Distributed Joint Fault Estimation for Multi-Agent Systems via Dynamic Event-Triggered Communication

Zeyuan Wang and Mohammed Chadli, *Senior Member, IEEE*

***Abstract*—This letter studies a novel distributed fault estimation framework for multi-agent systems under directed topology, subject to time-varying multiplicative and additive faults. Both actuator and sensor faults are simultaneously addressed by introducing an augmented system. A two-step design process is presented aimed at joint estimation of faults, system state, and exogenous disturbance, which involves an ℓth-order proportional-integral observer and a constrained least square estimator. Utilizing the relative output of neighbor information enhances the accuracy of fault and state estimation. Output sharing is realized by a dynamic event-triggered communication protocol, which effectively saves network resources. The design conditions of the observer are formulated as an optimization problem subject to linear matrix inequalities, ensuring guaranteed H-infinity performance of not only estimation error but also event error. Simulation results validate the effectiveness and feasibility of the proposed method.**

***Index Terms*—Distributed control, estimation, fault diagnosis, networked control systems.**

I. Introduction

TECHNOLOGICAL advances have led to increasing research and applications on multi-agent systems (MASs). In this context, agents collaborate through information exchange via networks, with distributed control and state estimation design. Examples include leader-following consensus [1], distributed localization of multi-vehicle [2], and unmanned aerial vehicle cooperative control [3], to name a few.

Due to the interconnection of MASs, the propagation of faults across agents can lead to safety risks, highlighting the significance of fault estimation within fault diagnosis systems. Various approaches have been investigated for fault estimation, including unknown input observer (UIO) for sensor faults [4], proportional-integrator observer (PIO) for actuator faults [5], reduced-order UIO for actuator faults, and intermediate-estimator-based methods for fault estimation and consensus tracking [6], to name a few. Simultaneous estimation of sensor and actuator faults presents more challenges. In [7], a mixed H_∞/H_- formulation is presented for compound fault detection. In addition, the authors in [8] and [9] develop a PIO-based observer for TS fuzzy system with unmeasurable decision variables, employing compensatory signals and H_∞ criteria.

While the studies mentioned above primarily concentrate on additive faults, research on multiplicative faults remains limited. Multiplicative faults bring increased complexity due to their coupled effects with control inputs. Notably, [10] proposes a neural network (NN)-based method for estimating actuator multiplicative faults. Reference [11] introduces the least squares (LS)-based estimation approach under conditions where the system's state is known. Reference [12] models multiplicative faults as a polytopic matrix and addresses them using a T-S observer. Besides, in [13], authors propose an adaptive NN-based approach for large-scale systems with multiplicative and additive faults. The research in [14] attempts to address the issue of compound faults under the formation problem where the external disturbance is not considered. In summary, the methodologies in existing studies are either limited to a single agent or focus solely on actuator or sensor faults. Consequently, compound fault estimation in MASs with both multiplicative and additive terms still remains an open problem.

We also notice that many articles utilize relative outputs as inputs for fault observers, such as in [4], [5], [7], [10], [15]. This method relies on continuous communication between agents and neighbors, leading to high network loads, particularly in large-scale MASs. One solution to this issue is event-triggered communication, which has drawn extensive research attention, as demonstrated in [16], [17]. However, it is notable that there is limited literature developing this technique in relative output signals for fault estimation, which constitutes a part of our study.

In summary, to the best of our knowledge, there is currently no existing solution that can simultaneously estimate multiplicative and additive compound faults in MASs under event-triggered communication networks, which is the objective of our work. Therefore, we outline our contributions as follows:

1. A novel joint fault estimation strategy is proposed for MAS under directed communication topology, which enables simultaneous estimation of time-varying actuator and sensor faults. Both multiplicative and additive faults are distinguished. This strategy consists of 1) a distributed ℓth-order proportional integral observer (PIO) utilizing local sensor information and

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The authors are with the IBISC, University of Paris-Saclay, 91020 Evry, France (e-mail: zeyuan.wang@universite-paris-saclay.fr; mchadli20@gmail.com).

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shared neighbor's output, and 2) a decentralized least square estimator, operating in a data-driven manner by online solving a constrained regression problem.

2. Unlike the existing studies [4], [5], [7], [10], [15] which relies on continuous neighbors' outputs, we develop a dynamic event-triggered communication protocol. This can facilitate adaptive data transmission, which can significantly save network resources.

3. A comprehensive robustness analysis based on H_∞ criteria is performed, establishing sufficient conditions for the design of robust observers. The design problem is formulated as a set of optimization problems subject to linear matrix inequalities (LMIs), with maximum robustness for estimation error and event error against unknown input.

The rest of this letter is organized as follows: Section II provides basic graph theory. Section III presents the problem formulation and a two-step fault estimation process. The detailed estimation framework, including the distributed PIO and LS estimator, is presented in Section IV. The numerical simulation is demonstrated in Section V, followed with a conclusion in Section VI.

Notations: Let $\mathbb{R}$, $\mathbb{N}^+$ denote the set of real numbers and the set of positive natural numbers, respectively. Given a matrix P, P^T denotes its transpose. $P < 0 (P \leq 0)$ denotes that P is negative definite (semi negative definite), and $P > 0 (P \geq 0)$ means $-P < 0 (-P \leq 0)$. I_n denotes an identity matrix of dimension n. $\otimes$ denotes the Kronecker product. Let $*$ denote the symmetric entries in a matrix. Denote $\mathrm{diag}(a_1, \ldots, a_N)$ a $N \times N$ diagonal matrix whose entries are $a_1, \ldots, a_N$.

II. Preliminaries

Consider a leader-following MAS with $N (N \in \mathbb{N}^+)$ followers and one leader. The communication topology is represented by a directed graph $\mathcal{G}_d = (\mathcal{V}, \mathcal{E})$ consisting of a vertex set $\mathcal{V} = \{v_1, \ldots, v_N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Follower agent i and the leader are represented as vertices v_i and v_0, respectively. Denote $\mathcal{N}_i$ the set of neighbors of agent i. The weighted adjacency matrix $\mathcal{A}_d = (a_{ij}) \in \mathbb{R}^{N \times N}$ of $\mathcal{G}_d$ is defined such that $a_{ii} = 0$, $a_{ij} = 1$ if v_i is connected to v_j (i.e., there exists a directed edge (v_i, v_j) from v_j to v_i) and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L}_d = (l_{ij}) \in \mathbb{R}^{N \times N}$ is defined as $l_{ii} = \sum_{i \neq j} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. Define $\bar{\mathcal{G}}_d = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ the augmented graph of $\mathcal{G}_d$, with $\bar{\mathcal{V}} = \mathcal{V} \cup \{v_0\}$ and $(v_i, v_0) \in \bar{\mathcal{E}}$ if agent i is connected to the leader. Define leader adjacency matrix $E_d = \mathrm{diag}(\varepsilon_1, \ldots, \varepsilon_N)$ as a diagonal matrix where its diagonal element $\varepsilon_i = 1$ if $(v_i, v_0) \in \bar{\mathcal{E}}$ otherwise $\varepsilon_i = 0$. Define $\mathcal{H} = \mathcal{L}_d + E_d$.

III. Problem Statement

Consider a MAS with N followers and one virtual leader with the following discrete-time dynamics:

$$\begin{aligned}
&\text{Followers } \forall i \in \{1, \ldots, N\}: \\
&\begin{cases} x_i(t+1) = Ax_i(t) + B\Theta_i(t)\,(u_i(t) + f_{ai}(t)) + Dd_i(t) \\ y_i(t) = Cx_i(t) + Hf_{si}(t) \end{cases} \\
&\text{Leader:} \quad x_0(t+1) = Ax_0(t), \quad y_0(t) = Cx_0(t)
\end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_x}$ and $y_i(t) \in \mathbb{R}^{n_y}$ are agent's state and output respectively. $u_i(t) \in \mathbb{R}^{n_u}$ is the control input. $\Theta_i(t) \in \mathbb{R}^{n_u \times n_u}$ is the multiplicative faults, $f_{ai}(t) \in \mathbb{R}^{n_u}$ and $f_{si}(t) \in \mathbb{R}^{n_s}$ are the actuator additive fault and sensor additive faults, respectively. $d_i(t)$ is exogenous disturbance. In particular, $\Theta_i(t) = \mathrm{diag}(\theta_i^1(t), \ldots, \theta_i^{n_u}(t))$, where $\theta_i^j \in [0, 1]$ represents the effectiveness of j-th actuator of agent i. $f_{ai} = (f_{ai}^1, \ldots, f_{ai}^{n_u})^T$, with $f_{ai}^j \in [\underline{f}_{ai}^j, \bar{f}_{ai}^j]$, $j = 1, \ldots, n_u$, where $\underline{f}_{ai}^j$ and $\bar{f}_{ai}^j$ are the knowing lower bound and upper bound of the additive fault f_{ai}^j. Note that only $\Theta_i(t)$ and $f_{ai}(t)$ are assumed to be slow time-varying. The objective is to estimate not only the state x_i but also the unknown faults Θ_i, f_{ai}, f_{si} as well as the disturbance d_i.

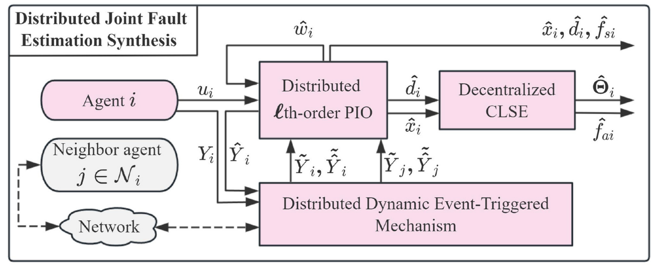

Fig. 1. Distributed Joint Fault Estimation Procedure.

In order to estimate the actuator and sensor faults simultaneously, we transform the sensor faults to be a part of the process fault by using an output filter $\xi_i(t+1) = E(\xi_i(t) - y_i(t))$ with a prescribed Schur matrix E. Define a new state $X_i(t) = \left(x_i^T(t)\ \xi_i^T(t)\right)^T \in \mathbb{R}^{n_x + n_y}$, and the new augmented system is

$$\begin{cases} X_i(t+1) = \tilde{A}X_i(t) + \tilde{B}u_i(t) + \tilde{F}w_i(t) \\ Y_i(t) = \tilde{C}X_i(t) \end{cases} \tag{2a}$$

$$\tilde{A} = \begin{pmatrix} A & 0 \\ -EC & E \end{pmatrix}, \ \tilde{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \ \tilde{F} = \begin{pmatrix} B & D & 0 \\ 0 & 0 & -EH \end{pmatrix} \tag{2b}$$

$$\tilde{C} = \begin{pmatrix} 0 & I_{n_y} \end{pmatrix}, \ w_i(t) = \begin{pmatrix} \bar{\Theta}_i(t)u_i(t) + \Theta_i(t)f_{ai}(t) \\ d_i(t) \\ f_{si}(t) \end{pmatrix} \tag{2c}$$

where $\bar{\Theta}_i(t) = \Theta_i(t) - I_{n_u}$. Note that the new variable $w_i(t)$ contains all unknown faults and disturbances of the original system (1), which acts as an unknown input of the augmented system (2a). However, there is a coupling effect between actuator multiplicative fault Θ_i and additive fault f_{ai} at the first term of w_i, making it non-trivial to be distinguished by a common observer.

To jointly estimate the multiplicative and additive faults, we propose a novel estimation framework consisting of a two-step design process:

1) *Step 1:* Design an ℓth-order proportional-integral observer (PIO) to realize robust estimation of the state x_i and the unknown input w_i by using the relative output of neighbors. A dynamic event-triggered communication design is also involved to avoid continuous communication.
2) *Step 2:* Design a constrained least square estimator (CLSE) to distinguish further the coupling variables Θ_i and f_{ai} based on the prior PIO estimation and system's model.

The PIO is distributed and relies on an event-triggered communication protocol, while the CLSE is decentralized. Fig. 1 is the block diagram of this joint estimation strategy.

The following assumptions hold in this letter.

Assumption 1: The unknown signal $w_i(t)$ is assumed to have bounded ℓth order derivatives ($\ell \in \mathbb{N}^+$). The dynamic of

$w_i(t)$ is described in (3).

$$\begin{cases} w_i(t+1) = w_i(t) + w_i^1(t) \\ w_i^1(t+1) = w_i^1(t) + w_i^2(t) \\ \dots \\ w_i^{\ell-1}(t+1) = w_i^{\ell-1}(t) + w_i^{\ell}(t), \quad \|w_i^{\ell}(t)\| \leq \bar{w} \end{cases} \tag{3}$$

Remark 1: Note that Assumption 1 guarantees that the disturbance $d_i(t)$ and the sensor fault $f_{si}(t)$ have bounded ℓth order derivative. In addition, $\Theta_i(t)$, $f_{ai}(t)$ and $u_i(t)$ and their 1st to ℓth order derivatives are also bounded.

Assumption 2: The graph $\mathcal{G}_d$ is fixed and directed, and the augmented graph $\bar{\mathcal{G}}_d$ contains a spanning tree with the leader agent being its root.

We recall the following lemma used in [1]:

Lemma 1: There exists a positive diagonal matrix $\Psi = \operatorname{diag}(\psi_1, \dots, \psi_N) > 0$ such that $\Psi\mathcal{H} + \mathcal{H}^T\Psi > 0$.

IV. Main Results

A. Step 1: PIO Description With Dynamic Event-Triggered Communication

The proposed ℓth-order proportional-integral observer (PIO) estimates simultaneously the faults and the state in the presence of the unknown disturbance, described as follows:

$$\begin{cases} \hat{X}_i(t+1) = \tilde{A}\hat{X}_i(t) + \tilde{B}u_i(t) + \tilde{F}\hat{w}_i(t) + K_P r_i(t) \\ \hat{Y}_i(t) = \tilde{C}\hat{X}_i(t) \\ \hat{w}_i(t+1) = K_I r_i(t) + \hat{w}_i(t) + \hat{w}_i^1(t) \\ \hat{w}_i^1(t+1) = K_I^1 r_i(t) + \hat{w}_i^1(t) + \hat{w}_i^2(t) \\ \dots \\ \hat{w}_i^{\ell-1}(t+1) = K_I^{\ell-1} r_i(t) + \hat{w}_i^{\ell-1}(t) \end{cases} \tag{4}$$

where $\hat{X}_i = \begin{pmatrix} \hat{x}_i^T & \hat{\xi}_i^T \end{pmatrix}^T$ is the estimated augmented state and $\hat{w}_i$ is the estimation of w_i, which contains the estimated disturbance $\hat{d}_i$ and estimated sensor faults $\hat{f}_{si}$. K_P, K_I^1, …, $K_I^{\ell-1}$ are observer gains to be designed. Note that the PIO can also give an estimation of first to $(\ell-1)$th order derivative of w_i, which provides more information about w_i. $r_i(t)$ is the relative output of neighbors defined as follows:

$$\begin{aligned} r_i(t) = \sum_{j\in\mathcal{N}_i} & a_{ij}[\tilde{Y}_j(t) - \tilde{Y}_i(t) \\ & - \left(\tilde{\hat{Y}}_j(t) - \tilde{\hat{Y}}_i(t)\right)] - \varepsilon_i\left(\tilde{Y}_i(t) - \tilde{\hat{Y}}_i(t)\right) \end{aligned} \tag{5}$$

where the event-based outputs $\tilde{Y}_i(t)$ and $\tilde{\hat{Y}}_i(t)$ $(i \in \{1, \dots, N\})$ are defined as follows:

$$\begin{aligned} &\text{At } t = t_k^i: \quad \tilde{Y}_i(t) = Y_i\left(t_k^i\right), \ \tilde{\hat{Y}}_i(t) = \hat{Y}_i\left(t_k^i\right) \\ &t \in [t_k^i, t_{k+1}^i): \tilde{Y}_i(t) = \tilde{Y}_i\left(t_k^i\right), \ \tilde{\hat{Y}}_i(t) = \tilde{\hat{Y}}_i\left(t_k^i\right) \end{aligned} \tag{6}$$

where t_k^i is the kth event of agent i, which will be defined in the sequel.

Remark 2: Different from [4], [5], [7], [10], [15], $r_i(t)$ is generated by event variables $\tilde{Y}_i(t)$ and $\tilde{\hat{Y}}_i(t)$, which avoids continuous data transmission.

Remark 3: $r_i(t)$ can provide more abundant information by introducing the graph connectivity information $\mathcal{H}$ in PIO design (see (7)), which can yield more accurate estimation (similar idea used in [5] but not with ℓth-order PIO). Besides, the leader's dynamic is not involved in (4), and only the connectivity information ε_i is used.

Note that except at the event moments, the event variables $\tilde{Y}_i(t)$, $\tilde{\hat{Y}}_i(t)$ differ from the true value $Y_i(t)$, $\hat{Y}_i(t)$. To measure this difference, we define the following event error h_i: $h_i(t) = \tilde{Y}_i(t) - \tilde{\hat{Y}}_i(t) - (Y_i(t) - \hat{Y}_i(t))$, and we denote $h = \begin{pmatrix} h_1^T & \dots & h_N^T \end{pmatrix}^T$. Define the output error $e_{Y_i} = Y_i - \hat{Y}_i$, which yields $r_i = \sum_j a_{ij}(h_j + e_{Y_j} - (h_i + e_{Y_i})) - \varepsilon_i(h_i + e_{Y_i})$). Define output error vector $e_Y = \begin{pmatrix} e_{Y_1}^T & \cdots & e_{Y_N}^T \end{pmatrix}^T$. The augmented state estimation error is defined as $e_{X_i} = X_i - \hat{X}_i$, $e_{w_i} = w_i - \hat{w}_i$, …, $e_{w_i^{\ell-1}} = w_i^{\ell-1} - \hat{w}_i^{\ell-1}$. We denote $e_X = \begin{pmatrix} e_{X_1}^T & \cdots & e_{X_N}^T \end{pmatrix}^T$, $e_w = \begin{pmatrix} e_{w_1}^T & \cdots & e_{w_N}^T \end{pmatrix}^T$, …, $e_{w^{\ell-1}} = \begin{pmatrix} e_{w_1^{\ell-1}}^T & \cdots & e_{w_N^{\ell-1}}^T \end{pmatrix}^T$, $w^{\ell} = \begin{pmatrix} (w_1^{\ell})^T & \cdots & (w_N^{\ell})^T \end{pmatrix}^T$. The final error vector is defined as $e = \begin{pmatrix} e_X^T \ e_w^T & \cdots & e_{w^{\ell-1}}^T \end{pmatrix}^T \in \mathbb{R}^{N(n_X + \ell n_w)}$, with $n_X = n_x + n_y$, and $n_w = n_u + n_d + n_s$. Define $r = \begin{pmatrix} r_1^T & \cdots & r_N^T \end{pmatrix}^T$, which yields that $r = -(\mathcal{H} \otimes I_{n_y})(h + e_Y)$.

With the notations above, we can obtain the following error dynamics:

$$e(t+1) = \mathcal{F}e(t) + \mathcal{K}(\mathcal{H} \otimes I)h(t) + B_w w^{\ell}(t) \tag{7}$$

$$\mathcal{F} = \mathcal{A} + \mathcal{K}\left(\mathcal{H} \otimes \tilde{C}\right)C_1 \tag{8}$$

$$\mathcal{A} = \begin{pmatrix} I_N \otimes \tilde{A} & I_N \otimes \tilde{F} & 0 & \cdots & 0 \\ 0 & I & I & 0 & \vdots \\ 0 & 0 & I & \ddots & 0 \\ \vdots & & \ddots & \ddots & I \\ 0 & 0 & \dots & 0 & I \end{pmatrix} \tag{9}$$

$$\mathcal{K} = \begin{pmatrix} I_N \otimes K_P \\ I_N \otimes K_I \\ I_N \otimes K_I^1 \\ \dots \\ I_N \otimes K_I^{\ell-1} \end{pmatrix}, C_1^T = \begin{pmatrix} I_{Nn_X} \\ 0 \\ \dots \\ 0 \end{pmatrix}, B_w = \begin{pmatrix} 0 \\ \dots \\ 0 \\ I_{Nn_w} \end{pmatrix} \tag{10}$$

Drawing insights from [18], the dynamic event-triggered mechanism (DETM) in this letter is proposed as

$$t_{k+1}^i = inf\left\{t | t > t_k^i, \ g_i(t) \geq 0\right\} \tag{11}$$

where $g_i(t) = -\eta_i(t) + \omega(h_i^T(t)U_1h_i(t) - e_{Y_i}^T(t)U_2e_{Y_i}(t))$. $U_1 > 0$, $U_2 > 0$ are two matrices to be designed. The auxiliary variable $\eta_i(t)$ is employed to prolong the inter-event time and is defined as $\eta_i(t+1) = \lambda\eta_i(t) - h_i^T(t)U_1h_i(t) + e_{Y_i}^T(t)U_2e_{Y_i}(t)$, with the initial condition $\eta_i(0) > 0$. λ and ω are two positive scalars satisfying $\lambda \in (0, 1)$ and $(\lambda - 1/\omega) > 0$. The event rule implies that $\forall t$, $g_i(t) < 0$, which yields

$$\forall t, \quad \eta_i(t+1) > \lambda\eta_i(t) - \eta_i(t)/\omega > 0 \tag{12}$$

Since $\lambda \in (0, 1)$ and $\eta_i(t) > 0$, it follows that

$$\begin{aligned} &\eta_i(t+1) - \eta_i(t) \\ &= (\lambda - 1)\eta_i(t) - h_i^T(t)U_1h_i(t) + e_{Y_i}^T(t)U_2e_{Y_i}(t) \\ &\leq -h_i^T(t)U_1h_i(t) + e_{Y_i}^T(t)U_2e_{Y_i}(t) \end{aligned} \tag{13}$$

Remark 4: The auxiliary variable η_i has its own dynamic and is proven to help to provide longer inter-event time than static event-triggered mechanism [18]. Furthermore, since we only consider the discrete-time system, the Zeno behavior is naturally excluded.

B. H_∞ Robustness Conditions for PIO Estimation

In this section, PIO robust estimation against the bounded disturbance is studied by considering the H_∞ performance. Since the dynamic of estimation error given in (7) is perturbed by unknown but bounded signal w^ℓ, the objective is to find sufficient conditions for robust PIO such that the errors $e(t), h(t)$ satisfy $\sum_{k=0}^{t}\|(e^T(k)\ \ h^T(k))^T\|^2 \leq \gamma^2\sum_{k=0}^{t}\|w^\ell(k)\|^2$ under the zero-initial condition, where the positive scalar γ should be as small as possible.

The following result gives LMI conditions for robust estimation.

Theorem 1: The estimation of the PIO (4) robustly converges to the state of the augmented system (2a), and H_∞ performance is guaranteed with an attenuation level γ, if there exists $P>0$, $U_1>0$, $U_2>0$, $L_P, L_I, L_I^1,\dots,L_I^{\ell-1}$ such that the following optimization problem holds:

$$\min \gamma > 0, \ \text{ subject to } \Sigma < 0 \tag{14}$$

where

$$\Sigma = \begin{pmatrix} -P & 0 & 0 & \Sigma_{14} & I & 0 & \Sigma_{17} \\ * & -I_N\otimes U_1 & 0 & \Sigma_{24} & 0 & I & 0 \\ * & * & -\gamma I & B_w^T P & 0 & 0 & 0 \\ * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & * & -I_N\otimes U_2 \end{pmatrix} \tag{15}$$

$$\Sigma_{14} = \mathcal{A}^T P + C_1^T\left((\mathcal{H}^T\Psi)\otimes\tilde{C}^T\right)\mathcal{L}^T \tag{16}$$

$$\Sigma_{17} = C_1^T\left(I\otimes(\tilde{C}^T U_2)\right),\ \ \Sigma_{24} = \left((\mathcal{H}^T\Psi)\otimes I\right)\mathcal{L}^T \tag{17}$$

$$\mathcal{L} = \begin{pmatrix} I_N\otimes L_P \\ I_N\otimes L_I \\ I_N\otimes L_I^1 \\ \cdots \\ I_N\otimes L_I^{\ell-1} \end{pmatrix}, P = \begin{pmatrix} \Psi\otimes P_{00} & \cdots & \Psi\otimes P_{0\ell} \\ \vdots & \ddots & \vdots \\ \Psi\otimes P_{\ell 0} & \cdots & \Psi\otimes P_{\ell\ell} \end{pmatrix} \tag{18}$$

and $\Psi>0$ satisfying Lemma 1. Then, the observer gains are obtained by

$$\begin{pmatrix} K_P \\ K_I \\ \cdots \\ K_I^{\ell-1} \end{pmatrix} = \left(\overline{P}\right)^{-1}\begin{pmatrix} L_P \\ L_I \\ \cdots \\ L_I^{\ell-1} \end{pmatrix},\ \text{ with } \overline{P} = \begin{pmatrix} P_{00} & \cdots & P_{0\ell} \\ \vdots & \ddots & \vdots \\ P_{\ell 0} & \cdots & P_{\ell\ell} \end{pmatrix} \tag{19}$$

Proof: Consider the following Lyapunov function

$$V(t) = e^T(t)Pe^T(t) + \sum_{i=1}^{N}\eta_i(t) \tag{20}$$

where P is in (18). Note that by (12) $\eta_i(t) > 0$ thus $\forall t$, $V(t)\geq 0$. To have the guaranteed H_∞ performance, the following condition should be satisfied:

$$J = V(t+1) - V(t) + \gamma^{-1}\chi^T(t)\chi(t) - \gamma\left(w^\ell(t)\right)^T w^\ell(t) < 0 \tag{21}$$

where $\chi = (e^T\ h^T)^T$. Note that the relation in (13) yields $\sum_i(\eta_i(t+1)-\eta_i(t)) \leq \sum_i(-h_i^T(t)U_1h_i(t) + e_{Y_i}^T(t)U_2e_{Y_i}(t)) = -h^T(t)(I_N\otimes U_1)h(t) + e_Y^T(t)(I_N\otimes U_2)e_Y(t)$. Replacing $V(t)$ by this inequality, and by (7), we obtain

$$\begin{aligned} J \leq & \left(\mathcal{F}e + \mathcal{K}(\mathcal{H}\otimes I)h + B_w w^\ell\right)^T P \\ & \times\left(\mathcal{F}e + \mathcal{K}(\mathcal{H}\otimes I)h + B_w w^\ell\right) - e^T Pe \\ & - h^T(I_N\otimes U_1)h + e_Y^T(I_N\otimes U_2)e_Y \\ & + \gamma^{-1}\left(e^Te + h^Th\right) - \gamma\left(w^\ell\right)^T w^\ell = \varsigma^T\Pi\varsigma \end{aligned} \tag{22}$$

by using the relation $e_Y = (I_N\otimes\tilde{C})\ C_1 e$. $\varsigma = (e^T\ h^T\ (w^\ell)^T)^T$, and $\Pi = \Pi_1+\Pi_2$, where $\Pi_1 = S^TP^{-1}S$, $S = \left(P\mathcal{F}\ P\mathcal{K}(\mathcal{H}\otimes I)\ PB_w\right)$, $\Pi_2 = \text{diag}(-P + C_1^T(I_N\otimes\tilde{C}^TU_2\tilde{C})C_1 + \gamma^{-1}I, -I_N\otimes U_1 + \gamma^{-1}I,\ -\gamma I)$.

Hence, $\Pi<0$ is a necessary and sufficient condition for $J<0$. By using the Schur complement, it is equivalent to $\Pi<0$ that

$$\begin{pmatrix} -P & 0 & 0 & \mathcal{F}^TP & I & 0 & \Sigma_{17} \\ * & -I_N\otimes U_1 & 0 & (\mathcal{H}^T\otimes I)\mathcal{K}^TP & 0 & I & 0 \\ * & * & -\gamma I & B_w^TP & 0 & 0 & 0 \\ * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & * & -I_N\otimes U_2 \end{pmatrix} < 0 \tag{23}$$

Using the expression in (8) yields $\mathcal{F}^TP = \mathcal{A}^TP + C_1^T(\mathcal{H}^T\otimes\tilde{C}^T)\mathcal{K}^TP$. Then, replacing P, $\mathcal{K}$ by (18) and (10), respectively, with the change of variables in (19) yields:

$$P\mathcal{K} = P\begin{pmatrix} I\otimes K_P \\ I\otimes K_I \\ \cdots \\ I\otimes K_I^{\ell-1} \end{pmatrix} = \begin{pmatrix} \Psi\otimes L_P \\ \Psi\otimes L_I \\ \cdots \\ \Psi\otimes L_I^{\ell-1} \end{pmatrix} = \mathcal{L}(\Psi\otimes I) \tag{24}$$

where $\mathcal{L}$ is defined in (18). Therefore $\mathcal{F}^TP = \mathcal{A}^TP + C_1^T((\mathcal{H}^T\Psi)\otimes\tilde{C}^T)\mathcal{L}^T$ and $(\mathcal{H}^T\otimes I)\ \mathcal{K}^TP = ((\mathcal{H}^T\Psi)\otimes I)\ \mathcal{L}^T$, which completes the proof. ∎

Remark 5: Note that the designed condition of Theorem 1 is in a linear form. Indeed, the LMI condition $\Sigma<0$ in (14) can be easily solved by numerical tools (such as the YALMIP software).

C. Step 2: CLSE Design

The constrained least square estimator (CLSE) is a data-driven estimator that further distinguishes multiplicative faults from additive faults. The CLSE is only for the estimation of Θ_i and f_{ai}. The sensor fault f_{si} and disturbance signal d_i could be directly estimated by PIO in the variable $\hat{w}_i$ (see Fig. 1). Suppose we have a well-designed PIO in step 1 and consider the following process:

$$x_i(t+1) - Ax_i(t) - Dd_i(t) = B\Theta_i(t)(u_i(t) + f_{ai}(t)) \tag{25}$$

Denote $z_i(t) = x_i(t) - Ax_i(t-1) - Dd_i(t-1)$, and $\Pi_i(t) = (B\bar{u}_i(t-1)B)$, where $\bar{u}_i = \text{diag}(u_i)$. Denote the fault vector:

$$\tilde{f}_i(t+1) = \left(\theta_i^1(t)\ \ \dots\ \ \theta_i^{n_u}(t)\ \ (\theta_i^1f_{ai}^1)(t)\ \ \dots\ \ (\theta_i^{n_u}f_{ai}^{n_u})(t)\right)^T \tag{26}$$

where $f_{ai}^1,\dots,f_{ai}^{n_u}$ are elements of additive faults f_{ai}. With the above notations, a concise expression is obtained: $z_i(t) =$

$\Pi_i(t)\tilde{f}_i(t)$. Then, estimating $\tilde{f}_i$ becomes a regression problem. In general, this problem can be solved by any parameter estimation methods, such as least squares, recursive least squares, or neural networks [19]. In this letter, we consider using a sliding-window least square method with boxed-constrained. A sliding window of length N_w is used to collect the most recent data from $t-N_w+1$ to t, with the assumption that $\tilde{f}_i$ is approximately a constant over $[t-N_w+1, t]$:

$$z_i^{N_w}(t) = \Pi_i^{N_w}(t)\tilde{f}_i(t) \tag{27}$$

where $z_i^{N_w}(t) = (z_i^T(t)\ z_i^T(t-1)\ \ldots\ z_i^T(t-N_w+1))^T$, $\Pi_i^{N_w}(t) = (\Pi_i^T(t)\ \Pi_i^T(t-1)\ \cdots\ \Pi_i^T(t-N_w+1))^T$.

Since the estimation error is marginal given a well-designed PIO, the optimal estimation of $\tilde{f}_i(t)$ is given by the following optimization problem with boxed constraint:

$$\hat{\tilde{f}}_i(t) = \arg\min_f \|\hat{z}_i^{N_w}(t) - \Pi_i^{N_w}(t)f\|^2 \quad \text{such that } f \in \left[\underline{\tilde{f}}_i, \overline{\tilde{f}}_i\right] \tag{28}$$

where $\hat{z}_i^{N_w}(t) = (\hat{z}_i^T(t)\ \hat{z}_i^T(t-1)\ \ldots\ \hat{z}_i^T(t-N_w+1))^T$, $\hat{z}_i(t) = \hat{x}_i(t) - A\hat{x}_i(t-1) - D\hat{d}_i(t-1)$, $\underline{\tilde{f}}_i = (0\ \ldots\ 0\ \min(0,\underline{f}_{ai}^1)\ \ldots\ \min(0,\underline{f}_{ai}^{n_u}))^T$, $\overline{\tilde{f}}_i = (1\ \ldots\ 1\ \max(0,\overline{f}_{ai}^1)\ \ldots\ \max(0,\overline{f}_{ai}^{n_u}))^T$. Note that $\hat{d}_i(t-1) = [0\ I_{n_d}\ 0]\hat{w}_i(t-1)$ and $\hat{x}_i(t) = [I_{n_x}\ 0]\hat{X}_i(t)$, where $\hat{w}_i$ and $\hat{X}_i$ are estimated by the PIO in Step 1. The variable flow between Step 1 and Step 2 is also illustrated in Fig. 1.

Remark 6: A warm-up stage is needed to collect at least N_w samples as the initialization of the CLSE.

Remark 7: The imposed constraints $\underline{\tilde{f}}_i$ and $\overline{\tilde{f}}_i$ can yield a more accurate estimation than unconstrained methods, such as in [11], [20]. Unfortunately, the well-known solution $\hat{\tilde{f}}_i(t) = (\Pi_i^{N_w}(t))^{\dagger}\hat{z}_i^{N_w}(t)$ or its recursive version [20] under unconstrained LS regression is no longer valid. To solve this constrained LS problem, we use the interior point algorithm with the MATLAB solver *"lsqlin"*.

Remark 8: Different from [11], where the LS is based on measurable states without disturbance, which is difficult to achieve in a real-world setting, our strategy provides a more general solution together with PIO design for joint estimation.

V. Numerical Examples

Consider a MAS with three follower agents and one leader described by the dynamic in [21]. A zero-order holder for discretization is employed with the sampling period $T_s = 0.001s$, which yields the following discrete dynamic for (1):

$$A = \begin{pmatrix} 0.999 & 0.001 & 0 & 0 \\ -0.001 & 1.000 & 0 & 0 \\ 0 & 0 & 1.000 & 0.003 \\ 0 & 0 & -0.003 & 1.000 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.001 & 0 \\ 0.001 & 0 \\ 0 & 0.001 \\ -0.001 & -0.001 \end{pmatrix}$$

and $C = \text{diag}(1, 0.5, 0.5, 1)$, $D = (0\ 0.01\ 0\ 0.01)^T$, $H = (0\ 0.1\ 0\ 0)^T$. The communication topology is described as

$$\mathcal{L}_d = \begin{pmatrix} 1.2 & 0 & -1.2 \\ -0.8 & 0.8 & 0 \\ -2 & 0 & 2 \end{pmatrix}, \quad \mathcal{D}_d = \text{diag}(1, 0, 0)$$

TABLE I
Fault Scenario Settings

$0 \le tT_s < 5$	no faults
$5 \le tT_s < 20$	$\theta_1^1 = 0.8 - 0.005(t-5)$, $\theta_1^2 = 0.2$ $f_{a1} = [0.1(t-5) + 0.05\sin(\frac{t-5}{2}), -0.02(t-5)]^T$
$20 \le tT_s < 30$	$\theta_1^1 = 0.725$, $\theta_1^2 = 0.2$ $f_{a1} = [1.5 + 0.05\sin(\frac{t-5}{2}), -0.02(t-5)]^T$ $f_{a2} = [-2 + 0.05\sin(\frac{(t-20}{2.5}), 0]^T$ $f_{s1} = 0.2$, $f_{s2} = -0.5 + 0.1\sin(\frac{t}{2})$
$tT_s \ge 30$	$\theta_1^1 = 0.725$, $\theta_1^2 = 0.2$, $\theta_2^1 = 1$, $\theta_2^2 = 0.5$ $f_{a1} = [1.5 + 0.05\sin(\frac{t-5}{2}), -0.02(t-5)]^T$ $f_{a2} = [-2 + 0.05\sin(\frac{(t-20}{2.5}), 3]^T$ $f_{s1} = 0.2$, $f_{s2} = -0.5 + 0.1\sin(\frac{t}{2})$

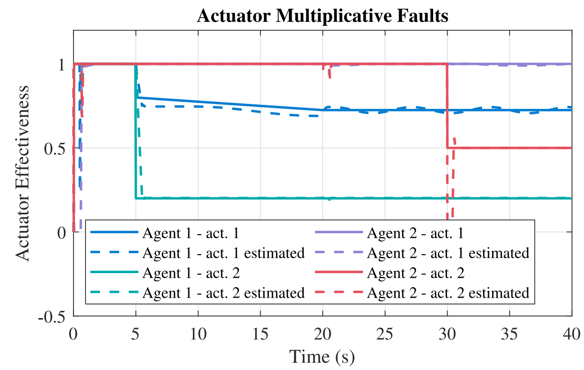

Fig. 2. Actuator multiplicative faults and estimation.

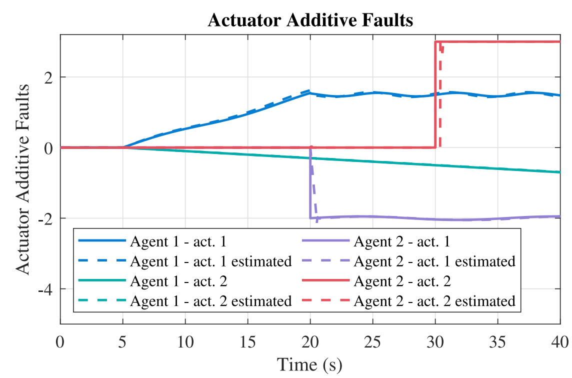

Fig. 3. Actuator additive faults and estimation.

Set the length of the sliding window as $N_w = 500$. Set $E = \text{diag}(0.8, 0.8)$. Table I describes non-identical faults in Agent 1 and Agent 2. The fault signals are illustrated in Fig. 2 - Fig. 4. In this example, we construct a 2nd-order PIO (4) by solving the LMI conditions in Theorem 1. The simulation results for faults estimation are depicted in Fig. 2, Fig. 3, and Fig. 4, which show that both the PIO (for sensor faults) and the CLSE (for actuator faults decoupling) have a validated estimation.

Meanwhile, the PIO offers an accurate estimation of the system state x_i and disturbance d_i, depicted in Fig. 5 and Fig. 6, respectively. Notice that sudden faults can cause some perturbation, such as in $t = 5$ and $t = 20$. Overall, the simulation results show a good simultaneous estimation of faults, states, and disturbances.

Furthermore, we conclude the inter-event time of each agent in Table II. Notice that for each agent, the inter-event time is much longer than the sampling period T_s, which shows a reduction in communication frequency.

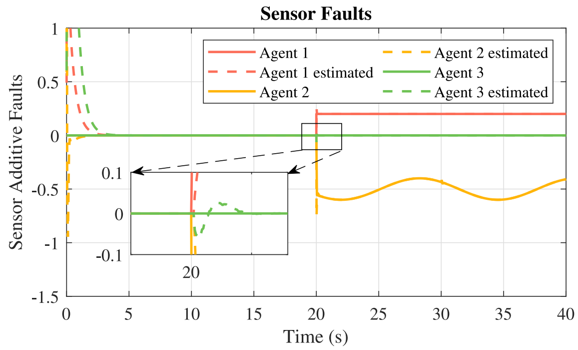

Fig. 4. Sensor faults and estimation.

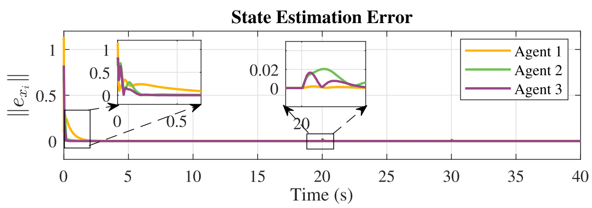

Fig. 5. State estimation error, $\|e_{x_i}\| = \|x_i - \hat{x}_i\|$, $i = 1, 2, 3$.

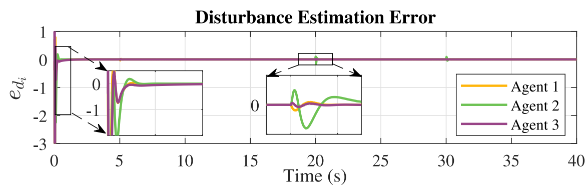

Fig. 6. Disturbance estimation error, $e_{d_i} = d_i - \hat{d}_i$, $i = 1, 2, 3$.

TABLE II
INTER-EVENT TIME (IET) IN MS

	Agent 1	Agent 2	Agent 3	T_s
Average IET	1.72	1.61	1.77	**1.00**
Maximum IET	**19.0**	**17.0**	**20.0**	

VI. Conclusion

In this letter, a novel distributed fault estimation framework for time-varying compound faults in MASs is proposed. The coupling effect of multiplicative and additive faults is addressed by the proposed two-step design process, including the ℓth-order PIO and the CLSE approach, subject to actuator and sensor faults. The PIO acts as a fundamental observer, providing state and disturbance estimation with guaranteed H-infinity performance w.r.t. estimation error and event error, while the CLSE distinguishes between multiplicative and additive faults based on PIO results. Additionally, the drawback of continuous communication is addressed by introducing a DETM, effectively reducing network bandwidth usage. Simulation results confirm the efficacy of the theoretical methods. Communication delays and a general error analysis of CLSE are not addressed in this letter, and they will be the focus of our future work.

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