



Standard Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus¹

This standard is issued under the fixed designation E111; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the Department of Defense.

1. Scope

1.1 This test method² covers the determination of Young's modulus, tangent modulus, and chord modulus of structural materials. This test method is limited to materials in which and to temperatures and stresses at which creep is negligible compared to the strain produced immediately upon loading and to elastic behavior.

1.2 Because of experimental problems associated with the establishment of the origin of the stress-strain curve described in 8.1, the determination of the initial tangent modulus (that is, the slope of the stress-strain curve at the origin) and the secant modulus are outside the scope of this test method.

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory requirements prior to use.*

2. Referenced Documents

2.1 ASTM Standards:³

- E4 Practices for Force Verification of Testing Machines
- E6 Terminology Relating to Methods of Mechanical Testing
- E8 Test Methods for Tension Testing of Metallic Materials
- E9 Test Methods of Compression Testing of Metallic Materials at Room Temperature

¹ This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.

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² This test method is a revision of E111 – 61 (1978), "Young's Modulus at Room Temperature" and includes appropriate requirements of E231 – 69 (1975), "Static Determination of Young's Modulus of Metals at Low and Elevated Temperatures" to permit the eventual withdrawal of the latter method. Method E231 is under the jurisdiction of ASTM-ASME Joint Committee on Effect of Temperature on the Property of Metals.

³ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

E21 Test Methods for Elevated Temperature Tension Tests of Metallic Materials

E83 Practice for Verification and Classification of Extensometer Systems

E231 Method for Static Determination of Young's Modulus of Metals at Low and Elevated Temperatures⁴

E1012 Practice for Verification of Test Frame and Specimen Alignment Under Tensile and Compressive Axial Force Application

2.2 *General Considerations*—While certain portions of the standards and practices listed are applicable and should be referred to, the precision required in this test method is higher than that required in general testing.

3. Terminology

3.1 Definitions:

3.1.1 *accuracy*—the degree of agreement between an accepted standard value of Young's modulus (the average of many observations made according to this method, preferably by many observers) and the value determined.

3.1.1.1 Increased accuracy is associated with decreased bias relative to the accepted standard value; two methods with equal bias relative to the accepted standard value have equal accuracy even if one method is more precise than the other. See also *bias* and *precision*.

3.1.1.2 The accepted standard value is the value of Young's modulus for the statistical universe being sampled using this method. When an accepted standard value is not available, accuracy cannot be established.

3.1.2 *bias, statistical*—a constant or systematic error in test results.

3.1.2.1 Bias can exist between the accepted standard value and a test result obtained from this test method, or between two test results obtained from this test method, for example, between operators or between laboratories.

3.1.3 *precision*—the degree of mutual agreement among individual measurements made under prescribed like conditions.

⁴ Withdrawn. Last approved version of this historical standard is referenced on www.astm.org.

3.1.4 *Young's modulus*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit (see Fig. 1a).

3.1.4.1 *tangent modulus*—the slope of the stress-strain curve at any specified stress or strain (see Fig. 1b).

3.1.4.2 *chord modulus*—the slope of the chord drawn between any two specified points on the stress-strain curve (see Fig. 1c).

3.2 For definitions of other terms used in this test method, refer to Terminology E6.

4. Summary of Test Method

4.1 A uniaxial force is applied to the test specimen and the force and strain are measured, either incrementally or continuously. The axial stress is determined by dividing the indicated force by the specimen's original cross-sectional area. The appropriate slope is then calculated from the stress-strain curve, which may be derived under conditions of either increasing or decreasing forces (increasing from preload to maximum applied force or decreasing from maximum applied force to preload).

5. Significance and Use

5.1 The value of Young's modulus is a material property useful in design for calculating compliance of structural

materials that follow Hooke's law when subjected to uniaxial loading (that is, the strain is proportional to the applied force).

5.2 For materials that follow nonlinear elastic stress-strain behavior, the value of tangent or chord modulus is useful in estimating the change in strain for a specified range in stress.

5.3 Since for many materials, Young's modulus in tension is different from Young's modulus in compression, it shall be derived from test data obtained in the stress mode of interest.

5.4 The accuracy and precision of apparatus, test specimens, and procedural steps should be such as to conform to the material being tested and to a reference standard, if available.

5.5 Precise determination of Young's modulus requires due regard for the numerous variables that may affect such determinations. These include (1) characteristics of the specimen such as orientation of grains relative to the direction of the stress, grain size, residual stress, previous strain history, dimensions, and eccentricity; (2) testing conditions, such as alignment of the specimen, speed of testing, temperature, temperature variations, condition of test equipment, ratio of error in applied force to the range in force values, and ratio of error in extension measurement to the range in extension values used in the determination; and (3) interpretation of data (see Section 9).

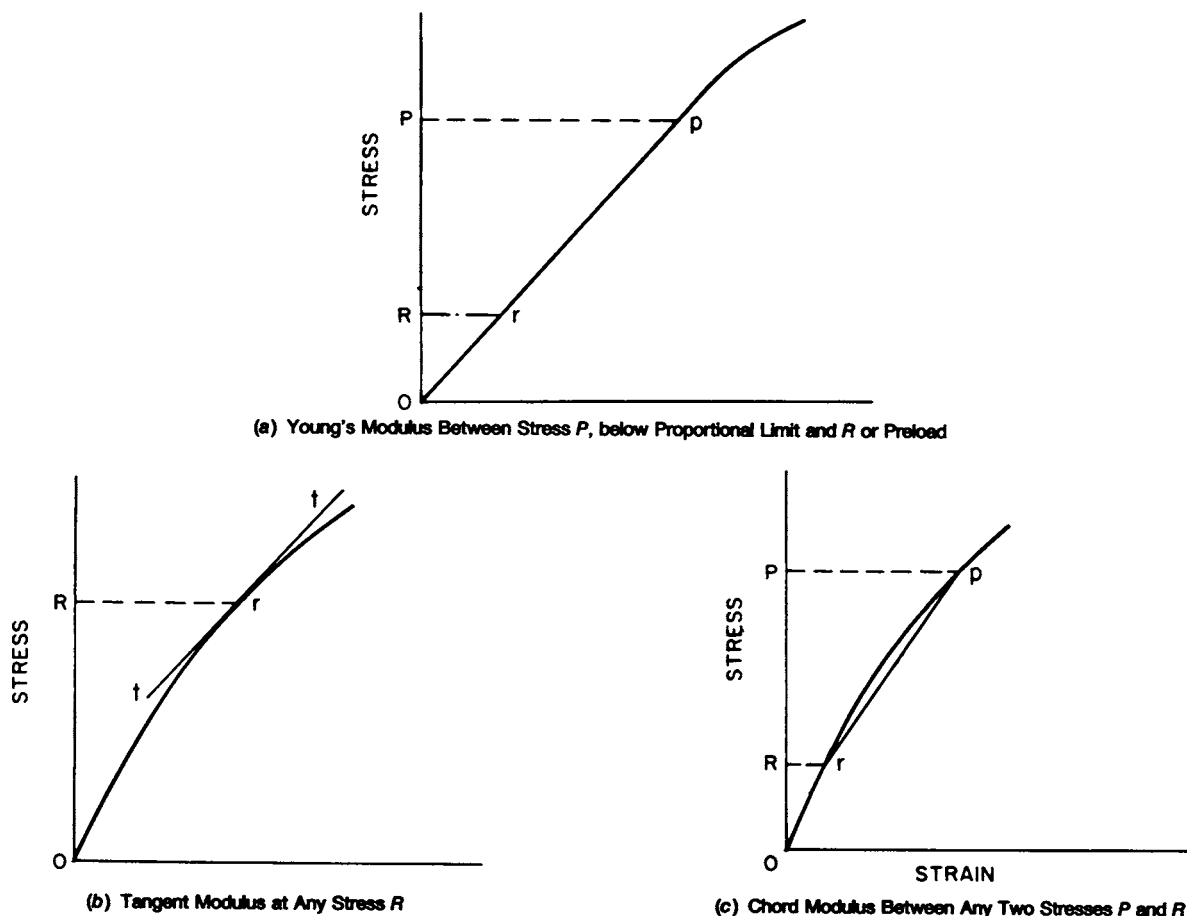


FIG. 1 Stress-Strain Diagrams Showing Straight Lines Corresponding to (a) Young's Modulus, (b) Tangent Modulus, and (c) Chord Modulus

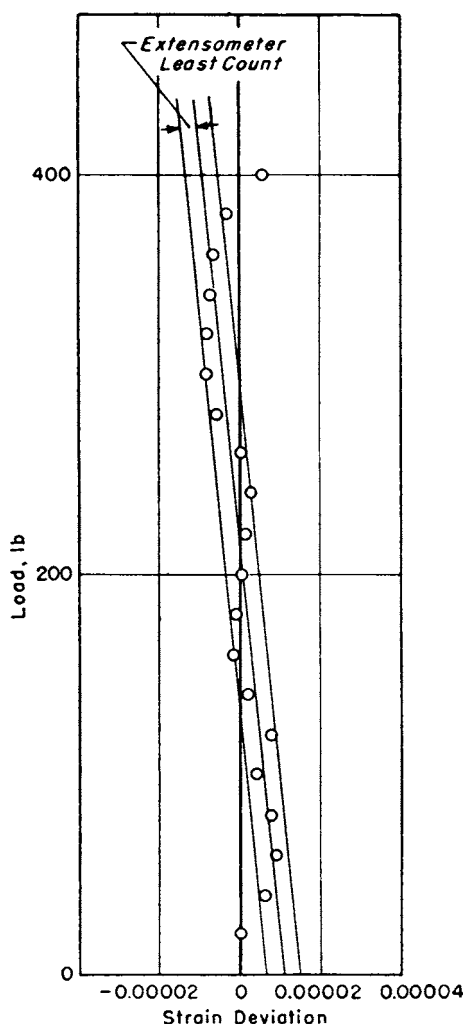


FIG. 2 Load-Deviation Graph

5.6 When the modulus determination is made at strains in excess of 0.25 %, correction should be made for changes in cross-sectional area and gage length, by substituting the instantaneous cross section and instantaneous gage length for the original values.

5.7 Compression results may be affected by barreling (see Test Methods E9). Strain measurements should therefore be made in the specimen region where such effects are minimal.

6. Apparatus

6.1 *Dead Weights*—Calibrated dead weights may be used. Any cumulative errors in the dead weights or the dead weight loading system shall not exceed 0.1 %.

6.2 *Testing Machines*—In determining the suitability of a testing machine, the machine shall be calibrated under conditions approximating those under which the determination is made. Corrections may be applied to correct for proven systematic errors.

6.3 *Loading Fixtures*—Grips and other devices for obtaining and maintaining axial alignment are shown in Test Methods E8 and E9. It is essential that the loading fixtures be properly designed and maintained. Procedures for verifying the align-

ment are described in detail in Practice E1012. The allowable bending as defined in Practice E1012 shall not exceed 5 %.

6.4 *Extensometers*—Class B-1 or better extensometers as described in Practice E83 shall be used. Corrections may be applied for proven systematic errors in strain and are not considered as a change in class of the extensometer. Either an averaging extensometer or the average of the strain measured by at least two extensometers arranged at equal intervals around the cross section be used. If two extensometers are used on other than round sections, they shall be mounted at ends of an axis of symmetry of the section. If a force-strain recorder, strain-transfer device, or strain follower is used with the extensometer, they shall be calibrated as a unit in the same manner in which they are used for determination of Young's modulus. The gage length shall be determined with an accuracy consistent with the precision expected from the modulus determination and from the extensometer.

NOTE 1—The accuracy of the modulus determination depends on the precision of the strain measurement. The latter can be improved by increasing the gage length. This may, however, present problems in maintaining specimen tolerances and temperature uniformity.

6.5 *Furnaces or Heating Devices*—When determining Young's modulus at elevated temperature, the furnace or heating device used shall be capable of maintaining a uniform temperature in the reduced section of the test specimen so that a variation of not more than $\pm 1.5^{\circ}\text{C}$ for temperatures up to and including 900°C , and not more than $\pm 3.0^{\circ}\text{C}$ for temperatures above 900°C , occurs. (Heating by self-resistance is not accepted.) Minimize temperature variations and control changes within the allowable limits, since differences in thermal expansion between specimen and extensometer parts may cause significant errors in apparent strain. An instrumented sample representative of the real test will demonstrate that the setup meets the above capabilities.

6.6 *Low-Temperature Baths and Refrigeration Equipment*—When determining Young's modulus at low temperatures, an appropriate low-temperature bath or refrigeration system is required to maintain the specimen at the specified temperature during testing. For a low-temperature bath, the lower tension rod or adapter may pass through the bottom of an insulated container and be welded or fastened to it to prevent leakage. For temperatures to about -80°C , chipped dry ice may be used to cool an organic solvent such as ethyl alcohol in the low-temperature bath. Other organic solvents having lower solidification temperatures, such as methylcyclohexane or isopentane, may be cooled with liquid nitrogen to temperatures lower than -80°C . Liquid nitrogen may be used to achieve a testing temperature of -196°C . Lower testing temperatures may be achieved with liquid hydrogen and liquid helium, but special containers or cryostats are required to provide for minimum heat leakage to permit efficient use of these coolants. When liquid hydrogen is used, special precautions must be taken to avoid explosions of hydrogen gas and air mixtures. If refrigeration equipment is used to cool the specimens with air as the cooling medium, it is desirable to have forced air circulation to provide uniform cooling.

NOTE 2—At low temperatures, when using a coolant bath, immersion-type extensometers are recommended.



6.7 Temperature measuring, controlling, and recording instruments shall be calibrated periodically against a secondary standard, such as a precision potentiometer. Lead-wire error should be checked with the lead wires in place as they normally are used.

7. Test Specimens

7.1 *Selection and Preparation of Specimens*—Special care shall be taken to obtain representative specimens which are straight and uniform in cross section. If straightening of the material for the specimen is required, the resultant residual stresses shall be removed by a subsequent stress relief heat treatment which shall be reported with the test results.

7.2 *Dimensions*—The recommended specimen length (and fillet radius in the case of tension specimens) is greater than the minimum requirements for general-purpose specimens. In addition, the ratio of length to cross section of compression specimens should be such as to avoid buckling (see Test Methods E9).

NOTE 3—For examples of tension and compression specimens, see Test Methods E8 and E9.

7.3 For tension specimens, the center lines of the grip sections and of the threads of threaded-end specimens shall be concentric with the center line of the gage section within close tolerances in order to obtain the degree of alignment required. If pin-loaded sheet-type specimens are used, the centers of the gripping holes shall be not more than 0.005 times the width of the gage section from its center line. For sheet-type specimens, it may be necessary to provide small tabs or notches for attaching the extensometer.

NOTE 4—The effect of eccentric loading may be illustrated by calculating the bending moment and stress thus added. For a standard 12.5-mm diameter specimen, the stress increase is 1.5 % for each 0.025 mm of eccentricity. This error increases to about 2.5 % per 0.025 mm for a 9-mm diameter specimen and to about 3.2 % per 0.025 mm for a 6-mm diameter specimen.

7.4 The length of the reduced section of tension specimens shall exceed the gage length by at least twice the diameter or twice the width. The length of compression specimens shall be in accordance with Test Methods E9, and all specimens shall have a uniform cross-sectional area throughout the gage length.

NOTE 5—If a general-purpose tension specimen such as those shown in Test Methods E8, having a small amount of taper in the reduced section is used, the average cross-sectional area for the gage length should be used in computing stress.

7.5 For compression specimens, the ends shall be flat, parallel and perpendicular to the lateral surfaces as specified in Test Methods E9.

7.6 This test method is intended to produce intrinsic materials properties. Therefore, the specimen needs to be free of residual stresses, which may require an annealing procedure at $T_m/3$ for 30 min to relieve the stresses in the material (where T_m is the melting point of the material in K). The procedure must be mentioned in the report section. If the intent of the test is to verify the performance of a product, the heat treatment procedure may be omitted. Record the condition of the material tested, including any heat treatment, in the test report.

8. Procedure

8.1 For most loading systems and test specimens, effects of backlash, specimen curvature, initial grip alignment, etc., introduce significant errors in the extensometer output when applying a small force to the test specimen. Measurements shall therefore be made from a small force or preload, known to be high enough to minimize these effects, to some higher applied force, still within either the proportional limit or elastic limit of the material. For linearly elastic materials, the slope of the straight-line portion of the stress-strain curve shall be established between the preload and the proportional limit to define Young's modulus. If the actual stress-strain curve is desired, this line can appropriately be shifted along the strain axis to coincide with the origin. For nonlinearly elastic materials the tangent or chord modulus may be established between the appropriate stress values on the stress strain curve.

8.2 *Measurement of Specimens*—Make the measurements for the determination of average cross-sectional area at the ends of the gage length and at least at one intermediate location. Use any means of measuring that is capable of producing area calculations within 1 % accuracy.

8.3 *Alignment*—Take special care to ensure as nearly axial loading as possible. The strain increments between the initial-load and the final-load measurement on opposite sides of the specimens should not differ from the average by more than 3 %.

8.4 *Soaking Time of Specimens at Testing Temperature*—After the specimen to be tested has reached the testing temperature, maintain the specimen at the testing temperature for a sufficient length of time to attain equilibrium conditions of the specimen and extensometer before applying force. Report the time to attain test temperature and the time at temperature before applying force.

NOTE 6—The recommended soak time at the test temperature is 1 hour per 25 mm (1 hour/inch) of specimen thickness or diameter.

If the temperature of the system is not uniform by the time loading of the specimen is started, variations in thermal expansion will be reflected in the modulus line. Furthermore, fluctuations in temperature of the extensometer extensions during testing which result from cycling of the furnace temperature or changes in the level of the cooling bath may also affect the slope of the modulus line.

8.5 *Speed of Testing*—The speed of testing shall be low enough that thermal effects of adiabatic expansion or contraction are negligible and that accurate determination of load and extension is possible, yet the speed shall be high enough that creep will be negligible. In loading with dead weights, avoid temporary overloading due to inertia of the weights. The strain rate should be reported.

NOTE 7—A minimum of three runs are recommended for each specimen. Care must be taken not to exceed the proportional limit in the case of Young's modulus, and the elastic limit in the case of the tangent or chord modulus. Report each of the three values or the average along with the method for getting them.

NOTE 8—It is recognized that Young's modulus, tangent modulus, or chord modulus for a given specimen may be determined along with yield strength and tensile strength using a single loading cycle. If modulus values are determined this way, report that only one loading cycle was used. Three cycles within the elastic region as recommended in Note 7,

can be used to determine the modulus, before straining the specimen into the plastic range to determine yield and tensile strengths.

8.6 Temperature Control—The average temperature over the specimen gage length shall not deviate from the indicated nominal test temperature by more than $\pm 2^\circ\text{C}$. In elevated-temperature tests, indicated temperature variations along the gage length of the specimen shall not exceed the following limits: up to and including $900 \pm 1.5^\circ\text{C}$, above $900 \pm 3.0^\circ\text{C}$. (See 6.5.) The test must be performed with the same setup and under similar conditions as those of the instrumented test described in 6.5.

NOTE 9—The terms “indicated nominal temperature” or “indicated temperature” mean the temperature that is indicated by the temperature-measuring device using good pyrometric practice.⁵

NOTE 10—It is recognized that actual temperatures may vary more than the indicated temperatures. The use of “indicated temperatures” for the limits of permissible variation in temperature are not to be construed as minimizing the importance of good practice and precise temperature control. All laboratories are obligated to keep the variation of indicated temperature from the actual temperature as small as is practical. Temperature changes during the test, within the allowable limits, can cause significant strain errors due to differences in thermal expansion of the test specimen and extensometer parts. Temperature changes should be minimized while making strain measurements.

8.7 In low-temperature testing in which the bath is cooled with dry ice or in which a refrigeration system is used, the temperature of the medium around the specimen shall be maintained at temperatures within 1.5°C of the specified temperature. Bath temperatures or the temperature of circulating air from a refrigeration system may be done with a copper-constantan thermocouple or a suitable thermometer. If the specimen is submerged in a bath at the boiling point of the bath, sufficient soaking time (see **Note 6**) must be allowed to provide equilibrium conditions. Specimens tested in boiling liquids must meet the temperature control requirements specified in 8.6.

8.7.1 Caution—The boiling point of a commercial liquid gas may not be the same as the published temperature for the pure liquid gas.

8.8 Temperature Measurement—The method of temperature measurement must be sufficiently sensitive and reliable to ensure that the temperature of the specimen is within the limits specified in 8.6 and 8.7. Thermocouples in conjunction with potentiometers or millivolt meters are generally used to measure temperatures. A discussion of temperature measurement and the use of thermocouples is given in Test Methods E21.

9. Interpretation of Data

9.1 If a plot of load-versus-extension (force versus elongation) is obtained by means of an autographic recorder, the value for Young’s modulus is obtained by determining the slope of the line for forces less than the force corresponding to the proportional limit. Choice of the lower force point depends on the limitations set forth in 8.1. Young’s modulus is calculated from the force increment and corresponding extension incre-

ment, between two points on the line as far apart as possible, by use of the following equation:

$$E = \left(\frac{\Delta_p}{A_o} \right) / \left(\frac{\Delta_c}{L_o} \right) \quad (1)$$

where:

Δ_p = force increment,
 A_o = original cross-sectional area,
 Δ_c = extension increment, and
 L_o = original gage length.

The precision of the value obtained for Young’s modulus will depend upon the precision of each of the values used in the calculation. It is suggested that the report include an estimate of the precision of the reported value of Young’s modulus based on the summation of the precisions of the respective values. When the modulus determination is made at strains in excess of 0.25 %, corrections shall be made for changes in cross-sectional area and gage length by substituting the instantaneous cross section and instantaneous gage length for the original values.

9.2 If the load-versus-extension data are obtained in numerical form, the errors introduced by plotting the data and fitting graphically a straight line to the experimental points are reduced by determining Young’s modulus as the slope of the straight line fitted to the appropriate data by the method of least squares. This method also permits statistical study of the data and therefore an evaluation of the variability of the modulus within the stress range employed. The equation for Young’s modulus fitted by the method of least squares (all data pairs having equal weight) is:

$$\text{Young's modulus, } E = (\Sigma(XY) - K\bar{X}\bar{Y}) / (\Sigma X^2 - K\bar{X}^2) \quad (2)$$

where:

Y = applied axial stress, and
 X = corresponding strain.

In terms of the measured load P_i and measured original cross-sectional area A_o and gage length L_o ,

$$X = \frac{\Delta_c}{L_o}$$

$$Y = \frac{\Delta_p}{A_o}$$

$$\bar{Y} = \frac{\Sigma Y}{K} = \text{average of } Y \text{ values}$$

$$\bar{X} = \frac{\Sigma X}{K} = \text{average of } X \text{ value}$$

K = number of X, Y data pairs and Σ = sum from 1 to K .

The coefficient of determination, r^2 , indicates the goodness of fit achieved in a single test. This coefficient is defined as follows:

$$r^2 = \left(\left[\Sigma XY - \frac{\Sigma X \Sigma Y}{K} \right]^2 \right) / \left(\left[\Sigma X^2 - \frac{(\Sigma X)^2}{K} \right] \left[\Sigma Y^2 - \frac{(\Sigma Y)^2}{K} \right] \right) \quad (3)$$

Values of r^2 close to 1.00 are desirable (see **Table 1**).

NOTE 11—Many programmable calculators have built-in programs for calculating the slopes of straight lines fitted to a number of data pairs and

⁵ For further information on temperature control and measurement, see *Panel Discussion on Pyrometric Practices, ASTM STP 178, 1955.*

**TABLE 1 Fitting of Straight Lines Coefficient of Variation of Slope (Percent) (V_1)**

| Data Pairs (K) | Sample Correlation Coefficients (r) | | | | |
|-------------------|---|---------|---------|---------|---------|
| | 0.90000 | 0.99000 | 0.99900 | 0.99990 | 0.99999 |
| 3 | ±48.4 | ±14.2 | ±4.47 | ±1.41 | ±0.447 |
| 5 | 27.9 | 8.22 | 2.58 | 0.816 | 0.258 |
| 10 | 17.1 | 5.03 | 1.58 | 0.500 | 0.158 |
| 20 | 11.4 | 3.35 | 1.05 | 0.333 | 0.105 |
| 30 | 9.1 | 2.69 | 0.84 | 0.267 | 0.084 |
| 50 | 6.9 | 2.05 | 0.64 | 0.204 | 0.064 |
| 100 | 4.8 | 1.44 | 0.45 | 0.142 | 0.045 |

their coefficient of determination. Details of the procedure may be found in standard textbooks on statistics or numerical analysis.^{6, 7, 8, 9}

Calculate the coefficient of variation of the slope as follows (see Table 1 for representative values):

$$V_1 = 100 \sqrt{\frac{\frac{1}{r^2} - 1}{K - 2}} \quad (4)$$

where:

V_1 = coefficient of variation, %

NOTE 12—Under normal circumstances the coefficient of variation should not be larger than 2 %; however with care, values less than 0.5 % have been found to be achievable in aluminum alloys.

9.3 In determining the stress range to be used in these calculations it is often helpful to examine the data by the strain deviation method.^{10,11,12} For this test method, random variations in the data are considered as variations in strain, a trial modulus is chosen so that the deviations will be small, and the strain deviations are calculated as follows: Strain deviation = strain – (stress/trial modulus). These deviations are plotted to a large scale as abscissas with the stresses or loads as ordinates (Fig. 2). The stress range for which data are used to obtain Young's modulus is determined by analyzing the random variations in strain deviation, typically from the least count of the extensometer (extensometer least count). This method is illustrated graphically in Fig. 2.

9.4 Young's modulus may also be determined by means of the deviation graph by fitting graphically a straight line to the appropriate points. From this line the deviation increment corresponding to a given stress increment can be read and substituted in the following equation:

$$\text{Young's modulus} = A / [(A/B) + C] \quad (5)$$

where:

A = stress increment,

B = trial modulus, and

C = deviation increment.

It is suggested that the strain corresponding to the spacing of the parallel lines be reported as measure of the variability of the data.

9.5 In the case of nonlinear elastic materials, the stress-strain curve may be obtained by fitting the load-versus-extension or load-versus-strain data pairs to a polynomial approximation⁷ and the chord modulus obtained in calculating the slope between two specified sets of data pairs below the elastic limit on the fitting curve. The choice of the lower of the two sets of data pairs depends on the limitations set forth in 8.1.

9.6 To establish confidence intervals for the regression line the following equation may be used:

$$\pm I = tV_1 \quad (6)$$

where:

I = percent of slope confidence interval,

V_1 = coefficient of variation, expressed in percent (see 9.2), and

t = t – statistic from standard tables at $K - 2$ degrees of freedom and confidence level selected.

Table 2 gives an example of representative values calculated using a 95 % confidence interval.

10. Report

10.1 Report the following information:

10.1.1 *Specimen Material*—Specimen material, alloy, heat treatment, mill batch number, grain direction, and other relevant material information.

10.1.2 *Specimen Configuration*—Sketch of the specimen configuration or reference to the specimen drawing.

10.1.3 *Specimen Dimensions*—Actual measured dimensions for the specimen.

10.1.4 *Test Fixture*—Description of the test fixture or reference to fixture drawings.

10.1.5 *Testing Machine and Extensometers*—Manufacturer, model, serial number, and force range of the testing machine and the extensometers.

10.1.6 *Speed of Testing*—Test rate and mode of control.

10.1.7 *Temperature*—Test temperature, time to attain test temperature and time at temperature before applying force.

TABLE 2 Fitting of Straight Lines for 95 % Confidence Interval Percentage Values of Slope Confidence Interval (I)

| Data Pairs (K) | t -Statistic | Sample Correlation Coefficients (r) | | | |
|----------------|----------------|---|---------|---------|---------|
| | | 0.99000 | 0.99900 | 0.99990 | 0.99999 |
| 3 | 12.71 | ±180 | ±56.8 | ±17.9 | ±5.7 |
| 5 | 3.182 | 26.2 | 8.2 | 2.6 | 0.8 |
| 10 | 2.306 | 11.6 | 3.6 | 1.2 | 0.4 |
| 20 | 2.101 | 7.0 | 2.2 | 0.7 | 0.2 |
| 30 | 2.048 | 5.5 | 1.7 | 0.5 | 0.17 |
| 50 | 2.011 | 4.1 | 1.29 | 0.41 | 0.129 |
| 100 | 1.984 | 2.8 | 0.89 | 0.28 | 0.089 |

⁶ Youden, W. J., *Statistical Methods for Chemists*, John Wiley and Sons, Inc., New York, NY, 1951, Ch. S, pp. 40–49.

⁷ Fröberg, C. E., *Introduction to Numerical Analysis*, Second Edition, Addison Wesley Publishing Co., Reading, MA, 1969, p. 335.

⁸ *Experimental Statistics*, NBS Handbook No. 91. May be obtained from Superintendent of Natrelle, M.G., Documents, U.S. Government Printing Office, Washington, DC 20402.

⁹ Bowker, A. H., and Lieberman, G. J., *Engineering Statistics*, Prentice Hall, Englewood Cliffs, NJ, 1959, pp. 331–333.

¹⁰ Smith, C. S., “Proportional Limit Tests on Copper Alloys,” *Proceedings, ASTM*, Vol 40, 1974, p. 864.

¹¹ McVetty, P. G., and Mochel, N. L., “The Tensile Properties of Stainless Iron and Other Alloys at Elevated Temperatures,” *Transactions, Am. Soc. Steel Treating*, Vol 11, 1927, pp. 78–92.

¹² Discussion by L. B. Tuckerman of paper by Templin, R. L., “The Determination and Significance of the Proportional Limit in the Testing of Metals,” *Proceedings, ASTM*, Vol 29, Part II, 1929, p. 538.



10.1.8 *Stress-Strain Diagram*—Stress-strain diagram with scales, specimen number, test data, rate, and other pertinent information.

10.1.9 *Young's Modulus, Tangent Modulus, Chord Modulus*—Modulus value and the method used to determine the value in accordance with Section 9.

11. Precision and Bias

11.1 *Precision*—The following parameters are reported to impact upon the precision of this test method:

11.1.1 Characteristics of the specimen such as orientation of grains relative to the axial stress, grain size, residual stress, previous strain history, dimensions, and eccentricity.

11.1.2 Testing conditions such as alignment of the specimen, speed of testing, temperature, temperature variations, conditions of test equipment, ratio of error in force to the range in force values, and ratio of error in extension measurement to the range in extension values.

11.1.3 Interpretation of data such as whether graphical or digital data were taken, calibration of recording or data-logging device, number of data pairs used to obtain slope of stress-

strain curve (see Table 1). One measure of the precision of Young's modulus is the confidence interval for the computed regression line as shown in 9.5. Details for obtaining such confidence intervals may be found in Refs. 8 and 10.

11.2 *Bias*—A statement of bias of this test method requires reference standard values for one or more materials based on many measurements. Such standard reference values are presently not available.

NOTE 13—While a large amount of published data on Young's modulus of various materials are available in the open literature, it is unlikely that these data had been determined by using the exact procedure described in this test method. This will require interlaboratory test programs utilizing the procedures of this test method on various materials. Therefore, at the present time, the bias of the test method is unknown. However, calibration standards are available for testing machines and measuring devices. Hence, the degree of agreement of an individual or average determination between testing machine and measuring device and calibration standard should be stated.

12. Keywords

12.1 chord modulus; stress-strain diagram; tangent modulus; Young's modulus

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