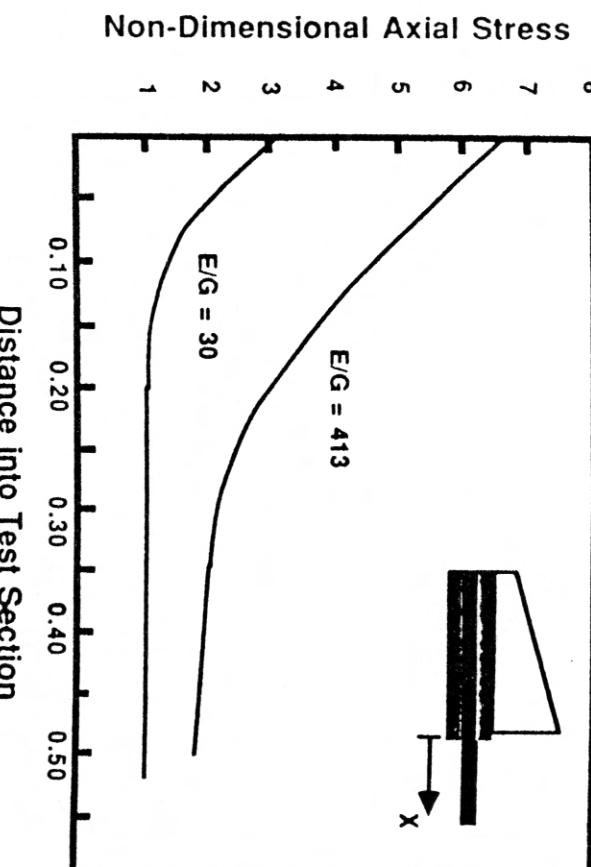


## COMPRESSION STRENGTH AND MODULUS

METHOD COMPARISON T300/5208

METHOD	MODULUS		STRENGTH	
	Msi	Gpa	Ksi	MPa
Sandwich Beam	20.3 ±1.04	139.0 ±8.72	222.0 ±40.5	1536.0 ±273
Pyrimidal Wedge	19.2 ±0.20	132.3 ±1.53	190.0 ±5.6	1308.0 ±38.9
Conical Wedge	20.3 ±0.45	139.6 ±3.21	250.0 ±31.3	1723.0 ±215

## VARIATION OF STRESS IN THE TEST SECTION

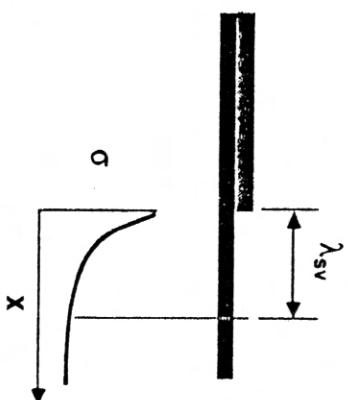


## DECAY OF STRESS CONCENTRATION

St. Venant's Approximation:

$$\lambda_{sv} \leq \frac{4.6t}{2\pi} \left( \frac{E_x}{G_{xz}} \right)^{1/2}$$

$$\frac{\lambda_F}{E} < \frac{t}{2} \left( \frac{F_x}{G_{xz}} \right)^{1/2}$$



$t$  = thickness

$E_x$  = Axial Young's Modulus

$G_{xz}$  = Interlaminar Shear Modulus

$$L \geq 2 \lambda^p E$$

$$\frac{L}{t} \geq \frac{1}{2} \left( \frac{E_K}{G_{yy,t}} \right)^{1/2}$$

## STABILITY REQUIREMENTS

### For Compressive Failure Prior to Buckling

$$\sigma_c = \frac{12}{\pi^2} \left( \frac{l}{l_i} \right)^2 \frac{E_z}{G_{zz}} + 1.2 \left( \frac{E_z}{G_{zz}} \right) \quad (5)$$

For linear-elastic material behavior, the critical buckling strain is defined by:

$$\epsilon_b = \frac{\sigma_c}{E_z} \quad (6)$$

Combining eqns (5) and (6), the critical buckling strain was found to be a function of the specimen geometry and material anisotropy. The relationship is given by:

$$\epsilon_b = \left[ \frac{12}{\pi^2} \left( \frac{l}{l_i} \right)^2 + 1.2 \left( \frac{E_z}{G_{zz}} \right) \right]^{-1} \quad (7)$$

Specimen geometries ensuring that material failure will occur prior to buckling-induced failure correspond to  $l/l_i$  ratios which make the critical buckling strain in eqn (7) greater than the anticipated failure strain for a given material system.

$$\frac{l}{l_i} = \left[ \frac{\pi^2 \left( \frac{1}{\epsilon_f} - 1.2 \left( \frac{E_z}{G_{zz}} \right) \right)}{12} \right]^{1/2} \quad (12)$$

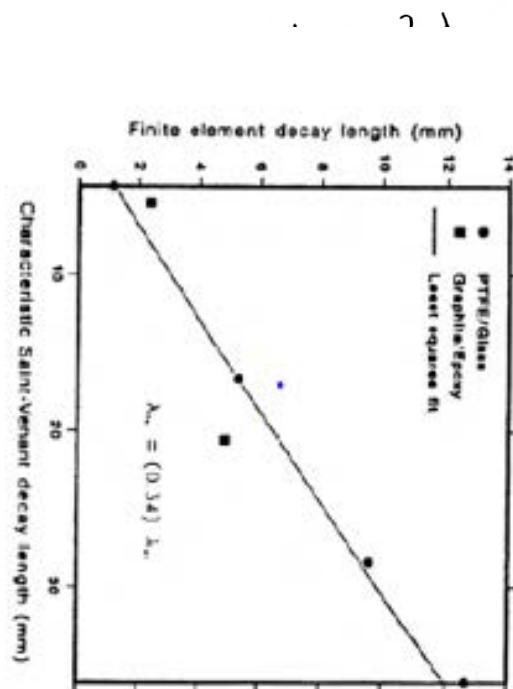


Figure (6) : Decay Length Correlation

Figure (6) : Decay Length Correlation

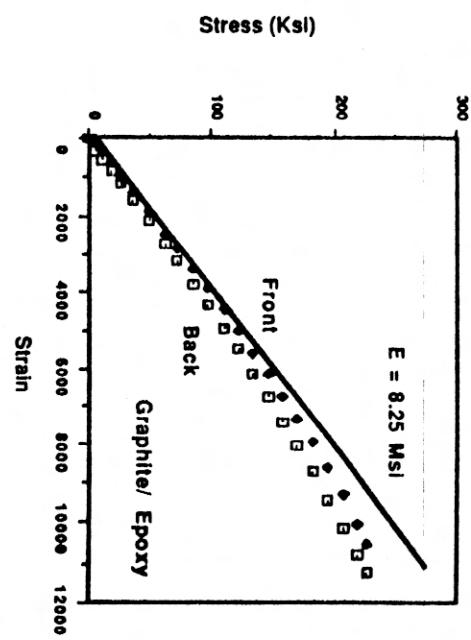
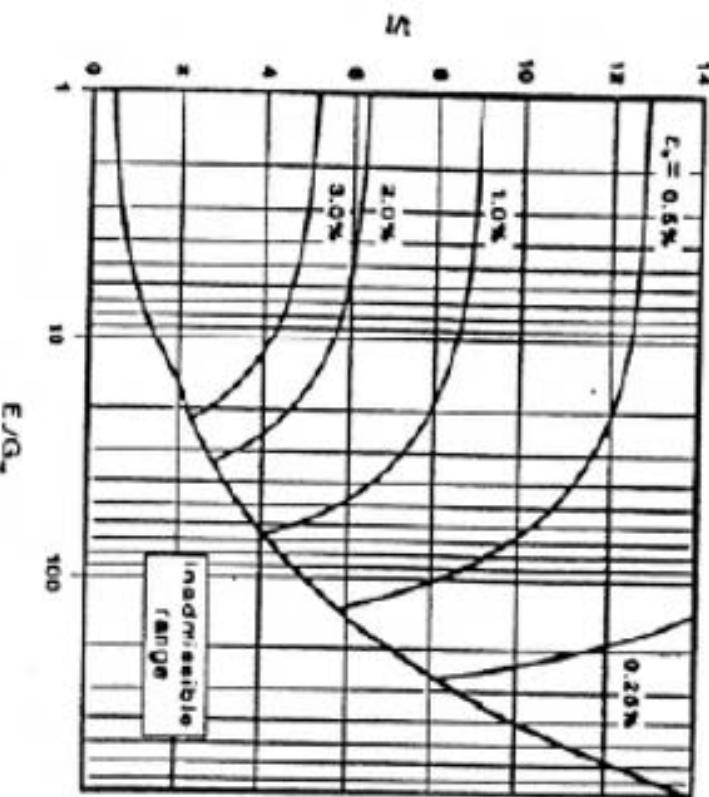


Figure 18 : Specimen Design for Accurate Modulus Determination and Stengel Measurement



## SHEAR TESTING

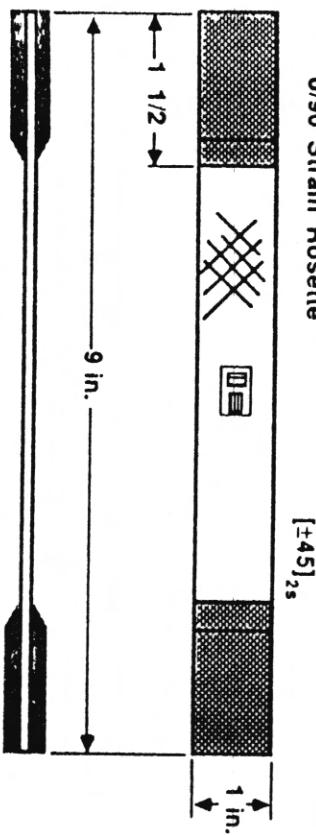
- Inplane Shear Stress-Strain Response
  - ASTM D3515
- Rail Shear Test for Inplane Shear
  - ASTM D4055
- 10° Off-Axis Test
- Short Beam Shear Test (Interlaminar Strength)
  - V-Notched Beam shear Tests
  - Iosipescu
  - Asymmetric Four Point Bending Test (AFPB)
  - Arcan

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## $\pm 45$ TENSION TEST

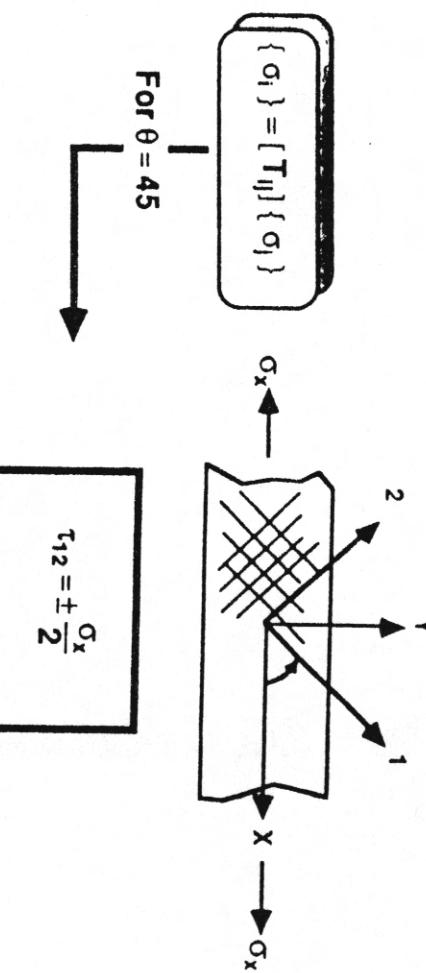
### Inplane Shear Coupon Geometry



0/90 Strain Rosette

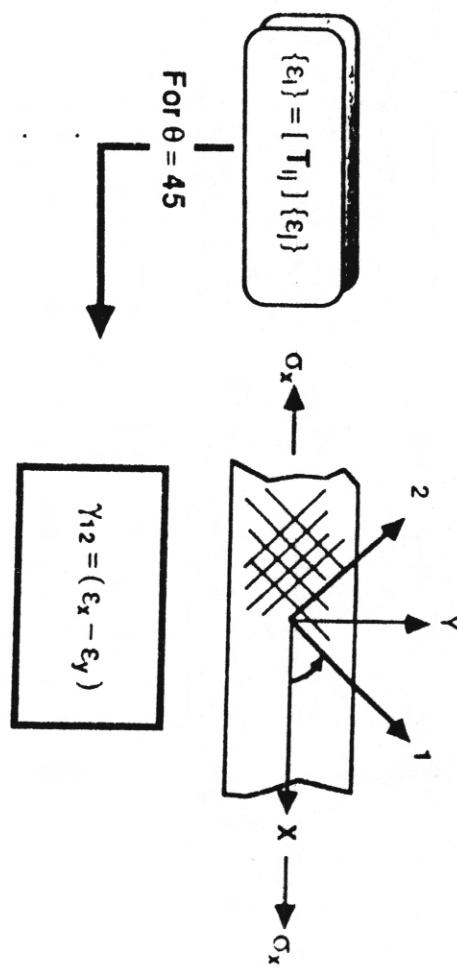
$[\pm 45]_{2s}$

## DATA REDUCTION



Strain Transformation

DATA REDUCTION

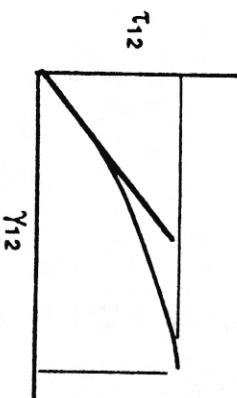


## DATA REDUCTION

### $\pm 45$ Inplane Shear

$$\tau_{12} = \pm \frac{\sigma_x}{2}$$

$$\gamma_{12} = (\varepsilon_x - \varepsilon_y)$$



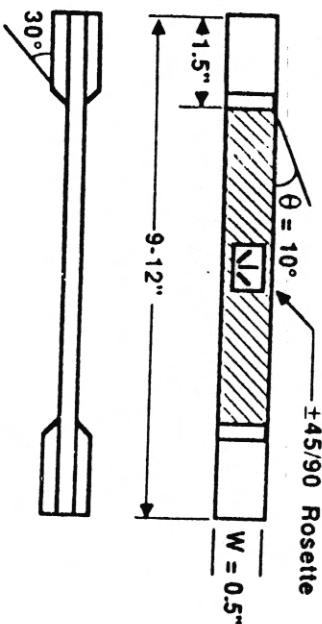
### Ultimate Strength

$$\tau_{12}^{\text{ult}} = \frac{P_{\text{ult}}}{2A}$$

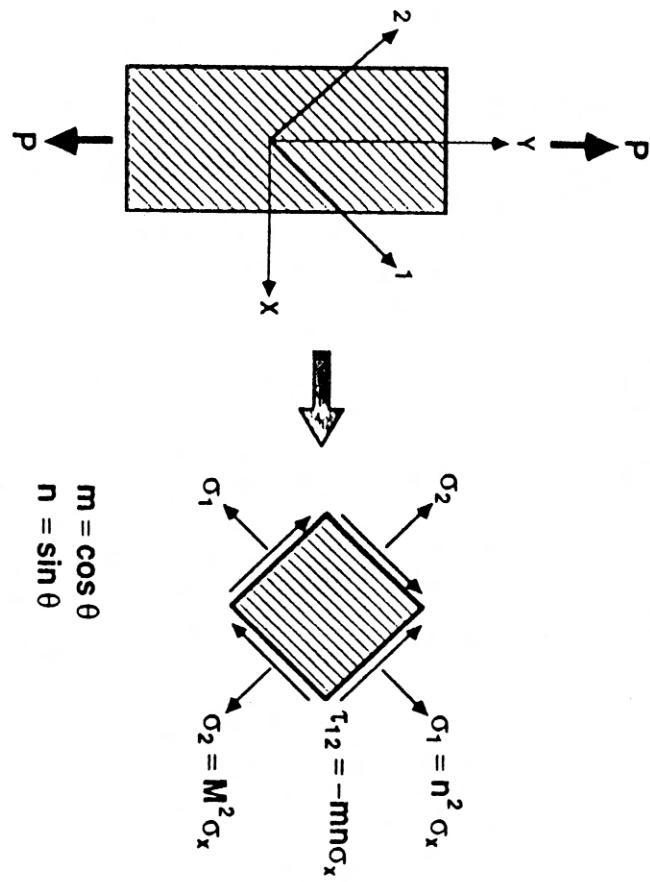
$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

### Modulus

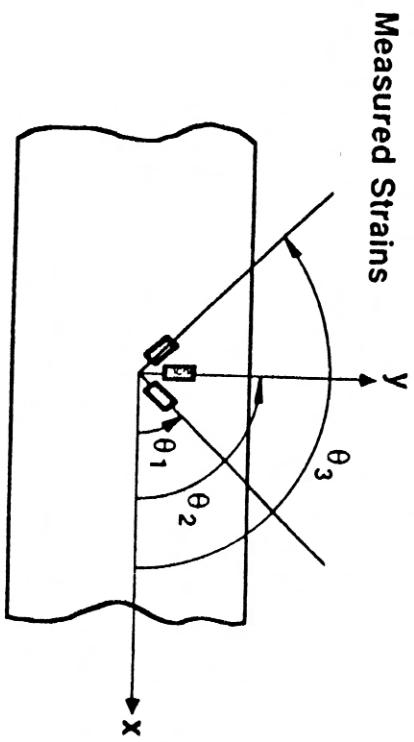
### $10^\circ$ Off Axis Test



## BIAXIAL STRESSES



## STRAIN ANALYSIS



## STRAIN ANALYSIS

### Determining Strain Components

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} m_1^2 & n_1^2 & m_1 n_1 \\ m_2^2 & n_2^2 & m_2 n_2 \\ m_3^2 & n_3^2 & m_3 n_3 \end{bmatrix}^{-1} \begin{Bmatrix} \varepsilon_{\theta_1} \\ \varepsilon_{\theta_2} \\ \varepsilon_{\theta_3} \end{Bmatrix}$$

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## STRAIN TRANSFORMATION

Knowing  $\varepsilon_x$ ,  $\varepsilon_y$  and  $\gamma_{xy}$ , we find.

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & (m^2 - n^2) \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$m = \cos \theta$$

$$n = \sin \theta$$

Thus

$$\gamma_{12} = -2mn \varepsilon_x + 2mn \varepsilon_y + (m^2 - n^2) \gamma_{xy}$$

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## DATA REDUCTION

### Strength

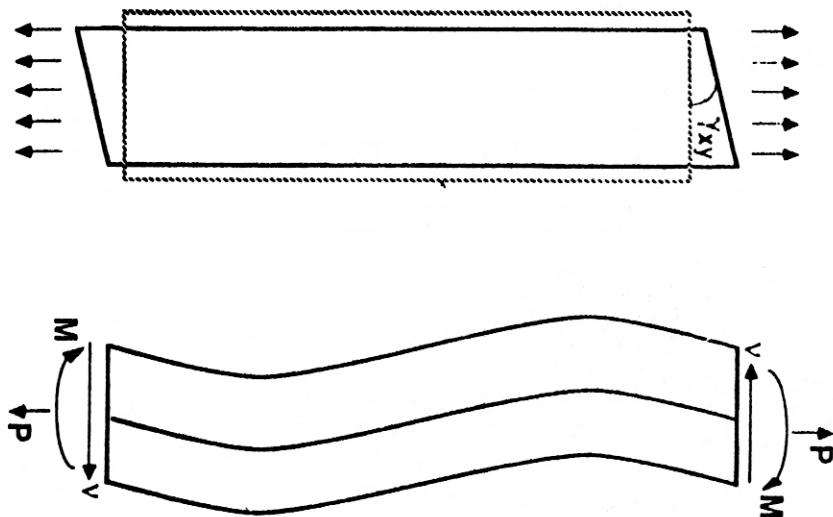
$$\tau_{12} = -mn \sigma_x^{ULT}$$

$$\sigma_x^{ULT} = P^{ULT}/A$$

### Modulus

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{-mn \sigma_x}{[-2mn \epsilon_x + 2mn \epsilon_y + (m^2 - n^2) \gamma_{xy}]}$$

OR AXIS SHEAR



## OFF-AXIS TENSION

For Uniaxial Tension

$$\varepsilon_x = \frac{1}{E_x} \sigma_x$$

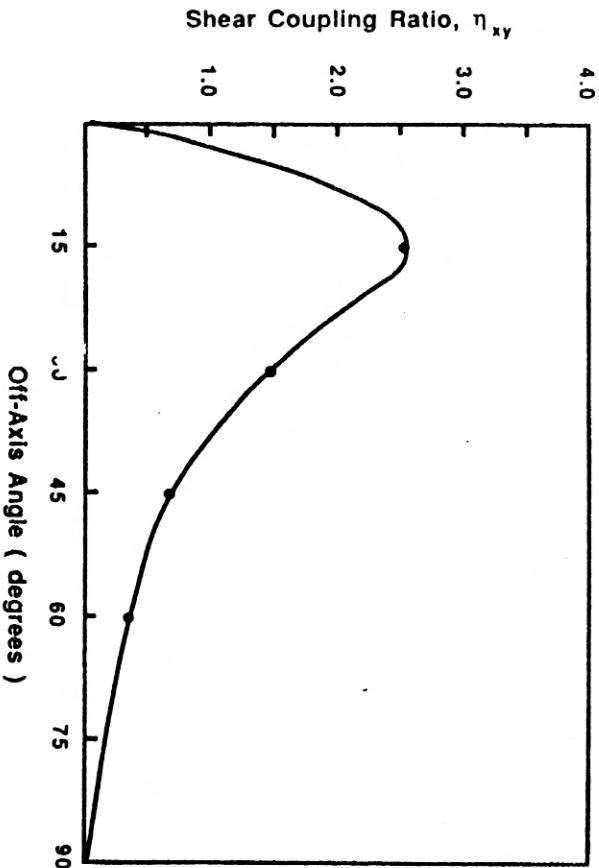
$$\varepsilon_y = -\frac{v_{xy}}{E_x} \sigma_x$$

$$\gamma_{xy} = \frac{\gamma_{xy}}{E_x} \sigma_x$$

where

$$\gamma_{xy} = \frac{S_{16}}{S_{11}}$$

### SHEAR COUPLING RESPONSE



# OFF-AXIS TEST SPECIMEN DESIGN

## Slenderness Ratio

Assume

$$E_x' = (1 - \eta) E_x^i$$

where

$E_x^i$  = measured apparent modulus

$\eta$  = error term

## SLENDERNESS RATIO

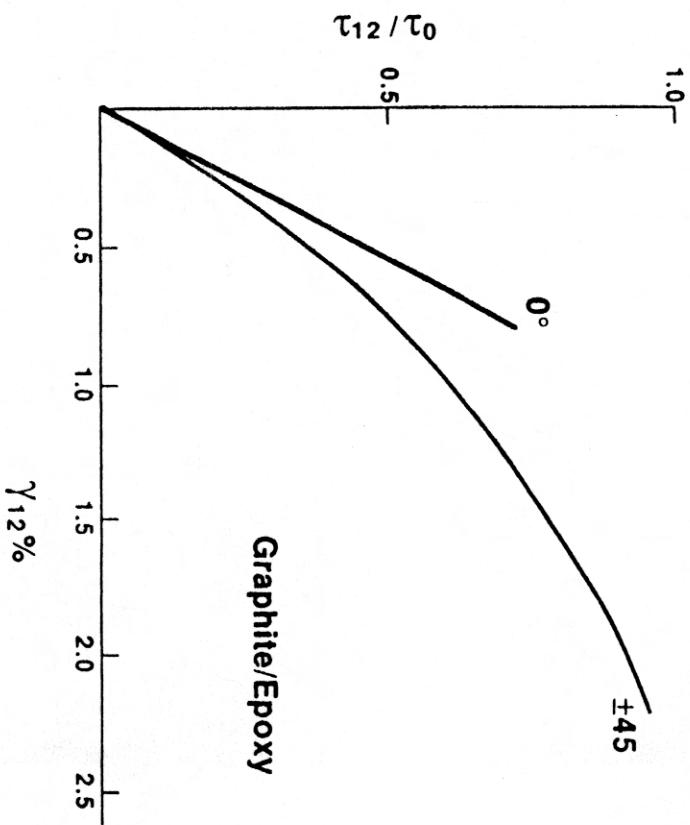
$$\eta = \frac{3 \eta_{xy}}{\left[ \frac{3E_x}{G_{xy}} + \frac{2L^2}{W^2} \right]}$$

$\eta \rightarrow 0$  if  $\eta_{xy} \rightarrow 0$

or

$L/W$  is large

# COMPARISON OF $\pm 45$ AND $10^\circ$ OFF-AXIS TEST RESULTS



## RAIL SHEAR TEST

- Two-Rail
- Three-Rail
- Measures Strength and Modulus

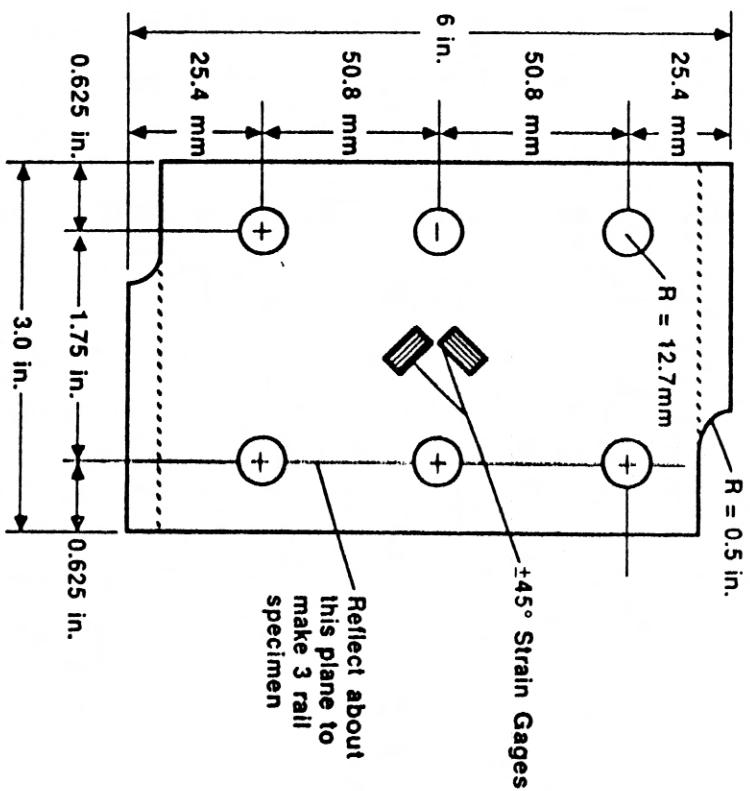
# RAIL SHEAR

## Test Specimen Design

- Unidirectional
  - $0^\circ$
  - $90^\circ$
- Laminate
  - $A_{16}, A_{26} = 0$

## TEST SPECIMEN

### Two Rail



## TEST SPECIMEN CONSIDERATIONS

- Length to width ratio for test section is 8:1.
- Bolts do not carry load.
- Rails act perfectly clamped.

### DATA REDUCTION

#### STRESS

$$\tau_{xy} = \frac{P}{bh} \quad (2\text{-rail})$$

$$\tau_{xy} = \frac{P}{2bh} \quad (3\text{-rail})$$

#### Shear Strength

$$\tau_{xy}^{ult} = \frac{P^{ult}}{bh}$$

#### STRAIN

$$\gamma_{xy} = \epsilon_{45} - \epsilon_{-45}$$

#### Shear Modulus

$$G_{xy} = \frac{\tau_{xy}}{\gamma_{xy}}$$