

Selective-Candidate Framework with Similarity Selection Rule for Evolutionary Optimization

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Abstract

This paper proposes a generic selective-candidate framework with similarity selection rule (SCSS) for performance enhancement of well-established evolutionary optimization algorithms. It is done by using a more efficient selective searching direction. In the SCSS framework, M ($M > 1$) candidates are generated from each current solution by M independent reproduction procedures. The winner is then determined by employing a similarity selection rule that achieves a balance between exploitation and exploration. This computationally light rule simultaneously considers the evolution status (fitness ranking information) of the current solution as well as its Euclidian distances to each of the M candidates. The SCSS framework can be easily applied to evolutionary algorithms or swarm intelligences. Experiments conducted with 60 benchmark functions show the superiority of SCSS in three classic, four state-of-the-art and four up-to-date algorithms.

Keywords: Similarity selection, exploitation and exploration, evolutionary algorithms, swarm intelligence, global optimization.

1. Introduction

Constructed on a population basis, evolutionary optimization explores a searching space by iteratively performing genetic operations (for evolutionary algorithms, EAs [1, 2]) or social learning processes (for swarm intelligences, SIs [3]). In order to maintain diversity in the search, the genetic operations and social learning processes commonly involve some levels of randomness. For example, parents are randomly selected in the mutation process of classic differential evolution (DE) [4]; normal distributions are utilized in

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evolution strategy (ES) [5]; and two randomly distributed numbers are used in the velocity update equation of particle swarm optimization (PSO) [6]. Due to the randomness, different reproduction trials performed on the same current solution may produce different offspring, bringing up different building blocks and leading to different searching performance. This is not just confined to the classic EAs [2, 4, 5] and SIs [6, 7], but to many of their variants (eg. improved EAs [8-19] and SIs [20, 21]), in which the reproduction procedure is usually performed to generate one offspring for each parent at a time. This results in candidate being determined somehow in a random manner. Moreover, advanced variants usually introduced additional operations and parameters to the classical methods, resulting in increasing factors that determine the location of offspring.

To meet this challenge, this paper proposes a generic selective-candidate framework with similarity selection rule (SCSS), which simultaneously considers the factors that affect the generation of candidates. SCSS firstly generates M ($M > 1$) candidates for each current solution by M independent reproduction procedures. Afterwards, these M candidates compete, with one of them becoming the final candidate for each current solution based on a selective rule. The big challenge here is that it should be effective and efficient. On the one hand, it should not involve objective function evaluation, which may cost additional function evaluations. While on the other hand, it is required to provide a potential excellent candidate for the next generation. To resolve these issues, a similarity selection (SS) rule is proposed which preliminarily measures the quality of a candidate before the function evaluation, by considering its Euclidian distance to the current solution as well as the fitness ranking information of the current solution. As shown later, SS is able to secure a balance between the exploitation and exploration capabilities. The main contributions of this work are summarized as follows.

(1) Different algorithms may be suitable for solving different optimization problems [22-24]. This paper provides a generic method for significant performance enhancement of some existing algorithms such as those in [10-17, 20].

(2) The similarity selection rule proposed in SCSS, in which superior solutions prefer closest candidates while inferior solutions favor farthest candidates strikes a balance between exploitation and exploration and consequently demonstrates remarkable performance enhancement of several start-of-the-art and top algorithms available in the literature.

The rest of this paper is organized as follows. Section 2 reviews the related works on the research of balanced exploitation and exploration capabilities. Section 3 describes the proposed framework. Experimental results and discussions are presented in section 4, while section 5 concludes this paper.

2. Exploitation and Exploration

Exploitation and exploration are two cornerstones of evolutionary optimization [25]. In 2013, Črepinšek et al. [26] conducted a comprehensive survey on the research works that focused on balancing the exploitation and exploration capabilities (EEC) of EAs. According to [26], the amount of exploitation and exploration in EAs is not only affected by the genetic operations, i.e. mutation, crossover, and selection, but also by the algorithmic parameter setting and the representation of individuals. In the following, recent related works that were not included in [26] are reviewed and summarized.

2.1 EEC controlled by genetic operations/social learning

Generally, genetic operations/social learning determines the evolution direction. For DE and ES, the genetic operations are mutation and crossover/recombination. While for PSO, the social learning procedures consist of the velocity and position update equations. With respect to the research on DE, in [27], Epitropakis et al. proposed a proximity-based (Pro) mutation operator, in which the probability of a parent involved in mutation is inversely proportional to its distance from the mutated individual. In [28], Gong et al. proposed a ranking-based (Rank) mutation operator, in which better parents are associated with higher selection probabilities for mutation. In [29], Zheng et al. proposed a collective information based (CIM) mutation operator, which combines the useful information provided by multiple promising solutions. In [30], Wang et al. developed a multiobjective sorting-based (MS) mutation operator for DE, which considers the fitness as well as the diversity of the parents selected for mutation. In [31], Zheng and Zhang introduced a jumping genes crossover operator to promote the population diversity for multi-objective optimization. Besides these newly designed genetic operations, EEC can also be controlled by the ensemble of multiple mutation strategies. In [32], Wu et al. proposed a multi-population based ensemble of mutation strategies approach to diversify the search. In [33, 34], mutation strategies are configured according to individuals' differences in fitness values. In [35], Gong et al. took advantage of multiple operators to generate multiple offspring that are used for selecting solutions by using a cheap surrogate model. Considering ES, covariance matrix adaptation ES (CMA-ES) [11] improves the convergence of ES by utilizing the covariance matrix information. Particle swarm CMA-ES (PS-CMA-ES) [36] combines the global exploration advantage of PSO and the local search ability of CMA-ES to compromise exploration and exploitation. In [15, 37], CMA-ES was combined with DE to take advantage of both methods. Regarding PSO, distance-based locally informed PSO (LIPS) [20] utilizes the local best, instead of the global best experience to update the velocities of particles and consequently exhibits promising performance in multimodal optimization. Orthogonal learning PSO (OLPSO) [21] employed an orthogonal learning strategy to construct a useful exemplar for exploitation. Heterogeneous comprehensive learning PSO (HCLPSO) [38] divides the entire swarm into two subpopulations including an exploration one and an exploitation one, with an exploration-enhanced and an

exploitation-enhanced velocity update strategy used in the corresponding subpopulation, respectively.

2.2 EEC controlled by parameter tuning

Parameters control the evolution scale. The common parameter in evolutionary optimization is the population size. In [14], Tanabe and Fukunaga enhanced the performance of success-history based adaptive DE (SHADE) [13] by adopting a linear population size reduction (LPSR) scheme. LPSR maintains a large population to promote diversity at the earlier evolution stages but a small population for exploitation at the latter stages. In [39], restart CMA-ES with increasing population size (IPOP-CMA-ES) increases the population size of CMA-ES and restarts the search when any one of the preset stopping criteria is met, which allows a more global search of CMA-ES and enhances the exploration ability. In [40], bi-population CMA-ES (BIPOP-CMA-ES) divides the population into two interlaced multi-start regimes with the same function evaluation budget. One regime has an increasing population size while population size of the other one does not change rapidly.

Apart from population size, there are also extra parameters introduced in a specific algorithm that may need fine-tuning. In [41], Draa et al. proposed to tune the mutation and crossover factors of DE according to sinusoidal formulas. In [13], Tanabe and Fukunaga improved the performance of JADE [12] by introducing a success-history based parameter adaptation (SHA) scheme, which can generate more diverse control parameters than that of JADE. In [16], sinusoidal tuning method [41] was combined with the parameter adaption method of SHADE [13]. In [17], the greediness parameter p of the “current-to-pbest/1” [12] mutation strategy was adjusted at different evolution stages to maintain different amounts of EEC.

2.3 EEC controlled by the combination of genetic operations/social learning and parameter tuning

There are also some works [42, 43] aimed at simultaneously controlling genetic operations/social learning and parameters. In [42], Mallipeddi et al. proposed to improve DE with an ensemble of parameters and mutation strategies. In [43], Wang et al. proposed to use three different mutation strategies combined with three different pairs of control parameters to generate solutions for selecting the fittest. These methods strike a balance between exploitation and exploration using two steps. The first step maintains a mutation strategy pool with diverse searching characteristics while the second step emphasizes exploitation by fitness-based reward [42] or greedy pre-selection [43].

Besides the above three categories, EEC can also be controlled by other mechanisms, such as restart [44, 45], population midpoint monitoring [46] and external archive [12].

The work presented in this paper falls into category 2.3. The similarity selection rule proposed in the SCSS framework focuses on the candidates, which reveals the combined effects of all the operations (i.e. evolution direction) and parameters (i.e. evolution scale). Meanwhile, it also takes the evolution status of each current

solution into consideration.

3 Proposed Framework

3.1 Motivations

Generally, the procedures¹ for EAs/SIs can be summarized as **Algorithm 1**. It is common that one candidate is generated from a current solution by applying a reproduction procedure. However, due to the random nature available in the operations and parameters, it is not guaranteed that the candidates will be located within promising search areas. It is obvious that, when the reproduction procedure is repeated for once more, the two candidates from the same current solution are likely to be different. Thus, a question naturally rises: which one is more beneficial for the search? To alleviate the possible adverse effect from randomness and improve the performance of these algorithms, we propose a generic selective-candidate framework with similarity selection rule (SCSS), in which M candidates ($M > 1$) are generated for each of the current solutions by M independent reproduction procedures. Afterwards, only one of them is selected by a proposed selective rule, to become the final competitor against each current solution.

Algorithm 1. General Procedures of EAs and SIs

- 1: Initialize population $X = \{X_1, X_2, \dots, X_{NP}\}$;
 - 2: **While** the stopping criteria are not met **Do**
 - 3: Determine the control parameters CP for genetic operations/social learning;
 - 4: Produce a new population Y via genetic operations/social learning on X ;
 - 5: Evaluate the fitness of Y ;
 - 6: Select solutions as new X from $X \cup Y$ to enter next iteration.
 - 7: **End While**
-

3.2 SCSS Framework

The pseudo-code of the proposed SCSS framework is presented in **Algorithm 2**, which consists of two components, i.e. multiple candidates generation and similarity selection (SS) rule.

3.2.1 Multiple Candidates Generation

As seen from Algorithm 2, SCSS framework performs M independent reproductions with M independent parameters (i.e. evolution scale) (line 5) and M independent operations (i.e. evolution direction) (line 7). Thus, for each current solution X_i , it owns a candidate pool $Y_i^m \{m = 1, 2, \dots, M\}$. Afterwards, one solution Y_i

¹ For brevity, a review of three typical algorithms, DE, ES and PSO is presented in the supplementary file.

from the corresponding M candidates will be selected for each X_i by SS rule (lines 14 and 18). Besides, the actual parameters used for each selected solution are determined (lines 15 and 19).

Algorithm 2. SCSS Framework

```

1: Initialize population  $X = \{X_1, X_2, \dots, X_{NP}\};$ 
2: While the stopping criteria are not met Do
3: Determine the fitness ranking  $rank(i)$  of each individual  $i \{i = 1, 2, \dots, NP\};$  // fitness ranking for similarity selection rule

    ----- Multiple Candidates Generation -----
4: For  $m = 1: M$ 
5:   Determine the control parameters  $CP^m$  for genetic operations/social learning, following the original design of the baseline;
6:   For  $i = 1: NP$  Do
7:     Produce new solution  $Y_i^m$  via genetic operations/social learning on  $X_i$ ;
8:     Calculate  $dist_i^m = \text{Euclidian distance}(Y_i^m, X_i);$  // similarity calculation for SS rule
9:   End For
10: End For

    ----- Similarity Selection Rule -----
11: For  $i = 1: NP$  Do
12:   If  $rank(i) \leq \text{ceil}(NP \times GD)$  //  $GD$  is a greedy degree parameter, which controls the trade-off of EEC
13:      $index = \arg \min_{m \in \{1, 2, \dots, M\}} (dist_i^m);$ 
14:      $Y_i = Y_i^{index};$ 
15:      $CP_i = CP_i^{index};$ 
16:   Else
17:      $index = \arg \max_{m \in \{1, 2, \dots, M\}} (dist_i^m);$ 
18:      $Y_i = Y_i^{index};$ 
19:      $CP_i = CP_i^{index};$ 
20:   End If
21: End For

22: Evaluate the fitness of  $Y$ ;
23: Select solutions as new  $X$  from  $X \cup Y$  to enter next iteration.
24: End While

```

3.2.2 Similarity Selection Rule

Apparently, the core issue of the SCSS framework is to determine the final competitor from M candidates. Due to different parameter settings combined with different genetic or different social learning procedures, the M candidates can be very different. Therefore, the way of selecting the competitors will directly affect the performance. An effective rule could bring performance enhancement while an inappropriate one may even deteriorate the performance. Moreover, the rule should be efficient, meaning that the computational load should be light.

In this paper, we propose a similarity selection (SS) rule, which simultaneously considers the fitness ranking information $rank(i)$ of current solution X_i and its Euclidian distance $dist_i^m$ to each of the M candidates Y_i^m , as shown in **Algorithm 2** (lines 11-21), where

$$dist_i^m = \sqrt{\sum_{j=1}^D (Y_{i,j}^m - X_{i,j})^2},$$

and D is the number of decision variables.

By adjusting SS, the amount of exploitation and exploration can be directly controlled. For instance, favoring candidates closest to the current solutions are exploitative while preferring the ones farthest to the current solutions could encourage exploration.

However, the appropriate choice of an SS rule for a specific algorithm depends on the EEC of the given algorithm. It is assumed that EEC could be represented by a searching radius (SRAD). A larger SRAD would result in a more explorative characteristic, and vice versa. The effects of SRAD on the performance of an algorithm are illustrated in Fig. 1. In Fig. 1 (a), optimizer 1 is very explorative with a large SRAD. Therefore, the search is very random and there are little risks suffered from local optima. However, this large SRAD would make the individuals such as 1 and 2 hard to refine. In Fig. 1(b), optimizer 2 is very exploitative with a small SRAD. In this case, individuals 1 and 2 focus more on local searches. It is difficult for them to move from basin I to basin II, which is important for diversity enhancement. Different from optimizers 1 and 2, in Fig. 1 (c), optimizer 3 has a balanced EEC with an appropriate SRAD. However, one drawback with this optimizer is that the SRAD is the same for the superior individual 1 and the inferior individual 2. For the superior individual 1, this SRAD may not be small enough for an efficient local search while for the inferior individual 2, the SRAD may not be large enough for it to jump from basin I to basin II. In Fig. 1(d), optimizer 4 is an improved version of optimizer 3 based on SCSS with $M = 2$. The possible candidates generated by SCSS could be close to the current solutions with a small SRAD (solid line circles in Fig.1(d)), such as A1, B1, C1 and D1, or away from with a large SRAD (dotted line circles in Fig. 1(d)), such as A2, B2, C2, and D2. Indeed, the SRAD size of optimizer 3 ranges between those of the dotted line circles and the solid line circles

of optimizer 4. Therefore, compared with the SRAD of optimizer 3, the solid line circles of optimizer 4 provide a smaller radius for local search while the dotted line circles could be large enough for basin-jumping.

Regarding different cases: 1) for an explorative optimizer with an SRAD that is too large (Fig.1(a)), the SRAD should be reduced to concentrate the search; 2) for an exploitative optimizer with an SRAD that is too small (Fig.1(b)), the SRAD should be enlarged to encourage exploration to new searching areas; and 3) for a well-performing optimizer with balanced EEC, different search tasks should be assigned to solutions with different potentials. On the one hand, since new best solutions are likely to be located in the area near the top solutions in the context of continuous landscape, the superior solutions are assigned with the exploitation task. They are then compared to the closest candidates to make them steadily exploit promising areas. While on the other hand, to prevent the population from rapid diversity loss, the inferior solutions are assigned with the farthest candidates.

In view of the above, two SS schemes are proposed as follows:

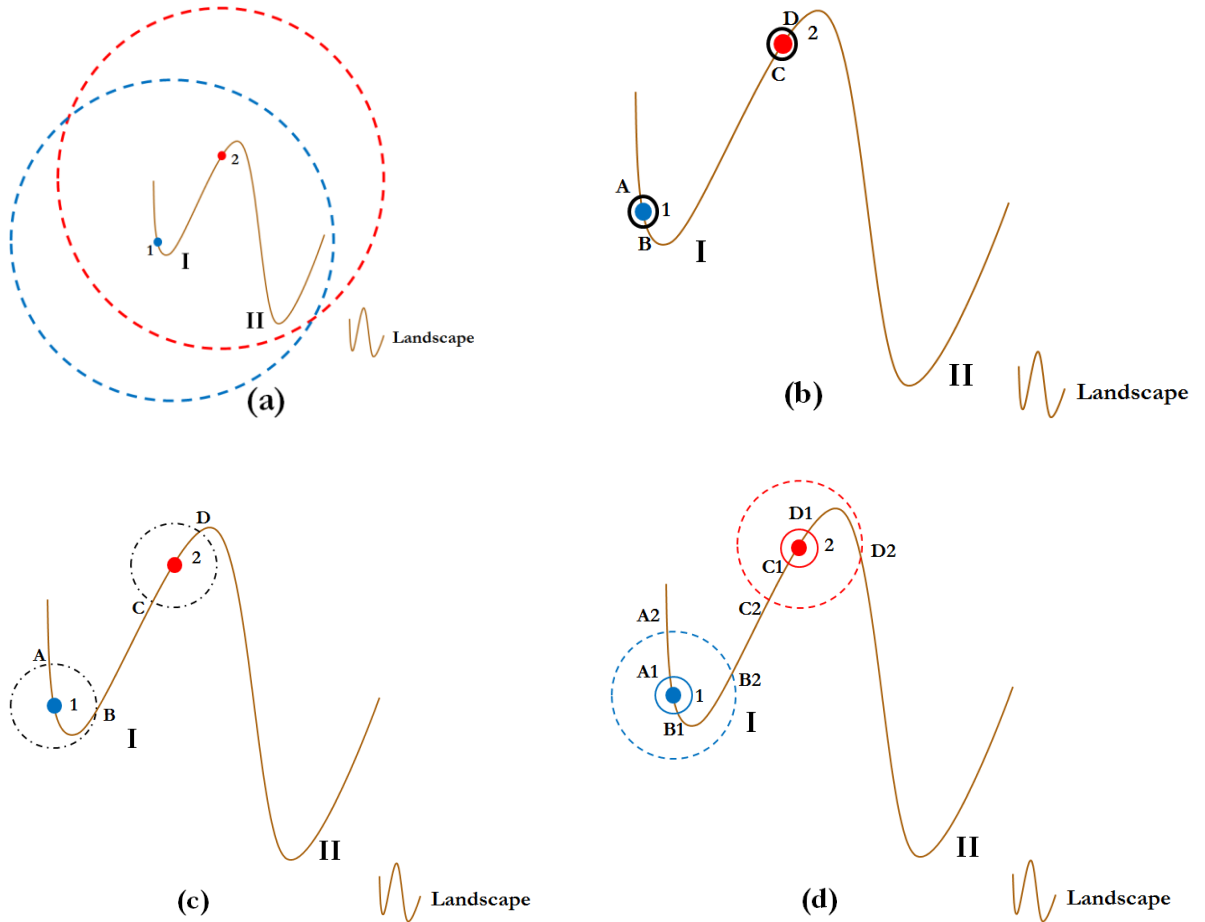


Fig.1 Illustration of the effects of SRAD on the performance of an algorithm.

Scheme 1: If $rank(i) \leq \text{ceil}(NP \times GD)$

Select the closest candidate from $Y_i^m \{m = 1, 2, \dots, M\}$ for individual X_i ;

Else

Select the farthest candidate from $Y_i^m \{m = 1, 2, \dots, M\}$ for individual X_i ;

End If

Scheme 2: If $rand_i(0,1) > rank(i)/NP$

Select the closest candidate from $Y_i^m \{m = 1, 2, \dots, M\}$ for individual X_i ;

Else

Select the farthest candidate from $Y_i^m \{m = 1, 2, \dots, M\}$ for individual X_i ;

End If

where $rank(i) \in \{1, 2, \dots, NP\}$ is the fitness ranking of individual X_i and $rank(i)=1$ is the best. $rand_i(0,1)$ is a uniformly distributed random number within (0,1) for individual $X_i \{i = 1, 2, \dots, NP\}$.

In Scheme 1, the proportion of top individuals preferring the closest candidates is controlled by a greedy degree parameter GD in the range [0,1]. Specifically, the superior $GD \times 100\%$ selects the nearest candidates while the inferior $(1 - GD) \times 100\%$ portion selects the farthest candidates. The larger the GD value is, the more exploitative Scheme 1 becomes.

In Scheme 2, higher ranked individuals are associated with higher probabilities in using the closest candidates, while lower ranked ones are likely to utilize the farthest candidates. Scheme 2 is proposed for the reason that it is parameterless. As shown later in Section 4, it works well for most of the advanced EA and SI variants.

Based on Algorithm 2, the SCSS variants for the original EAs and SIs can be easily implemented. The work flow of three SCSS variants, i.e. SCSS-DE, SCSS-ES and SCSS-PSO for the classic DE, ES, and PSO is given in Algorithms S1, S2 and S3 in the supplementary file respectively.

3.2.3 Time Complexity

This subsection discusses the time complexity of the proposed method. Considering DE as an example, its time complexity is $O(NP \cdot D \cdot G_{\max})$, where G_{\max} is the maximum number of generations. In SCSS-DE, the complexity of fitness ranking and Euclidian distance calculation for each generation is $O(NP \cdot \log_2 NP)$ and $O(M \cdot NP \cdot D)$, respectively. Besides, the complexity of M reproductions is $O(M \cdot NP \cdot D)$. Since $\log_2 NP \ll D$, the overall complexity is $O(M \cdot NP \cdot D \cdot G_{\max})$. As investigated in Section 4, $M = 2 \ll NP$ is sufficient for advanced DEs, such as the JADE [12] and L-SHADE [14] algorithms. Thus, the complexity of advanced SCSS-DEs remains $O(NP \cdot D \cdot G_{\max})$.

4 Simulation

In this section, the effectiveness of the proposed SCSS framework and its working mechanism are investigated through comprehensive experiments conducted using the CEC2014 [47] and CEC2017 [48] benchmark function sets. Each function set consists of 30 functions with diverse mathematic characteristics, such as unimodal, multimodal, hybrid and composition. Since the CEC function suits are with bounded constraints, to make the comparison fair, the constraint handling technique adopted in the SCSS variants is kept the same as the corresponding baselines. The solution error value, defined as $f(x) - f(x^*)$, is used to measure the performance of the compared algorithms, where $f(x)$ is the smallest fitness obtained after $10^4 \times D$ function evaluations and $f(x^*)$ is the fitness of the global optimal x^* . Following [47, 48], solution error values smaller than 10^{-8} are considered as zero. For each test function, 51 independent runs are performed, while the mean and standard deviations of the solution error values are reported. Besides, in order to draw statistically sounded conclusions, Wilcoxon signed-rank test [49] with 5% significance level is applied to compare the performance. The symbols “−”, “=” and “+” represent that the baseline algorithms perform significantly worse than, similar to or better than the corresponding SCSS variants, respectively. The significant ones are highlighted in **bold**.

4.1 Performance Enhancement of Classic EAs and SIs

In this subsection, the proposed SCSS framework is first integrated with three classic EAs and SIs, i.e. DE and ES from EA family and PSO from SIs. Performance of the resulting variants, SCSS-DE, SCSS-ES and SCSS-PSO are compared with the baseline algorithm, respectively.

Parameters settings for the compared algorithms are summarized as follows:

DE and SCSS-DE: $NP = 100$, $F = 0.7$, $CR = 0.5$;

ES and SCSS-ES: $\mu = 25$, $\lambda = 100$, intermediate recombination is used;

PSO and SCSS-PSO: $NP = 20$, $w = 0.9$, $c_1 = 2.0$, and $c_2 = 2.0$;

In addition, regarding the SS rule, Scheme 1 with $GD = 1$ and $M = 2$ are adopted in the three SCSS variants. These settings are based on the experimental findings given later in Section. 4.3. The comparison results on 30- D and 50- D CEC2014 functions are summarized in Fig.2.

From Fig. 2, the effectiveness of the proposed SCSS framework on all the considered algorithms can be observed. In the total 180 cases, SCSS variants win in 125 ($=21+26+15+22+27+14$) cases and only lose in 1 case. Specifically, in the 30- D cases, SCSS-DE and SCSS-ES perform significantly better than their corresponding baseline on 21 and 26 functions and lose on one and no function, respectively. SCSS-PSO wins PSO on 15 functions and ties on 15 functions. In the 50- D case, SCSS-DE, SCSS-ES, and SCSS-PSO win the baselines on 22, 27 and 14 functions and the rests are tie. It should be remarked that, since the classic algorithms use fixed parameter settings, these performance improvements are attributed to the control of the

randomness of the reproduction operations by SCSS, such as the random selection of parents for mutation and dimension-wise crossover in DE. In summary, the proposed SCSS framework significantly enhances the performance of these basic algorithms.

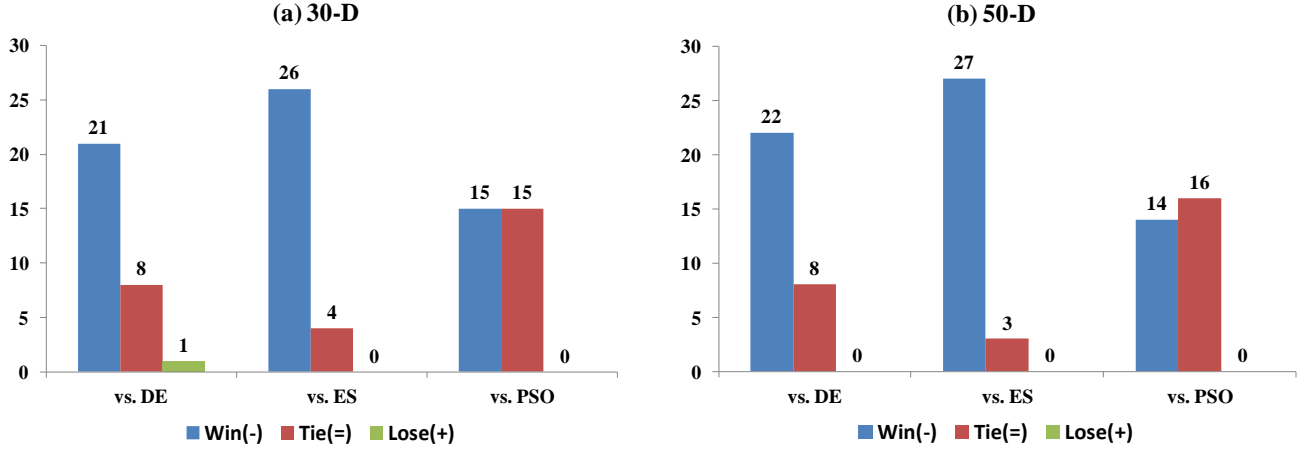


Fig.2 Comparison results of three SCSS-based classic algorithms with the baselines on CEC2014 test functions: (a) 30-D, (b) 50-D.

4.2 Performance Enhancement of Advanced EAs and SIs

Thanks to the efforts by EA and SI researchers, the performance of the classic algorithms had been greatly improved by many advanced variants. Thus, it is more important that our proposed method could also further enhance these algorithms. For demonstration, SCSS is incorporated into four advanced baselines, namely JADE [12], SHADE [13], CMA-ES [11] and LIPS [20]. Parameter settings for the compared algorithms are set the same as those recommended in their original literature. Additionally, for the SCSSs, in SCSS-JADE, SCSS-SHADE, and SCSS-LIPS, Scheme 2 is utilized as the SS rule while in SCSS-CMA-ES, Scheme 1 with $GD = 0$ is adopted. The reproduction times M is set to 2 for SCSS-JADE and SCSS-SHADE, 4 for SCSS-LIPS and 5 for SCSS-CMA-ES. These settings are based on the parameter sensitivity analyses given in Section. 4.3.

The experimental results on 30-D and 50-D CEC2014 functions are shown in Table S1 and Table S2 respectively, in the supplementary file and further summarized in Fig. 3.

As observed from Fig. 3, SCSS also exhibits remarkable improvements on the advanced algorithms. Out of the total 240 cases, SCSS wins in 134 ($=14+14+17+23+16+11+13+26$) cases and loses in 17 ($=1+0+5+2+1+0+5+3$) cases. More specifically, for the advanced DEs, i.e. JADE and SHADE, SCSS improves their performance on 55 functions and is inferior on 2 functions. For CMA-ES, SCSS wins in 17 and 13 cases and loses in 5 cases on the 30-D and 50-D functions, respectively. For the advanced PSO

algorithm, i.e. LIPS, SCSS-LIPS is superior on more than 20 functions and inferior on far fewer functions in both 30- D and 50- D cases.

Considering the diverse mathematic properties of the test functions, it can be concluded that SCSS consistently work well on various types of functions, including unimodal, multimodal, hybrid and composition.

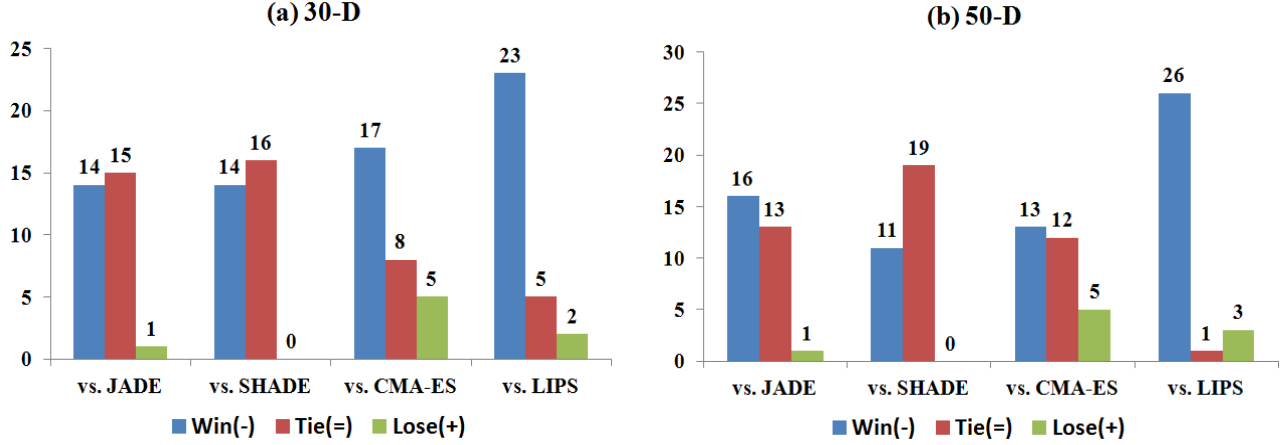


Fig.3 Comparison results of four SCSS-based advanced algorithms with the baselines on CEC2014 test functions: (a) 30-D, (b) 50-D.

4.3 Working Mechanism of SS Rule

4.3.1 Influence of SS rule on the performance of SCSS

The performance sensitivity of SCSS to the SS rule is firstly investigated. Performance of the seven SCSSs, i.e. SCSS-DE, SCSS-ES, SCSS-PSO, SCSS-JADE, SCSS-SHADE, SCSS-CMA-ES and SCSS-LIPS with different SS rule (i.e. Scheme 1 with six GD values, i.e. 0, 0.2, 0.4, 0.6, 0.8, 1 and Scheme 2) are compared with those of the baseline algorithms, respectively. The M value for all the SCSS variants in this experiment is set to 2. Table S3 in the supplementary file and Fig. 4 present the comparison results “-/+” and P-N values (defined as the number of “-” minus the number of “+”) respectively.

From Fig.4, the followings can be observed:

1) For the classic algorithms, including DE, ES, and PSO, SCSS variants adopting larger GD values perform better than those with smaller ones. The reason lies in that the classic algorithms are explorative and lack in exploitation (the case in Fig.1(a)). Large GD values could encourage exploitation to remedy the blindness of the search. While small GD values, such as $GD=0$, make the algorithms even more explorative and deteriorate the performance, as can be observed from Fig.4.

2) For the advanced algorithms, Scheme 2 is the best choice for SCSS-SHADE and SCSS-LIPS and the third best choice for SCSS-JADE. For SCSS-JADE and SCSS-SHADE, the performance of SCSSs

significantly degenerates when GD is too large ($GD = 1$) or too small ($GD = 0$). It is because JADE and SHADE themselves maintain relatively balanced EEC (the case in Fig.1 (c)). $GD = 1$ would over-emphasize exploitation and make the algorithms too greedy while a over-explorative setting $GD = 0$ may deteriorate the performance on test functions which need more exploitation.

3) For SCSS-CMA-ES, Scheme 1 with $GD = 0$ achieves the best performance, which indicates that the original CMA-ES (the case in Fig. 1(b)) needs more exploration in order to improve performance. This observation is in consistent with the statements in some CMA-ES literature, such as PS-CMA-ES [36] and IPOP-CMA-ES [39] that CMA-ES could benefit from enhanced exploration capability when solving the difficult CEC benchmarks.

In conclusion, the choice of a best SS rule depends on the EEC of the baselines even though Scheme 2 consistently performs significantly better than or similar to the baselines. Nevertheless, for an optimizer with relatively balanced EEC, Scheme 2 is recommended.

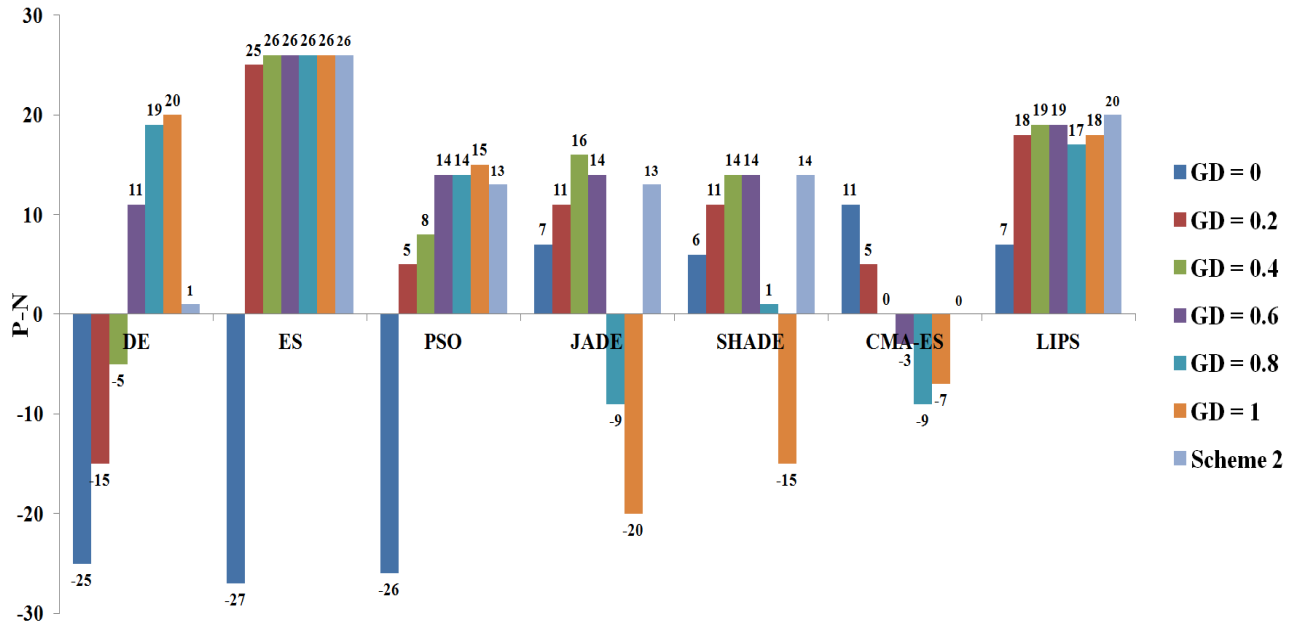


Fig.4 P-N values of SCSS variants with different SS rules against the baselines on 30-D CEC2014 test functions. (P-N value = the number of functions that SCSS variant outperforms the baseline — the number of functions that SCSS variant loses to the baseline).

4.3.2 Behavior of SS rule

In the proposed SCSS framework, the selection of the closest or farthest candidates is conducted based on the fitness ranking of the current solutions. In this way, SCSS adjusts the level of exploration/exploitation according to their potentials. In the experiment conducted on SCSS-DE ($GD = 1$) and SCSS-SHADE (Scheme 2), SS rule is compared with the randomly selecting (RS) manner (i.e. selecting manner in the

baseline algorithm). The total distance TD between the selected candidates and the current solutions against the rank on 30- D functions F5 and F13 in the median run is shown in Fig. 5.

From Fig. 5, we have the following observations: 1) on the explorative DE, SS enhances the exploitation on all the ranks, resulting in smaller TD values than that of RS; 2) on SHADE with relative balanced EEC, for ranks smaller than $NP/2 = 50$, SS yields smaller TD compared to RS, resulting in more exploitation. While for ranks larger than 50, it is the opposite case; 3) on SHADE, for RS, TD does not vary much with the rank while for SS, TD significantly increases with the rank. Since the searching radius $SRAD$ can be roughly calculated as $TD/\text{Max_Gen}$, where Max_Gen is the maximum number of generations and it is the same for SHADE and SCSS-SHADE, $SRAD \propto TD$. This means that $SRAD$ increases with the rank in SS while it is the same in RS. In other words, SS is a finer strategy; 4) the smaller TD values of SHADE compared to that of DE reveal that SHADE is more exploitative than DE. Therefore, unlike the case in SCSS-DE, enlarge GD in SCSS-SHADE may make the algorithm over-exploitative and deteriorate the performance, as can be observed from Fig.4.

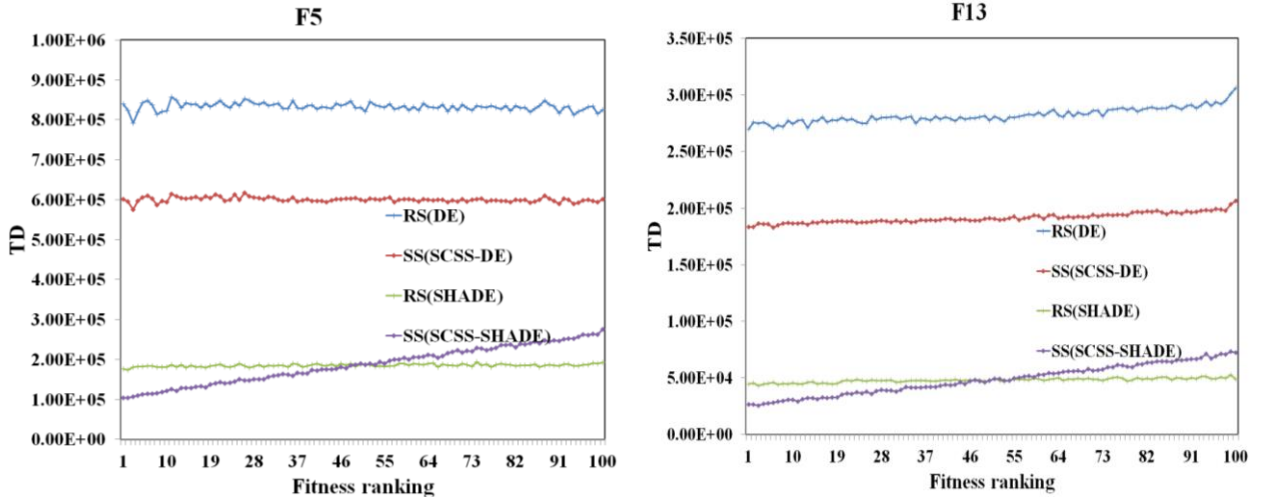


Fig.5 TD against the rank on 30- D CEC2014 functions F5 and F13. (The similar phenomena can be observed on all the CEC functions)

To further show the superiority of SS (Scheme 2), it is compared with the opposite version, as follows:

If $\text{rand}_i(0,1) > \text{rank}(i)/NP$

Select the farthest candidate from $Y_i^m \{m = 1, 2, \dots, M\}$ for individual X_i ;

Else

Select the closest candidate from $Y_i^m \{m = 1, 2, \dots, M\}$ for individual X_i ;

End If

Except this difference, other settings are unaltered. As examples, experiments were conducted on SCSS-JADE and SCSS-SHADE. As seen from Table 1 and Table S4, the standard SCSS algorithms exhibit significantly better performance than the variants with the opposite SS rule. In addition, comparing Table S1

with Table S4, it is also observed that these variants perform significantly worse than the baselines, which indicates that the opposite version is an inappropriate selective rule. This confirms the illustrations given in Section 3.2.2 and Fig. 1.

Table 1 Comparison results of SCSS-JADE and SCSS-SHADE with the opposite SS rule on 30-D CEC2014 test functions

| -/=/+ | |
|--------------------------------|--------|
| SCSS-JADE_oppo vs. SCSS-JADE | 24/5/1 |
| SCSS-SHADE_oppo vs. SCSS-SHADE | 21/8/1 |

4.3.3 Combined effects of operations and parameters by SS rule

SS rule considers the candidates that reveal the combined effects of operations and parameters, which makes SCSS a general framework that can be easily applied to various types of EAs and SIs. The effects of SCSS on the randomness of operations and parameters of the previously considered algorithms are summarized as follows.

- 1) For the three classic algorithms DE, ES and PSO, since the parameters are fixed during the entire evolution process, SCSS only focuses on the operations;
- 2) For the advanced DEs, i.e. JADE and SHADE, except the operations, since different reproduction procedure m may use different F and CR , SCSS reveals their combined effects;
- 3) In the advanced ES, i.e. CMA-ES, new individuals are generated from the center of best solutions by following a normal distribution. Thus, in SCSS-CMA-ES, different normal distributions are sampled in different reproduction procedures;
- 4) In the advanced PSO, i.e. LIPS, SCSS uses different independently generated φ_j in the position update equation, which is a uniformly distributed random number ranged in $[0, 4.1/neighborhood\ size]$ for each dimension j [20].

4.4 Performance Sensitivity to M

In SCSS, M ($M > 1$) reproduction procedures should be performed. Indeed, if M is set to 1, SCSS variants degenerate to baselines. Apparently, the performance of the SCSS is influenced by M . The SCSS variants with five different M values, i.e. $M = 2, 3, 4, 5$ and 10 are investigated. Except M , other parameter settings for the compared algorithms are set the same as those used previously in Sections 4.1 and 4.2. Performance comparisons of the SCSS variants with the baselines on 30-D CEC2014 functions are summarized in Table S5 and Fig.6. In addition, to show the dynamic performance variation with increasing M , the performance of the SCSS variants using adjacent M settings are also compared with each other, as shown in Table S6 and Fig.7.

From Fig.6, it can be observed that all of the M settings significantly improve the performance of the baselines except SCSS-JADE and SCSS-SHADE with $M = 10$.

In Fig.7, for clarity, the algorithms are divided into two categories. Category 1 includes the SCSS variants which may perform significantly better with $M > 2$ than with $M = 2$, while category 2 lists the SCSS variants which perform similarly or even worse with increased M values.

In category 1, it is observed that the performance of DE and ES consistently improves when M increases. In this paper, we only investigate cases up to $M=10$ because these classic algorithms are significantly inferior to the advanced algorithms. Moreover, increasing M will increase the computational complexity of the algorithm. For CMA-ES and LIPS, SCSS variants with $M = 5$ and $M = 4$ show the best performance, respectively. It is noticed that in the SCSS-CMA-ES, GD is set to 0, thus, larger M values would make the algorithm more explorative.

In category 2, enlarging M does not bring significant performance improvements. On the contrary, it may even significantly degrade the performance, eg. $M > 4$ for JADE and SHADE, or $M > 2$ for PSO. The reason is that different from those in category 1 (eg. DE, ES and LIPS), JADE, SHADE and PSO include elite individuals in their reproduction processes. Specifically, the top individuals used in the “current-to- $pbest/1$ ” mutation strategy of JADE and SHADE and the global best $gbest$ used in the velocity update equation of PSO. Thus, the algorithm with too large an M value is potentially too greedy, making the algorithms stuck in local optima.

Overall, it can be seen that the appropriate M value is relatively small for the advanced variants.

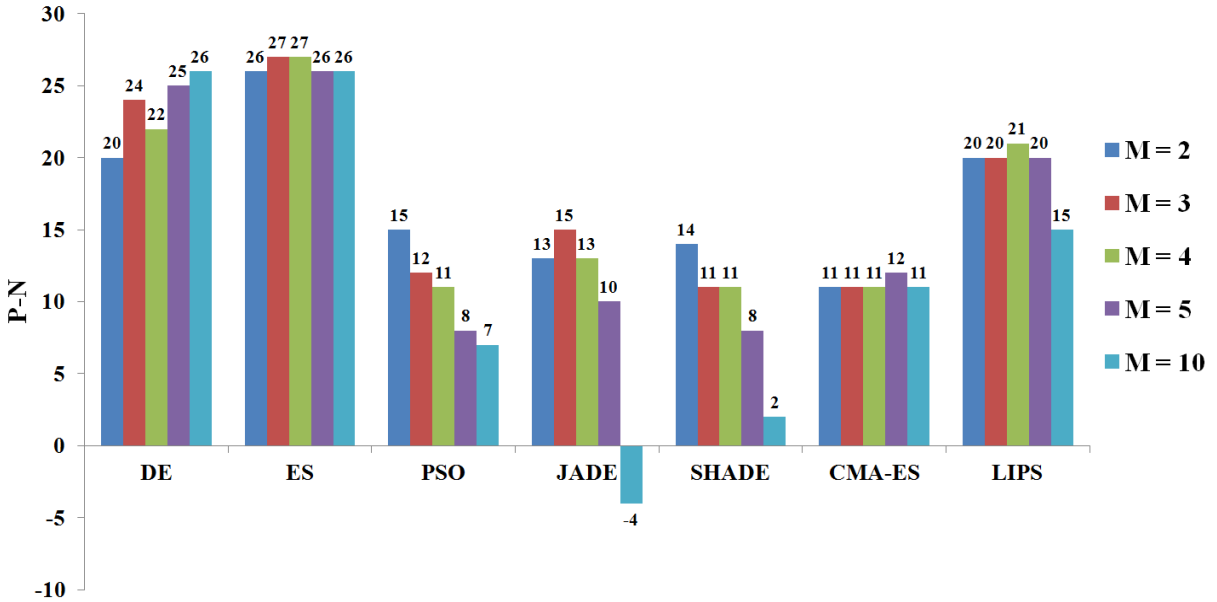


Fig.6 P-N values of SCSS variants with different M settings against the baselines. (P-N value = the number of functions that SCSS variant outperforms the baseline — the number of functions that SCSS variant loses to the baseline).

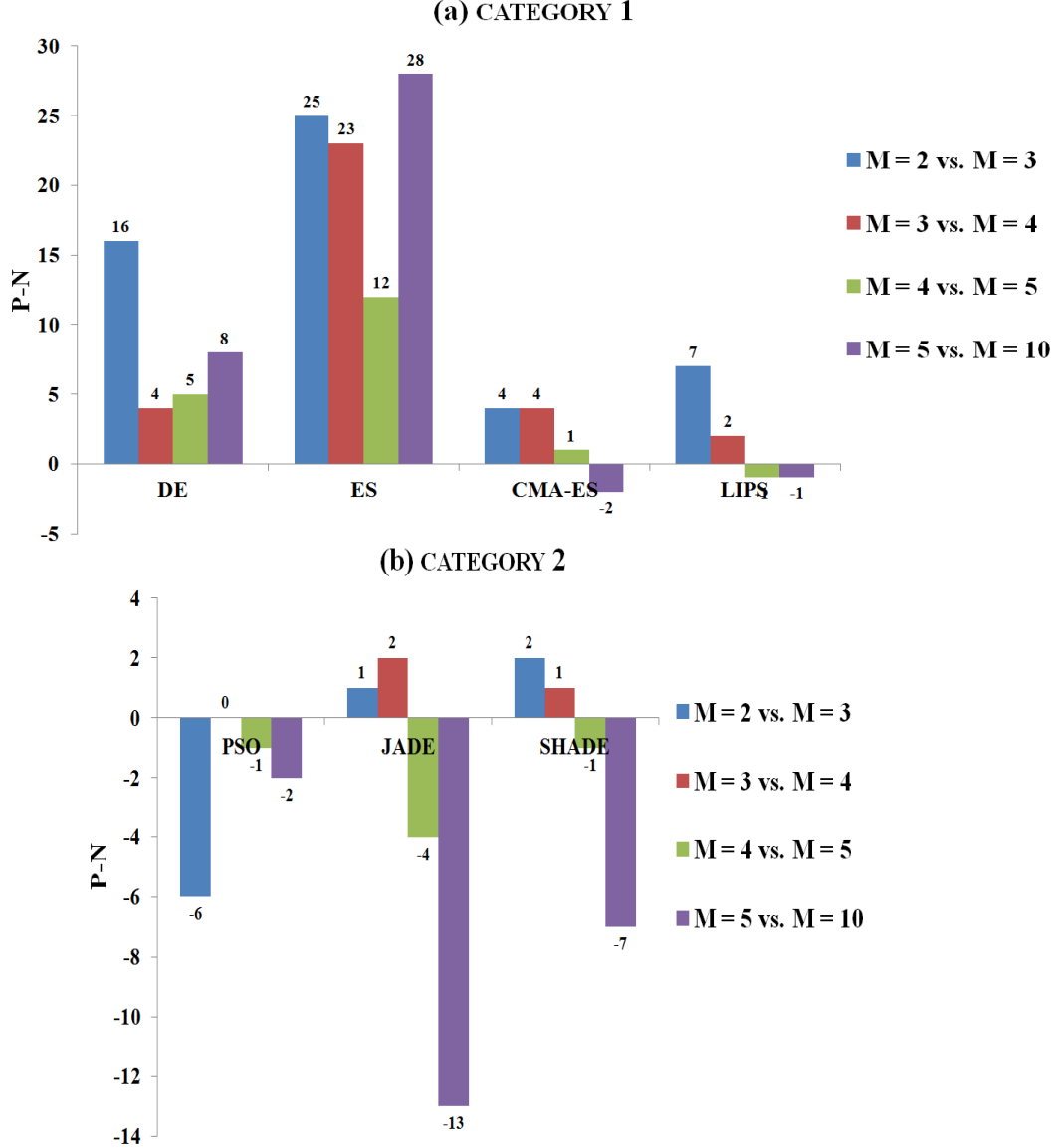


Fig.7 P-N values between SCSS variants (A vs. B) with adjacent M settings (P-N value = the number of functions that B outperforms A — the number of functions that B loses to A).

4.5 Application in Top Methods from CEC Competitions

From Sections 4.3 and 4.4, it can be concluded that advanced SCSS-DEs with Scheme 2 and $M = 2$, SCSS-CMA-ES with Scheme 1 ($GD = 0$) and $M = 5$ exhibit promising performance. In this subsection, to demonstrate the flexibility, SCSS is further applied with these settings to four highly competitive algorithms from the CEC competitions. Among them, L-SHADE [14] is the winner of the CEC2014 competition, UMOEA-II [15] and L-SHADE_EpSin [16] are the joint-winner of the CEC2016 competition and jSO [17] is the one of the best-performing algorithms in the CEC2017 competition. Their source codes are available at <http://www.ntu.edu.sg/home/epnsugan/>. Parameter settings for the top algorithms are set the same as the

original literature.

From Table S7, Table S8 and Fig.8, it can be concluded that SCSS also enhances the performance of these top methods. Out of the total 240 cases, SCSSs wins in 88 ($=10+9+8+7+18+10+13+13$) cases and loses in 12 ($=2+1+0+2+2+3+0+2$) cases. Specifically, in the 30-*D* case, SCSS-L-SHADE, SCSS-UMOEa-II, SCSS-L-SHADE_EpSin, and SCSS-jSO perform significantly better than the corresponding baseline in 10, 9, 8 and 7 cases and underperform in 2, 1, 0 and 2 cases, respectively. In the 50-*D* case, the performance improvements are more significant. SCSS-L-SHADE, SCSS-UMOEa-II, SCSS-L-SHADE_EpSin and SCSS-jSO exhibit superior performance on 18, 10, 13 and 13 functions respectively and are inferior on far fewer functions.

Fig.9 shows the convergence plot of SCSS-L-SHADE versus L-SHADE on six selected 50-*D* CEC2014 functions. As observed, SCSS-L-SHADE exhibits better convergence performance than L-SHADE. In conclusion, these performance enhancements indicate that the proposed SCSS framework is a better alternative for these top algorithms.

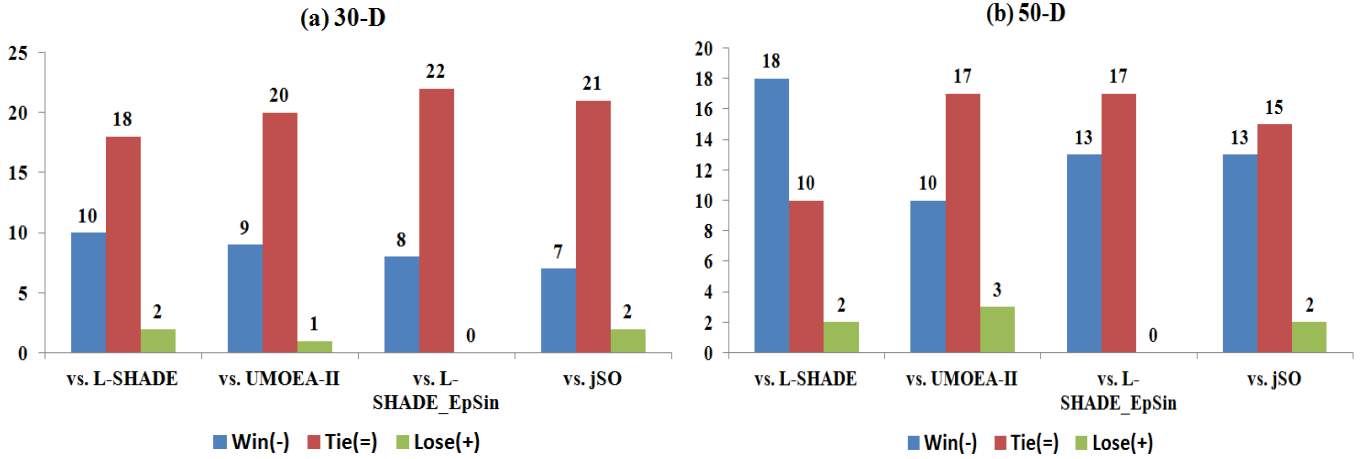


Fig.8 Comparison results of four SCSS-based top algorithms with the baselines on CEC2014 test functions: (a) 30-D, (b) 50-D.

4.6 Performance on CEC2017 Test Suit

To assess the performance of SCSS on a wider variety of functions, in this subsection, we further test the advanced SCSS variants on the newly developed CEC2017 test suit [48]. Parameter settings for the algorithms are the same as those used in Sections 4.2 and 4.5. Tables S9-S12 present the experimental results on 30-*D* and 50-*D* functions and Table 2 summarizes the comparison results.

From Table 2, it is clear that SCSS also significantly improves the performance of the baselines on the CEC2017 functions. In the total 480 cases, SCSS wins in 225 cases, ties in 240 cases and loses in 15 cases.

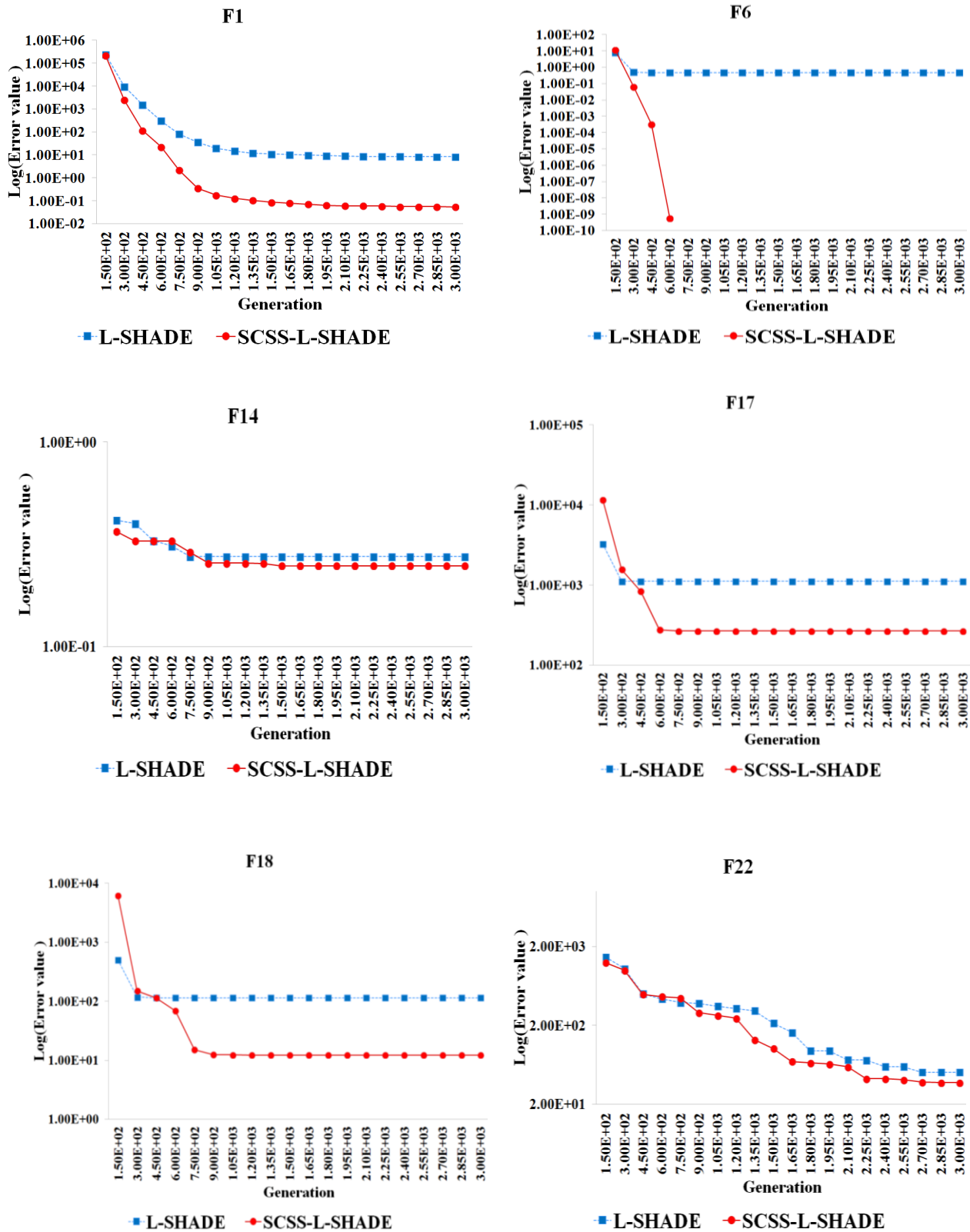


Fig.9 Convergence plot of SCSS-L-SHADE versus L-SHADE on six selected 50-D CEC2014 functions in the median run. (Note: On F6, SCSS-L-SHADE reaches the global optimal at generation 750)

Table 2 Comparison results of SCSS variants with the baselines on CEC2017 test suit

| -/=/+ | 30-D | 50-D |
|--------------------------------------|------------|---------|
| JADE vs. SCSS-JADE | 19/11/0 | 18/10/2 |
| SHADE vs. SCSS-SHADE | 7/23/0 | 11/19/0 |
| CMA-ES vs. SCSS-CMA-ES | 18/11/1 | 16/14/0 |
| LIPS vs. SCSS-LIPS | 28/1/1 | 28/1/1 |
| L-SHADE vs. SCSS- L-SHADE | 9/18/3 | 15/15/0 |
| UMOEa-II vs. SCSS-UMOEa-II | 3/24/3 | 14/14/2 |
| L-SHADE_EpSin vs. SCSS-L-SHADE_EpSin | 7/21/2 | 13/17/0 |
| jSO vs. SCSS-jSO | 7/23/0 | 12/18/0 |
| Total | 225/240/15 | |

4.7 Scalability Study

To study scalability, the SCSS framework is also tested on 100-D CEC2017 functions. The four top methods are selected for the experiment and the parameters are set the same as those used previously.

As shown in Table S13 and Fig.10, SCSS still yields remarkable performance improvements on the higher dimensional functions, which are much more difficult than the lower ones. In the total of 120 cases, SCSS outperforms in 70 (=20+14+16+20) cases and underperforms in 6 (=2+2+0+2) cases. These improvements should be attributed to the balanced exploitation and exploration maintained by the SS rule.

Furthermore, the overall performance of the considered algorithms are compared according to multiple problem Wilcoxon's test [50] and Friedman's test [50]. Based on multiple problems Wilcoxon's test, Table 3 shows that the SCSS variants perform significantly better than the corresponding baselines at $\alpha = 0.05$. With respect to the Friedman's test, Table 4 indicates that SCSS-jSO is the best-performing algorithm, which achieves the smallest ranking value of 2.76, followed by SCSS-L-SHADE_EpSin.

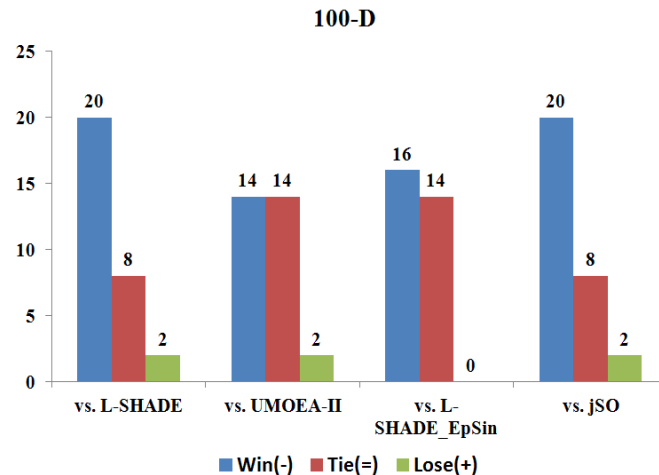


Fig.10 Comparison results of four SCSS-based top algorithms with the baselines on 100-D CEC2017 test functions.

Table 3 Comparison results of the top SCSS variants with the baselines on 30-D, 50-D and 100-D CEC2017 benchmark set according to multi-problem Wilcoxon's test

| | $R+$ | $R-$ | p -value | $\alpha = 0.05$ |
|---|--------|--------|----------------|-----------------|
| SCSS-L-SHADE vs. L-SHADE | 3235.0 | 770.0 | 0.0E+00 | Yes |
| SCSS-UMOEa-II vs. UMOEA-II | 3052.5 | 952.5 | 1.7E-05 | Yes |
| SCSS-L-SHADE_EpSin vs. L-SHADE_EpSin | 3077.0 | 1018.0 | 3.4E-05 | Yes |
| SCSS-jSO vs. jSO | 3710.5 | 384.5 | 0.0E+00 | Yes |

Table 4 Overall performance ranking of the considered algorithms on 30-D, 50-D and 100-D CEC2017 benchmark set by Friedman's test

| Algorithm | Ranking |
|--------------------|-------------|
| SCSS-jSO | 2.76 |
| SCSS-L-SHADE_EpSin | 3.60 |
| jSO | 3.88 |
| L-SHADE_EpSin | 4.33 |
| SCSS-L-SHADE | 4.57 |
| SCSS-UMOEa-II | 5.19 |
| L-SHADE | 5.67 |
| UMOEa-II | 5.96 |

5 Conclusion

To address the potential adverse effect of randomness in evolutionary algorithms, a selective-candidate framework with similarity selection rule (SCSS) is proposed in this paper. In SCSS, each current solution owns a pool of M candidates generated by M reproduction procedures. The final candidate is then determined from the pool by a selective rule. In order to provide an effective and efficient selective rule, a similarity selection method based on fitness rank and Euclidian distance measure was designed. It was subsequently shown that a good balance between exploration and exploitation capabilities could be obtained. This was confirmed through comprehensive experiments conducted on the CEC2014 and CEC2017 test suits, which demonstrated the success of the design on several classic, advanced and top algorithms from EA and SI families.

In the future, we plan to investigate the performance of SCSS on other metaheuristics. The supplementary document and MATLAB demo codes of SCSS can be downloaded from <https://zsxhomepage.github.io/>.

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Supplemental file of “Selective-Candidate Framework with Similarity Selection Rule for Evolutionary Optimization”

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Review of Evolutionary Algorithms and Swarm Intelligences

We briefly review and present the flow of three popular EAs and SIs including DE, ES and PSO and then give the general procedures.

1. DE

Differential evolution (DE) as proposed by Storn and Price [1] is a simple yet powerful EA. At each generation G , three genetic operations, namely mutation, crossover, and selection are included.

Initialization: Given a D -dimensional minimization problem, DE starts with a population $\mathbf{P}_0 = \{\mathbf{X}_{1,0}, \mathbf{X}_{2,0}, \dots, \mathbf{X}_{NP,0}\}$ of NP individuals which is uniformly sampled from the entire searching space.

Mutation: Mutation in DE is performed by combining a basic vector with one or more difference vectors to generate a mutant vector $\mathbf{V}_{i,G}$ $\{i = 1, 2, \dots, NP\}$. The classic “rand/1” mutation strategy is formulated as follows.

$$\mathbf{V}_{i,G} = \mathbf{X}_{r1,G} + F \times (\mathbf{X}_{r2,G} - \mathbf{X}_{r3,G}) \quad (1)$$

where r_1, r_2 and r_3 are three distinct integers within $[1, NP]$ and are different from the index i , while F is a mutation factor between 0 and 1.

Crossover: After mutation, crossover is performed between the mutant vector $\mathbf{V}_{i,G}$ and the current vector $\mathbf{X}_{i,G}$ to generate a trial vector $\mathbf{U}_{i,G}$ as follows.

$$u_{i,j,G} = \begin{cases} v_{i,j,G} & \text{if } \text{rand}_j(0,1) \leq CR \text{ or } j = j_{rand} \\ x_{i,j,G} & \text{otherwise} \end{cases} \quad (2)$$

where $\text{rand}_j(0,1)$ is a uniform random number in $(0, 1)$, j_{rand} is a randomly generated integer from $[1, D]$, and CR is a crossover factor within $[0,1]$.

Selection: Selection compares the fitness of $\mathbf{U}_{i,G}$ with that of the corresponding $\mathbf{X}_{i,G}$ and selects the better one to enter into the next generation.

$$\mathbf{X}_{i,G+1} = \begin{cases} \mathbf{U}_{i,G} & \text{if } f(\mathbf{U}_{i,G}) \leq f(\mathbf{X}_{i,G}) \\ \mathbf{X}_{i,G} & \text{otherwise} \end{cases} \quad (3)$$

2. ES

Evolution strategy (ES) first appeared in 1964 at the Technical University of Berlin (TUB), and was used to solve hydrodynamic problems [2]. Different versions of ES have been proposed since this first version. Generally, ES can be categorized according to the number of parents and offspring involved in each generation. (1+1)-ES includes only one parent, which generates one offspring for each generation by means of Gaussian mutation. $(\mu + 1)$ -ES uses μ ($\mu > 1$) parents to generate one offspring per generation. $(\mu + \lambda)$ -ES utilizes μ parents to generate λ ($\lambda > \mu$) offspring and then chooses μ individuals from the $(\mu + \lambda)$ individuals to enter next generation, while (μ, λ) -ES chooses μ individuals only from the λ offspring.

Initialization: Given a D -dimensional minimization problem, ES starts with an initial population $\mathbf{P}_0 = \{\mathbf{X}_{1,0}, \mathbf{X}_{2,0}, \dots, \mathbf{X}_{\mu,0}\}$ of μ individuals. Each individual $\mathbf{X}_{i,0} = [x_{i,1,0}, x_{i,2,0}, \dots, x_{i,D,0}, \sigma_{i,1,0}, \sigma_{i,2,0}, \dots, \sigma_{i,D,0}]$, ($i = 1, 2, \dots, \mu$) has D variables and D independent standard deviations. The initial standard deviation $\sigma_{i,0}$ is calculated as

$$\sigma_{i,0} = \frac{\Delta X_i}{\sqrt{D}} \quad (4)$$

where ΔX_i is the Euclidian distance between $\mathbf{X}_{i,0}$ and the fittest individual in the initial population.

Recombination: At each generation G , recombination is performed on two randomly selected individuals to produce a new individual $\mathbf{X}\mathbf{R}_{i,G}$ $\{i = 1, 2, \dots, \lambda\}$. Different recombination strategies are specified as follows:

$$\mathbf{X}\mathbf{R}_{i,j,G} = \begin{cases} x_{p,j,G}, \text{ without recombination} \\ x_{p,j,G} \text{ or } x_{q,j,G}, \text{ discrete recombination} \\ x_{p,j,G} + \mathcal{X} \cdot (x_{q,j,G} - x_{p,j,G}), \text{ intermediate recombination} \end{cases} \quad (5)$$

where p and q are the two distinct integers uniformly selected from the set $\{1, 2, \dots, \mu\}$, $j = 1, 2, \dots, D$ is the dimension to be recombined and χ is a constant value usually set to 0.5 [3].

Mutation: Following recombination, mutation is performed to generate λ mutant individuals $\mathbf{XM}_{i,G}\{i = 1, 2, \dots, \lambda\}$ as described by the following:

$$\sigma_{i,j,G} = \sigma_{i,j,G} \cdot \exp(\tau' \cdot N(0,1) + \tau \cdot N_i(0,1)) \quad (6)$$

$$xm_{i,j,G} = xr_{i,j,G} + N(0, \sigma_{i,j,G}) \quad (7)$$

where $j = 1, 2, \dots, D$, $N(0,1)$ and $N_i(0,1)$ are two normal distributions, τ' and τ are constants usually set as unity.

Selection: Select μ fittest individuals from the set of $\mu + \lambda$ individuals $((\mu + \lambda)$ -ES), or from the set of λ offspring produced by mutation $((\mu, \lambda)$ -ES).

3. PSO

Particle swarm optimization (PSO) as proposed by Kennedy and Eberhart [4] imitates the swarm behavior of animals, such as birds flocking and fish schooling. Given a D -dimensional minimization problem, PSO explores the searching space by utilizing a swarm of NP particles with each particle associated with a velocity vector $\mathbf{V}_i = [v_{i1}, v_{i2}, \dots, v_{iD}]$ and a position vector $\mathbf{X}_i = [x_{i1}, x_{i2}, \dots, x_{iD}]$, $i = 1, 2, \dots, NP$. During the searching process, each individual historical best position vector is recorded in $\mathbf{pbest}_i = [p_{i1}, p_{i2}, \dots, p_{iD}]$ and the global best position vector is stored in $\mathbf{gbest} = [g_1, g_2, \dots, g_D]$. Based on \mathbf{pbest}_i and \mathbf{gbest} , particles update their velocity and position at each iteration by using Eq. (8) and (9) respectively:

$$v_{ij} = w \times v_{ij} + c_1 \times r_{1j} \times (\mathbf{pbest}_{ij} - x_{ij}) + c_2 \times r_{2j} \times (\mathbf{gbest}_j - x_{ij}) \quad (8)$$

$$x_{ij} = x_{ij} + v_{ij} \quad (9)$$

where w is the inertia weight, c_1 and c_2 are the acceleration constants, which are commonly set to 2.0. r_{1j} and r_{2j} are two uniformly distributed random numbers within (0, 1) for each dimension j . The updated velocity $|v_{ij}|$ on each dimension is bounded by a maximum value V_{MAXj} . If $|v_{ij}|$ exceeds V_{MAXj} , then it is set as $\text{sign}(v_{ij}) V_{MAXj}$.

4. General Procedures

From above, the general procedures for EAs and SIs is summarized as **Algorithm 1**.

Algorithm 1. General Procedures of EAs and SIs

- 1: Initialize population $\mathbf{X} = \{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_{NP}\}$;
 - 2: **While** the stopping criteria are not met **Do**
 - 3: Determine the control parameters CP for genetic operations or social learning;
 - 4: Produce a new population \mathbf{Y} via genetic operations or social learning on \mathbf{X} ;
 - 5: Evaluate the fitness of \mathbf{Y} ;
 - 6: Select solutions as new \mathbf{X} from $\mathbf{X} \cup \mathbf{Y}$ to enter next iteration.
 - 7: **End While**
-

- [1] R. Storn and K. Price, Differential evolution—A simple and efficient adaptive scheme for global optimization over continuous spaces, Berkeley, CA, Tech. Rep., 1995, tech. Rep. TR-95-012.
- [2] T. Bäck and H.-P. Schwefel, An overview of evolutionary algorithms for parameter optimization, *Evol. Comput.*, 1 (1993) 1–23.
- [3] T. Bäck, *Evolutionary Algorithms in Theory and Practice*. London, U.K.: Oxford Univ. Press, 1996.
- [4] J. Kennedy and R. C. Eberhart, Particle swarm optimization, in *Proc. IEEE Int. Conf. Neural Netw.*, 4 (1995) 1942–1948.

SCSS variants:

The arrows “ \Leftarrow ” highlight the differences between the SCSS variants and the baseline algorithms.

Algorithm S1. SCSS-DE

```

1: Set the population size  $NP$ , initialize the population  $P_0 = \{X_{1,0}, X_{2,0}, \dots, X_{NP,0}\}$ , set  $F$  and  $CR$ , set the generation counter  $G = 0$ ;
2: Set  $GD$ ;  $\Leftarrow$ 
3: While the stopping criteria are not met Do
4: Determine the fitness ranking  $rank(i)$  of each individual  $i$   $\{i = 1, 2, \dots, NP\}$ ;  $\Leftarrow$ 
5: For  $m = 1: M$   $\Leftarrow$ 
6:   For  $i = 1: NP$  Do
-----Mutation-----
7:   Generate a mutant vector  $V_{i,G}^m$  using Eq. (1);
-----Crossover-----
8:   Generate a trial vector  $U_{i,G}^m$  using Eq. (2);
9:    $dist_i^m$  = Euclidian distance ( $U_{i,G}^m, X_{i,G}$ );  $\Leftarrow$ 
10:  End For  $\Leftarrow$ 
11: End For  $\Leftarrow$ 
12: For  $i = 1: NP$  Do
13:   If  $rank(i) \leq \text{ceil}(NP \times GD)$   $\Leftarrow$ 
14:      $index = \arg \min_{m \in \{1, 2, \dots, M\}} (dist_i^m)$ ;  $\Leftarrow$ 
15:      $U_{i,G} = U_{i,G}^{index}$ ;  $\Leftarrow$ 
16:   Else  $\Leftarrow$ 
17:      $index = \arg \max_{m \in \{1, 2, \dots, M\}} (dist_i^m)$ ;  $\Leftarrow$ 
18:      $U_{i,G} = U_{i,G}^{index}$ ;  $\Leftarrow$ 
19:   End If  $\Leftarrow$ 
20: End For
21: Evaluate the fitness of  $U_{i,G} \{i = 1, 2, \dots, NP\}$ ;
-----Selection-----
22: For  $i = 1: NP$  Do
23:   If  $f(U_{i,G}) \leq f(X_{i,G})$ 
24:      $X_{i,G+1} = U_{i,G}$ ;
25:   Else
26:      $X_{i,G+1} = X_{i,G}$ ;
27:   End If
28: End For
29:  $G = G + 1$ ;
30: End While

```

Algorithm S2. SCSS-ES

```

1: Set the population size  $\mu$ , initialize the population  $P_0 = \{X_{1,0}, X_{2,0}, \dots, X_{\mu,0}\}$ , set the generation counter  $G = 0$ ;
2: Set  $GD$ ;  $\Leftarrow$ 
3: While the stopping criteria are not met Do
4: Determine the fitness ranking  $RANK(k)$  of each individual  $k$   $\{k = 1, 2, \dots, \mu\}$ ;  $\Leftarrow$ 
5: For  $i = 1: \lambda$  Do
-----Recombination-----
6: Randomly choose  $p$  and  $q$ , use the  $p$ th and  $q$ th individuals from  $P_G$  to generate a new individual  $XR_{i,G}$  with the

```

recombination strategy, i.e. Eq. (5);

```

7: Calculate the fitness  $rank(i)$  of individual  $i \{i = 1, 2, \dots, \lambda\}$  as  $(RANK(p) + RANK(q))/2$ ;  $\Leftarrow$ 
8: End For
9: For  $m = 1: M$   $\Leftarrow$ 
10:   For  $i = 1: \lambda$  Do
-----Mutation-----
11:   Use Eq. (6) and (7) to mutate the individual  $XR_{i,G}$  produced by recombination and generate a mutant individual  $XM_{i,G}^m$ ;
12:    $dist_i^m$  = Euclidian distance ( $XM_{i,G}^m, XR_{i,G}$ );  $\Leftarrow$ 
13:   End For
14: End For  $\Leftarrow$ 
15: For  $i = 1: \lambda$  Do
16:   If  $rank(i) \leq \text{ceil}(\lambda \times GD)$   $\Leftarrow$ 
17:      $index = \arg \min_{m \in \{1, 2, \dots, M\}} (dist_i^m)$ ;  $\Leftarrow$ 
18:      $XM_{i,G} = XM_{i,G}^{index}$ ;  $\Leftarrow$ 
19:   Else
20:      $index = \arg \max_{m \in \{1, 2, \dots, M\}} (dist_i^m)$ ;  $\Leftarrow$ 
21:      $XM_{i,G} = XM_{i,G}^{index}$ ;  $\Leftarrow$ 
22:   End If  $\Leftarrow$ 
23: End For
24: Evaluate the fitness of all the new individuals  $XM_{i,G} \{i = 1, 2, \dots, \lambda\}$ ;
-----Selection-----
25: Select  $\mu$  fittest individuals  $X_{i,G} \{i = 1, 2, \dots, \mu\}$  from the  $\mu + \lambda$  individuals to form a new population  $P_{G+1}$ .
26:  $G = G + 1$ ;
27: End While

```

Algorithm S3. SCSS-PSO

```

1: Set the swarm size  $NP$ , initialize positions  $X = \{X_1, X_2, \dots, X_{NP}\}$ , initialize velocities  $V = \{V_1, V_2, \dots, V_{NP}\}$ , record each particle's historical best position in  $pbest_i$  and the global best position in  $gbest$ , set  $w$ ,  $c_1$  and  $c_2$ , set iteration counter  $IT = 0$ ;
2: Set  $GD$ ;  $\Leftarrow$ 
3: While the stopping criteria are not met Do
4: Determine the fitness ranking  $rank(i)$  of each particle  $i \{i = 1, 2, \dots, NP\}$ ;
5: For  $m = 1: M$   $\Leftarrow$ 
6:   For  $i = 1: NP$  Do
7:     For  $j = 1: D$  Do
8:       Update  $v_{ij}^m$  using Eq. (8);
9:       Adjust  $v_{ij}^m$  if it exceeds  $V_{MAXj}$ ;
10:      Update  $x_{ij}^m$  using Eq. (9);
11:    End For
12:     $dist_i^m$  = Euclidian distance ( $X_i^m, pbest_i$ );  $\Leftarrow$ 
13:    End For
14:  End For  $\Leftarrow$ 
15:  For  $i = 1: NP$  Do
16:    If  $rank(i) \leq \text{ceil}(NP \times GD)$   $\Leftarrow$ 
17:       $index = \arg \min_{m \in \{1, 2, \dots, M\}} (dist_i^m)$ ;  $\Leftarrow$ 

```

```

18:       $X_i = X_i^{index};$                                  $\Leftarrow$ 
19:  Else                                                 $\Leftarrow$ 
20:       $index = \arg \max_{m \in \{1, 2, \dots, M\}} (dist_i^m);$      $\Leftarrow$ 
21:       $X_i = X_i^{index};$                                  $\Leftarrow$ 
22:  End If                                               $\Leftarrow$ 
23: End For
24: For  $i = 1: NP$  Do
25:   Evaluate the fitness of the new position  $X_i$ ;
26:   If  $f(X_i) \leq f(pbest_i)$ 
27:      $pbest_i = X_i$ ;
28:   End If
29:   If  $f(X_i) \leq f(gbest)$ 
30:      $gbest = X_i$ ;
31:   End If
32: End For
33:  $IT = IT + 1$ ;
34: End While

```

Remark 1: In SCSS framework, the control parameters that are actually used, CP_i of Y_i should be determined (lines 15 and 19 in Algorithm 2) for the reason that different reproduction procedure m may use different CP and the CP may have further usages. For example, in the well-known JADE and SHADE algorithms, control parameters F and CR are generated according to Cauchy and normal distributions, respectively and after selection, the successful CP are archived to determine new location parameters of Cauchy and normal distributions. Thus, in SCSS, the generations of F and CR are independent in each reproduction procedure m and the successful CP that are actually used is archived. In Algorithms S1 and S3, this is not shown because the classic DE and PSO use pre-defined fixed CP , i.e. F and CR in DE and w , c_1 and c_2 in PSO.

Remark 2: In PSO, the personal best position of each particle is regarded as a current solution for the similarity calculation (line 12 in Algorithm S3).

Remark 3: Different from the one-to-one reproduction procedures in DE and PSO, λ offspring is generated by using μ parents in ES. Therefore, we treat the λ new individuals \mathbf{XR} produced by recombination as the current solutions, and their fitness rankings are calculated to be the average ranking of the p th and q th individuals used to perform recombination (lines 6 and 7 in Algorithm S2).

TABLE CAPTIONS

TABLE S1 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **ADVANCED ALGORITHMS** WITH THE BASELINES ON 30-D CEC2014 BENCHMARK SET

TABLE S2 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **ADVANCED ALGORITHMS** WITH THE BASELINES ON 50-D CEC2014 BENCHMARK SET

TABLE S3 COMPARISONS RESULTS OF SCSS VARIANTS WITH DIFFERENT SS RULES AGAINST THE BASELINES ON 30-D CEC2014 TEST FUNCTIONS ($M = 2$ FOR ALL THE SCSS VARIANTS, BEST ENTRIES ARE HIGHLIGHTED)

TABLE S4 PERFORMANCE COMPARISONS OF SCSS-JADE AND SCSS-SHADE WITH THE OPPOSITE SS RULE ON 30-D CEC2014 BENCHMARK SET

TABLE S5 PERFORMANCE COMPARISON OF SCSS VARIANTS WITH DIFFERENT M SETTINGS WITH THE BASELINES (BEST ENTRIES ARE HIGHLIGHTED)

TABLE S6 PERFORMANCE COMPARISON BETWEEN SCSS VARIANTS WITH ADJACENT M SETTINGS

TABLE S7 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **TOP ALGORITHMS** WITH THE BASELINES ON 30-D CEC2014 BENCHMARK SET

TABLE S8 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **TOP ALGORITHMS** WITH THE BASELINES ON 50-D CEC2014 BENCHMARK SET

TABLE S9 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **ADVANCED ALGORITHMS** WITH THE BASELINES ON 30-D CEC2017 BENCHMARK SET

TABLE S10 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **ADVANCED ALGORITHMS** WITH THE BASELINES ON 50-D CEC2017 BENCHMARK SET

TABLE S11 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **TOP ALGORITHMS** WITH THE BASELINES ON 30-D CEC2017 BENCHMARK SET

TABLE S12 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **TOP ALGORITHMS** WITH THE BASELINES ON 50-D CEC2017 BENCHMARK SET

TABLE S13 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED **TOP ALGORITHMS** WITH THE BASELINES ON 100-D CEC2017 BENCHMARK SET

TABLE S1 PERFORMANCE (MEAN(STD)) COMPARISONS OF FOUR SCSS-BASED ADVANCED ALGORITHMS WITH THE BASELINES ON 30-D CEC2014 BENCHMARK SET

| | | JADE | SCSS-JADE | SHADE | SCSS-SHADE | CMA-ES | SCSS-CMA-ES | LIPS | SCSS-LIPS |
|-----------------------------|-----------|------------------------------|----------------------------|-----------------------|----------------------------|------------------------------|----------------------------|------------------------------|----------------------------|
| Unimodal Functions | cec14 F1 | 2.04E+03 = (2.59E+03) | 1.47E+03 (2.14E+03) | 1.61E+03 = (2.04E+03) | 1.50E+03 (2.68E+03) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 2.84E+07 - (2.65E+07) | 5.42E+06 (6.50E+06) |
| | cec14 F2 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 2.58E+03 = (4.30E+03) | 5.84E+03 (8.14E+03) |
| | cec14 F3 | 2.08E-05 - (1.13E-04) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 3.93E+03 - (3.64E+03) | 2.13E+03 (1.95E+03) |
| Simple Multimodal Functions | cec14 F4 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 2.74E+02 - (1.13E+02) | 1.40E+02 (6.49E+01) |
| | cec14 F5 | 2.03E+01 - (3.12E-02) | 2.03E+01 (7.09E-02) | 2.02E+01 - (2.78E-02) | 2.01E+01 (2.29E-02) | 2.00E+01 + (3.27E-05) | 2.13E+01 (5.20E-01) | 2.00E+01 + (8.23E-05) | 2.09E+01 (4.90E-02) |
| | cec14 F6 | 8.76E+00 = (2.72E+00) | 7.33E+00 (3.86E+00) | 6.42E+00 - (3.15E+00) | 4.12E+00 (3.37E+00) | 4.12E+01 - (9.58E+00) | 4.19E+00 (5.18E+00) | 1.48E+01 - (2.70E+00) | 7.72E+00 (2.24E+00) |
| | cec14 F7 | 3.38E-04 = (1.71E-03) | 1.93E-04 (1.38E-03) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 1.64E-03 = (3.51E-03) | 1.59E-03 (4.45E-03) | 1.59E-03 = (4.86E-03) | 2.37E-03 (4.57E-03) |
| | cec14 F8 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 4.08E+02 - (8.57E+01) | 2.31E+02 (2.00E+02) | 5.35E+01 - (1.26E+01) | 2.64E+01 (6.79E+00) |
| | cec14 F9 | 2.58E+01 - (3.62E+00) | 2.13E+01 (4.82E+00) | 2.10E+01 - (3.81E+00) | 1.92E+01 (3.44E+00) | 6.35E+02 - (1.23E+02) | 2.17E+02 (2.74E+02) | 6.29E+01 - (1.82E+01) | 3.62E+01 (8.74E+00) |
| | cec14 F10 | 4.49E-03 + (1.05E-02) | 9.39E-03 (1.52E-02) | 5.31E-03 = (1.01E-02) | 7.76E-03 (1.17E-02) | 4.92E+03 - (7.43E+02) | 3.49E+03 (1.10E+03) | 1.97E+03 - (4.14E+02) | 9.61E+02 (2.63E+02) |
| | cec14 F11 | 1.66E+03 - (2.67E+02) | 1.54E+03 (2.28E+02) | 1.48E+03 = (2.35E+02) | 1.50E+03 (2.02E+02) | 5.10E+03 - (8.25E+02) | 3.58E+03 (1.15E+03) | 2.54E+03 - (4.39E+02) | 2.02E+03 (4.10E+02) |
| | cec14 F12 | 2.60E-01 - (4.06E-02) | 2.27E-01 (4.87E-02) | 2.10E-01 - (2.67E-02) | 1.68E-01 (2.45E-02) | 3.76E-01 - (4.02E-01) | 2.40E-01 (1.01E+00) | 1.78E-01 = (4.81E-02) | 7.59E-01 (1.02E+00) |
| | cec14 F13 | 2.10E-01 - (3.53E-02) | 1.85E-01 (3.68E-02) | 2.23E-01 - (3.61E-02) | 2.04E-01 (3.18E-02) | 2.62E-01 + (7.72E-02) | 4.24E-01 (1.46E-01) | 3.06E-01 - (6.43E-02) | 2.75E-01 (5.22E-02) |
| | cec14 F14 | 2.24E-01 = (3.09E-02) | 2.32E-01 (3.71E-02) | 2.27E-01 - (3.04E-02) | 2.09E-01 (3.26E-02) | 3.71E-01 + (9.68E-02) | 5.66E-01 (2.97E-01) | 2.45E-01 + (3.56E-02) | 3.10E-01 (7.15E-02) |
| | cec14 F15 | 3.11E+00 - (4.17E-01) | 2.86E+00 (3.22E-01) | 2.97E+00 - (3.67E-01) | 2.59E+00 (3.03E-01) | 3.49E+00 = (7.56E-01) | 3.21E+00 (6.63E-01) | 1.08E+01 - (3.87E+00) | 3.92E+00 (8.93E-01) |
| | cec14 F16 | 9.49E+00 = (3.17E+01) | 9.34E+00 (4.29E+01) | 9.51E+00 = (3.99E+01) | 9.50E+00 (4.24E+01) | 1.43E+01 - (4.33E-01) | 1.38E+01 (7.44E-01) | 1.15E+01 - (4.96E-01) | 1.06E+01 (4.65E-01) |
| Hybrid Functions | cec14 F17 | 1.24E+03 - (3.35E+02) | 8.28E+02 (3.47E+02) | 9.44E+02 - (3.12E+02) | 5.78E+02 (2.32E+02) | 1.56E+03 = (4.64E+02) | 1.71E+03 (3.84E+02) | 2.89E+05 - (3.04E+05) | 1.86E+05 (2.99E+05) |
| | cec14 F18 | 2.11E+02 - (8.15E+02) | 4.72E+01 (2.34E+01) | 3.44E+01 - (1.74E+01) | 2.05E+01 (1.20E+01) | 1.35E+02 + (4.50E+01) | 1.78E+02 (7.13E+01) | 4.88E+02 = (7.08E+02) | 4.92E+02 (9.08E+02) |
| | cec14 F19 | 4.52E+00 - (6.74E-01) | 4.01E+00 (8.54E-01) | 3.95E+00 = (4.72E-01) | 3.84E+00 (6.58E-01) | 1.01E+01 - (2.11E+00) | 6.74E+00 (1.58E+00) | 2.54E+01 - (2.49E+01) | 8.85E+00 (2.76E+00) |
| | cec14 F20 | 2.02E+03 = (2.81E+03) | 1.88E+03 (2.44E+03) | 1.09E+01 - (4.61E+00) | 8.41E+00 (3.45E+00) | 2.89E+02 - (1.01E+02) | 1.49E+02 (5.45E+01) | 1.47E+04 = (7.71E+03) | 1.23E+04 (7.41E+03) |
| | cec14 F21 | 4.07E+03 - (1.89E+04) | 2.41E+02 (1.15E+02) | 2.13E+02 = (1.01E+02) | 1.90E+02 (1.12E+02) | 1.04E+03 - (3.50E+02) | 8.64E+02 (3.05E+02) | 1.11E+05 - (8.42E+04) | 4.26E+04 (5.58E+04) |
| | cec14 F22 | 1.30E+02 = (6.92E+01) | 1.10E+02 (6.90E+01) | 6.36E+01 = (4.93E+01) | 7.12E+01 (6.10E+01) | 3.07E+02 - (2.29E+02) | 1.16E+02 (1.10E+02) | 3.27E+02 - (1.20E+02) | 2.28E+02 (1.10E+02) |
| Composition Functions | cec14 F23 | 3.15E+02 = (4.02E-13) | 3.15E+02 (4.02E-13) | 3.15E+02 = (4.02E-13) | 3.15E+02 (4.02E-13) | 3.15E+02 + (3.15E-12) | 3.15E+02 (2.57E-11) | 3.24E+02 - (5.26E+00) | 3.16E+02 (5.73E-01) |
| | cec14 F24 | 2.26E+02 = (3.11E+00) | 2.25E+02 (3.27E+00) | 2.24E+02 = (1.01E+00) | 2.24E+02 (1.21E+00) | 2.33E+02 - (6.83E+00) | 2.26E+02 (6.96E+00) | 2.39E+02 - (4.83E+00) | 2.33E+02 (5.09E+00) |
| | cec14 F25 | 2.05E+02 - (2.18E+00) | 2.03E+02 (6.04E-01) | 2.04E+02 - (1.04E+00) | 2.03E+02 (4.63E-01) | 2.04E+02 - (2.42E+00) | 2.03E+02 (5.20E-01) | 2.16E+02 - (3.59E+00) | 2.11E+02 (1.97E+00) |
| | cec14 F26 | 1.00E+02 - (3.77E-02) | 1.00E+02 (3.56E-02) | 1.00E+02 - (3.26E-02) | 1.00E+02 (3.42E-02) | 1.31E+02 - (1.37E+02) | 1.26E+02 (1.58E+02) | 1.32E+02 - (4.40E+01) | 1.09E+02 (2.68E+01) |
| | cec14 F27 | 3.60E+02 = (5.07E+01) | 3.44E+02 (5.09E+01) | 3.16E+02 = (3.71E+01) | 3.21E+02 (4.03E+01) | 4.40E+02 - (2.10E+02) | 3.40E+02 (3.93E+01) | 6.03E+02 - (1.66E+02) | 4.79E+02 (9.74E+01) |
| | cec14 F28 | 7.99E+02 = (2.34E+01) | 8.01E+02 (1.64E+01) | 7.95E+02 = (1.99E+01) | 7.93E+02 (2.17E+01) | 4.43E+03 - (3.23E+03) | 1.25E+03 (1.41E+03) | 1.78E+03 - (3.95E+02) | 1.12E+03 (1.70E+02) |
| | cec14 F29 | 7.33E+02 - (1.60E+01) | 7.20E+02 (7.10E+01) | 7.25E+02 - (1.02E+01) | 7.12E+02 (5.40E+01) | 7.88E+02 = (9.18E+01) | 8.00E+02 (1.45E+02) | 1.34E+04 - (5.19E+04) | 1.29E+03 (2.46E+02) |
| | cec14 F30 | 1.55E+03 = (6.33E+02) | 1.53E+03 (6.34E+02) | 1.45E+03 - (6.13E+02) | 1.19E+03 (3.57E+02) | 2.30E+03 - (5.50E+02) | 1.58E+03 (5.95E+02) | 3.84E+04 - (2.59E+04) | 1.08E+04 (6.59E+03) |
| -/=/+ | | 14/15/1 | | 14/16/0 | | 17/8/5 | | 23/5/2 | |

TABLE S2 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED ADVANCED ALGORITHMS WITH THE BASELINES
ON 50-D CEC2014 BENCHMARK SET

| | | JADE | SCSS-JADE | SHADE | SCSS-SHADE | CMA-ES | SCSS-CMA-ES | LIPS | SCSS-LIPS |
|-----------------------------|-----------|------------------------------|----------------------------|-----------------------|----------------------------|------------------------------|----------------------------|------------------------------|----------------------------|
| Unimodal Functions | cec14 F1 | 1.88E+04 = (1.26E+04) | 1.97E+04 (1.52E+04) | 2.24E+04 = (1.14E+04) | 2.66E+04 (1.09E+04) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 1.29E+08 - (7.81E+07) | 8.45E+06 (1.32E+07) |
| | cec14 F2 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 7.57E+02 + (1.40E+03) | 1.72E+03 (2.71E+03) |
| | cec14 F3 | 3.06E+03 - (2.03E+03) | 2.01E+03 (2.98E+03) | 3.13E-06 - (1.39E-05) | 1.02E-07 (3.42E-07) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 1.67E+04 - (6.05E+03) | 1.14E+04 (5.51E+03) |
| Simple Multimodal Functions | cec14 F4 | 1.37E+01 = (3.36E+01) | 2.32E+01 (4.20E+01) | 2.81E+01 - (4.30E+01) | 3.08E+01 (4.60E+01) | 3.28E+01 = (4.68E+01) | 1.35E+01 (3.42E+01) | 7.09E+02 - (3.77E+02) | 2.08E+02 (5.28E+01) |
| | cec14 F5 | 2.04E+01 - (3.27E-02) | 2.02E+01 (2.06E-01) | 2.02E+01 - (2.34E-02) | 2.02E+01 (2.30E-02) | 2.00E+01 + (1.77E-06) | 2.14E+01 (3.67E-01) | 2.00E+01 + (1.49E-05) | 2.11E+01 (3.62E-02) |
| | cec14 F6 | 1.59E+01 = (6.47E+00) | 1.67E+01 (6.84E+00) | 6.87E+00 = (5.99E+00) | 5.35E+00 (4.96E+00) | 7.68E+01 - (1.08E+01) | 1.74E+01 (1.85E+01) | 3.71E+01 - (4.26E+00) | 2.33E+01 (3.96E+00) |
| | cec14 F7 | 4.15E-03 = (5.75E-03) | 2.42E-03 (4.81E-03) | 1.59E-03 = (3.91E-03) | 1.69E-03 (4.22E-03) | 5.32E-04 = (2.22E-03) | 6.77E-04 (2.42E-03) | 5.88E-03 - (1.93E-02) | 7.25E-04 (2.57E-03) |
| | cec14 F8 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 7.39E+02 - (1.09E+02) | 6.12E+02 (2.31E+02) | 1.44E+02 - (1.89E+01) | 6.73E+01 (1.23E+01) |
| | cec14 F9 | 5.43E+01 = (7.72E+00) | 3.86E+01 (8.83E+00) | 4.03E+01 = (5.05E+00) | 3.95E+01 (5.80E+00) | 1.13E+03 - (2.41E+02) | 5.88E+02 (4.78E+02) | 1.81E+02 - (2.84E+01) | 1.08E+02 (2.14E+01) |
| | cec14 F10 | 1.05E-02 = (9.47E-03) | 1.25E-02 (1.56E-02) | 5.14E-03 = (8.35E-03) | 9.06E-03 (1.30E-02) | 8.43E+03 - (9.42E+02) | 7.21E+03 (1.17E+03) | 4.33E+03 - (5.04E+02) | 2.52E+03 (4.62E+02) |
| | cec14 F11 | 3.82E+03 - (2.72E-02) | 3.53E+03 (2.87E+02) | 3.65E+03 = (3.25E+02) | 3.55E+03 (3.46E+02) | 8.23E+03 - (9.32E+02) | 7.25E+03 (1.10E+03) | 5.15E+03 - (4.95E+02) | 4.20E+03 (6.68E+02) |
| | cec14 F12 | 2.61E-01 - (3.01E-02) | 2.14E-01 (7.30E-02) | 2.07E-01 - (2.79E-02) | 1.71E-01 (2.59E-02) | 2.71E-01 - (2.55E-01) | 7.63E-02 (4.56E-01) | 2.63E-01 - (7.48E-02) | 6.84E-01 (1.12E+00) |
| | cec14 F13 | 3.13E-01 - (4.70E-02) | 2.75E-01 (3.91E-02) | 3.20E-01 = (3.32E-02) | 3.12E-01 (4.02E-02) | 3.48E-01 + (7.71E-02) | 8.08E-01 (1.59E-01) | 4.31E-01 = (5.93E-02) | 4.12E-01 (5.70E-02) |
| | cec14 F14 | 3.00E-01 = (2.93E-02) | 3.18E-01 (9.22E-02) | 2.86E-01 = (6.25E-02) | 2.69E-01 (4.02E-02) | 4.43E-01 + (2.50E-01) | 1.26E+00 (4.03E-01) | 2.71E-01 + (3.14E-02) | 3.48E-01 (1.19E-01) |
| | cec14 F15 | 7.27E+00 - (8.65E-01) | 5.94E+00 (6.97E-01) | 6.35E+00 - (7.66E-01) | 5.66E+00 (5.90E-01) | 6.41E+00 = (1.25E+00) | 6.02E+00 (1.20E+00) | 7.62E+01 - (4.32E+01) | 1.20E+01 (2.95E+00) |
| | cec14 F16 | 1.77E+01 = (5.34E-01) | 1.80E+01 (1.05E+00) | 1.79E+01 = (4.14E-01) | 1.79E+01 (3.62E-01) | 2.38E+01 = (5.19E-01) | 2.40E+01 (6.18E-01) | 2.05E+01 - (6.41E-01) | 1.94E+01 (6.37E-01) |
| | cec14 F17 | 2.29E+03 = (6.74E+02) | 2.53E+03 (7.80E+02) | 2.74E+03 = (8.65E+02) | 2.74E+03 (8.27E+02) | 2.69E+03 = (6.15E+02) | 2.60E+03 (5.98E+02) | 4.00E+06 - (5.97E+06) | 7.38E+05 (1.42E+06) |
| | cec14 F18 | 1.64E+02 = (4.16E+01) | 1.66E+02 (4.06E+01) | 1.47E+02 = (4.44E+01) | 1.39E+02 (4.31E+01) | 2.30E+02 + (4.57E+01) | 2.67E+02 (7.08E+01) | 3.26E+02 - (1.64E+02) | 2.53E+02 (7.75E+01) |
| | cec14 F19 | 1.48E+01 - (5.97E+00) | 1.06E+01 (5.22E+00) | 1.63E+01 - (7.08E+00) | 1.28E+01 (4.48E+00) | 1.84E+01 - (2.57E+00) | 1.46E+01 (2.30E+00) | 5.78E+01 - (2.86E+01) | 4.25E+01 (2.26E+01) |
| Hybrid Functions | cec14 F20 | 8.19E+03 - (6.72E+03) | 1.99E+03 (4.70E+03) | 1.92E+02 - (6.69E+01) | 1.10E+02 (4.37E+01) | 4.44E+02 - (1.22E+02) | 2.71E+02 (8.53E+01) | 3.02E+04 - (1.09E+04) | 1.91E+04 (7.19E+03) |
| | cec14 F21 | 1.29E+03 - (4.85E+02) | 2.36E+04 (1.61E+05) | 1.40E+03 - (4.92E+02) | 1.01E+03 (3.33E+02) | 1.70E+03 = (4.32E+02) | 1.62E+03 (3.71E+02) | 5.78E+05 - (4.16E+05) | 1.71E+05 (1.07E+05) |
| | cec14 F22 | 4.78E+02 - (1.66E+02) | 3.76E+02 (1.61E+02) | 3.76E+02 = (1.18E+02) | 3.38E+02 (1.09E+02) | 4.19E+02 - (2.61E+02) | 3.20E+02 (2.11E+02) | 8.43E+02 - (2.08E+02) | 5.69E+02 (1.88E+02) |
| | cec14 F23 | 3.44E+02 + (4.55E-13) | 3.44E+02 (5.16E-13) | 3.44E+02 = (4.31E-13) | 3.44E+02 (4.50E-13) | 3.44E+02 = (2.32E-05) | 3.44E+02 (2.38E-05) | 3.77E+02 - (1.34E+01) | 3.50E+02 (1.83E+00) |
| | cec14 F24 | 2.74E+02 = (2.05E+00) | 2.75E+02 (1.89E+00) | 2.73E+02 = (1.93E+00) | 2.72E+02 (1.89E+00) | 3.67E+02 - (5.44E+02) | 2.76E+02 (2.43E+00) | 2.95E+02 - (6.01E+00) | 2.80E+02 (3.16E+00) |
| | cec14 F25 | 2.23E+02 - (3.19E+00) | 2.11E+02 (6.51E+00) | 2.18E+02 - (5.01E+00) | 2.11E+02 (6.05E+00) | 2.05E+02 - (9.61E-01) | 2.05E+02 (2.18E-01) | 2.40E+02 - (8.81E+00) | 2.25E+02 (4.59E+00) |
| Composition Functions | cec14 F26 | 1.04E+02 - (1.95E+01) | 1.00E+02 (8.92E-02) | 1.02E+02 - (1.40E+01) | 1.00E+02 (5.89E-02) | 1.17E+02 + (5.81E+01) | 1.09E+02 (4.04E+01) | 1.66E+02 - (4.65E+01) | 1.36E+02 (4.82E+01) |
| | cec14 F27 | 4.65E+02 - (5.76E+01) | 4.35E+02 (5.42E+01) | 3.91E+02 = (4.89E+01) | 3.79E+02 (4.65E+01) | 5.33E+02 - (1.06E+02) | 4.57E+02 (7.00E+01) | 1.39E+03 - (1.29E+02) | 9.91E+02 (8.80E+01) |
| | cec14 F28 | 1.15E+03 - (3.72E+01) | 1.12E+03 (3.47E+01) | 1.13E+03 = (4.00E+01) | 1.11E+03 (3.05E+01) | 7.61E+03 - (5.87E+03) | 4.39E+03 (2.98E+03) | 4.52E+03 - (7.42E+02) | 2.55E+03 (3.27E+02) |
| | cec14 F29 | 8.81E+02 = (5.80E+01) | 8.94E+02 (9.69E+01) | 9.01E+02 = (6.55E+01) | 9.02E+02 (6.54E+01) | 8.86E+02 = (6.70E+01) | 8.94E+02 (8.74E+01) | 8.33E+06 - (4.37E+07) | 2.09E+03 (5.43E+02) |
| | cec14 F30 | 9.78E+03 - (7.82E+02) | 9.26E+03 (8.07E+02) | 9.35E+03 - (6.62E+02) | 8.87E+03 (6.64E+02) | 9.31E+03 = (7.96E+02) | 9.45E+03 (1.09E+03) | 2.84E+05 - (1.17E+05) | 6.41E+04 (2.21E+04) |
| -/+ | | 16/13/1 | | 11/19/0 | | 13/12/5 | | 26/1/3 | |

TABLE S3 COMPARISONS RESULTS OF SCSS VARIANTS WITH DIFFERENT SS RULES AGAINST THE BASELINES ON 30-D CEC2014 TEST FUNCTIONS ($M = 2$ FOR ALL THE SCSS VARIANTS, BEST ENTRIES ARE HIGHLIGHTED)

| -/+ (P-N) | Scheme 1 | | | | | | Scheme 2 |
|-----------|---------------------|--------------|---------------------|---------------------|--------------------|--------------------|---------------------|
| | $GD=0$ | $GD=0.2$ | $GD=0.4$ | $GD=0.6$ | $GD=0.8$ | $GD=1.0$ | |
| DE | 0/5/25(-25) | 1/13/16(-15) | 2/21/7(-5) | 11/19/0(11) | 19/11/0 (19) | 21/8/1(20) | 5/21/4(1) |
| ES | 0/3/27 (-27) | 25/5/0 (25) | 26/4/0 (26) | 26/4/0 (26) | 26/4/0 (26) | 26/4/0 (26) | 26/4/0 (26) |
| PSO | 0/4/26 (-26) | 10/15/5(5) | 10/18/2(8) | 14/16/0(14) | 14/16/0(14) | 15/15/0(15) | 13/17/0(13) |
| JADE | 14/9/7 (7) | 15/11/4 (11) | 19/8/3 (16) | 15/14/1 (14) | 5/11/14 (-9) | 2/6/22 (-20) | 14/15/1 (13) |
| SHADE | 12/12/6 (6) | 14/13/3 (11) | 15/14/1 (14) | 14/16/0 (14) | 5/21/4 (1) | 3/9/18 (-15) | 14/16/0 (14) |
| CMA-ES | 13/15/2 (11) | 6/23/1 (5) | 0/30/0 (0) | 1/25/4 (-3) | 1/19/10 (-9) | 1/21/8 (-7) | 2/26/2 (0) |
| LIPS | 16/5/9 (7) | 22/4/4 (18) | 22/5/3 (19) | 22/5/3 (19) | 21/5/4 (17) | 20/8/2 (18) | 23/4/3 (20) |

TABLE S4 PERFORMANCE COMPARISONS OF SCSS-JADE AND SCSS-SHADE WITH OPPOSITE SS RULE ON 30-D CEC2014 BENCHMARK SET

| | SCSS-JADE_oppo | SCSS-JADE | SCSS-SHADE_oppo | SCSS-SHADE | | SCSS-JADE_oppo | SCSS-JADE | SCSS-SHADE_oppo | SCSS-SHADE |
|-----------|------------------------------|----------------------------|------------------------------|----------------------------|-----------|-----------------------|----------------------------|-----------------------|----------------------------|
| cec14 F1 | 1.81E+05 - (1.28E+06) | 1.47E+03 (2.14E+03) | 2.96E+03 - (2.97E+03) | 1.50E+03 (2.68E+03) | cec14 F16 | 9.91E+00 - (2.48E-01) | 9.34E+00 (4.29E-01) | 9.70E+00 - (3.76E-01) | 9.50E+00 (4.24E-01) |
| cec14 F2 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | cec14 F17 | 2.85E+05 - (4.29E+05) | 8.28E+02 (3.47E+02) | 1.28E+03 - (3.31E+02) | 5.78E+02 (2.32E+02) |
| cec14 F3 | 2.90E+00 - (3.03E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | cec14 F18 | 2.85E+03 - (3.60E+03) | 4.72E+01 (2.34E+01) | 7.89E+01 - (2.74E+01) | 2.05E+01 (1.20E+01) |
| cec14 F4 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | cec14 F19 | 4.86E+00 - (7.86E-01) | 4.01E+00 (8.54E-01) | 4.33E+00 - (4.82E-01) | 3.84E+00 (6.58E-01) |
| cec14 F5 | 2.03E+01 - (2.70E-02) | 2.03E+01 (7.09E-02) | 2.02E+01 - (2.61E-02) | 2.01E+01 (2.29E-02) | cec14 F20 | 3.53E+03 - (2.22E+03) | 1.88E+03 (2.44E+03) | 2.50E+01 - (1.35E+01) | 8.41E+00 (3.45E+00) |
| cec14 F6 | 1.24E+01 - (1.20E+01) | 7.33E+00 (3.86E+00) | 7.78E+00 - (2.87E+00) | 4.12E+00 (3.37E+00) | cec14 F21 | 7.95E+04 - (8.65E+04) | 2.41E+02 (1.15E+02) | 4.46E+02 - (1.96E+02) | 1.90E+02 (1.12E+02) |
| cec14 F7 | 0.00E+00 = (0.00E+00) | 1.93E-04 (1.38E-03) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | cec14 F22 | 1.64E+02 - (7.95E+01) | 1.10E+02 (6.90E+01) | 9.88E+01 - (5.80E+01) | 7.12E+01 (6.10E+01) |
| cec14 F8 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | cec14 F23 | 3.15E+02 = (2.48E-11) | 3.15E+02 (4.02E-13) | 3.15E+02 = (4.02E-13) | 3.15E+02 (4.02E-13) |
| cec14 F9 | 3.91E+01 - (5.50E+00) | 2.13E+01 (4.82E+00) | 2.76E+01 - (3.80E+00) | 1.92E+01 (3.44E+00) | cec14 F24 | 2.26E+02 - (3.01E+00) | 2.25E+02 (3.27E+00) | 2.24E+02 = (1.03E+00) | 2.24E+02 (1.21E+00) |
| cec14 F10 | 2.45E-03 + (6.77E-03) | 9.39E-03 (1.52E-02) | 3.27E-03 + (7.65E-03) | 7.76E-03 (1.17E-02) | cec14 F25 | 2.05E+02 - (2.05E+00) | 2.03E+02 (6.04E-01) | 2.05E+02 - (1.99E+00) | 2.03E+02 (4.63E-01) |
| cec14 F11 | 2.24E+03 - (1.84E+02) | 1.54E+03 (2.28E+02) | 1.95E+03 - (2.02E+02) | 1.50E+03 (2.02E+02) | cec14 F26 | 1.00E+02 - (3.71E-02) | 1.00E+02 (3.56E-02) | 1.00E+02 - (3.07E-02) | 1.00E+02 (3.42E-02) |
| cec14 F12 | 3.76E-01 - (3.71E-02) | 2.27E-01 (4.87E-02) | 3.28E-01 - (2.69E-02) | 1.68E-01 (2.45E-02) | cec14 F27 | 3.61E+02 - (5.23E+01) | 3.44E+02 (5.09E+01) | 3.19E+02 - (4.00E+01) | 3.21E+02 (4.03E+01) |
| cec14 F13 | 2.59E-01 - (3.58E-02) | 1.85E-01 (3.68E-02) | 2.47E-01 - (3.04E-02) | 2.04E-01 (3.18E-02) | cec14 F28 | 8.15E+02 - (1.91E+01) | 8.01E+02 (1.64E+01) | 7.96E+02 = (1.88E+01) | 7.93E+02 (2.17E+01) |
| cec14 F14 | 2.46E-01 - (3.02E-02) | 2.32E-01 (3.71E-02) | 2.41E-01 - (2.56E-02) | 2.09E-01 (3.26E-02) | cec14 F29 | 1.28E+03 - (4.43E+02) | 7.20E+02 (7.10E+01) | 7.34E+02 - (1.92E+01) | 7.12E+02 (5.40E+01) |
| cec14 F15 | 4.30E+00 - (4.90E-01) | 2.86E+00 (3.22E-01) | 3.76E+00 - (4.39E-01) | 2.59E+00 (3.03E-01) | cec14 F30 | 1.97E+03 - (6.55E+02) | 1.53E+03 (6.34E+02) | 1.54E+03 - (5.46E+02) | 1.19E+03 (3.57E+02) |
| -/+ | 24/5/1 | | 21/8/1 | | | | | | |

TABLE S5 PERFORMANCE COMPARISON OF SCSS VARIANTS WITH DIFFERENT M SETTINGS
WITH THE BASELINES (BEST ENTRIES ARE HIGHLIGHTED)

| -/=/+ (P-N) | $M = 2$ | $M = 3$ | $M = 4$ | $M = 5$ | $M = 10$ |
|-------------|---------------------|---------------------|--------------------|--------------------|--------------------|
| DE | 21/8/1 (20) | 25/4/1 (24) | 23/6/1 (22) | 26/3/1 (25) | 27/2/1 (26) |
| ES | 26/4/0 (26) | 27/3/0 (27) | 27/3/0 (27) | 27/2/1 (26) | 27/2/1 (26) |
| PSO | 15/15/0 (15) | 13/16/1(12) | 12/17/1(11) | 10/18/2(8) | 10/17/3 (7) |
| JADE | 14/15/1 (13) | 16/13/1 (15) | 14/15/1 (13) | 13/14/3 (10) | 8/10/12 (-4) |
| SHADE | 14/16/0 (14) | 13/15/2 (11) | 14/13/3 (11) | 12/14/4 (8) | 12/8/10 (2) |
| CMA-ES | 13/15/2 (11) | 15/11/4 (11) | 15/11/4 (11) | 17/8/5 (12) | 17/7/6 (11) |
| LIPS | 23/4/3 (20) | 23/4/3 (20) | 23/5/2(21) | 23/4/3 (20) | 20/5/5 (15) |

TABLE S6 PERFORMANCE COMPARISON BETWEEN SCSS VARIANTS
WITH ADJACENT M SETTINGS

| CATEGORY 1 | | | | |
|-------------|----------------------|----------------------|----------------------|-----------------------|
| -/=/+ (P-N) | $M = 2$ v.s. $M = 3$ | $M = 3$ v.s. $M = 4$ | $M = 4$ v.s. $M = 5$ | $M = 5$ v.s. $M = 10$ |
| DE | 17/12/1 (16) | 5/24/1 (4) | 6/23/1 (5) | 12/14/4 (8) |
| ES | 25/5/0 (25) | 23/7/0(23) | 12/18/0 (12) | 28/2/0 (28) |
| CMA-ES | 8/18/4 (4) | 7/20/3(4) | 1/29/0 (1) | 5/18/7 (-2) |
| LIPS | 8/21/1 (7) | 2/28/0 (2) | 0/29/1 (-1) | 3/23/4 (-1) |
| CATEGORY 2 | | | | |
| -/=/+ (P-N) | $M = 2$ v.s. $M = 3$ | $M = 3$ v.s. $M = 4$ | $M = 4$ v.s. $M = 5$ | $M = 5$ v.s. $M = 10$ |
| PSO | 1/22/7 (-6) | 1/28/1 (0) | 0/29/1 (-1) | 0/28/2 (-2) |
| JADE | 3/25/2 (1) | 5/22/3 (2) | 4/18/8 (-4) | 5/7/18 (-13) |
| SHADE | 4/24/2 (2) | 4/23/3 (1) | 7/15/8 (-1) | 5/13/12 (-7) |

TABLE S7 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED TOP ALGORITHMS WITH THE BASELINES
ON 30-D CEC2014 BENCHMARK SET

| | | L-SHADE | SCSS- L-SHADE | UMOEa-II | SCSS- UMOEa-II | L-SHADE_ EpSin | SCSS- L-SHADE_ EpSin | jSO | SCSS- jSO |
|--------------------------------|-----------|--|--------------------------------------|--|--------------------------------------|--------------------------|--------------------------------------|--|--------------------------------------|
| Unimodal Functions | cec14 F1 | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) |
| | cec14 F2 | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) |
| | cec14 F3 | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) |
| Simple Multimodal Functions | cec14 F4 | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) |
| | cec14 F5 | 2.01E+01 - (3.46E-02) | 2.01E+01 (5.37E-02) | 2.00E+01 = (1.03E-03) | 2.00E+01 = (4.78E-05) | 2.01E+01 - (2.98E-02) | 2.01E+01 (4.75E-02) | 2.09E+01 = (8.04E-02) | 2.09E+01 = (4.80E-02) |
| | cec14 F6 | 9.01E-03 = (6.43E-02) | 9.01E-03 = (6.43E-02) | 1.99E-01 = (1.35E+00) | 4.24E-06 = (1.86E-05) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 8.61E-06 = (3.52E-05) | 1.02E-02 = (7.27E-02) |
| | cec14 F7 | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) |
| | cec14 F8 | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 = (0.00E+00) |
| | cec14 F9 | 7.22E+00 = (1.33E+00) | 7.38E+00 = (1.63E+00) | 8.97E+00 = (1.79E+00) | 9.03E+00 = (2.07E+00) | 1.31E+01 - (1.94E+00) | 1.24E+01 (2.15E+00) | 8.76E+00 - (1.97E+00) | 7.57E+00 (1.62E+00) |
| | cec14 F10 | 5.72E-03 = (1.11E-02) | 7.35E-03 = (1.37E-02) | 1.63E-03 = (5.65E-03) | 4.08E-03 = (8.35E-03) | 4.49E-03 = (9.60E-03) | 4.90E-03 = (1.07E-02) | 1.43E+00 = (1.02E+00) | 1.64E+00 = (9.94E-01) |
| | cec14 F11 | 1.23E+03 = (1.92E+02) | 1.24E+03 = (1.85E+02) | 1.41E+03 = (3.01E+02) | 1.43E+03 = (3.18E+02) | 1.14E+03 = (2.09E+02) | 1.16E+03 = (2.03E+02) | 1.20E+03 = (2.73E+02) | 1.26E+03 = (2.45E+02) |
| | cec14 F12 | 1.73E-01 = (2.13E-02) | 1.65E-01 = (3.01E-02) | 1.01E-01 = (5.51E-02) | 1.08E-01 = (6.90E-02) | 1.54E-01 = (2.30E-02) | 1.46E-01 = (2.77E-02) | 4.17E-01 + (4.93E-01) | 9.00E-01 = (7.61E-01) |
| | cec14 F13 | 1.05E-01 = (1.56E-02) | 1.08E-01 = (1.56E-02) | 1.14E-01 = (1.81E-02) | 1.09E-01 = (2.15E-02) | 1.34E-01 - (1.64E-02) | 1.24E-01 (1.61E-02) | 1.37E-01 + (2.24E-02) | 1.52E-01 = (3.04E-02) |
| | cec14 F14 | 2.38E-01 - (2.69E-02) | 1.90E-01 (2.41E-02) | 2.29E-01 - (2.52E-02) | 2.10E-01 (3.27E-02) | 1.93E-01 = (2.91E-02) | 1.93E-01 = (2.44E-02) | 2.26E-01 = (4.08E-02) | 2.30E-01 = (3.63E-02) |
| | cec14 F15 | 2.28E+00 - (2.93E-01) | 2.16E+00 (2.47E-01) | 2.44E+00 = (4.60E-01) | 2.29E+00 = (5.34E-01) | 2.37E+00 - (2.41E-01) | 2.24E+00 (2.91E-01) | 2.37E+00 - (2.73E-01) | 2.13E+00 (3.37E-01) |
| | cec14 F16 | 8.51E+00 + (3.61E-01) | 8.65E+00 = (4.00E-01) | 9.15E+00 + (5.25E-01) | 9.57E+00 = (6.20E-01) | 8.30E+00 = (4.58E-01) | 8.26E+00 = (3.76E-01) | 8.58E+00 = (7.71E-01) | 8.60E+00 = (7.27E-01) |
| Hybrid Functions | cec14 F17 | 2.09E+02 - (1.13E+02) | 8.89E+01 (4.59E+01) | 1.29E+02 - (7.85E+01) | 7.77E+01 (4.25E+01) | 1.94E+02 - (8.71E+01) | 1.42E+02 (8.41E+01) | 6.38E+01 = (2.31E+01) | 6.22E+01 = (2.13E+01) |
| | cec14 F18 | 6.89E+00 - (3.23E+00) | 3.01E+00 (1.50E+00) | 4.85E+00 - (1.76E+00) | 3.89E+00 (1.47E+00) | 6.02E+00 = (2.44E+00) | 5.68E+00 = (2.09E+00) | 2.14E+00 = (1.23E+00) | 2.19E+00 = (1.17E+00) |
| | cec14 F19 | 3.75E+00 - (5.74E-01) | 3.08E+00 (6.64E-01) | 2.69E+00 - (6.23E-01) | 2.23E+00 (6.65E-01) | 2.63E+00 = (8.21E-01) | 2.78E+00 = (6.45E-01) | 2.04E+00 = (7.16E-01) | 1.86E+00 = (6.30E-01) |
| | cec14 F20 | 2.84E+00 = (1.04E+00) | 2.59E+00 = (1.07E+00) | 3.57E+00 = (1.41E+00) | 3.72E+00 = (1.34E+00) | 2.34E+00 = (1.06E+00) | 2.67E+00 = (1.18E+00) | 2.04E+00 = (8.67E-01) | 1.97E+00 = (8.07E-01) |
| | cec14 F21 | 9.08E+01 - (7.29E+01) | 3.33E+01 (5.40E+01) | 7.84E+01 - (7.25E+01) | 2.43E+01 (4.11E+01) | 9.09E+01 = (7.94E+01) | 9.96E+01 = (8.91E+01) | 2.86E+01 = (4.42E+01) | 1.18E+01 = (8.29E+00) |
| | cec14 F22 | 2.45E+01 - (3.35E+00) | 2.31E+01 (2.00E+00) | 3.43E+01 - (2.47E+01) | 2.54E+01 (4.05E+00) | 5.17E+01 - (5.09E+01) | 3.76E+01 (3.85E+01) | 2.91E+01 - (2.45E+01) | 2.31E+01 (3.73E+00) |
| Composition Functions | cec14 F23 | 3.15E+02 = (4.02E-13) | 3.15E+02 = (3.18E-13) | 3.15E+02 = (4.02E-13) | 3.15E+02 = (4.02E-13) | 3.15E+02 = (4.02E-13) | 3.15E+02 = (4.16E-13) | 3.15E+02 = (4.16E-13) | 3.15E+02 = (4.02E-13) |
| | cec14 F24 | 2.24E+02 - (1.46E+00) | 2.22E+02 (3.44E+00) | 2.24E+02 - (1.95E+00) | 2.22E+02 (4.63E+00) | 2.11E+02 = (1.10E+01) | 2.11E+02 = (1.10E+01) | 2.09E+02 - (1.08E+01) | 2.02E+02 (5.83E+00) |
| | cec14 F25 | 2.03E+02 - (5.33E-02) | 2.03E+02 (4.10E-02) | 2.03E+02 - (3.95E-02) | 2.03E+02 (4.46E-02) | 2.03E+02 = (3.95E-02) | 2.03E+02 = (3.24E-02) | 2.03E+02 = (2.75E-02) | 2.03E+02 = (2.60E-02) |
| | cec14 F26 | 1.00E+02 = (1.47E-02) | 1.00E+02 = (1.38E-02) | 1.00E+02 = (1.92E-02) | 1.00E+02 = (1.98E-02) | 1.00E+02 - (1.25E-02) | 1.00E+02 (1.64E-02) | 1.00E+02 = (2.13E-02) | 1.00E+02 = (2.44E-02) |
| | cec14 F27 | 3.00E+02 + (1.25E-13) | 3.00E+02 = (2.16E-13) | 3.02E+02 = (1.40E+01) | 3.02E+02 = (1.40E+01) | 3.00E+02 - (1.85E-13) | 3.00E+02 (9.09E-14) | 3.00E+02 = (2.30E-13) | 3.00E+02 = (1.23E-05) |
| | cec14 F28 | 8.35E+02 = (1.83E+01) | 8.33E+02 = (1.96E+01) | 8.39E+02 = (1.42E+01) | 8.35E+02 = (1.53E+01) | 8.37E+02 = (1.56E+01) | 8.37E+02 = (1.81E+01) | 8.25E+02 - (2.15E+01) | 8.16E+02 (1.94E+01) |
| | cec14 F29 | 7.16E+02 = (2.52E+00) | 7.15E+02 = (1.55E+00) | 7.17E+02 - (3.10E+00) | 7.16E+02 (2.28E+00) | 7.22E+02 = (1.17E+01) | 7.20E+02 = (6.36E+00) | 7.16E+02 - (2.07E+00) | 7.15E+02 (1.17E+00) |
| | cec14 F30 | 1.40E+03 = (6.66E+02) | 1.37E+03 = (6.31E+02) | 9.28E+02 = (3.55E+02) | 9.35E+02 = (4.83E+02) | 1.46E+03 = (6.33E+02) | 1.51E+03 = (6.72E+02) | 6.20E+02 - (1.67E+02) | 5.70E+02 (1.73E+02) |
| -/+ | | 10/18/2 | | 9/20/1 | | 8/22/0 | | 7/21/2 | |

Note: The structural bias that affects the performance of UMOEA-II and L-SHADE_EpSin were removed according to the suggestions in [5]. In detail, in UMOEA-II and SCSS-UMOEa-II, the mutation strategy $V_{i,G} = F_i \times X_{r1,G} + (X_{r2,G} - X_{r3,G})$ was modified as $V_{i,G} = X_{r1,G} + (X_{r2,G} - X_{r3,G})$ by setting $F_i = 1$. In L-SHADE_EpSin and SCSS-L-SHADE_EpSin, the local search procedures were skipped.

[5] A. P. Piotrowski and J. J. Napiorkowski, Some metaheuristics should be simplified, Inf. Sci, 427 (2018) 32-62.

TABLE S8 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED TOP ALGORITHMS WITH THE BASELINES
ON 50-D CEC2014 BENCHMARK SET

| | | L-SHADE | SCSS- L-SHADE | UMOEa-II | SCSS- UMOEa-II | L-SHADE_ EpSin | SCSS- L-SHADE_ EpSin | jSO | SCSS- jSO |
|--------------------------------|-----------|--|--------------------------------------|--|--------------------------------------|--------------------------|--------------------------------------|--|--------------------------------------|
| Unimodal Functions | cec14 F1 | 9.71E+02 - (1.66E+03) | 1.04E+02 (5.89E+02) | 1.17E-03 - (9.11E-04) | 5.83E-04 (3.83E-04) | 1.33E-02 - (7.34E-02) | 5.13E-05 (3.62E-04) | 1.49E+01 - (3.06E+01) | 1.59E+00 (2.80E+00) |
| | cec14 F2 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| | cec14 F3 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| Simple Multimodal Functions | cec14 F4 | 8.23E+01 = (3.38E+01) | 7.62E+01 (4.00E+01) | 2.69E+01 + (4.42E+01) | 5.00E+01 (4.95E+01) | 5.65E+01 - (4.83E+01) | 4.12E+01 (4.81E+01) | 5.02E+01 = (4.93E+01) | 5.79E+01 (4.86E+01) |
| | cec14 F5 | 2.03E+01 - (3.08E-02) | 2.02E+01 (8.40E-02) | 2.00E+01 = (6.24E-04) | 2.00E+01 (4.88E-06) | 2.03E+01 - (3.24E-02) | 2.02E+01 (7.18E-02) | 2.11E+01 = (5.59E-02) | 2.11E+01 (5.17E-02) |
| | cec14 F6 | 9.14E-02 - (2.74E-01) | 5.69E-02 (2.45E-01) | 3.49E-01 - (4.91E-01) | 8.13E-02 (3.21E-01) | 2.04E-04 - (2.15E-04) | 2.14E-05 (4.97E-05) | 3.80E-03 - (5.50E-03) | 3.66E-02 (1.44E-01) |
| | cec14 F7 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| | cec14 F8 | 3.64E-08 - (3.14E-08) | 2.37E-08 (4.23E-08) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 3.53E-09 = (2.52E-08) | 0.00E+00 (0.00E+00) | 0.00E+00 + (0.00E+00) | 1.82E-09 (6.41E-09) |
| | cec14 F9 | 1.26E+01 - (2.44E+00) | 1.18E+01 (2.02E+00) | 1.60E+01 - (4.61E+00) | 1.39E+01 (3.94E+00) | 3.03E+01 - (5.20E+00) | 1.90E+01 (5.72E+00) | 1.59E+01 - (3.69E+00) | 1.13E+01 (2.93E+00) |
| | cec14 F10 | 1.72E-01 - (5.24E-02) | 1.38E-01 (5.18E-02) | 1.30E+00 + (1.19E+00) | 3.53E+00 (2.29E+00) | 4.17E-02 = (2.19E-02) | 3.73E-02 (1.78E-02) | 9.92E+00 = (3.90E+00) | 8.40E+00 (3.24E+00) |
| | cec14 F11 | 3.42E+03 - (3.46E+02) | 3.28E+03 (3.38E+02) | 3.94E+03 = (7.60E+02) | 3.93E+03 (6.03E+02) | 3.09E+03 = (3.06E+02) | 3.00E+03 (3.23E+02) | 3.22E+03 = (3.37E+02) | 3.26E+03 (3.75E+02) |
| | cec14 F12 | 2.44E-01 - (3.53E-02) | 2.11E-01 (3.26E-02) | 1.63E-01 = (1.06E-01) | 1.68E-01 (1.06E-01) | 2.16E-01 - (2.70E-02) | 1.99E-01 (2.81E-02) | 3.69E-01 = (4.10E-01) | 7.48E-01 (7.45E-01) |
| | cec14 F13 | 1.60E-01 - (1.74E-02) | 1.50E-01 (2.08E-02) | 1.63E-01 = (2.40E-02) | 1.60E-01 (2.33E-02) | 2.06E-01 - (2.08E-02) | 1.90E-01 (2.35E-02) | 1.92E-01 = (2.83E-02) | 2.01E-01 (4.22E-02) |
| | cec14 F14 | 3.23E-01 - (4.96E-02) | 2.49E-01 (9.34E-02) | 3.01E-01 - (2.29E-02) | 2.63E-01 (2.99E-02) | 1.89E-01 - (2.33E-02) | 1.84E-01 (3.13E-02) | 2.91E-01 - (4.34E-02) | 2.73E-01 (4.15E-02) |
| | cec14 F15 | 5.30E+00 - (5.66E-01) | 4.99E+00 (4.75E-01) | 5.39E+00 = (1.04E+00) | 5.13E+00 (1.06E+00) | 5.68E+00 - (4.74E-01) | 5.04E+00 (5.05E-01) | 5.18E+00 - (4.85E-01) | 4.68E+00 (6.92E+00) |
| | cec14 F16 | 1.69E+01 + (4.35E-01) | 1.71E+01 (4.88E-01) | 1.84E+01 + (7.63E-01) | 1.86E+01 (6.65E-01) | 1.67E+01 - (3.44E-01) | 1.65E+01 (4.28E-01) | 1.70E+01 + (9.41E-01) | 1.73E+01 (7.30E-01) |
| | cec14 F17 | 1.63E+03 - (3.52E+02) | 5.59E+02 (2.32E+02) | 1.11E+03 - (3.60E+02) | 3.94E+02 (1.81E+02) | 3.60E+02 = (1.60E+02) | 3.51E+02 (1.39E+02) | 3.51E+02 - (1.70E+02) | 1.76E+02 (1.11E+02) |
| | cec14 F18 | 1.05E+02 - (1.38E+01) | 2.30E+01 (6.42E+00) | 5.70E+01 - (2.14E+01) | 1.56E+01 (4.28E+00) | 1.89E+01 = (6.40E+00) | 1.83E+01 (6.76E+00) | 1.08E+01 - (3.24E+00) | 7.21E+00 (2.16E+00) |
| | cec14 F19 | 8.11E+00 + (1.87E+00) | 9.64E+00 (1.45E+00) | 8.17E+00 = (2.20E+00) | 7.66E+00 (2.39E+00) | 9.99E+00 - (8.84E-01) | 9.76E+00 (8.22E-01) | 9.25E+00 - (8.19E-01) | 8.56E+00 (7.29E-01) |
| | cec14 F20 | 1.45E+01 - (3.75E+00) | 7.96E+00 (1.96E+00) | 1.34E+01 - (3.52E+00) | 9.33E+00 (1.30E+00) | 6.04E+00 = (2.23E+00) | 5.93E+00 (1.86E+00) | 5.67E+00 = (1.95E+00) | 5.17E+00 (1.71E+00) |
| | cec14 F21 | 5.59E+02 - (1.62E+02) | 3.42E+02 (1.11E+02) | 4.38E+02 - (1.27E+02) | 3.49E+02 (1.32E+02) | 3.25E+02 = (9.65E+01) | 3.08E+02 (1.05E+02) | 3.03E+02 - (9.88E+01) | 2.36E+02 (8.45E+01) |
| | cec14 F22 | 1.03E+02 = (7.30E+01) | 9.95E+01 (7.03E+01) | 1.81E+02 = (8.35E+01) | 1.93E+02 (1.19E+02) | 9.35E+01 - (6.13E+01) | 6.34E+01 (5.00E+01) | 1.51E+02 - (1.00E+02) | 1.03E+02 (8.34E+01) |
| Composition Functions | cec14 F23 | 3.44E+02 = (3.20E-13) | 3.44E+02 (3.46E-13) | 3.44E+02 = (4.67E-13) | 3.44E+02 (4.73E-13) | 3.44E+02 = (2.93E-13) | 3.44E+02 (3.18E-13) | 3.44E+02 = (3.03E-13) | 3.44E+02 (3.46E-13) |
| | cec14 F24 | 2.75E+02 - (4.98E-01) | 2.74E+02 (1.13E+00) | 2.75E+02 - (8.57E-01) | 2.75E+02 (7.27E-01) | 2.68E+02 = (1.23E+00) | 2.68E+02 (1.50E+00) | 2.72E+02 - (1.80E+00) | 2.70E+02 (2.18E+00) |
| | cec14 F25 | 2.05E+02 - (3.48E-01) | 2.05E+02 (2.33E-01) | 2.05E+02 = (2.98E-01) | 2.05E+02 (3.00E-01) | 2.05E+02 = (1.39E-01) | 2.05E+02 (9.28E-02) | 2.05E+02 - (1.82E-01) | 2.05E+02 (1.35E-01) |
| | cec14 F26 | 1.00E+02 = (1.98E-02) | 1.00E+02 (1.66E-02) | 1.00E+02 = (2.50E-02) | 1.00E+02 (2.05E-02) | 1.00E+02 - (4.98E-02) | 1.00E+02 (3.46E-02) | 1.00E+02 = (2.37E-02) | 1.00E+02 (3.87E-02) |
| | cec14 F27 | 3.42E+02 - (2.68E+01) | 3.35E+02 (2.17E+01) | 3.34E+02 - (3.31E+01) | 3.23E+02 (2.59E+01) | 3.17E+02 = (2.28E+01) | 3.25E+02 (2.34E+01) | 3.10E+02 - (1.85E+01) | 3.10E+02 (1.84E+01) |
| | cec14 F28 | 1.13E+03 = (3.69E+01) | 1.12E+03 (3.09E+01) | 1.12E+03 = (2.83E+01) | 1.11E+03 (2.69E+01) | 1.14E+03 = (3.72E+01) | 1.14E+03 (3.83E+01) | 1.09E+03 = (2.81E+01) | 1.08E+03 (3.04E+01) |
| | cec14 F29 | 8.04E+02 = (3.34E+01) | 8.02E+02 (3.22E+01) | 8.05E+02 = (4.27E+01) | 7.95E+02 (3.95E+01) | 8.05E+02 = (2.77E+01) | 8.13E+02 (4.03E+01) | 8.04E+02 = (4.11E+01) | 8.03E+02 (4.48E+01) |
| | cec14 F30 | 8.59E+03 = (4.15E+02) | 8.53E+03 (3.14E+02) | 8.62E+03 = (4.71E+02) | 8.64E+03 (5.04E+02) | 8.50E+03 = (3.71E+02) | 8.60E+03 (4.33E+02) | 8.38E+03 = (3.90E+02) | 8.30E+03 (3.38E+02) |
| -/+/- | | 18/10/2 | | 10/17/3 | | 13/17/0 | | 13/15/2 | |

TABLE S9 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED ADVANCED ALGORITHMS WITH THE BASELINES
ON 30-D CEC2017 BENCHMARK SET

| | | JADE | SCSS-JADE | SHADE | SCSS-SHADE | CMA-ES | SCSS-CMA-ES | LIPS | SCSS-LIPS |
|-----------------------------|-----------|-----------------------|----------------------------|-----------------------|----------------------------|------------------------------|----------------------------|------------------------------|----------------------------|
| Unimodal Functions | cec17 F1 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 8.03E+02 + (1.45E+03) | 2.73E+03 (4.18E+03) |
| | cec17 F2 | 1.58E-05 = (8.56E-06) | 1.70E-05 (9.99E-06) | 1.77E-05 = (1.03E-05) | 1.39E-05 (8.49E-06) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 2.33E+01 - (9.02E+01) | 1.87E-03 (1.95E-04) |
| | cec17 F3 | 1.18E+04 - (1.92E+04) | 7.74E+02 (5.53E+03) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 1.60E+04 - (7.66E+03) | 7.74E+03 (3.55E+03) |
| Simple Multimodal Functions | cec17 F4 | 5.18E+01 = (2.08E+01) | 5.14E+01 (2.06E+01) | 5.47E+01 = (1.62E+01) | 5.29E+01 (1.76E+01) | 3.99E+01 + (2.74E+01) | 4.30E+01 (2.55E+01) | 1.64E+02 - (9.39E+01) | 1.11E+02 (4.93E+01) |
| | cec17 F5 | 2.83E+01 - (4.01E+00) | 2.17E+01 (4.50E+00) | 1.99E+01 = (3.24E+00) | 1.97E+01 (3.18E+00) | 6.58E+02 - (2.22E+02) | 1.34E+02 (2.26E+02) | 6.43E+01 - (1.35E+01) | 3.43E+01 (9.30E+00) |
| | cec17 F6 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 9.91E+01 - (1.56E+01) | 3.99E+01 (4.70E+01) | 8.27E+00 - (5.05E+00) | 4.58E-01 (5.87E-01) |
| | cec17 F7 | 5.61E+01 - (3.87E+00) | 5.19E+01 (4.41E+00) | 5.09E+01 - (3.87E+00) | 4.92E+01 (2.84E+00) | 3.66E+03 - (1.11E+03) | 2.71E+02 (8.12E+02) | 9.77E+01 - (2.10E+01) | 7.32E+01 (1.09E+01) |
| | cec17 F8 | 2.84E+01 - (5.00E+00) | 2.39E+01 (4.09E+00) | 2.16E+01 = (3.42E+00) | 2.07E+01 (3.64E+00) | 5.79E+02 - (1.43E+02) | 1.60E+02 (2.05E+02) | 6.23E+01 - (1.31E+01) | 3.58E+01 (8.35E+00) |
| | cec17 F9 | 2.13E-02 = (9.01E-02) | 7.02E-03 (2.43E-02) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 1.37E+04 - (3.23E+03) | 5.85E+03 (7.13E+03) | 6.01E+02 - (4.21E+02) | 2.07E+01 (2.53E+01) |
| | cec17 F10 | 1.88E+03 - (2.70E+02) | 1.79E+03 (2.39E+02) | 1.73E+03 = (2.71E+02) | 1.72E+03 (2.46E+02) | 4.93E+03 - (5.98E+02) | 4.05E+03 (1.01E+03) | 2.80E+03 - (4.44E+02) | 2.15E+03 (3.40E+02) |
| Hybrid Functions | cec17 F11 | 3.37E+01 - (2.26E+01) | 2.28E+01 (2.00E+01) | 2.10E+01 = (2.53E+01) | 2.13E+01 (2.47E+01) | 1.67E+02 - (5.67E+01) | 1.20E+02 (3.97E+01) | 1.99E+02 - (1.41E+02) | 8.58E+01 (4.31E+01) |
| | cec17 F12 | 1.48E+03 = (8.87E+02) | 1.30E+03 (7.31E+02) | 2.03E+03 - (2.68E+03) | 1.20E+03 (5.83E+02) | 1.51E+03 = (3.69E+02) | 1.55E+03 (3.41E+02) | 1.85E+06 - (6.00E+06) | 1.78E+05 (2.11E+05) |
| | cec17 F13 | 4.36E+01 = (2.16E+01) | 3.92E+01 (1.61E+01) | 3.84E+01 - (1.76E+01) | 2.68E+01 (1.20E+01) | 1.57E+03 = (7.42E+02) | 1.35E+03 (7.07E+02) | 5.74E+03 - (5.63E+03) | 2.78E+03 (4.82E+03) |
| | cec17 F14 | 9.70E+03 - (1.12E+04) | 2.05E+03 (7.03E+03) | 2.73E+01 = (5.83E+00) | 2.61E+01 (4.08E+00) | 1.85E+02 = (5.74E+01) | 1.66E+02 (5.33E+01) | 1.40E+04 - (1.13E+04) | 8.81E+03 (2.02E+04) |
| | cec17 F15 | 1.94E+03 - (3.78E+03) | 1.14E+02 (6.60E+02) | 1.32E+01 = (9.70E+00) | 1.05E+01 (5.76E+00) | 3.09E+02 = (1.32E+02) | 2.83E+02 (1.36E+02) | 2.35E+03 - (3.05E+03) | 1.40E+03 (2.16E+03) |
| | cec17 F16 | 3.92E+02 - (1.27E+02) | 3.27E+02 (1.28E+02) | 2.91E+02 - (1.16E+02) | 2.43E+02 (1.35E+02) | 5.92E+02 - (2.96E+02) | 3.36E+02 (2.36E+02) | 7.30E+02 - (2.21E+02) | 4.78E+02 (1.61E+02) |
| | cec17 F17 | 8.33E+01 - (2.86E+01) | 7.21E+01 (2.09E+01) | 4.83E+01 = (1.29E+01) | 5.10E+01 (9.63E+00) | 2.80E+02 - (2.03E+02) | 1.45E+02 (9.83E+01) | 2.89E+02 - (1.19E+02) | 1.52E+02 (6.88E+01) |
| | cec17 F18 | 5.06E+04 - (7.16E+04) | 7.69E+03 (3.87E+04) | 7.32E+01 - (4.20E+01) | 3.43E+01 (1.53E+01) | 2.07E+02 = (8.94E+01) | 1.98E+02 (7.43E+01) | 1.71E+05 - (1.53E+05) | 1.16E+05 (6.72E+04) |
| | cec17 F19 | 1.88E+03 - (4.75E+03) | 1.20E+01 (6.37E+00) | 7.83E+00 = (3.06E+00) | 7.40E+00 (2.40E+00) | 2.04E+02 - (8.72E+01) | 1.73E+02 (6.95E+01) | 1.55E+03 = (1.99E+03) | 1.61E+03 (3.30E+03) |
| | cec17 F20 | 9.72E+01 - (5.22E+01) | 7.83E+01 (4.58E+01) | 6.23E+01 = (3.64E+01) | 5.43E+01 (3.33E+01) | 1.38E+03 - (3.73E+02) | 2.05E+02 (1.65E+02) | 3.21E+02 - (1.02E+02) | 1.83E+02 (7.84E+01) |
| Composition Functions | cec17 F21 | 2.28E+02 - (4.78E+00) | 2.22E+02 (4.93E+00) | 2.21E+02 = (3.13E+00) | 2.20E+02 (3.86E+00) | 4.92E+02 - (2.67E+02) | 3.03E+02 (1.56E+02) | 2.65E+02 - (1.55E+01) | 2.39E+02 (9.85E+00) |
| | cec17 F22 | 1.00E+02 = (2.56E-05) | 1.39E+02 (2.76E+02) | 1.00E+02 = (1.00E-13) | 1.00E+02 (1.00E-13) | 5.70E+03 - (1.03E+03) | 3.05E+03 (2.50E+03) | 1.58E+02 - (4.06E+02) | 1.00E+02 (2.11E-13) |
| | cec17 F23 | 3.75E+02 - (6.33E+00) | 3.71E+02 (6.99E+00) | 3.68E+02 = (4.87E+00) | 3.66E+02 (5.71E+00) | 1.99E+03 - (8.26E+02) | 6.46E+02 (6.87E+02) | 4.45E+02 - (3.32E+01) | 3.91E+02 (1.08E+01) |
| | cec17 F24 | 4.40E+02 - (4.90E+00) | 4.36E+02 (5.27E+00) | 4.38E+02 - (3.82E+00) | 4.36E+02 (3.77E+00) | 4.74E+02 = (9.73E+01) | 4.57E+02 (1.09E+01) | 5.00E+02 - (2.83E+01) | 4.49E+02 (1.02E+01) |
| | cec17 F25 | 3.87E+02 - (1.86E-01) | 3.87E+02 (1.72E-01) | 3.87E+02 - (1.38E-01) | 3.87E+02 (1.33E-01) | 3.87E+02 - (2.74E+00) | 3.87E+02 (2.71E-02) | 4.29E+02 - (2.71E+01) | 3.99E+02 (1.32E+01) |
| | cec17 F26 | 1.19E+03 - (1.51E+02) | 1.16E+03 (8.12E+01) | 1.12E+03 = (6.24E+01) | 1.09E+03 (6.26E+01) | 1.20E+03 - (4.75E+02) | 1.20E+03 (3.22E+02) | 1.47E+03 - (8.10E+02) | 1.14E+03 (5.73E+02) |
| | cec17 F27 | 5.01E+02 = (7.16E+00) | 5.03E+02 (7.65E+00) | 5.02E+02 = (5.62E+00) | 5.02E+02 (4.92E+00) | 8.04E+02 - (1.74E+03) | 4.86E+02 (1.08E+01) | 6.12E+02 - (2.52E+01) | 5.56E+02 (1.69E+01) |
| | cec17 F28 | 3.41E+02 = (5.64E+01) | 3.34E+02 (5.44E+01) | 3.34E+02 = (5.47E+01) | 3.30E+02 (4.90E+01) | 3.51E+02 = (6.13E+01) | 3.42E+02 (5.34E+01) | 5.00E+02 - (9.70E+01) | 3.90E+02 (7.31E+01) |
| | cec17 F29 | 4.85E+02 - (2.28E+01) | 4.74E+02 (1.52E+01) | 4.63E+02 = (2.62E+01) | 4.65E+02 (1.66E+01) | 7.88E+02 - (1.84E+02) | 6.36E+02 (1.25E+02) | 9.73E+02 - (1.78E+02) | 7.05E+02 (7.69E+01) |
| | cec17 F30 | 2.79E+03 = (2.00E+03) | 2.13E+03 (1.42E+02) | 2.10E+03 = (1.27E+02) | 2.08E+03 (1.39E+02) | 2.22E+03 = (2.09E+02) | 2.19E+03 (2.20E+02) | 1.19E+05 - (1.81E+05) | 1.20E+04 (5.61E+03) |
| -/+ | | 19/11/0 | | 7/23/0 | | 18/11/1 | | 28/1/1 | |

TABLE S10 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED ADVANCED ALGORITHMS WITH THE BASELINES
ON 50-D CEC2017 BENCHMARK SET

| | | JADE | SCSS-JADE | SHADE | SCSS-SHADE | CMA-ES | SCSS-CMA-ES | LIPS | SCSS-LIPS |
|-----------------------------|-----------|------------------------------|----------------------------|-----------------------|----------------------------|-----------------------|----------------------------|------------------------------|----------------------------|
| Unimodal Functions | cec17 F1 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 1.17E+03 + (2.02E+03) | 2.89E+03 (4.25E+03) |
| | cec17 F2 | 4.21E-05 + (1.21E-05) | 4.93E-05 (1.63E-05) | 5.08E-05 = (1.48E-05) | 5.41E-05 (1.87E-05) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 7.62E+02 - (7.84E+02) | 3.25E-03 (4.46E-04) |
| | cec17 F3 | 1.42E+04 - (3.38E+04) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 9.27E+04 - (2.23E+04) | 6.53E+04 (1.57E+04) |
| Simple Multimodal Functions | cec17 F4 | 5.46E+01 = (5.18E+01) | 5.37E+01 (5.01E+01) | 6.40E+01 = (5.03E+01) | 5.50E+01 (4.53E+01) | 4.34E+01 = (4.79E+01) | 3.61E+01 (4.31E+01) | 6.66E+02 - (3.39E+02) | 2.52E+02 (7.79E+01) |
| | cec17 F5 | 5.18E+01 - (9.01E+00) | 3.98E+01 (9.33E+00) | 4.35E+01 - (5.40E+00) | 3.89E+01 (6.36E+00) | 1.03E+03 - (1.78E+02) | 6.32E+02 (4.78E+02) | 1.68E+02 - (2.62E+01) | 1.00E+02 (2.00E+01) |
| | cec17 F6 | 0.00E+00 + (0.00E+00) | 5.77E-07 (2.18E-06) | 1.59E-06 = (2.26E-06) | 1.67E-06 (1.87E-06) | 9.54E+01 - (1.04E+01) | 7.49E+01 (3.66E+01) | 2.41E+01 - (5.43E+00) | 4.92E+00 (2.13E+00) |
| | cec17 F7 | 9.89E+01 - (8.16E+00) | 8.94E+01 (8.04E+00) | 8.91E+01 - (5.48E+00) | 8.60E+01 (5.82E+00) | 6.42E+03 - (1.55E+03) | 1.65E+03 (2.74E+03) | 3.74E+02 - (6.09E+01) | 1.74E+02 (2.69E+01) |
| | cec17 F8 | 5.43E+01 - (8.64E+00) | 4.17E+01 (8.53E+00) | 4.21E+01 = (6.54E+00) | 4.10E+01 (7.27E+00) | 1.09E+03 - (2.12E+02) | 5.94E+02 (4.60E+02) | 1.74E+02 - (3.49E+01) | 1.02E+02 (1.71E+01) |
| | cec17 F9 | 1.44E+00 = (1.52E+00) | 1.46E+00 (1.26E+00) | 3.87E-01 = (3.94E-01) | 3.55E-01 (4.33E-01) | 3.08E+04 = (5.49E+03) | 2.64E+04 (1.16E+04) | 4.44E+03 - (1.45E+03) | 8.85E+02 (5.90E+02) |
| | cec17 F10 | 3.70E+03 - (3.77E+02) | 3.49E+03 (3.97E+02) | 3.48E+03 = (3.77E+02) | 3.43E+03 (3.50E+02) | 8.04E+03 - (9.92E+02) | 7.19E+03 (1.22E+03) | 5.14E+03 - (6.66E+02) | 4.24E+03 (6.02E+02) |
| Hybrid Functions | cec17 F11 | 1.57E+02 - (5.18E+01) | 1.32E+02 (3.61E+01) | 8.67E+01 - (2.71E+01) | 6.88E+01 (1.66E+01) | 2.88E+02 - (6.63E+01) | 2.08E+02 (5.01E+01) | 2.35E+03 - (2.45E+03) | 2.58E+02 (8.87E+01) |
| | cec17 F12 | 7.02E+03 = (6.81E+03) | 6.57E+03 (3.92E+03) | 5.66E+03 = (3.09E+03) | 6.95E+03 (4.86E+03) | 2.66E+03 = (6.49E+02) | 2.64E+03 (6.45E+02) | 1.35E+07 - (4.17E+07) | 1.84E+06 (1.55E+06) |
| | cec17 F13 | 2.52E+02 = (1.52E+02) | 2.10E+02 (1.23E+02) | 2.94E+02 - (1.94E+02) | 1.33E+02 (5.36E+01) | 2.55E+03 = (7.76E+02) | 2.28E+03 (7.63E+02) | 6.58E+03 - (3.64E+03) | 1.16E+03 (7.74E+02) |
| | cec17 F14 | 6.91E+04 - (1.19E+05) | 5.09E+03 (2.12E+04) | 1.82E+02 - (4.59E+01) | 8.43E+01 (2.75E+01) | 3.16E+02 = (7.64E+01) | 2.97E+02 (9.08E+01) | 1.32E+05 - (3.30E+05) | 2.61E+04 (2.66E+04) |
| | cec17 F15 | 1.13E+03 - (2.51E+03) | 1.92E+02 (9.30E+01) | 2.52E+02 - (1.05E+02) | 1.28E+02 (5.77E+01) | 4.88E+02 = (1.68E+02) | 4.84E+02 (1.20E+02) | 1.97E+03 - (1.89E+03) | 8.09E+02 (6.53E+02) |
| | cec17 F16 | 9.06E+02 - (1.65E+02) | 7.24E+02 (1.67E+02) | 7.26E+02 = (1.83E+02) | 7.44E+02 (1.31E+02) | 9.06E+02 - (3.97E+02) | 5.49E+02 (3.04E+02) | 1.44E+03 - (3.37E+02) | 9.12E+02 (2.46E+02) |
| | cec17 F17 | 6.40E+02 - (1.59E+02) | 5.52E+02 (1.55E+02) | 4.78E+02 = (1.37E+02) | 4.90E+02 (1.25E+02) | 9.86E+02 - (2.57E+02) | 5.71E+02 (2.25E+02) | 1.16E+03 - (2.11E+02) | 7.70E+02 (1.70E+02) |
| | cec17 F18 | 1.82E+05 - (4.33E+05) | 1.59E+02 (1.54E+02) | 1.38E+02 - (8.50E+01) | 1.10E+02 (7.29E+01) | 3.60E+02 = (1.23E+02) | 3.31E+02 (1.07E+02) | 1.21E+06 - (2.22E+06) | 3.56E+05 (2.38E+05) |
| | cec17 F19 | 9.41E+02 - (2.46E+03) | 1.19E+02 (4.55E+01) | 1.14E+02 - (4.32E+01) | 7.53E+01 (3.39E+01) | 2.71E+02 = (1.30E+02) | 2.43E+02 (7.61E+01) | 3.34E+03 = (4.99E+03) | 3.26E+03 (5.11E+03) |
| | cec17 F20 | 4.74E+02 - (1.35E+02) | 3.97E+02 (1.28E+02) | 3.46E+02 = (1.19E+02) | 3.27E+02 (9.96E+01) | 2.37E+03 - (5.04E+02) | 8.23E+02 (8.32E+02) | 6.79E+02 - (1.67E+02) | 4.60E+02 (1.57E+02) |
| Composition Functions | cec17 F21 | 2.54E+02 - (1.03E+01) | 2.41E+02 (8.60E+00) | 2.44E+02 = (6.19E+00) | 2.42E+02 (7.15E+00) | 7.97E+02 - (4.85E+02) | 4.13E+02 (3.21E+02) | 3.60E+02 - (3.55E+01) | 3.01E+02 (1.72E+01) |
| | cec17 F22 | 3.68E+03 - (1.67E+03) | 3.41E+03 (1.45E+03) | 3.50E+03 = (1.50E+03) | 3.27E+03 (1.57E+03) | 9.11E+03 - (1.09E+03) | 7.94E+03 (1.30E+03) | 4.55E+03 - (2.41E+03) | 3.92E+03 (1.87E+03) |
| | cec17 F23 | 4.79E+02 - (1.09E+01) | 4.65E+02 (1.01E+01) | 4.66E+02 - (8.46E+00) | 4.60E+02 (8.48E+00) | 3.18E+03 - (6.79E+02) | 1.20E+03 (1.18E+03) | 7.13E+02 - (6.14E+01) | 5.59E+02 (2.46E+01) |
| | cec17 F24 | 5.40E+02 - (8.46E+00) | 5.29E+02 (6.59E+00) | 5.35E+02 - (8.93E+00) | 5.30E+02 (6.90E+00) | 7.00E+02 - (2.49E+02) | 5.72E+02 (2.19E+01) | 7.71E+02 - (7.71E+01) | 6.05E+02 (1.99E+01) |
| | cec17 F25 | 5.23E+02 = (3.28E+01) | 5.20E+02 (3.62E+01) | 5.15E+02 = (3.61E+01) | 5.08E+02 (3.75E+01) | 5.02E+02 = (3.32E+01) | 4.94E+02 (2.97E+01) | 9.66E+02 - (2.15E+02) | 6.35E+02 (4.87E+01) |
| | cec17 F26 | 1.63E+03 - (1.22E+02) | 1.50E+03 (1.34E+02) | 1.45E+03 - (9.07E+01) | 1.41E+03 (9.53E+01) | 1.90E+03 - (5.02E+02) | 1.76E+03 (5.10E+02) | 3.87E+03 - (6.48E+02) | 2.19E+03 (6.09E+02) |
| | cec17 F27 | 5.58E+02 = (2.58E+01) | 5.55E+02 (2.94E+01) | 5.37E+02 = (1.88E+01) | 5.31E+02 (1.33E+01) | 7.55E+02 - (1.17E+03) | 4.76E+02 (1.37E+01) | 1.19E+03 - (9.61E+01) | 8.66E+02 (6.62E+01) |
| | cec17 F28 | 4.91E+02 = (2.25E+01) | 4.94E+02 (2.11E+01) | 4.82E+02 = (2.44E+01) | 4.85E+02 (2.38E+01) | 4.70E+02 = (2.01E+01) | 4.64E+02 (1.60E+01) | 1.49E+03 - (4.96E+02) | 6.25E+02 (5.57E+01) |
| | cec17 F29 | 4.60E+02 = (6.92E+01) | 4.72E+02 (7.48E+01) | 4.38E+02 = (5.83E+01) | 4.46E+02 (5.42E+01) | 1.04E+03 - (2.96E+02) | 6.93E+02 (1.73E+02) | 2.02E+03 - (3.35E+02) | 1.12E+03 (1.80E+02) |
| | cec17 F30 | 6.64E+05 = (9.01E+04) | 6.56E+05 (8.03E+04) | 6.57E+05 = (7.82E+04) | 6.54E+05 (6.50E+04) | 7.86E+05 = (1.45E+05) | 7.87E+05 (1.72E+05) | 3.31E+07 - (1.45E+07) | 4.90E+06 (1.58E+06) |
| -/+/- | | 18/10/2 | | 11/19/0 | | 16/14/0 | | 28/1/1 | |

TABLE S11 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED TOP ALGORITHMS WITH THE BASELINES
ON 30-D CEC2017 BENCHMARK SET

| | | L-SHADE | SCSS- L-SHADE | UMOEa-II | SCSS- UMOEa-II | L-SHADE_ EpSin | SCSS- L-SHADE_ EpSin | jSO | SCSS- jSO |
|--------------------------------|-----------|--|--------------------------------------|--|--------------------------------------|--|--------------------------------------|--------------------------|--------------------------------------|
| Unimodal Functions | cec17 F1 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| | cec17 F2 | 4.06E-09 - (8.59E-09) | 0.00E+00 (0.00E+00) | 4.14E-08 = (5.51E-08) | 3.23E-08 (5.00E-08) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 6.65E-08 = (9.56E-08) | 9.39E-08 (9.54E-08) |
| | cec17 F3 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| Simple Multimodal Functions | cec17 F4 | 5.86E+01 = (3.75E-14) | 5.86E+01 (3.27E-14) | 5.86E+01 = (4.90E-14) | 5.87E+01 (7.78E-01) | 5.86E+01 = (2.88E-14) | 5.86E+01 (2.93E-14) | 5.86E+01 = (2.13E-14) | 5.86E+01 (2.41E-14) |
| | cec17 F5 | 7.02E+00 = (1.52E+00) | 7.61E+00 (1.58E+00) | 8.29E+00 = (2.19E+00) | 8.54E+00 (2.06E+00) | 1.22E+01 - (1.60E+00) | 1.06E+01 (2.43E+00) | 8.32E+00 - (1.74E+00) | 7.49E+00 (1.80E+00) |
| | cec17 F6 | 3.38E-09 = (1.98E-08) | 1.14E-08 (3.73E-08) | 1.81E-08 = (8.05E-08) | 6.71E-09 (2.74E-08) | 8.05E-09 = (3.25E-08) | 0.00E+00 (0.00E+00) | 9.39E-09 = (3.29E-08) | 1.74E-08 (4.45E-08) |
| | cec17 F7 | 3.79E+01 + (1.18E+00) | 3.91E+01 (2.03E+00) | 4.04E+01 = (2.73E+00) | 4.06E+01 (2.68E+00) | 4.35E+01 - (2.48E+00) | 4.19E+01 (2.75E+00) | 3.84E+01 - (1.83E+00) | 3.75E+01 (1.33E+00) |
| | cec17 F8 | 7.11E+00 = (1.58E+00) | 8.09E+00 (2.13E+00) | 8.45E+00 = (1.86E+00) | 8.54E+00 (2.36E+00) | 1.35E+01 - (1.50E+00) | 1.26E+01 (2.46E+00) | 8.81E+00 - (2.17E+00) | 7.57E+00 (2.04E+00) |
| | cec17 F9 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| | cec17 F10 | 1.41E+03 = (2.31E+02) | 1.44E+03 (2.33E+02) | 1.69E+03 = (3.17E+02) | 1.63E+03 (3.04E+02) | 1.35E+03 = (1.90E+02) | 1.28E+03 (2.38E+02) | 1.49E+03 = (2.66E+02) | 1.54E+03 (2.18E+02) |
| Hybrid Functions | cec17 F11 | 3.73E+01 - (2.91E+01) | 3.36E+01 (2.90E+01) | 1.34E+01 = (2.02E+01) | 1.53E+01 (2.34E+01) | 1.58E+01 = (2.30E+01) | 1.97E+01 (2.55E+01) | 9.87E+00 = (1.89E+01) | 6.46E+00 (1.39E+01) |
| | cec17 F12 | 1.04E+03 - (3.37E+02) | 6.95E+02 (3.16E+02) | 8.28E+02 - (3.18E+02) | 2.84E+02 (1.85E+02) | 4.03E+02 = (2.22E+02) | 3.77E+02 (2.15E+02) | 1.66E+02 - (8.86E+01) | 8.34E+01 (7.27E+01) |
| | cec17 F13 | 1.92E+01 - (4.61E+00) | 1.73E+01 (4.88E+00) | 1.53E+01 = (6.24E+00) | 1.61E+01 (5.99E+00) | 1.42E+01 = (6.02E+00) | 1.54E+01 (5.86E+00) | 1.60E+01 = (5.76E+00) | 1.63E+01 (4.50E+00) |
| | cec17 F14 | 2.19E+01 + (1.22E+00) | 2.22E+01 (3.11E+00) | 2.22E+01 = (3.42E+00) | 2.22E+01 (4.58E+00) | 2.13E+01 = (4.65E+00) | 2.26E+01 (1.20E+00) | 2.20E+01 = (1.08E+00) | 2.14E+01 (3.19E+00) |
| | cec17 F15 | 3.54E+00 - (1.56E+00) | 2.80E+00 (1.34E+00) | 3.30E+00 - (1.70E+00) | 2.83E+00 (2.22E+00) | 2.41E+00 = (1.44E+00) | 2.58E+00 (1.61E+00) | 1.26E+00 = (8.34E-01) | 1.03E+00 (8.73E-01) |
| | cec17 F16 | 4.00E+01 = (2.74E+01) | 3.43E+01 (1.48E+01) | 9.31E+01 = (9.08E+01) | 7.11E+01 (8.16E+01) | 5.09E+01 - (4.44E+01) | 3.12E+01 (3.38E+01) | 6.50E+01 = (6.92E+01) | 5.02E+01 (6.73E+01) |
| | cec17 F17 | 3.29E+01 = (6.27E+00) | 3.44E+01 (5.90E+00) | 4.07E+01 + (8.68E+00) | 4.46E+01 (1.00E+01) | 2.83E+01 = (6.47E+00) | 2.91E+01 (5.86E+00) | 3.45E+01 - (7.04E+00) | 3.17E+01 (7.19E+00) |
| | cec17 F18 | 2.23E+01 - (1.28E+00) | 2.04E+01 (2.79E+00) | 2.15E+01 = (6.94E-01) | 2.13E+01 (7.26E-01) | 2.13E+01 = (9.45E-01) | 2.13E+01 (9.30E-01) | 2.08E+01 = (3.79E-01) | 1.95E+01 (4.82E+00) |
| | cec17 F19 | 5.96E+00 = (1.87E+00) | 5.90E+00 (2.05E+00) | 6.38E+00 = (1.91E+00) | 7.13E+00 (2.35E+00) | 5.24E+00 = (1.63E+00) | 5.10E+00 (1.87E+00) | 4.53E+00 = (1.90E+00) | 4.06E+00 (1.43E+00) |
| | cec17 F20 | 3.01E+01 = (5.93E+00) | 2.99E+01 (4.37E+00) | 4.27E+01 = (9.05E+00) | 3.97E+01 (7.88E+00) | 2.83E+01 = (7.68E+00) | 2.60E+01 (5.45E+00) | 3.01E+01 = (8.53E+00) | 2.75E+01 (7.25E+00) |
| Composition Functions | cec17 F21 | 2.08E+02 = (1.65E+00) | 2.08E+02 (1.53E+00) | 2.09E+02 = (2.11E+00) | 2.10E+02 (2.43E+00) | 2.12E+02 - (2.62E+00) | 2.10E+02 (2.50E+00) | 2.09E+02 - (1.93E+00) | 2.08E+02 (2.04E+00) |
| | cec17 F22 | 1.00E+02 = (9.20E-14) | 1.00E+02 (1.00E-13) | 1.00E+02 = (1.39E-13) | 1.00E+02 (1.87E-13) | 1.00E+02 = (1.00E-13) | 1.00E+02 (1.00E-13) | 1.00E+02 = (9.20E-14) | 1.00E+02 (1.00E-13) |
| | cec17 F23 | 3.54E+02 = (3.16E+00) | 3.54E+02 (2.98E+00) | 3.54E+02 = (4.25E+00) | 3.54E+02 (3.85E+00) | 3.55E+02 = (2.86E+00) | 3.55E+02 (3.71E+00) | 3.51E+02 - (3.46E+00) | 3.50E+02 (3.15E+00) |
| | cec17 F24 | 4.28E+02 = (1.58E+00) | 4.28E+02 (1.87E+00) | 4.28E+02 + (2.39E+00) | 4.29E+02 (2.35E+00) | 4.29E+02 - (2.73E+00) | 4.27E+02 (2.07E+00) | 4.26E+02 = (2.38E+00) | 4.26E+02 (3.06E+00) |
| | cec17 F25 | 3.87E+02 - (1.97E-02) | 3.87E+02 (1.26E-02) | 3.87E+02 - (2.43E-02) | 3.87E+02 (1.71E-02) | 3.87E+02 = (5.91E-03) | 3.87E+02 (5.70E-03) | 3.87E+02 = (5.99E-03) | 3.87E+02 (6.30E-03) |
| | cec17 F26 | 9.85E+02 - (3.55E+01) | 9.65E+02 (3.66E+01) | 9.51E+02 = (3.60E+01) | 9.52E+02 (4.31E+01) | 9.55E+02 - (3.92E+01) | 9.35E+02 (4.45E+01) | 9.30E+02 = (3.65E+01) | 9.25E+02 (4.04E+01) |
| | cec17 F27 | 5.07E+02 = (4.03E+00) | 5.06E+02 (5.63E+00) | 5.03E+02 = (4.75E+00) | 5.01E+02 (6.09E+00) | 5.05E+02 = (4.52E+00) | 5.05E+02 (4.34E+00) | 4.97E+02 = (6.63E+00) | 4.95E+02 (7.76E+00) |
| | cec17 F28 | 3.39E+02 = (5.61E+01) | 3.27E+02 (4.88E+01) | 3.20E+02 = (4.37E+01) | 3.26E+02 (4.74E+01) | 3.06E+02 + (2.63E+01) | 3.24E+02 (4.66E+01) | 3.13E+02 = (3.54E+01) | 3.02E+02 (1.60E+01) |
| | cec17 F29 | 4.36E+02 + (7.53E+00) | 4.42E+02 (1.15E+01) | 4.38E+02 + (1.62E+01) | 4.45E+02 (1.19E+01) | 4.29E+02 + (6.34E+00) | 4.35E+02 (8.65E+00) | 4.32E+02 = (1.58E+01) | 4.27E+02 (2.42E+01) |
| | cec17 F30 | 1.99E+03 - (5.56E+01) | 1.97E+03 (4.32E+01) | 1.97E+03 = (3.05E+01) | 1.98E+03 (3.66E+01) | 1.99E+03 = (7.24E+01) | 1.99E+03 (5.68E+01) | 1.97E+03 = (1.68E+01) | 1.97E+03 (1.11E+01) |
| -/=/+ | | 9/18/3 | | 3/24/3 | | 7/21/2 | | 7/23/0 | |

TABLE S12 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED TOP ALGORITHMS WITH THE BASELINES
ON 50-D CEC2017 BENCHMARK SET

| | | L-SHADE | SCSS- L-SHADE | UMOEa-II | SCSS- UMOEa-II | L-SHADE_ EpSin | SCSS- L-SHADE_ EpSin | jSO | SCSS- jSO |
|--------------------------------|-----------|--------------------------|--------------------------------------|--|--------------------------------------|--------------------------|--------------------------------------|--------------------------|--------------------------------------|
| Unimodal Functions | cec17 F1 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| | cec17 F2 | 5.01E-06 - (3.12E-06) | 1.66E-06 (9.79E-07) | 1.37E-05 - (6.95E-06) | 6.55E-06 (4.16E-06) | 2.23E-07 - (1.36E-07) | 9.62E-08 (6.14E-08) | 1.38E-05 = (8.23E-06) | 1.48E-05 (8.26E-06) |
| | cec17 F3 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 3.00E-10 + (2.14E-09) | 1.54E-08 (2.31E-08) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| Simple Multimodal Functions | cec17 F4 | 7.23E+01 - (4.94E+01) | 7.34E+01 (5.05E+01) | 7.22E+01 = (4.97E+01) | 8.27E+01 (5.36E+01) | 5.04E+01 = (4.38E+01) | 4.51E+01 (3.97E+01) | 5.85E+01 = (4.56E+01) | 4.87E+01 (4.11E+01) |
| | cec17 F5 | 1.19E+01 = (2.46E+00) | 1.20E+01 (1.99E+00) | 1.61E+01 - (4.55E+00) | 1.43E+01 (3.11E+00) | 2.90E+01 - (6.65E+00) | 1.94E+01 (6.64E+00) | 1.56E+01 - (2.65E+00) | 1.26E+01 (2.70E+00) |
| | cec17 F6 | 7.12E-08 - (2.58E-07) | 2.22E-08 (6.76E-08) | 1.66E-04 - (5.76E-04) | 1.16E-07 (2.28E-07) | 2.57E-07 - (3.41E-07) | 4.20E-08 (6.98E-08) | 4.10E-07 = (5.52E-07) | 2.85E-07 (5.12E-07) |
| | cec17 F7 | 6.50E+01 = (2.23E+00) | 6.46E+01 (2.12E+00) | 7.04E+01 = (5.17E+00) | 6.85E+01 (5.14E+00) | 7.98E+01 - (7.02E+00) | 7.15E+01 (5.69E+00) | 6.66E+01 - (3.10E+00) | 6.33E+01 (2.66E+00) |
| | cec17 F8 | 1.21E+01 = (2.39E+00) | 1.17E+01 (2.56E+00) | 1.58E+01 = (4.09E+00) | 1.43E+01 (4.17E+00) | 3.07E+01 - (3.99E+00) | 1.96E+01 (6.59E+00) | 1.69E+01 - (3.43E+00) | 1.20E+01 (2.67E+00) |
| | cec17 F9 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| | cec17 F10 | 3.32E+03 - (2.81E+02) | 3.12E+03 (3.27E+02) | 3.75E+03 = (5.99E+02) | 3.64E+03 (5.22E+02) | 3.07E+03 - (2.91E+02) | 2.89E+03 (2.90E+02) | 3.21E+03 - (3.78E+02) | 3.05E+03 (3.63E+02) |
| Hybrid Functions | cec17 F11 | 4.80E+01 - (6.64E+00) | 3.37E+01 (4.65E+00) | 4.42E+01 - (9.48E+00) | 3.16E+01 (4.51E+00) | 2.75E+01 = (2.01E+00) | 2.71E+01 (2.06E+00) | 2.66E+01 - (3.13E+00) | 2.50E+01 (4.12E+00) |
| | cec17 F12 | 2.07E+03 = (5.21E+02) | 2.10E+03 (4.81E+02) | 2.17E+03 = (5.36E+02) | 2.01E+03 (4.99E+02) | 1.38E+03 = (3.79E+02) | 1.36E+03 (3.67E+02) | 1.61E+03 - (4.42E+02) | 1.29E+03 (3.66E+02) |
| | cec17 F13 | 6.52E+01 - (2.98E+01) | 5.09E+01 (2.89E+01) | 4.69E+01 - (1.73E+01) | 3.56E+01 (1.57E+01) | 3.76E+01 = (2.60E+01) | 4.29E+01 (2.23E+01) | 3.17E+01 = (2.01E+01) | 2.60E+01 (2.09E+01) |
| | cec17 F14 | 3.06E+01 - (3.73E+00) | 2.48E+01 (2.30E+00) | 2.85E+01 - (3.30E+00) | 2.70E+01 (2.35E+00) | 2.71E+01 = (2.68E+00) | 2.67E+01 (2.57E+00) | 2.50E+01 = (2.34E+00) | 2.51E+01 (2.46E+00) |
| | cec17 F15 | 4.53E+01 - (1.40E+01) | 2.77E+01 (3.82E+00) | 3.45E+01 - (6.42E+00) | 2.69E+01 (3.14E+00) | 2.51E+01 = (3.17E+00) | 2.39E+01 (2.44E+00) | 2.37E+01 - (2.77E+00) | 2.12E+01 (1.81E+00) |
| | cec17 F16 | 3.76E+02 = (1.36E+02) | 3.49E+02 (1.17E+02) | 4.58E+02 = (1.68E+02) | 4.07E+02 (1.69E+02) | 3.31E+02 - (1.25E+02) | 2.68E+02 (1.16E+02) | 4.77E+02 = (1.36E+02) | 4.45E+02 (1.55E+02) |
| | cec17 F17 | 2.32E+02 = (6.72E+01) | 2.04E+02 (9.33E+01) | 3.14E+02 = (1.18E+02) | 3.01E+02 (1.07E+02) | 2.40E+02 - (6.48E+01) | 2.04E+02 (8.12E+01) | 2.93E+02 = (1.10E+02) | 2.61E+02 (1.04E+02) |
| | cec17 F18 | 5.06E+01 - (1.72E+01) | 2.80E+01 (3.87E+00) | 3.26E+01 - (7.70E+00) | 2.60E+01 (2.90E+00) | 2.53E+01 = (2.70E+00) | 2.46E+01 (2.15E+00) | 2.46E+01 - (2.42E+00) | 2.24E+01 (1.14E+00) |
| | cec17 F19 | 3.50E+01 - (1.39E+01) | 1.71E+01 (3.01E+00) | 2.08E+01 - (3.32E+00) | 1.70E+01 (3.00E+00) | 1.62E+01 = (3.11E+00) | 1.56E+01 (2.97E+00) | 1.42E+01 - (2.73E+00) | 1.17E+01 (2.65E+00) |
| | cec17 F20 | 1.56E+02 = (4.95E+01) | 1.72E+02 (6.37E+01) | 2.60E+02 = (1.20E+02) | 2.80E+02 (1.16E+02) | 1.35E+02 - (5.03E+01) | 1.07E+02 (2.47E+01) | 1.17E+02 = (6.45E+01) | 1.14E+02 (6.57E+01) |
| Composition Functions | cec17 F21 | 2.16E+02 - (2.26E+00) | 2.14E+02 (2.74E+00) | 2.20E+02 - (5.20E+00) | 2.18E+02 (4.64E+00) | 2.30E+02 - (6.27E+00) | 2.20E+02 (6.07E+00) | 2.17E+02 - (2.73E+00) | 2.14E+02 (3.27E+00) |
| | cec17 F22 | 2.84E+03 = (1.53E+03) | 3.33E+03 (8.42E+02) | 2.82E+03 = (2.11E+03) | 2.78E+03 (2.16E+03) | 1.54E+03 = (1.62E+03) | 2.10E+03 (1.46E+03) | 1.07E+03 = (1.61E+03) | 1.63E+03 (1.79E+03) |
| | cec17 F23 | 4.33E+02 - (4.04E+00) | 4.30E+02 (4.60E+00) | 4.42E+02 - (8.43E+00) | 4.37E+02 (7.54E+00) | 4.43E+02 - (6.60E+00) | 4.35E+02 (7.00E+00) | 4.30E+02 - (6.16E+00) | 4.26E+02 (6.54E+00) |
| | cec17 F24 | 5.12E+02 - (3.01E+00) | 5.11E+02 (2.81E+00) | 5.12E+02 = (4.82E+00) | 5.11E+02 (3.86E+00) | 5.13E+02 - (5.58E+00) | 5.08E+02 (4.57E+00) | 5.08E+02 = (4.54E+00) | 5.07E+02 (3.77E+00) |
| | cec17 F25 | 4.82E+02 - (4.55E+00) | 4.81E+02 (3.57E+00) | 4.82E+02 - (6.18E+00) | 4.81E+02 (2.33E+00) | 4.80E+02 = (1.44E-02) | 4.81E+02 (3.52E+00) | 4.81E+02 - (2.32E+00) | 4.81E+02 (3.15E+00) |
| | cec17 F26 | 1.21E+03 - (4.31E+01) | 1.17E+03 (3.93E+01) | 1.21E+03 = (6.22E+01) | 1.19E+03 (5.77E+01) | 1.27E+03 - (7.63E+01) | 1.18E+03 (1.08E+02) | 1.13E+03 = (4.90E+01) | 1.12E+03 (5.07E+01) |
| | cec17 F27 | 5.43E+02 = (2.15E+01) | 5.38E+02 (1.56E+01) | 5.36E+02 - (1.67E+01) | 5.31E+02 (1.78E+01) | 5.33E+02 = (1.56E+01) | 5.28E+02 (1.16E+01) | 5.14E+02 = (1.01E+01) | 5.10E+02 (1.37E+01) |
| | cec17 F28 | 4.64E+02 - (1.51E+01) | 4.60E+02 (5.68E+00) | 4.73E+02 - (2.25E+01) | 4.64E+02 (1.55E+01) | 4.60E+02 = (6.84E+00) | 4.60E+02 (6.84E+00) | 4.59E+02 = (3.03E-13) | 4.59E+02 (3.32E-13) |
| | cec17 F29 | 3.53E+02 = (1.08E+01) | 3.57E+02 (1.44E+01) | 3.62E+02 + (1.91E+01) | 3.84E+02 (1.93E+01) | 3.49E+02 = (9.11E+00) | 3.49E+02 (1.14E+01) | 3.65E+02 = (1.52E+01) | 3.65E+02 (1.40E+01) |
| | cec17 F30 | 6.68E+05 = (8.12E+04) | 6.51E+05 (8.03E+04) | 6.68E+05 = (1.02E+05) | 6.38E+05 (5.48E+04) | 6.50E+05 = (6.32E+04) | 6.72E+05 (8.23E+04) | 6.08E+05 = (3.03E+04) | 6.04E+05 (2.57E+04) |
| -/+ | | 15/15/0 | | 14/14/2 | | 13/17/0 | | 12/18/0 | |

TABLE S13 PERFORMANCE COMPARISONS OF FOUR SCSS-BASED TOP ALGORITHMS WITH THE BASELINES
ON 100-D CEC2017 BENCHMARK SET

| | | L-SHADE | SCSS- L-SHADE | UMOEa-II | SCSS- UMOEa-II | L-SHADE_ EpSin | SCSS- L-SHADE_ EpSin | jSO | SCSS- jSO |
|--------------------------------|-----------|--|--------------------------------------|--|--------------------------------------|--------------------------|--------------------------------------|--|--------------------------------------|
| Unimodal Functions | cec17 F1 | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) |
| | cec17 F2 | 3.16E-04 + (5.07E-05) | 3.41E-04 (5.81E-05) | 9.66E-05 = (1.75E-05) | 9.31E-05 (1.35E-05) | 1.58E-04 - (4.22E-05) | 1.38E-04 (4.25E-05) | 3.10E-04 + (5.45E-05) | 3.66E-04 (6.77E-05) |
| | cec17 F3 | 5.47E-06 + (6.19E-06) | 1.07E-03 (1.73E-03) | 2.84E-06 + (3.01E-06) | 6.60E-06 (4.57E-06) | 5.35E-09 - (1.11E-08) | 2.20E-10 (1.57E-09) | 2.71E-06 + (2.72E-06) | 1.52E-04 (1.69E-04) |
| Simple Multimodal Functions | cec17 F4 | 2.01E+02 - (7.69E+00) | 2.00E+02 (8.00E+00) | 1.87E+02 = (4.03E+01) | 1.93E+02 (3.12E+01) | 2.04E+02 = (9.79E+00) | 2.05E+02 (1.11E+01) | 1.94E+02 = (2.35E+01) | 1.96E+02 (1.09E+01) |
| | cec17 F5 | 3.78E+01 - (7.64E+00) | 2.69E+01 (6.48E+00) | 3.53E+01 - (7.62E+00) | 2.79E+01 (7.14E+00) | 6.06E+01 - (7.15E+00) | 4.15E+01 (6.26E+00) | 4.29E+01 - (7.17E+00) | 2.84E+01 (5.43E+00) |
| | cec17 F6 | 1.37E-03 - (8.75E-04) | 5.37E-04 (4.36E-04) | 8.12E-03 - (5.54E-03) | 2.61E-03 (2.27E-03) | 3.51E-05 - (1.38E-05) | 9.41E-06 (5.14E-06) | 1.61E-04 - (4.30E-04) | 1.68E-05 (1.18E-05) |
| | cec17 F7 | 1.51E+02 - (4.80E+00) | 1.38E+02 (4.48E+00) | 1.41E+02 - (9.72E+00) | 1.36E+02 (9.40E+00) | 1.67E+02 - (9.13E+00) | 1.45E+02 (5.70E+00) | 1.41E+02 - (6.94E+00) | 1.27E+02 (4.53E+00) |
| | cec17 F8 | 3.92E+01 - (5.48E+00) | 2.75E+01 (5.11E+00) | 3.60E+01 - (7.09E+00) | 2.78E+01 (7.23E+00) | 5.73E+01 - (9.38E+00) | 3.87E+01 (6.26E+00) | 4.31E+01 - (5.58E+00) | 2.99E+01 (5.62E+00) |
| | cec17 F9 | 1.56E-01 - (2.22E-01) | 1.42E-02 (6.64E-02) | 5.35E-01 - (5.13E-01) | 9.17E-02 (1.35E-01) | 0.00E+00 = (0.00E+00) | 0.00E+00 (0.00E+00) | 4.60E-02 - (1.11E-01) | 0.00E+00 (0.00E+00) |
| | cec17 F10 | 1.14E+04 - (6.11E+02) | 1.05E+04 (4.67E+02) | 1.19E+04 = (1.25E+03) | 1.13E+04 (1.59E+03) | 1.05E+04 - (5.15E+02) | 9.57E+03 (4.63E+02) | 9.71E+03 - (6.59E+02) | 9.23E+03 (6.08E+02) |
| | cec17 F11 | 3.86E+02 - (9.53E+01) | 1.54E+02 (5.30E+01) | 4.27E+02 - (1.03E+02) | 1.58E+02 (4.12E+01) | 4.16E+01 = (2.39E+01) | 4.26E+01 (2.91E+01) | 1.06E+02 - (3.82E+01) | 7.21E+01 (3.10E+01) |
| | cec17 F12 | 2.37E+04 = (1.05E+04) | 2.25E+04 (8.53E+03) | 4.52E+03 = (8.56E+02) | 4.86E+03 (1.42E+03) | 5.28E+03 - (1.39E+03) | 4.62E+03 (7.33E+02) | 2.05E+04 - (1.06E+04) | 1.41E+04 (8.02E+03) |
| | cec17 F13 | 1.36E+03 - (8.06E+02) | 2.45E+02 (7.34E+01) | 3.60E+02 - (1.47E+02) | 1.64E+02 (4.77E+01) | 7.92E+01 = (2.87E+01) | 8.36E+01 (3.44E+01) | 1.60E+02 - (4.19E+01) | 1.12E+02 (2.79E+01) |
| Hybrid Functions | cec17 F14 | 2.55E+02 - (3.25E+01) | 1.01E+02 (2.01E+01) | 2.35E+02 - (3.25E+01) | 7.25E+01 (1.56E+01) | 5.13E+01 = (8.93E+00) | 4.86E+01 (6.46E+00) | 6.28E+01 - (1.18E+01) | 3.95E+01 (4.08E+00) |
| | cec17 F15 | 2.50E+02 = (4.87E+01) | 2.59E+02 (4.34E+01) | 2.67E+02 - (5.38E+01) | 2.21E+02 (4.82E+01) | 7.28E+01 = (3.14E+01) | 7.73E+01 (2.83E+01) | 1.64E+02 - (4.20E+01) | 9.73E+01 (3.56E+01) |
| | cec17 F16 | 1.79E+03 - (2.58E+02) | 1.55E+03 (2.39E+02) | 1.67E+03 = (4.55E+02) | 1.64E+03 (4.27E+02) | 1.55E+03 - (2.51E+02) | 1.31E+03 (2.61E+02) | 1.84E+03 = (3.15E+02) | 1.74E+03 (2.99E+02) |
| | cec17 F17 | 1.20E+03 - (2.21E+02) | 1.04E+03 (2.00E+02) | 1.36E+03 = (3.13E+02) | 1.28E+03 (2.62E+02) | 1.16E+03 - (1.72E+02) | 9.23E+02 (1.76E+02) | 1.26E+03 - (2.63E+02) | 1.13E+03 (2.20E+02) |
| | cec17 F18 | 2.15E+02 = (4.60E+01) | 2.11E+02 (5.33E+01) | 2.35E+02 = (6.29E+01) | 2.16E+02 (4.72E+01) | 7.92E+01 = (2.19E+01) | 7.59E+01 (1.83E+01) | 1.76E+02 - (4.05E+01) | 1.11E+02 (3.07E+01) |
| | cec17 F19 | 1.77E+02 - (2.31E+01) | 1.63E+02 (2.46E+01) | 1.76E+02 - (2.65E+01) | 1.52E+02 (2.50E+01) | 5.22E+01 = (6.65E+00) | 5.09E+01 (5.78E+00) | 1.07E+02 - (2.14E+01) | 5.22E+01 (5.72E+00) |
| | cec17 F20 | 1.57E+03 - (2.42E+02) | 1.50E+03 (1.79E+02) | 1.93E+03 = (3.61E+02) | 1.89E+03 (3.11E+02) | 1.44E+03 - (1.96E+02) | 1.23E+03 (1.89E+02) | 1.38E+03 = (2.44E+02) | 1.29E+03 (2.12E+02) |
| | cec17 F21 | 2.69E+02 - (5.81E+00) | 2.59E+02 (4.38E+00) | 2.56E+02 = (6.84E+00) | 2.55E+02 (6.49E+00) | 2.83E+02 - (1.41E+01) | 2.64E+02 (5.61E+00) | 2.64E+02 - (6.56E+00) | 2.49E+02 (5.18E+00) |
| | cec17 F22 | 1.19E+04 - (5.24E+02) | 1.12E+04 (6.26E+02) | 1.27E+04 = (1.81E+03) | 1.25E+04 (1.61E+03) | 1.08E+04 - (5.90E+02) | 9.54E+03 (5.05E+02) | 1.07E+04 - (6.27E+02) | 1.01E+04 (6.70E+02) |
| | cec17 F23 | 5.68E+02 = (7.98E+00) | 5.67E+02 (7.15E+00) | 5.70E+02 = (9.40E+00) | 5.70E+02 (1.34E+01) | 5.98E+02 - (7.21E+00) | 5.92E+02 (6.32E+00) | 5.69E+02 = (1.37E+01) | 5.67E+02 (1.14E+01) |
| Composition Functions | cec17 F24 | 9.19E+02 - (8.98E+00) | 9.12E+02 (8.61E+00) | 9.22E+02 - (8.89E+00) | 9.16E+02 (1.16E+01) | 9.37E+02 - (2.15E+01) | 9.08E+02 (8.10E+00) | 9.01E+02 - (1.04E+01) | 8.96E+02 (7.84E+00) |
| | cec17 F25 | 7.46E+02 = (3.47E+01) | 7.44E+02 (3.50E+01) | 7.49E+02 - (2.76E+01) | 7.29E+02 (3.77E+01) | 6.93E+02 = (4.53E+01) | 6.89E+02 (4.55E+01) | 7.18E+02 = (3.87E+01) | 7.13E+02 (4.26E+01) |
| | cec17 F26 | 3.41E+03 - (1.02E+02) | 3.31E+03 (9.92E+01) | 3.42E+03 - (9.37E+01) | 3.32E+03 (9.49E+01) | 3.24E+03 - (2.51E+02) | 3.06E+03 (9.06E+01) | 3.20E+03 - (8.46E+01) | 3.12E+03 (9.03E+01) |
| | cec17 F27 | 6.58E+02 - (1.38E+01) | 6.47E+02 (1.57E+01) | 6.41E+02 - (1.79E+01) | 6.32E+02 (1.61E+01) | 5.92E+02 = (1.37E+01) | 5.90E+02 (1.81E+01) | 5.86E+02 - (2.05E+01) | 5.77E+02 (2.28E+01) |
| | cec17 F28 | 5.28E+02 = (2.19E+01) | 5.34E+02 (2.30E+01) | 5.18E+02 + (3.80E+01) | 5.28E+02 (3.07E+01) | 5.15E+02 = (1.95E+01) | 5.22E+02 (2.30E+01) | 5.29E+02 = (2.78E+01) | 5.25E+02 (2.86E+01) |
| | cec17 F29 | 1.53E+03 = (1.92E+02) | 1.48E+03 (1.83E+02) | 1.40E+03 = (2.46E+02) | 1.48E+03 (2.33E+02) | 1.23E+03 = (1.62E+02) | 1.21E+03 (1.42E+02) | 1.33E+03 - (2.02E+02) | 1.25E+03 (1.82E+02) |
| | cec17 F30 | 2.43E+03 - (1.45E+02) | 2.34E+03 (1.32E+02) | 2.36E+03 = (1.26E+02) | 2.36E+03 (1.53E+02) | 2.34E+03 = (1.35E+02) | 2.37E+03 (1.92E+02) | 2.31E+03 = (1.23E+02) | 2.27E+03 (1.06E+02) |
| -/+ | | 20/8/2 | | 14/14/2 | | 16/14/0 | | 20/8/2 | |