# Improved Dynamic Event-Triggered Consensus Control for Multi-Agent Systems with Designable Inter-Event Time*

Zeyuan Wang[1] and Mohammed Chadli[2]

***Abstract*— This paper considers the leader-following consensus control for linear multi-agent systems. Two improved dynamic event-triggered control frameworks are proposed. The first one is based on a moving average approach, whereas the second is a fully-distributed control scheme based on a well-chosen Lyapunov function with rigorous proof of adjustable inter-event time. The proposed methods involve model-based estimation and clock-like auxiliary dynamic variables to increase the inter-event time as long as possible eventually. Compared to the static event-triggered strategy and the existing state-of-the-art dynamic event-triggered mechanism, the proposed approach significantly reduces the communication frequency while still guaranteeing asymptotic convergence. Numerical simulations demonstrate the validity of the proposed theoretical results.**

## I. INTRODUCTION

The research and application of multi-agent technology originated in the 1980s and gained widespread development in the mid-1990s. Today, it has become an unavoidable topic in distributed artificial intelligence. In recent decades, due to the development of cluster control such as unmanned aerial vehicle coordination, underwater cooperation, and robot formation control [1], consensus issues of multi-agent systems (MASs) have become the focus for many researchers. One of the fascinating topics is leader-following consensus [2], also known as model reference consensus [3], where a group of agents need to achieve consensus with the leader agents. It has been studied widely, for linear MASs [2] [4], nonlinear MASs [5], homogeneous or heterogeneous agents [6] under different scenarios.

In a time-triggered control mechanism, the control update and data transmission are implemented periodically. For MASs with a large number of agents, this could result in a higher burden of communication networks and higher consumption of computing resources [7]. This problem motivated a event-triggered mechanisms (ETM), which enables a minimum rate of data sampling, communication, and control update while the system performance is still guaranteed. This strategy was first introduced in [8], then gains significant development and is applied in different dynamic systems [9] [10]. The ETM was first used to handle the consensus problem of MASs in [11], then has been developed for generic linear MASs [12] [13] and nonlinear MASs [14], under different constraints such as communication delay [15], external disturbance [16], and actuator faults [17].

Based on the principle of the ETM, a dynamic event-triggered mechanism (DETM) was proposed to provide a more flexible parameter tuning approach and to increase the minimum inter-event time (MIET) [18]. By introducing an auxiliary dynamic variable (ADV), the additional dynamic results in a more intelligent event-triggering decision which enables a lower sampling rate while still keeping the performance [19]. The DETM is applied in consensus control of MASs in many different scenarios [20] [21]. The ADV gives more freedom to select a preferred (minimum) inter-event time by choosing appropriate parameters in the controller design phase; thus, the Zeno behavior of MAS is naturally excluded, though the rigorous proof of the MIET is still tricky without enforcing a dwell-time [22].

This work is inspired by [22], [20] and [23], where DETMs are realized based on clock-like ADVs. However, in the state-of-the-art research [20], a bad ADV will fail to achieve leader-following consensus because of a discontinuous Lyapunov function chosen. The DETM only guarantees stability between two consecutive events but not globally. Based on the above discussion, this paper proposes two improved DETMs to increase the inter-event time and eliminate the disadvantages of [20]. We demonstrate firstly an improvement by enforcing a global decay rate of the discontinuous Lyapunov function using a moving average approach. The second relies on choosing a new continuous Lyapunov function and gives rigorous proof of leader-following consensus and MIET. The MIET could be explicitly tuned by varying the upper bound of ADVs. Numerical examples are given to validate the proposed DETMs and demonstrate the effect of the parameter, and the comparison with the state-of-the-art results [20].

The rest of the paper is structured as follows. Section II presents preliminaries, including notations and graph theory. Section III contains the problem formulation and assumptions. The detailed design of two improved DETMs and the rigorous proof is presented in Section IV. Section V illustrates the proposed strategies' effectiveness by numerical simulation and the discussion of the results. Section VI is a brief conclusion.

## II. PRELIMINARIES

Given a matrix $\boldsymbol{S}$, $\boldsymbol{S}^T$ denotes its transpose. If $\boldsymbol{S}$ is a square matrix, $\lambda_{\min}(\boldsymbol{S})$ and $\lambda_{\max}(\boldsymbol{S})$ denote the minimum and maximum eigenvalues of $\boldsymbol{S}$ respectively, and $\boldsymbol{S}^{-1}$ represents its inverse. $\boldsymbol{S} < 0(\leq 0)$ denotes that $\boldsymbol{S}$ is negative

*This work was supported by the China Scholarship Council under Grant 202206020096

[1]Zeyuan Wang is with University Paris-Saclay, Univ Evry, IBISC, 91020 Evry, France zeyuan.wang@universite-paris-saclay.fr

[2]Mohammed Chadli is with University Paris-Saclay, Univ Evry, IBISC, 91020 Evry, France mohammed.chadli@univ-evry.fr

definite (negative semi-definite), and $\boldsymbol{S} > 0(\geq 0)$ means $-\boldsymbol{S} < 0(\leq 0)$. Denote $\boldsymbol{I}_n$ an $n$-dimensional identity matrix. $\otimes$ is the Kronecker product. $||\cdot||$ is the $\ell_2$-norm for a vector or the spectrum norm for a matrix.

The communication topology of $N$ follower agents is represented by an undirected weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with a vertex set $\mathcal{V} = \{v_1, ..., v_N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Follower agent $i$ and the leader agent are represented as vertices $v_i$ and $v_0$, respectively. Denote $\mathcal{N}_i$ the set of neighbors of agent $i$. The adjacency matrix $\boldsymbol{\mathcal{A}} = (a_{ij}) \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined such that $a_{ii} = 0$, $a_{ij} = a_{ji} > 0$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = a_{ji} = 0$ otherwise. Define the Laplacian matrix $\boldsymbol{\mathcal{L}} = (l_{ij}) \in \mathbb{R}^{N\times N}$ as $l_{ij} = -a_{ij}, i \neq j$ and $l_{ii} = \sum_{i\neq j} a_{ij}$. Define $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ as the augmented graph of $\mathcal{G}$, with $\bar{\mathcal{V}} = \mathcal{V} \cup \{v_0\}$ and $\bar{\mathcal{E}} = \mathcal{E} \cup \Delta\mathcal{E}$, where $(v_i, v_0) \in \Delta\mathcal{E}$ if agent $i$ is connected to the leader. Define $\boldsymbol{\mathcal{D}} = diag(d_1, ..., d_N)$ as a diagonal matrix where its diagonal element $d_i > 0$ if $(v_i, v_0) \in \bar{\mathcal{E}}$ otherwise $d_i = 0$. Define $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{D}}$.

## III. Problem Statement

Consider a linear MAS with $N$ follower agents and one leader represented by

$$\begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}\boldsymbol{x}_i(t) + \boldsymbol{B}\boldsymbol{u}_i(t), i = 1, ..., N \\ \dot{\boldsymbol{x}}_0(t) = \boldsymbol{A}\boldsymbol{x}_0(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}_i(t)$ is follower agents' state and $\boldsymbol{x}_0(t)$ is the leader's state. $\boldsymbol{A} \in \mathbb{R}^{n\times n}, \boldsymbol{B} \in \mathbb{R}^{n\times m}$, $\boldsymbol{u}_i(t) \in \mathbb{R}^m$ is the control input. Define the consensus error of agent $i$ as $\boldsymbol{\varepsilon}_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{x}_0(t)$ and $\boldsymbol{\varepsilon}(t) = [\boldsymbol{\varepsilon}_i^T(t), ..., \boldsymbol{\varepsilon}_N^T(t)]^T$. The goal of this study is to design an event-based control protocol such that all follower agents and the leader achieve the consensus, i.e., $\lim_{t\to\infty} ||\boldsymbol{\varepsilon}_i(t)|| = 0, \forall i \in \{1, ..., N\}$.

The proposed event-triggered control input $u_i(t)$ of agent $i$ is defined as

$$\begin{cases} \boldsymbol{u}_i(t) = \boldsymbol{K}\boldsymbol{z}_i(t) \\ \boldsymbol{z}_i(t) = \sum\limits_{j\in\mathcal{N}_i} a_{ij}(\hat{\boldsymbol{x}}_j(t) - \hat{\boldsymbol{x}}_i(t)) + d_i(\boldsymbol{x}_0(t) - \hat{\boldsymbol{x}}_i(t)) \end{cases} \tag{2}$$

where $\boldsymbol{K} \in \mathbb{R}^{m\times n}$ is the gain of the feedback control, $\hat{\boldsymbol{x}}_i(t), i \in \{1, ..., N\}$ is defined as:

$$\hat{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t_k^i)e^{\boldsymbol{A}(t-t_k^i)}, t \in [t_k^i, t_{k+1}^i) \tag{3}$$

where $\boldsymbol{x}_i(t_k^i)$ is the value of $\boldsymbol{x}_i(t)$ at the last triggering moment $t_k^i$. This feedback control law is fully distributed because it only contains local information of the agent and its neighbors. Define $\boldsymbol{e}_i(t) = \hat{\boldsymbol{x}}_i(t) - \boldsymbol{x}_i(t)$ and $\boldsymbol{e}(t) = [\boldsymbol{e}_1^T(t), ..., \boldsymbol{e}_N^T(t)]^T$.

The objective of this article is to design a DETM to determine the triggering moment $t_k^i$ under the control law (2). The data transmission process of agent $i$ ($i \in \{1, ...N\}$) is described as follows: at the triggering moment $t_k^i$, agent $i$ broadcasts its current state $x_i(t_k^i)$ to its neighbors. Simultaneously, agent $i$ updates the value of $\hat{x}_i(t_k^i)$ to $x_i(t_k^i)$, thereby updating the value of the control law in Equation (2). When agent $i$ receives the value of $x_j(t)$ from a neighbor agent $j$ at the neighbor's triggering moments $t = t_{k'}^j$, the value of $\hat{x}_j(t_{k'}^j)$ in equation (2) is also updated to $x_j(t_{k'}^j)$.

*Remark 1:* The term $\hat{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t_k^i)e^{\boldsymbol{A}(t-t_k^i)}$ is a kind of model-based estimation [24]. Since the trivial estimation $\hat{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t_k^i)$ may differ from $\boldsymbol{x}_i(t)$ very quickly, the exponential term $e^{\boldsymbol{A}}$ is used to reduce $\boldsymbol{e}_i(t)$ thus usually increases the inter-event time.

By definition, we can deduce $\hat{\boldsymbol{x}}_i(t) = \boldsymbol{e}_i(t) + \boldsymbol{x}_i(t)$, and $\boldsymbol{x}_i(t) = \boldsymbol{x}_0(t) + \boldsymbol{\varepsilon}_i(t)$, thus $\boldsymbol{z}_i(t) = \sum a_{ij}(\boldsymbol{e}_j(t) - \boldsymbol{e}_i(t) + \boldsymbol{\varepsilon}_j(t) - \boldsymbol{\varepsilon}_i(t)) + d_i(-\boldsymbol{e}_i(t) - \boldsymbol{\varepsilon}_i(t))$. The following relations are shown by calculating the derivatives of $\boldsymbol{\varepsilon}(t)$ and $\boldsymbol{e}(t)$:

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{A} - \boldsymbol{\mathcal{H}} \otimes \boldsymbol{B}\boldsymbol{K})\boldsymbol{\varepsilon}(t) - (\boldsymbol{\mathcal{H}} \otimes \boldsymbol{B}\boldsymbol{K})\boldsymbol{e}(t) \\ \dot{\boldsymbol{e}}(t) = (\boldsymbol{I}_N \otimes \boldsymbol{A} + \boldsymbol{\mathcal{H}} \otimes \boldsymbol{B}\boldsymbol{K})\boldsymbol{e}(t) + (\boldsymbol{\mathcal{H}} \otimes \boldsymbol{B}\boldsymbol{K})\boldsymbol{\varepsilon}(t) \end{cases} \tag{4}$$

The following assumptions hold in this paper:

*Assumption 1:* The pair $(\boldsymbol{A}, \boldsymbol{B})$ is stabilizable.

*Assumption 2:* The communication topology graph between $N$ follower agents is weighted, undirected and fixed.

*Assumption 3:* The augmented graph $\bar{\mathcal{G}}$ contains a spanning tree with the leader agent being its root.

We recall the following lemmas due to [2].

*Lemma 1:* Under assumptions 2 and 3, $\boldsymbol{\mathcal{H}} > 0$.

*Lemma 2:* Based on assumption 1 and Lemma 1, given $\delta_{\min} = \lambda_{\min}(\boldsymbol{\mathcal{H}})$, there exists a solution $\boldsymbol{P} > 0$ for the following matrix inequality

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P} - 2\delta_{\min}\boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{P} + \delta_{\min}\boldsymbol{I}_n < 0 \tag{5}$$

## IV. Dynamic event-triggered control design

### A. $\ell$-step Moving Average Adaptive Event-triggered Control

The DETM proposed in the state-of-the-art research [20] only guarantees the stability of the MAS between two consecutive events by choosing a discontinuous Lyapunov function which has an abrupt change at triggering moments. This could lead to divergence of the error system (4) since the global stability has not been proved. The idea of improvement is to add additional constraints to enforce a global decrease of the discontinuous Lyapunov function. We revisit the results in [20] as the following lemma:

*Lemma 3:* Under assumptions 1, 2, 3 and the control input (2) with $\boldsymbol{K} = \boldsymbol{B}^T\boldsymbol{P}$, the error system (4) is stable in any $[t_{k-1}^i, t_k^i)$ with the event-triggered rule defined as:

$$t_k^i \triangleq \inf\left\{t > t_{k-1}^i \mid \theta_i(t) \leq 0\right\}, \theta_i(t_{k-1}^i) = \bar{\theta}_i > 0$$
$$\dot{\theta}_i(t) \triangleq \begin{cases} \omega_i(t) - \tau_i & \text{if } ||\boldsymbol{e}_i(t)|| \neq 0 \text{ or } ||\boldsymbol{z}_i(t)|| \neq 0 \\ -\tau_i & \text{otherwise} \end{cases} \tag{6}$$

where $\tau_i > 0$ is a constant scalar and

$$\omega_i(t) \triangleq -\frac{(\alpha\theta_i + 2\delta\beta + (\theta_i - 1)^2)||\boldsymbol{e}_i||^2 + (2\varepsilon\theta_i + \beta^2)||\boldsymbol{z}_i||^2}{\eta||\boldsymbol{e}_i||^2 + ||\boldsymbol{z}_i||^2} \tag{7}$$

where $\alpha = ||\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P}||$, $\delta = \lambda_{\max}(\boldsymbol{\mathcal{H}})$, $\beta = ||\boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{P}||$, $\eta = \lambda_{\min}(\boldsymbol{P})$, $\epsilon = |\lambda_{\max}(\frac{\boldsymbol{A}+\boldsymbol{A}^T}{2})|$, and $\boldsymbol{P}$ is the solution of (5). And $\{t_{k+1}^i - t_k^i\}_{k\in N^+}, \forall i \in 1, ..., N$ is lower bounded.

The upper bound value $\bar{\theta}_i$ of $\theta_i$ is fixed. This lemma is proved by choosing a Lyapunov function $V(t)$, which is discontinuous at triggering moments when $\theta_i(t)$ is reset to $\bar{\theta}_i$ [20]. This could lead to a steep increase of $V(t)$ at discontinuous points; thus, only the stability between two consecutive events (but not globally) is guaranteed. The following theorem proposes a constraint of $\bar{\theta}_i$ which is not fixed anymore but varies depending on system states.

*Theorem 1:* The MAS (1) will reach leader-following consensus asymptotically using the event-triggered rule defined in Lemma 3 and the following adaptive $\bar{\theta}_i(t_k^i)$ rule:

$$\bar{\theta}_i(t_k^i) = \begin{cases} \frac{F}{\|\boldsymbol{z}_i(t_k^i)\|^2}, & \text{if } \|\boldsymbol{z}_i(t_k^i)\|^2 \geq \frac{F}{\bar{\theta}_i(0)} \\ \bar{\theta}_i(0), & \text{otherwise} \end{cases} \tag{8}$$

where $F$ is defined as $F = \boldsymbol{\varepsilon}_i^T(t_{k-\ell}^i)\boldsymbol{P}\boldsymbol{\varepsilon}_i(t_{k-\ell}^i)e^{-\rho(t_k^i - t_{k-1}^i)} - \boldsymbol{\varepsilon}_i^T(t_k^i)\boldsymbol{P}\boldsymbol{\varepsilon}_i(t_k^i) + \bar{\theta}_i(t_{k-\ell}^i)e^{-\rho(t_k^i - t_{k-1}^i)}\|\boldsymbol{z}_i(t_{k-\ell}^i)\|^2$ , $\bar{\theta}_i(0)$ is a positive scalar, $\ell \in \mathbb{N}^+$ is the moving average steps, and the decay rate $\rho > 0$.

*Proof:* Due to page limits, we provide a brief proof with the same Lyapunov function in Lemma 3 [20]: $V = \boldsymbol{\varepsilon}^T(\boldsymbol{I}_N \otimes \boldsymbol{P})\boldsymbol{\varepsilon} + \boldsymbol{e}^T(\boldsymbol{\Theta} \otimes \boldsymbol{P})\boldsymbol{e} + \boldsymbol{z}^T(\boldsymbol{\Theta} \otimes \boldsymbol{I}_n)\boldsymbol{z}$ and $\boldsymbol{\Theta} = diag(\theta_1, ..., \theta_N) > 0$. Decompose $V$ to N parts: $V = \sum_i^N V_i$ with $V_i = \boldsymbol{\varepsilon}_i^T P \boldsymbol{\varepsilon}_i + \theta_i \boldsymbol{e}_i^T \boldsymbol{P} \boldsymbol{e}_i + \theta_i ||\boldsymbol{z}_i||^2$. Notice that the term $\theta_i ||\boldsymbol{z}_i||^2$ is discontinuous at $t = t_k^i, \forall k \in \mathbb{N}^+$. Substituting (8) in $V_i$ gives

$$V_i(t_k^i) \leq V_i(t_{k-\ell}^i)e^{-\rho(t_k^i - t_{k-1}^i)} \tag{9}$$

which implies that the sequence $\{V_i(t_k^i)\}_{k\in\mathbb{N}}$ converges to 0. Due to Lemma (3), $\dot{V}(t) < 0$ between consecutive triggering moments, and for $\forall k \geq \ell + 1$, $\lim_{t\to(t_k)^+} V(t) < \lim_{t\to(t_{k-\ell})^+} V(t)$, it turns out that $V(t)$ converge to 0. ■

*Remark 2:* For practical engineering, in the initiation stage when $k \leq \ell$, we let $\bar{\theta}_i(t_{k-\ell}^i) = \bar{\theta}_i(0)$, $\boldsymbol{\varepsilon}_i(t_{k-\ell}^i) = \boldsymbol{\varepsilon}_i(0)$ and $\boldsymbol{z}_i(t_{k-\ell}^i) = \boldsymbol{z}_i(0)$. If $F$ is negative, we replace $\frac{F}{\|\boldsymbol{z}_i(t_k^i)\|^2}$ with $\frac{\max(F,\tau')}{\|\boldsymbol{z}_i(t_k^i)\|^2}$ in (8), where $\tau'$ is a tiny positive value, indicating that the controller needs to be triggered immediately at this time.

*Remark 3:* Notice that if $\ell = 1$, (9) becomes a direct comparison between $V_i(t_k^i)$ and $V_i(t_{k-1}^i)$. If $\ell \geq 2$, by adding $\sum_{n=k-\ell+1}^{k-1} V_i(t_n^i)$ on the left and right of (9), and divide both sides by $\ell$, we have $\text{VMA}_i(t_k^i, \ell) < \text{VMA}_i(t_{k-1}^i, \ell)$, where $\text{VMA}_i(t_k^i, \ell)$ is defined as $\text{VMA}_i(t_k^i, \ell) = \left(V_i(t_k^i) + V_i(t_{k-1}^i) + ... + V_i(t_{k-\ell+1}^i)\right)/\ell$, which is the moving average of $\{V_i(t_k^i)\}_{k\in\mathbb{N}}$ with $\ell$ steps. The sequence $\{V_i(t_k^i)\}_{k\in\mathbb{N}}$ may increase locally, but in a global view (represented by $\text{VMA}(t_k^i, \ell)$), it keeps a decreasing trend by a moving horizon of length $\ell$.

This essential strategy provides more tolerance to small perturbations of $V$ and eventually increases the inter-event time as long as possible while possibly sacrificing some performance temporarily. We will show in the section V-A the effectiveness of this strategy. Moreover, compared to the original DETM in [20], the instability problem is well addressed by varying $\bar{\theta}_i$ in an adaptive manner.

*Remark 4:* Since (2) is distributed but (8) is not (because it contains non-local information $\boldsymbol{\varepsilon}_i$, Theorem 1 is only semi-distributed. In the following section IV-B, we propose another DETM which is fully distributed.

### *B. Improved Dynamic Event-triggered Control with Adjustable Inter-event Time*

*Theorem 2:* Suppose assumptions 1, 2 and 3 are satisfied. Consider the control law (2) with $\boldsymbol{K} = \boldsymbol{B}^T\boldsymbol{P}$, the MAS (1) will reach leader-following consensus asymptotically with the event-triggered rule defined as follows:

$$\begin{aligned} t_k^i &\triangleq \inf\left\{t > t_{k-1}^i \mid \theta_i(t) \leq 0\right\}, \theta_i(t_{k-1}^i) = \bar{\theta}_i \\ \dot{\theta}_i(t) &= \begin{cases} \min(\omega_i(t), 0) - \tau_i & \text{if } ||\boldsymbol{e}_i(t)|| \neq 0 \\ -\tau_i & \text{otherwise} \end{cases} \end{aligned} \tag{10}$$

where $\bar{\theta}_i$ is a positive scalar, $\tau_i > 0$, and $\omega_i(t)$ is defined as:

$$\begin{aligned} \omega_i(t) \triangleq &- \frac{(\frac{k}{\delta_{\min}} - \delta_{\min})||\boldsymbol{e}_i||^2 + \boldsymbol{e}_i^T(2\delta_{\max}\boldsymbol{M}_1 + \theta_i\boldsymbol{M}_2)\boldsymbol{e}_i}{\boldsymbol{e}_i^T\boldsymbol{P}\boldsymbol{e}_i} \\ &- \frac{2(1-\theta_i)\,\boldsymbol{e}_i^T\boldsymbol{M}_1\boldsymbol{z}_i}{\boldsymbol{e}_i^T\boldsymbol{P}\boldsymbol{e}_i} - \frac{\delta_{\min}}{\boldsymbol{e}_i^T\boldsymbol{P}\boldsymbol{e}_i}\left(\frac{1}{k} - \frac{1}{\delta_{\max}^2}\right)||\boldsymbol{z}_i||^2 \end{aligned} \tag{11}$$

where $\delta_{\min} = \lambda_{\min}(\boldsymbol{\mathcal{H}})$, $\delta_{\max} = \lambda_{\max}(\boldsymbol{\mathcal{H}})$, $k > \delta_{\max}^2$, $\boldsymbol{M}_1 = \boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{P}$, $\boldsymbol{M}_2 = \boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P}$, and $\boldsymbol{P}$ is the solution of (5).

*Proof:* The proof consists of showing the stability of the error system (4) by Lyapunov theorem. Consider a continuous Lyapunov function $V(t)$:

$$V(t) = \underbrace{\boldsymbol{\varepsilon}^T(t)(\boldsymbol{I}_N \otimes \boldsymbol{P})\boldsymbol{\varepsilon}(t)}_{V_1(t)} + \underbrace{\boldsymbol{e}^T(t)(\boldsymbol{\Theta}(t) \otimes \boldsymbol{P})\boldsymbol{e}(t)}_{V_2(t)} \tag{12}$$

where $\boldsymbol{\Theta}(t) = diag(\theta_1(t), ..., \theta_N(t)) > 0$ and $\dot{\boldsymbol{\Theta}}(t) < 0$ by definition of (10), then $V(t) \geq 0$. Note that $V(t)$ is a continuous function, which is different from the literature [20]. Therefore the condition that $\dot{V}(t) < 0$ between any two consecutive events is sufficient to prove the convergence of $V(t)$ over the whole time horizon.

By calculating the derivative of $V_1$, we have

$$\dot{V}_1 = \boldsymbol{\varepsilon}^T(\boldsymbol{I}_N \otimes \boldsymbol{M}_2 - 2\boldsymbol{\mathcal{H}} \otimes \boldsymbol{M}_1)\boldsymbol{\varepsilon} - 2\boldsymbol{\varepsilon}^T(\boldsymbol{\mathcal{H}} \otimes \boldsymbol{M}_1)\boldsymbol{e} \tag{13}$$

By using (5), $\boldsymbol{I}_N \otimes \boldsymbol{M}_2 - 2\boldsymbol{\mathcal{H}} \otimes \boldsymbol{M}_1 \leq \boldsymbol{I}_N \otimes \boldsymbol{M}_2 - 2\delta_{\min}\boldsymbol{I}_N \otimes \boldsymbol{M}_1 = \boldsymbol{I}_N \otimes (\boldsymbol{M}_2 - 2\delta_{\min}\boldsymbol{M}_1) \leq -\delta_{\min}\boldsymbol{I}_{Nn}$, then $\dot{V}_1 \leq -\delta_{\min}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} - 2\boldsymbol{\varepsilon}^T(\boldsymbol{\mathcal{H}} \otimes \boldsymbol{M}_1)\boldsymbol{e}$. Define $\boldsymbol{z}(t) = [\boldsymbol{z}_1^T(t), ..., \boldsymbol{z}_N^T(t)]^T$. Notice that $\boldsymbol{\varepsilon} = -(\boldsymbol{\mathcal{H}}^{-1} \otimes \boldsymbol{I}_n)\boldsymbol{z} - \boldsymbol{e}$, then we obtain

$$\begin{aligned} \dot{V}_1 \leq &- \delta_{\min}(\boldsymbol{z}^T(\boldsymbol{\mathcal{H}}^{-1} \otimes \boldsymbol{I}_n)^2\boldsymbol{z} + \boldsymbol{e}^T\boldsymbol{e} + 2\boldsymbol{e}^T(\boldsymbol{\mathcal{H}}^{-1} \otimes \boldsymbol{I}_n)\boldsymbol{z}) \\ &+ \boldsymbol{z}^T(\boldsymbol{I}_N \otimes 2\boldsymbol{M}_1)\boldsymbol{e} + \boldsymbol{e}^T(\boldsymbol{\mathcal{H}} \otimes 2\boldsymbol{M}_1)\boldsymbol{e} \end{aligned} \tag{14}$$

We use the following inequalities:

$$-\boldsymbol{z}^T(\boldsymbol{\mathcal{H}}^{-1} \otimes \boldsymbol{I}_n)^2\boldsymbol{z} \leq -\frac{1}{\delta_{\max}^2}\boldsymbol{z}^T\boldsymbol{z} \tag{15}$$

$$\boldsymbol{e}^T(\boldsymbol{\mathcal{H}} \otimes 2\boldsymbol{M}_1)\boldsymbol{e} \leq \boldsymbol{e}^T(\delta_{\max}\boldsymbol{I}_N \otimes 2\boldsymbol{M}_1)\boldsymbol{e} \tag{16}$$

Perform matrix diagonalization of $\boldsymbol{\mathcal{H}}^{-1}$ to $\boldsymbol{\mathcal{H}}^{-1} = \boldsymbol{\Gamma}^T\boldsymbol{\Lambda}\boldsymbol{\Gamma}$ with $\boldsymbol{\Gamma}$ an orthogonal matrix and $\boldsymbol{\Lambda}$ a diagonal matrix. Denote

$\tilde{\boldsymbol{e}}_i = (\boldsymbol{\Gamma}\otimes \boldsymbol{I}_n)\boldsymbol{e}_i$ and $\tilde{\boldsymbol{z}}_i = (\boldsymbol{\Gamma}\otimes \boldsymbol{I}_n)\boldsymbol{z}_i$. Denote $\lambda_i$ the eigenvalue of $\boldsymbol{\mathcal{H}}$. Therefore,

$$\begin{aligned}&-2\boldsymbol{e}^T(\boldsymbol{\mathcal{H}}^{-1}\otimes \boldsymbol{I}_n)\boldsymbol{z} = -2\tilde{\boldsymbol{e}}^T(\boldsymbol{\Lambda}\otimes\boldsymbol{I}_n)\tilde{\boldsymbol{z}} = \sum_i -2\frac{1}{\lambda_i}\tilde{\boldsymbol{e}}_i^T\tilde{\boldsymbol{z}}_i \\ &\le \sum_i(\frac{k}{\lambda_i^2}\tilde{\boldsymbol{e}}_i^T\tilde{\boldsymbol{e}}_i + \frac{1}{k}\tilde{\boldsymbol{z}}_i^T\tilde{\boldsymbol{z}}_i), \quad \text{with } k>0 \\ &\le \frac{k}{\delta_{\min}^2}\tilde{\boldsymbol{e}}^T\tilde{\boldsymbol{e}} + \frac{1}{k}\tilde{\boldsymbol{z}}^T\tilde{\boldsymbol{z}} = \frac{k}{\delta_{\min}^2}\boldsymbol{e}^T\boldsymbol{e}+\frac{1}{k}\boldsymbol{z}^T\boldsymbol{z}\end{aligned} \tag{17}$$

Substitute inequalities (15),(16) and (17) into (14) and we get

$$\begin{aligned}\dot{V}_1 \le& (\frac{\delta_{\min}}{k} - \frac{\delta_{\min}}{\delta_{\max}^2})\boldsymbol{z}^T\boldsymbol{z} + (\frac{k}{\delta_{\min}} - \delta_{\min})\boldsymbol{e}^T\boldsymbol{e} \\ &+ \boldsymbol{z}^T(\boldsymbol{I}_N\otimes 2\boldsymbol{M}_1)\boldsymbol{e} + 2\delta_{\max}\boldsymbol{e}^T(\boldsymbol{I}_N\otimes\boldsymbol{M}_1)\boldsymbol{e}\end{aligned} \tag{18}$$

By calculating the derivative of $V_2$, and using $\boldsymbol{z} = -(\boldsymbol{\mathcal{H}}\otimes \boldsymbol{I}_n)(\boldsymbol{\varepsilon}+\boldsymbol{e})$ and (4), we get

$$\dot{V}_2 = \boldsymbol{e}^T(\boldsymbol{\Theta}\otimes\boldsymbol{M}_2)\boldsymbol{e} - 2\boldsymbol{e}^T(\boldsymbol{\Theta}\otimes\boldsymbol{M}_1)\boldsymbol{z} + \boldsymbol{e}^T(\dot{\boldsymbol{\Theta}}\otimes\boldsymbol{P})\boldsymbol{e} \tag{19}$$

Substitute (18) and (19) into $\dot{V}(t)$ and we have

$$\begin{aligned}\dot{V} =& \dot{V}_1 + \dot{V}_2 \\ \le& (\frac{\delta_{\min}}{k} - \frac{\delta_{\min}}{\delta_{\max}^2})\boldsymbol{z}^T\boldsymbol{z} + (\frac{k}{\delta_{\min}} - \delta_{\min})\boldsymbol{e}^T\boldsymbol{e} \\ &+ \boldsymbol{z}^T(\boldsymbol{I}_N\otimes 2\boldsymbol{M}_1)\boldsymbol{e} + 2\delta_{\max}\boldsymbol{e}^T(\boldsymbol{I}_N\otimes\boldsymbol{M}_1)\boldsymbol{e} \\ &+ \boldsymbol{e}^T(\boldsymbol{\Theta}\otimes\boldsymbol{M}_2)\boldsymbol{e} - 2\boldsymbol{e}^T(\boldsymbol{\Theta}\otimes\boldsymbol{M}_1)\boldsymbol{z} + \boldsymbol{e}^T(\dot{\boldsymbol{\Theta}}\otimes\boldsymbol{P})\boldsymbol{e} \\ =& \sum_i(\frac{\delta_{\min}}{k} - \frac{\delta_{\min}}{\delta_{\max}^2})\boldsymbol{z}_i^T\boldsymbol{z}_i + (\frac{k}{\delta_{\min}} - \delta_{\min})\boldsymbol{e}_i^T\boldsymbol{e}_i \\ &+ \boldsymbol{e}_i^T(2\delta_{\max}\boldsymbol{M}_1 + \theta_i\boldsymbol{M}_2)\boldsymbol{e}_i + \dot{\theta}_i\boldsymbol{e}_i^T\boldsymbol{P}\boldsymbol{e}_i \\ &+ 2(1-\theta_i)\boldsymbol{e}_i^T\boldsymbol{M}_1\boldsymbol{z}_i\end{aligned} \tag{20}$$

Replace $\dot{\theta}_i$ by (10), we have $\dot{V} < 0$, which means $\lim_{t\to\infty}||\boldsymbol{\varepsilon}_i(t)|| = 0$ and the MAS (1) reaches leader-following consensus asymptotically. ∎

*Remark 5:* Notice that this strategy is fully distributed; thus, each agent decides the moments of data transmission according to its local information, which facilitates algorithm implementation. It does not require additional data forwarding equipment.

*Corollary 1:* From Theorem 2, the inter-event time of agent $i$ is lower bounded by

$$t_{\min}^i = \int_{h=0}^{\bar{\theta}_i} \frac{dh}{\max(c_0 + c_1 h + c_2 h^2, 0) + \tau_i} \tag{21}$$

where $c_0 = (2\delta_{\max}\beta + k/\delta_{\min} - \delta_{\min} + 1/b)/\eta$, $c_1 = (\alpha - 2/b)/\eta$, $c_2 = 1/(\eta b)$, $\alpha = ||\boldsymbol{M}_2||$, $\beta = ||\boldsymbol{M}_1||$, $\eta = \lambda_{\min}(\boldsymbol{P})$ and $b = \delta_{\min}(1/\delta_{\max}^2 - 1/k)/\beta^2$.

*Proof:* Using Cauchy–Schwarz inequality, we have the following expression:

$$-2(1-\theta_i)\boldsymbol{e}_i^T\boldsymbol{M}_1\boldsymbol{z}_i \ge -\frac{1}{b}(1-\theta_i)^2||\boldsymbol{e}_i||^2 - b\beta^2||\boldsymbol{z}_i||^2, b>0 \tag{22}$$

Also notice that the following inequalities hold:

$$\begin{aligned}-\boldsymbol{e}_i^T(2\delta_{\max}\boldsymbol{M}_1 + \theta_i\boldsymbol{M}_2)\boldsymbol{e}_i &\ge -(2\delta_{\max}\beta + \alpha\theta_i)||\boldsymbol{e}_i||^2 \\ \boldsymbol{e}_i^T\boldsymbol{P}\boldsymbol{e}_i &\ge \eta||\boldsymbol{e}_i||^2\end{aligned} \tag{23}$$

By Substituting (22) and (23) into (11), and consider that $k > \delta_{\max}^2$, $b = \delta_{\min}(1/\delta_{\max}^2 - 1/k)/\beta^2 > 0$, we have $\omega_i(t) \ge -\frac{2\delta_{\max}\beta + \alpha\theta_i + k/\delta_{\min} - \delta_{\min}}{\eta} - \frac{(\theta_i - 1)^2}{\eta b} = -(c_0 + c_1\theta_i + c_2\theta_i^2)$. then we conclude that the time required for $\theta_i$ descending from $\bar{\theta}_i$ to 0 is lower bounded by $t_{\min}^i = \int_{h=\bar{\theta}_i}^{0}\frac{dh}{\min(-(c_0+c_1h+c_2h^2),0)-\tau_i}$, which is equivalent to (21). ∎

Corollary 1 implies that the Zeno behavior of the proposed DETM is excluded. By adjusting the value of $\bar{\theta}_i$, we can vary the MIET, and keep the communication frequency consistently lower than $1/t_{\min}^i$. Increasing the value of $\bar{\theta}_i$ enables the inter-event time as long as possible not to overload the communication network. Indeed, a smaller $\bar{\theta}_i$ could also be designed to have better surveillance and a higher data transmission rate if the network allows it. Therefore, by designing an appropriate parameter $\bar{\theta}_i$, the control performance and the communication frequency could achieve a good compromise.

## V. Numerical Examples and Discussions

To demonstrate the proposed improved DETM strategies, we perform simulations using the same MAS model in [20]. The matrices $\boldsymbol{A}$, $\boldsymbol{B}$ are defined as $\boldsymbol{A} = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix}0\\1\end{pmatrix}$. Fig. 1 presents the communication topology of agents. The initial condition of the leader and follower agents is $\boldsymbol{x}_0(0) = [1\ 1]^T$ and $\boldsymbol{x}_i(0) = [0.5i + 10\ \ i + 2.5]^T, i \in 1,...,10$. The matrix $\boldsymbol{P}$ is calculated according to (5): $\boldsymbol{P} = \begin{pmatrix}23 & 9.5\\ 9.5 & 8\end{pmatrix}$.

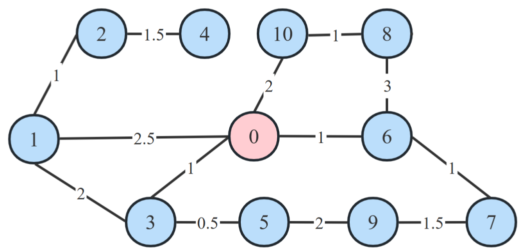


Fig. 1. Communication topology

### A. $\ell$-step Moving Average Adaptive Event-triggered Control

This section presents simulation results using $\ell$-step moving average DETM from Theorem 1. The initial value of $\bar{\theta}_i$ is $\bar{\theta}_i = 1000, i \in 1,...,10$. Notice that this value of $\bar{\theta}_i$ in the literature [20] will drive the error system (4) unstable, though this critical remark was not reported by the authors. The simulation parameters are chosen as $\rho = 0.1$ and $\ell = 3$. The trajectories of consensus error and $\theta_i(t)$ are shown in Fig. 2 and Fig. 3, respectively.

In Fig. 3, events occur at the discontinuous points where $\theta_i$ is reset to $\bar{\theta}_i$, which shows discontinuous and clock-like behavior of $\theta_i$. The value of $\bar{\theta}_i$ changes adaptively according

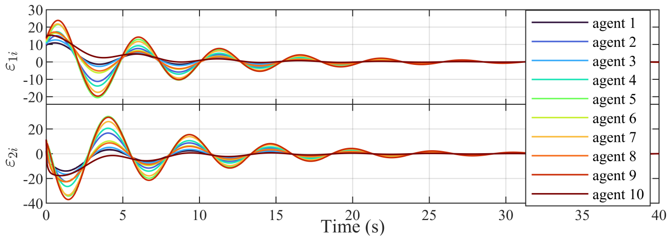


Fig. 2. Consensus error $\varepsilon_{1i}$ and $\varepsilon_{2i}, i \in 1, \ldots, 10$ with decay rate $\rho = 0.1$ and moving average steps $\ell = 3$

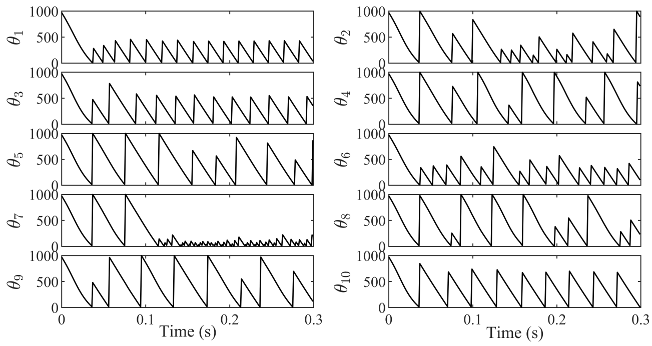


Fig. 3. Time evolution of auxiliary dynamic variables $\theta_i(t)$ with decay rate $\rho = 0.1$ and moving average steps $\ell = 3$

to (8). Fig. 4 plots the evolution of $\bar{\theta}_7$ for agent 7 when event occurs, showing the adaptive change of $\bar{\theta}_7$.

Table I presents how varying moving average steps $\ell$ and $\bar{\theta}_i$ affects the performance. By increasing moving average steps or $\bar{\theta}_i$, the inter-event time could be enlarged significantly, i.e., from 2.232 ms ($\bar{\theta}_i = 100$ and $\ell = 1$) to 30.243 ms ($\bar{\theta}_i = 1000$ and $\ell = 100$), which is higher than the results in [20] and the consensus is also guaranteed.

TABLE I
INTER-EVENT TIME (IN MS) USING DIFFERENT MOVING AVERAGE STEP $\ell$, WITH SAME DECAY RATE $\rho = 0.1$

| Case | $\bar{\theta}_i(0) = 100$ | $\bar{\theta}_i(0) = 500$ | $\bar{\theta}_i(0) = 1000$ |
|---|---|---|---|
| $\ell = 1$ | 2.232 | 4.019 | 4.821 |
| $\ell = 3$ | 2.501 | 4.901 | 6.135 |
| $\ell = 10$ | 2.633 | 5.378 | 7.666 |
| $\ell = 100$ | 2.817 | 12.479 | 30.243 |
| Reference [20] | 4.184 | 20.605 | unstable |

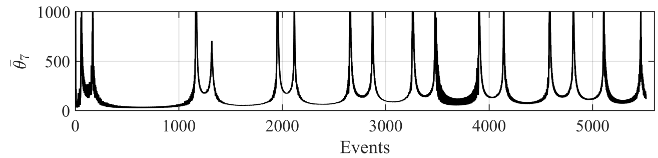


Fig. 4. Evolution of $\bar{\theta}_7$ with triggered events

### B. Improved Dynamic Event-triggered Control with Adjustable Inter-event Time

This section presents simulation results using DETM given in Theorem 2. The $\bar{\theta}_i$ is set to 1 and $k = 100 > \delta_{\max}(\boldsymbol{\mathcal{H}})$. Fig. 5 illustrates the evolution of consensus error and estimation error, respectively. The discontinuity in Fig. 5 (b) represents the event-triggered moment and corresponds to the discontinuity in Fig. 6. The periodic-like evolution of $\theta_i$ and $\boldsymbol{e}_i$ represents that at every triggering moment, the auxiliary variable is reset to its maximum value $\bar{\theta}_i$ and the estimation error returns to 0.

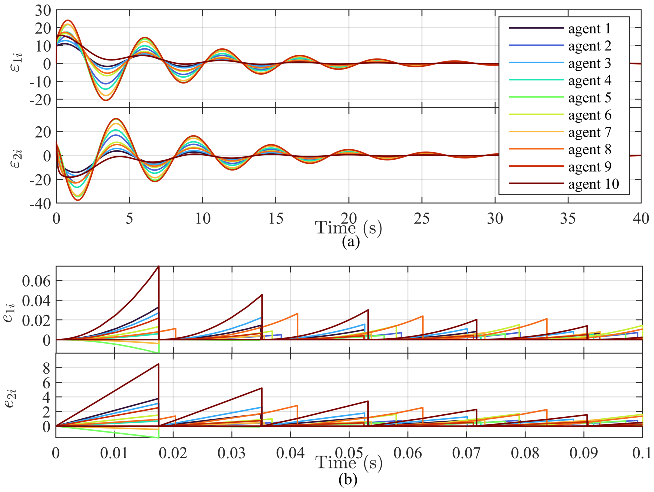


Fig. 5. (a) Consensus error $\varepsilon_{1i}$ and $\varepsilon_{2i}$ and (b) estimation error $e_{1i}$ and $e_{2i}, i \in 1, \ldots, 10$ with $\bar{\theta}_i = 1$

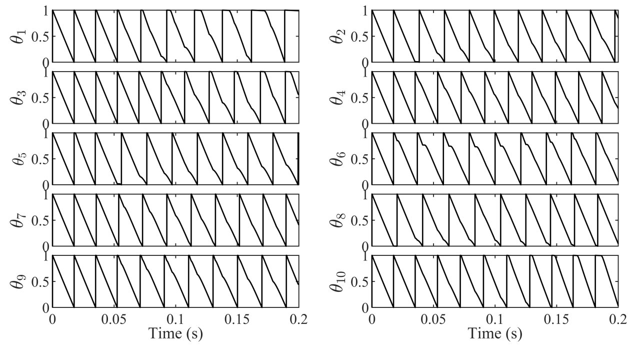


Fig. 6. Time evolution of auxiliary dynamic variables $\theta_i$ with $\bar{\theta}_i = 1$

Table II conclude the inter-event time using different event-triggered strategies: DETM from Theorem 2, DETM from the literature [20] and static event-triggered mechanism (SETM). For SETM, the Lyapunov function is the same as in the proof of Theorem 2 by taking $\boldsymbol{\Theta} = \boldsymbol{I}_N, \dot{\boldsymbol{\Theta}} = \boldsymbol{0}$, which finally leads to a static event-triggered rule. According to this table, the proposed DETM has the longest average inter-event time and always guarantees the asymptotic leader-following consensus, which shows a tremendous improvement compared to the SETM or the DETM in the literature [20].

Fig. 7 plots $t_{\min}$ with different $\bar{\theta}_i$ using (21). The proposed DETM prevents Zeno behavior as long as $\bar{\theta}_i > 0$. By adjusting $\bar{\theta}_i$, we can control the MIET and guarantee that the network communication is consistently below a designated frequency.

TABLE II

INTER-EVENT TIME (IN MS) USING DIFFERENT $\bar{\theta}_i$ UNDER DIFFERENT ETM STRATEGIES: MEAN VALUES (MEAN), MINIMUM VALUES (MIN) AND MAXIMUM VALUES (MAX) AMONG 10 FOLLOWER AGENTS

| Case | DETM using Theorem 2 | | | | |
|---|---|---|---|---|---|
| | $\bar{\theta}_i = 0.01$ | $\bar{\theta}_i = 0.1$ | $\bar{\theta}_i = 1$ | $\bar{\theta}_i = 500$ | $\bar{\theta}_i = 1000$ |
| Mean | **10.022** | **18.242** | **21.035** | **22.124** | **22.126** |
| Min | **10.022** | **16.611** | **18.727** | 19.231 | **19.241** |
| Max | **10.022** | **20.661** | **23.753** | **34.483** | **34.484** |
| Case | DETM in Reference [20] | | | | |
| | $\bar{\theta}_i = 0.01$ | $\bar{\theta}_i = 0.1$ | $\bar{\theta}_i = 1$ | $\bar{\theta}_i = 500$ | $\bar{\theta}_i = 1000$ |
| Mean | 0.0004 | 0.0042 | 0.042 | 20.542 | |
| Min | 0.0004 | 0.0041 | 0.042 | **20.408** | Unstable |
| Max | 0.0004 | 0.0043 | 0.042 | 20.661 | |
| Case | Static Event-triggered Mechanism | | | | |
| Mean | 0.228 | Min | 0.116 | Max | 1.613 |

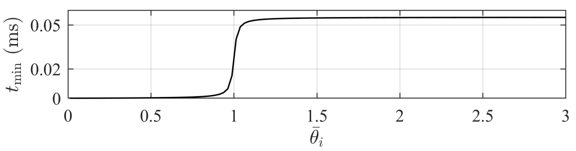


Fig. 7. Minimum inter-event time $t_{\min}$ with different $\bar{\theta}_i$ using (21)

## VI. CONCLUSIONS

This paper investigates the leader-following consensus problem of MASs and proposes two improved DETM strategies that ensure asymptotic consensus and prevent Zeno behavior. By using the moving average method to enforce a global convergence, a successful solution to the instability problem is proposed. To eliminate the drawbacks of the discontinuous Lyapunov function, another fully distributed DETM algorithm with inter-event time designable for all agents is developed. It has been shown that the performance in terms of inter-event time has been significantly improved compared to the reference results and static event-triggered control. An explicit expression of minimum inter-event time is also given to enable more flexible tuning. Further works will aim at observer-based consensus control.

## REFERENCES


[1] J. A. V. Trejo, "Fault tolerant methods design for a fleet of autonomous vehicles against faults/failures based on multi-agent systems," Ph.D. dissertation, Universite de Lorraine, Lorraine, France, Jul 2021.

[2] W. Ni and D. Cheng, "Leader-following consensus of multi-agent systems under fixed and switching topologies," *Systems & Control Letters*, vol. 59, no. 3–4, p. 209–217, Mar 2010.

[3] J. A. Vazquez Trejo, D. Rotondo, M. A. Medina, and D. Theilliol, "Observer-based event-triggered model reference control for multi-agent systems," in *2020 International Conference on Unmanned Aircraft Systems (ICUAS)*. Athens, Greece: IEEE, Sep 2020, p. 421–428.

[4] R. Yang, L. Liu, and G. Feng, "Leader-following output consensus of heterogeneous uncertain linear multiagent systems with dynamic event-triggered strategy," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 3, p. 1626–1637, Mar 2022.

[5] Q. Han, Y.-S. Zhou, Y.-X. Tang, X.-G. Tuo, and P. He, "Event-triggered finite-time sliding mode control for leader-following second-order nonlinear multi-agent systems," *IEEE Open Journal of Intelligent Transportation Systems*, vol. 3, pp. 570–579, 2022.

[6] H. Guo, M. Meng, and G. Feng, "Mean square leader-following consensus of heterogeneous multi-agent systems with markovian switching topologies and communication delays," *International Journal of Robust and Nonlinear Control*, vol. 33, no. 1, pp. 355–371, 2023.

[7] A. Qiu, A. W. Al-Dabbagh, and T. Chen, "A tradeoff approach for optimal event-triggered fault detection," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 3, pp. 2111–2121, 2019.

[8] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, p. 1680–1685, 2007.

[9] W. Qi, C. Zhang, G. Zong, S.-F. Su, and M. Chadli, "Finite-time event-triggered stabilization for discrete-time fuzzy markov jump singularly perturbed systems," *IEEE Transactions on Cybernetics*, pp. 1–10, 2022.

[10] Y. Wu, X. Yang, H. Yan, M. Chadli, and Y. Wang, "Adaptive fuzzy event-triggered sliding mode control for uncertain euler-lagrange systems with performance specifications," *IEEE Transactions on Fuzzy Systems*, pp. 1–13, 2022.

[11] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, p. 1291–1297, May 2012.

[12] W. Hu, L. Liu, and G. Feng, "Consensus of linear multi-agent systems by distributed event-triggered strategy," *IEEE Transactions on Cybernetics*, vol. 46, no. 1, p. 148–157, Jan 2016.

[13] I. Ahmed, M. Rehan, N. Iqbal, and C. K. Ahn, "A novel event-triggered consensus approach for generic linear multi-agents under heterogeneous sector-restricted input nonlinearities," *IEEE Transactions on Network Science and Engineering*, p. 1–10, 2023.

[14] J. Liu, Y. Zhang, Y. Yu, and C. Sun, "Fixed-time leader–follower consensus of networked nonlinear systems via event/self-triggered control," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 11, p. 5029–5037, Nov 2020.

[15] V. T. J. Antonio, G. Adrien, A.-M. Manuel, P. Jean-Christophe, C. Laurent, R. Damiano, and T. Didier, "Event-triggered leader-following formation control for multi-agent systems under communication faults: Application to a fleet of unmanned aerial vehicles," *Journal of Systems Engineering and Electronics*, vol. 32, no. 5, pp. 1014–1022, 2021.

[16] X. Ruan, J. Feng, C. Xu, and J. Wang, "Observer-based dynamic event-triggered strategies for leader-following consensus of multi-agent systems with disturbances," *IEEE Transactions on Network Science and Engineering*, vol. 7, no. 4, p. 3148–3158, Oct 2020.

[17] J. A. V. Trejo, M. Chadli, D. Rotondo, M. A. Medina, and D. Theilliol, "Event-triggered fault-tolerant leader-following control for homogeneous multi-agent systems," *IFAC-PapersOnLine*, vol. 55, no. 6, p. 79–84, 2022.

[18] A. Girard, "Dynamic triggering mechanisms for event-triggered control," *IEEE Transactions on Automatic Control*, vol. 60, no. 7, p. 1992–1997, Jul 2015.

[19] X. Ge, Q.-L. Han, X.-M. Zhang, and D. Ding, "Dynamic event-triggered control and estimation: A survey," *International Journal of Automation and Computing*, vol. 18, no. 6, p. 857–886, Dec 2021.

[20] X. Wu, B. Mao, X. Wu, and J. Lu, "Dynamic event-triggered leader-follower consensus control for multiagent systems," *SIAM Journal on Control and Optimization*, vol. 60, no. 1, p. 189–209, Feb 2022.

[21] Y. Yang, D. Yue, and C. Xu, "Dynamic event-triggered leader-following consensus control of a class of linear multi-agent systems," *Journal of the Franklin Institute*, vol. 355, no. 15, p. 7706–7734, Oct 2018.

[22] J. Berneburg and C. Nowzari, "Robust dynamic event-triggered coordination with a designable minimum interevent time," *IEEE Transactions on Automatic Control*, vol. 66, no. 8, pp. 3417–3428, 2021.

[23] X. Chu, N. Huang, and Z. Sun, "Event triggering control for dynamical systems with designable inter-event times," *IFAC-PapersOnLine*, vol. 53, no. 2, p. 6410–6415, 2020.

[24] E. Garcia, Y. Cao, and D. W. Casbeer, "Decentralized event-triggered consensus with general linear dynamics," *Automatica*, vol. 50, no. 10, p. 2633–2640, Oct 2014.