一　加速度为恒矢量时质点的运动方程

已知一质点作平面运动，其加速度$\vec{a}$ 为恒矢量，有

$$\vec{a}=a_x\vec{i}+a_y\vec{j}$$

$$\vec{a}=\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \qquad \int_{\vec{v}_0}^{\vec{v}}\mathrm{d}\vec{v}=\int_0^t\vec{a}\,\mathrm{d}t$$

积分可得　$\vec{v}=\vec{v}_0+\vec{a}t$

写成分量式　$v_x=v_{0x}+a_xt$　$v_y=v_{0y}+a_yt$

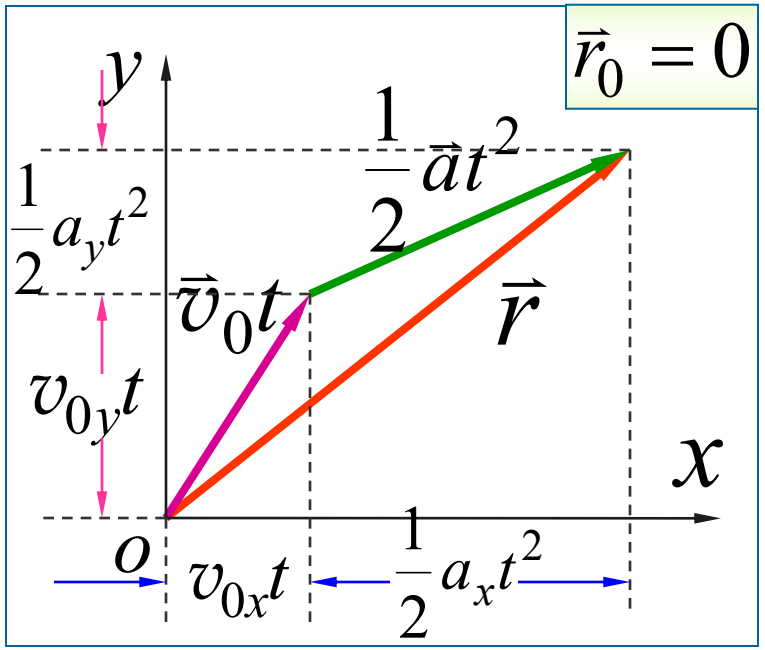
$$\vec{a}=a_x\vec{i}+a_y\vec{j} \qquad \vec{v}=\vec{v}_0+\vec{a}t$$

$$\mathrm{d}\vec{r}=\vec{v}\mathrm{d}t \qquad \int_{\vec{r}_0}^{\vec{r}}\mathrm{d}\vec{r}=\int_0^t(\vec{v}_0+\vec{a}t)\mathrm{d}t$$

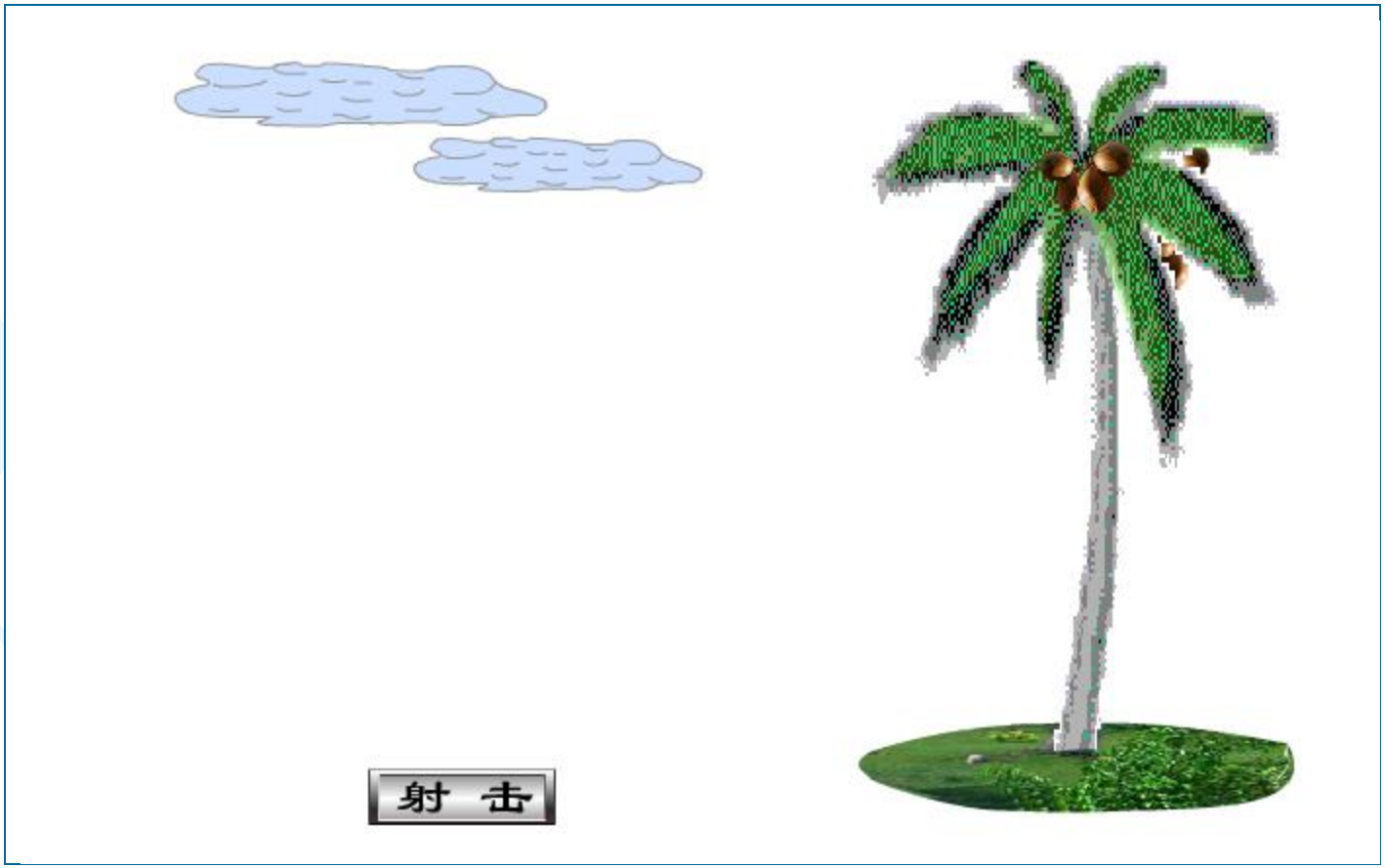
积分可得　$$\vec{r}-\vec{r}_0=\vec{v}_0t+\frac{1}{2}\vec{a}t^2$$

写成分量式为

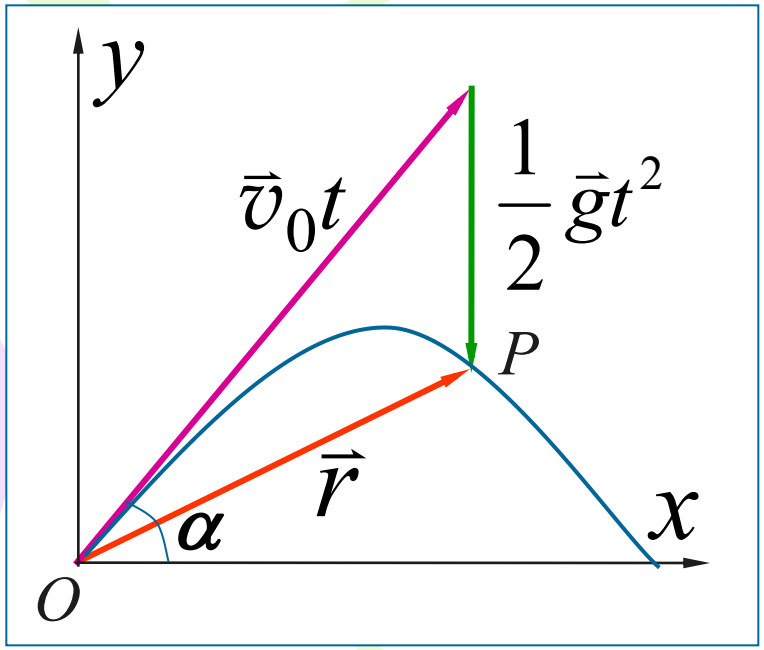
$$\begin{cases}x-x_0=v_{0x}t+\dfrac{1}{2}a_xt^2\\ y-y_0=v_{0y}t+\dfrac{1}{2}a_yt^2\end{cases}$$



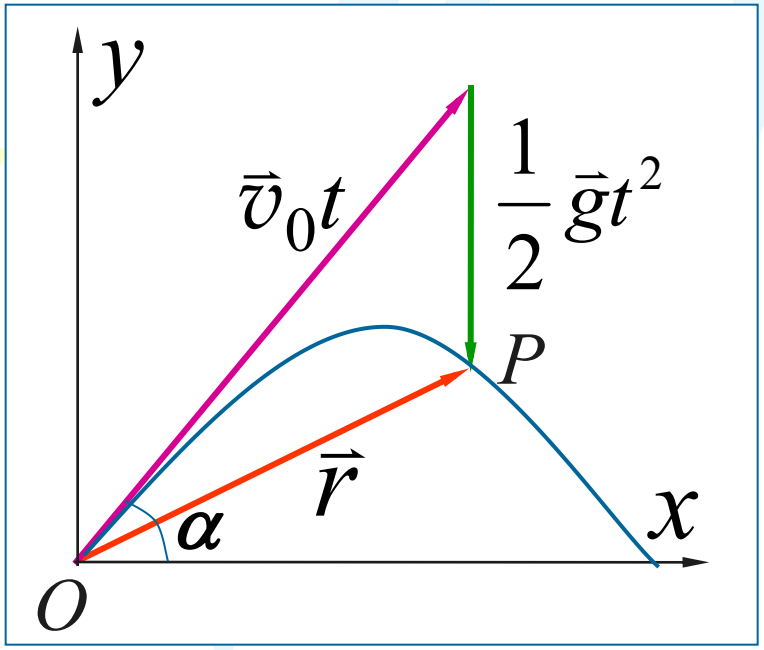
二 斜抛运动



当子弹从枪口射出时，椰子刚好从树上由静止自由下落. 试说明为什么子弹总可以射中椰子？


y
$\vec{v}_0 t$
$\frac{1}{2}\vec{g}t^2$
P
$\vec{r}$
α
x
O

例4 如图一抛体在地球表面附近，从原点O以初速$\vec{v}_0$沿与水平面上Ox轴的正向成α角抛出．如略去抛体在运动过程中空气的阻力作用，求抛体运动的轨迹方程和最大射程.

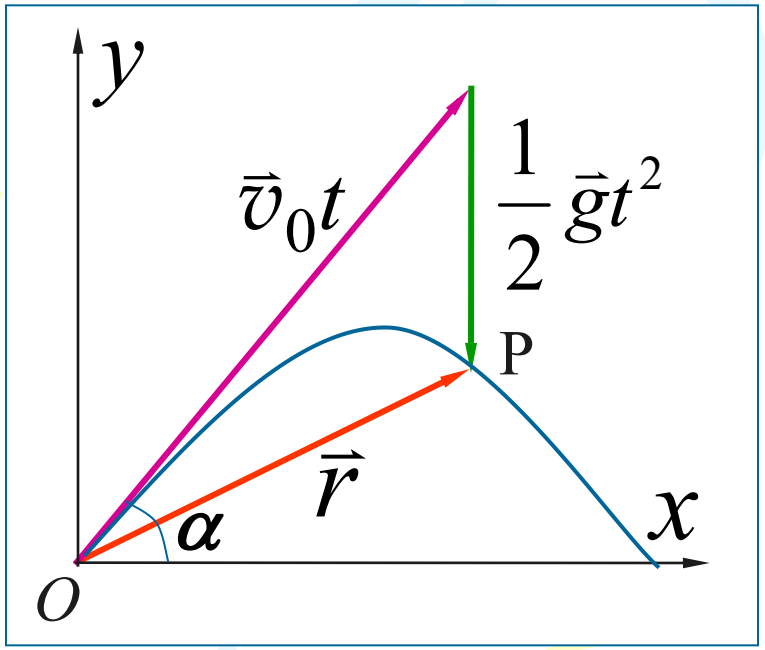


解

$$\vec{a}=\vec{a}_y=\vec{g}=-g\vec{j}$$

$$\vec{a}_x=0$$

$$\vec{r}=\vec{v}_0t+\frac{1}{2}\vec{g}t^2$$



按已知条件，t=0时，有

$$\begin{cases}v_{0x}=v_0\cos\alpha\\ v_{0y}=v_0\sin\alpha\end{cases}$$

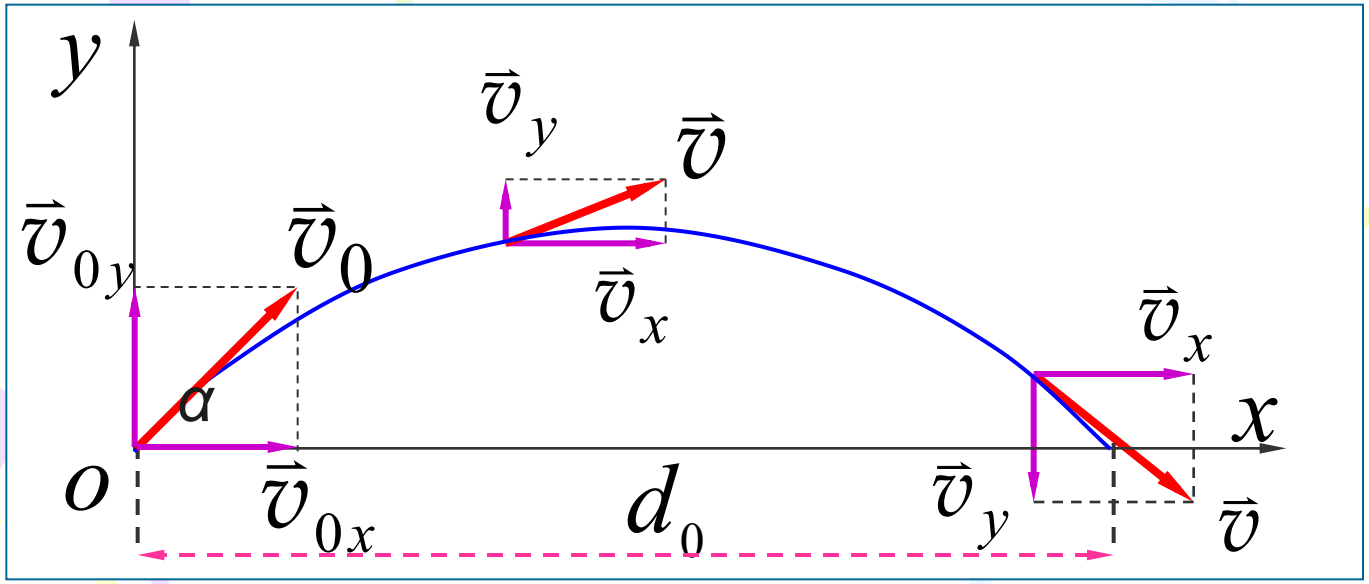
$$\begin{cases}a_x=0\\ a_y=-g\end{cases}$$

解得：

$$x = v_0 \cos\alpha \cdot t, \quad y = v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2$$

轨迹方程为：

$$y = x \tan\alpha - \frac{g}{2v_0^2 \cos^2 \alpha} x^2$$



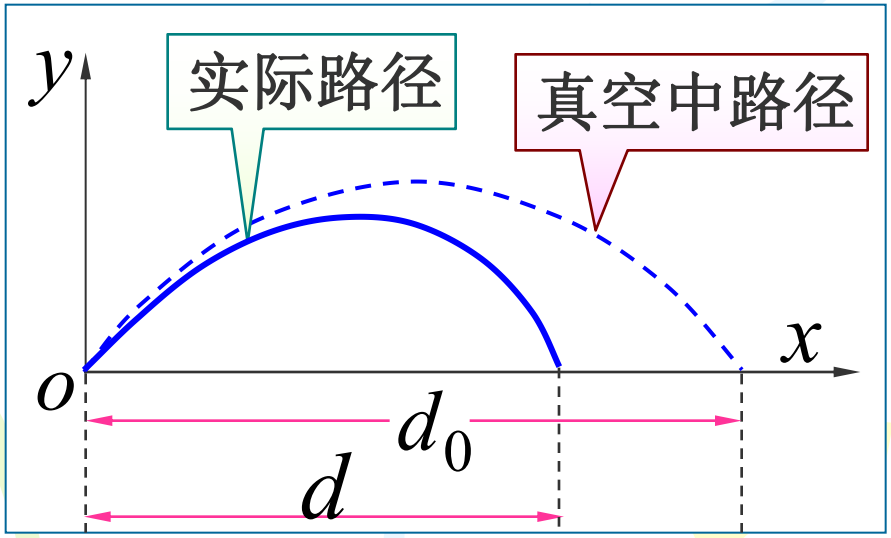
求最大射程

$$d_0 = \frac{2v_0^2}{g}\sin\alpha\cos\alpha,\quad \frac{\mathrm{d}d_0}{\mathrm{d}\alpha} = \frac{2v_0^2}{g}\cos 2\alpha = 0$$

当 $\alpha = \dfrac{\pi}{4}$,

$$d_{0\mathrm{m}} = v_0^2/g$$

由于空气阻力，实际射程小于最大射程.

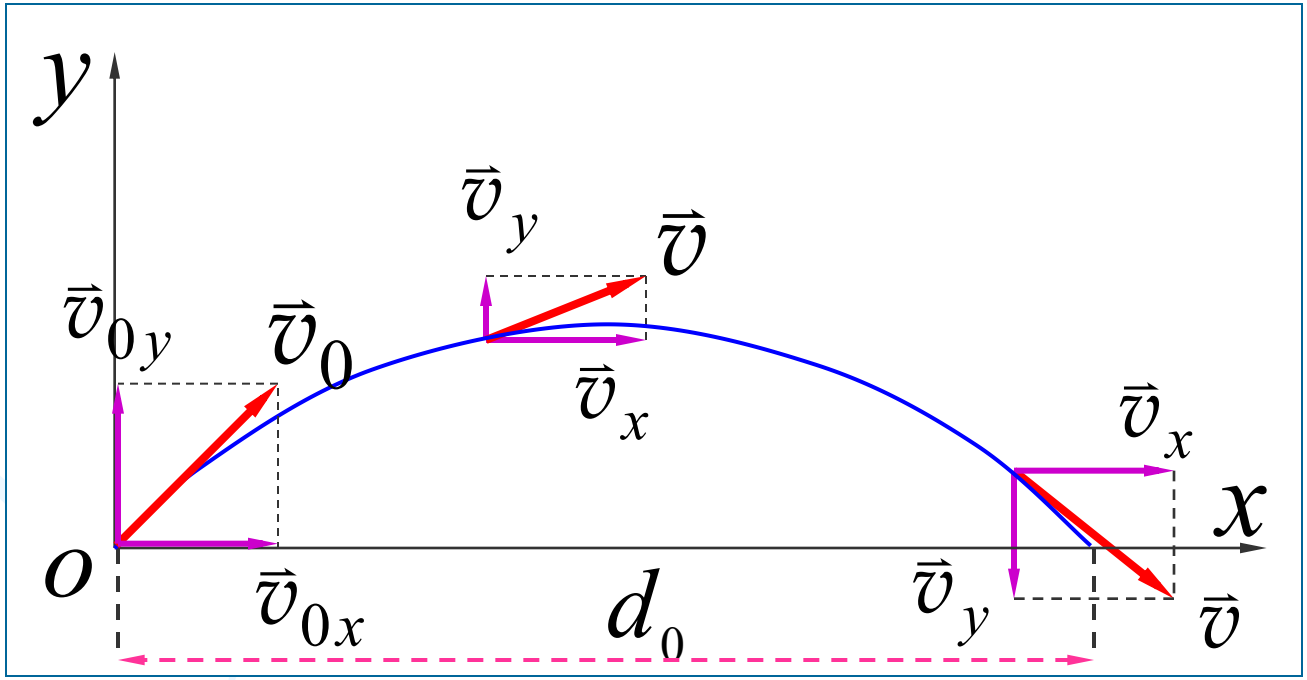


END

求斜抛运动的轨迹方程和最大射程

已知 $a_x = 0 \quad a_y = -g, \quad t = 0$ 时 $x_0 = y_0 = 0$

$$v_{0x} = v_0 \cos\alpha \qquad v_{0y} = v_0 \sin\alpha$$



$$x = v_0 \cos\alpha \cdot t \qquad y = v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2$$

$$x = v_0 \cos\alpha \cdot t \qquad y = v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2$$

消去方程中的参数 t 得轨迹

$$y = x\tan\alpha - \frac{y}{2v_0^2 \cos^2\alpha} x^2$$

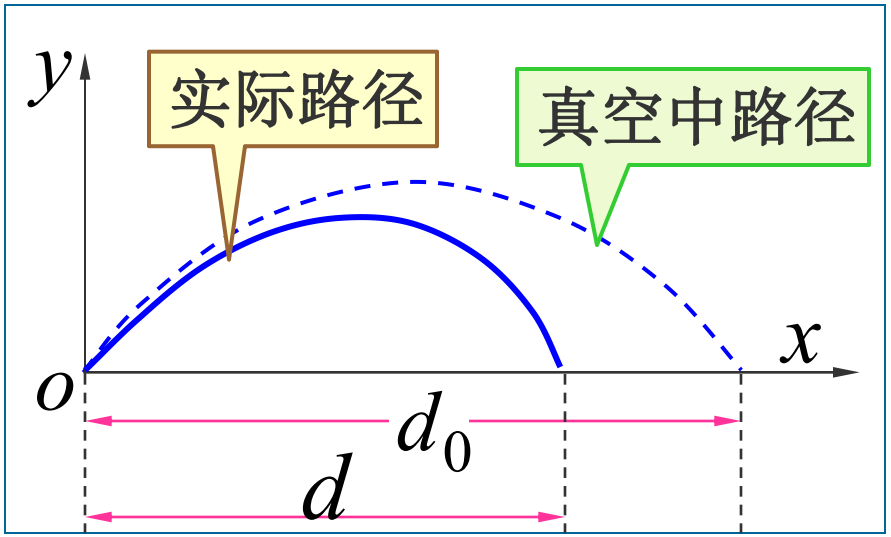
求最大射程

$$d_0 = \frac{2v_0^2}{g} \sin\alpha \cos\alpha$$

$$\frac{\mathrm{d}d_0}{\mathrm{d}\alpha} = \frac{2v_0^2}{g} \cos 2\alpha = 0$$

$$\alpha = \pi/4$$

最大射程 $d_{0\mathrm{m}} = v_0^2/g$



由于空气阻力，实际射程小于最大射程.