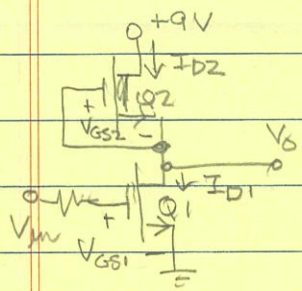
EECE 2412 HW #8 Solutions Sp '16 #1 ①

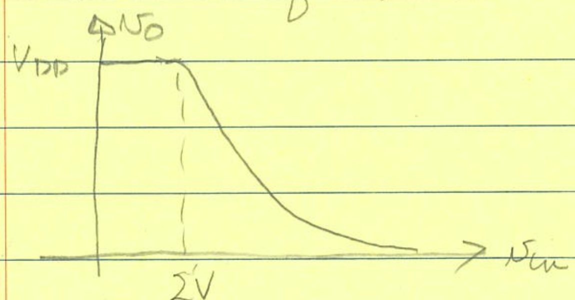
PROBLEM A



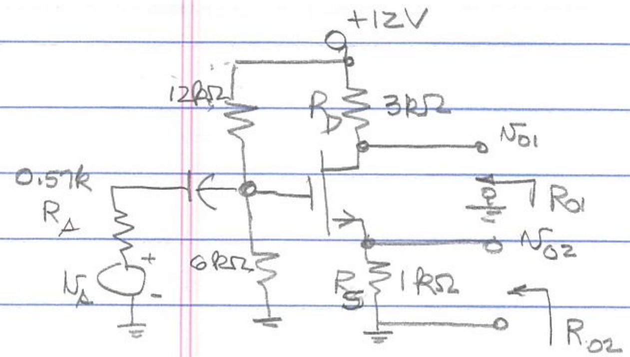
Observations: $I_{D1} = I_{D2}$, $V_{DS1} + V_{DS2} = V_{DD}$

$V_{GS1} = V_{in}$; $V_{GS2} = 0$, $V_{DS1} = V_o$

Based on the characteristics, for $V_{in} \le 2V$, where $V_{t1} = 2V$, Q1 is cutoff so $I_{D1} = 0$. Thus, $I_{D2} = 0$. On the Q2 characteristics, for $V_{GS2} = 0$, the only place $I_{D2} = 0$ is the origin. Then $V_{DS2} = 0$ and $V_{DS1} = V_{DD}$. As V_{in} increases slightly above 2 V, Q1 begins to conduct slightly but is still less than 800μA. Thus, Q2 operates "near" the origin and V_{DS2} is small so $V_{DS1} = V_o$ = a bit less than V_{DD}. When $V_{in} > 4V$, Q1 is <u>capable</u> of conducting at >800μA However, Q2 limits the current to 800μA and V_{DS1} becomes "small" and V_{DS2} begins to approach V_{DD}. This process continues as V_{in} increases beyond 4V. The transfer characteristic is

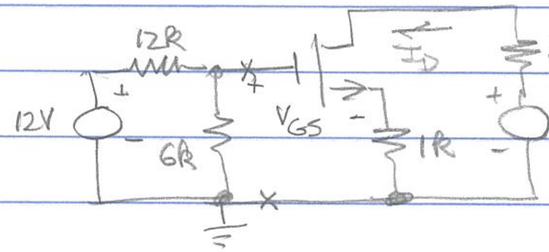


PROBLEM B



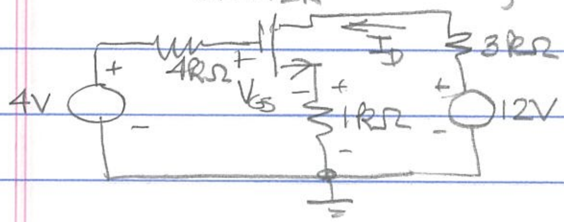
$I_D = 0.8(V_{GS}-2)^2$ mA $V_A = 60k\Omega$

The dc circuit is



Get Thèvenin equivalent to the left of X–X.

$V_{TH} = \frac{6R}{6R+12R}\times 12 = 4V$; $R_{TH} = 6R \| 12R = 4R\Omega$.



$V_{GS} = 4 - 1R \times I_D\,(mA) = 4 - 1 \cdot I_D$

Substitute this for V_{GS} in I_D equation and get

$0.8(4 - I_D - 2)^2 = I_D$ all currents in mA

$0.8(2 - I_D)^2 = I_D$ or $0.8(4 - 4I_D + I_D^2) = I_D$

and $4 - 4I_D + I_D^2 = 1.25\,I_D$ and $I_D^2 - 5.25\,I_D + 4 = 0$

$I_D = \frac{5.25 \pm \sqrt{(5.25)^2 - 16}}{2}$ and $I_D = 0.925$ mA; or $I_D = 4.325$mA

$I_D = 4.325$ mA is not physically possible as $3\times 4.325 = 12.975$ which is higher than the supply voltage. Also, $I_D \times 1R = 4.325V$ makes $V_{GS} = 4 - 4.325 = -0.325V$, well below the threshold voltage cutting the MOSFET off! Must select $I_D = 0.925$ mA

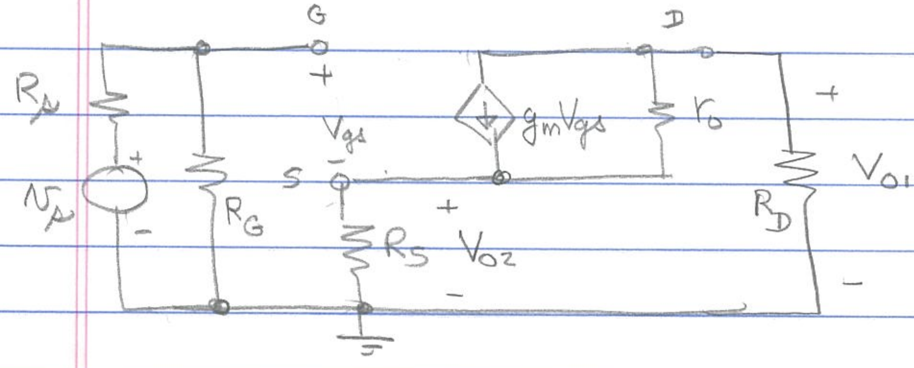
KVL for the drain loop is

$-12 + I_D \times 3R + V_{CE} + I_D \times 1R = 0$ or $V_{CE} = 12 - 4\,I_D$

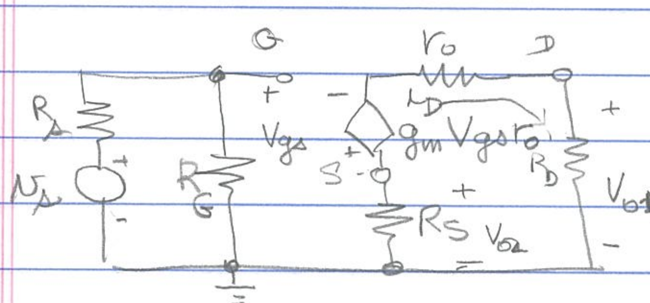
$V_{CE} = 12 - 4\times 0.925 = 8.3\,V$

PROBLEM C

1. The small-signal model of the common-source stage in Prob. B is



Best way to proceed is to convert $g_m V_{gs}$ in parallel with r_o to its voltage-source series R.



$V_{gs} = V_G - V_S$

$V_G = \frac{R_G}{R_G+R_s} v_s$

$V_S = -i_D R_S$

$$V_{GS} = \frac{R_G v_s}{R_G+R_s} + i_D R_S = \alpha v_s + i_D R_S$$

KVL Drain loop: $r_o g_m V_{gs} + i_D(r_o + R_D + R_S) = 0$ and

$$-r_o g_m \alpha v_s + i_D\left[r_o + R_D + R_S(1+g_m r_o)\right] = 0$$

$$i_D = +\frac{g_m r_o \alpha v_s}{r_o + R_D + R_S(1+g_m r_o)}$$

$$v_{O1} = -i_D R_D = \frac{-g_m r_o \alpha R_D v_s}{r_o + R_D + R_S(1+g_m r_o)}$$

$$v_{O2} = i_D R_S = \frac{g_m r_o \alpha v_s R_S}{r_o + R_D + R_S(1+g_m r_o)}$$

These are the literal answers to (4) and (5). (6) and (7) are both yes.

2. $A_{v1} = \left.\frac{v_{O1}}{v_s}\right|_{R_S=0} = \frac{-g_m r_o \alpha R_D}{r_o + R_D}$

3. $A_{v2} = \left.\frac{v_{O2}}{v_s}\right|_{R_D=0} = \frac{g_m r_o \alpha R_S}{r_o + R_S(1+g_m r_o)}$

The numerical answers to Problem C use

$$g_m = 2\sqrt{K\, I_{DQ}} = 2\sqrt{0.8 \times 0.925} = \underbrace{1.72}_{mA/V^2}; \quad r_o = 60\,k\Omega$$

$$\alpha = \frac{R_G}{R_G + R_A} = \frac{4k}{4k + 0.57k} = 0.875$$

2) $$A_{V1} = \frac{-1.72 \times 60 \times 0.875 \times 3}{60 + 3 + 1(1 + 1.72 \times 60)} = -4.3$$

3) $$A_{V2} = \frac{1.72 \times 60 \times 0.875 \times 1}{60 + 1(1 + 1.72 \times 60)} = 0.55$$

4) $$A_{V3} = \frac{-1.72 \times 60 \times 0.875 \times 3}{60 + 3 + 1(1 + 1.72 \times 60)} = -1.62$$

5) $$A_{V4} = \frac{1.72 \times 60 \times 0.875 \times 1}{60 + 3 + 1(1 + 1.72 \times 60)} = 0.54$$

PROBLEM D

To obtain R_{O1} and R_{O2} we can use Thèvenin's theorem. The Thèvenin resistance is V_{TH}/I_{SC}. We already have V_{TH} for both. To obtain I_{SC}, make $R_D = 0$ and find $I_D|_{R_D=0}$ to get R_{O1}; for R_{O2} make $R_S = 0$ and get $I_D|_{R_S=0}$. Then take the appropriate ratio!

$$V_{TH1} = -g_m r_o \alpha R_D v_s / [r_o + R_D + R_S(1 + g_m r_o)]$$

$$I_D|_{R_D=0} = -g_m r_o \alpha v_s / [r_o + R_S(1 + g_m r_o)]$$

$$R_{O1} = V_{TH1} / I_D|_{R_D=0} = R_D[r_o + R_S(1 + g_m r_o)] / [r_o + R_D + R_S(1 + g_m r_o)]$$

This is simply $R_D \,\|\, [r_o + R_S(1 + g_m r_o)]$

$$V_{TH2} = g_m r_o \alpha R_S v_s / [r_o + R_D + R_S(1 + g_m r_o)]$$

$$I_D|_{R_S=0} = g_m r_o \alpha v_s / [r_o + R_D]$$

$$R_{O2} = V_{TH2} / I_D|_{R_S=0} = R_S(r_o + R_D) / [r_o + R_D + R_S(1 + g_m r_o)]$$

Rewriting R_{o2} as

$$R_{o2} = \frac{R_S(r_o + R_D)}{r_o + R_D + R_S + g_m r_o R_S} = \frac{R_S(r_o+R_D)}{r_o+R_D+R_S} \times \frac{1}{1 + \frac{g_m r_o R_S}{r_o+R_D+R_S}}$$

The first term is $R_S \| (r_o+R_D)$ and the second illustrates the impedance reduction of the feedback provided by R_S. Note that R_S is part of the input loop affecting V_{gs} and R_S is part of the output loop indicating feedback.

Numerically, the values are

$$R_{o1} = 3 \| [60+3+1(1+1.72\times60)] = 3 \| 167.2 = 2.95\,k\Omega$$

$$R_{o2} = [1 \| (60+3)] \times \frac{1}{1+\frac{1.72\times60\times1}{60+3+1}} = 0.984 \times \frac{1}{2.61} = 0.376\,k\Omega$$

Observe: For R_{o1} the effect of the feedback makes the term in the [ ] larger so that the parallel combination is more closely $3\,k\Omega = R_D$. For R_{o2}, the effect of feedback reduces the output resistance. The resistance seen by R_D in R_{o1} increases because of the feedback provided by R_S.