

# PRIMA: What, Why, How, and So What?

Zaikun Zhang

The Hong Kong Polytechnic University

SIOPT 2023, Seattle

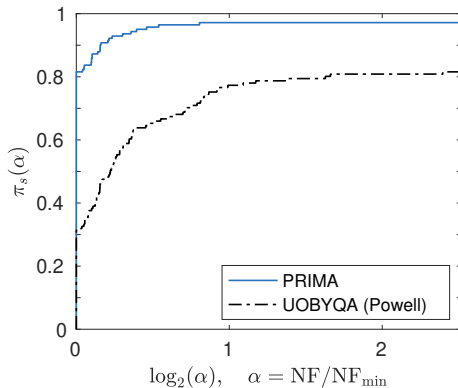
# What is this talk about?

Topics of this talk:

- ① A new DFO solver named PRIMA
- ② The mathematics behind it

Let us start with some tests of PRIMA based on the CUTEst problems.

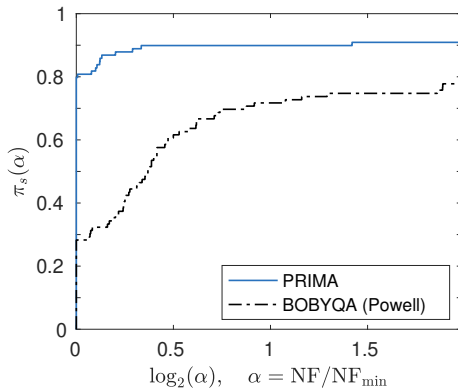
# Performance of PRIMA compared with Powell's solvers



PRIMA v.s. UOBYQA

(unconstrained problems, at most 100 variables)

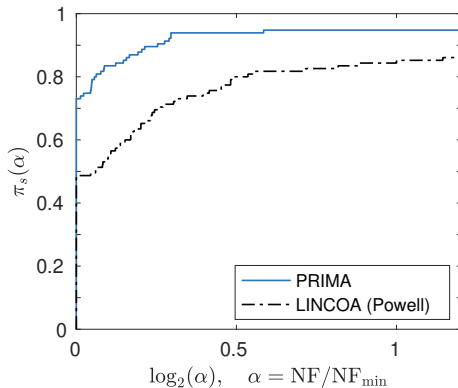
# Performance of PRIMA compared with Powell's solvers



PRIMA v.s. BOBYQA

(bound-constrained problems, at most 200 variables)

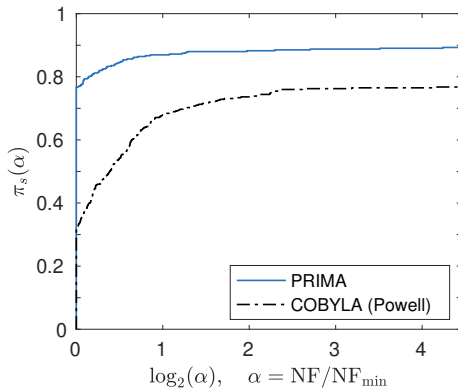
# Performance of PRIMA compared with Powell's solvers



PRIMA v.s. LINCOA

(linearly constrained problems, at most 200 variables, 20,000 constraints)

# Performance of PRIMA compared with Powell's solvers



PRIMA v.s. COBYLA

(nonlinearly constrained problems, at most 100 variables, 10,000 constraints)

Topics of this talk:

- ① ~~A new DFO solver named PRIMA~~

PRIMA is not a new solver but a **re**-implementation of Powell's solvers.

- ② ~~The mathematics behind it~~

There is no mathematics in this talk.

# PRIMA: Reference Implementation for Powell's Methods with Modernization and Amelioration

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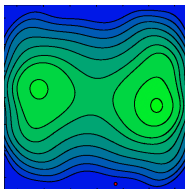
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Dedicated to the late Professor **M. J. D. Powell** FRS (1936–2015)

Funding: Hong Kong RGC grants 253012/17P, 153054/20P, and 153066/21P.



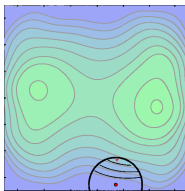
# Trust-region DFO methods based on interpolation models



$$x_{k+1} \approx x_k + \underset{\|d\| \leq \Delta_k}{\arg \min} M_k(x_k + d)$$

- $M_k$  is the trust-region model (surrogate)
  - $M_k(x) \approx f(x)$  around  $x_k$
  - $M_k$  interpolates  $f$  on a set  $\mathcal{X}_k$  consisting of previous iterates
- $\|d\| \leq \Delta_k$  is the trust-region constraint
  - If “things work well”, increase  $\Delta_k$
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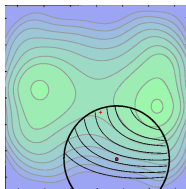
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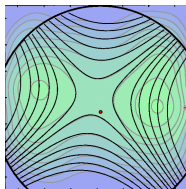
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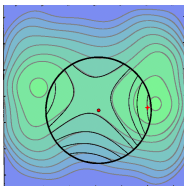
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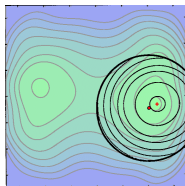
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# Maintenance of the interpolation set

The interpolation set  $\mathcal{X}_k$  must be updated with care.

- $\mathcal{X}_k$  must **reuse previous iterates** as much as possible.
- The **geometry** of  $\mathcal{X}_k$  must ensure the well-conditioning of the problem

$$M_k(x) = f(x), \quad x \in \mathcal{X}_k.$$

- Normally,  $\mathcal{X}_{k+1} = (\mathcal{X}_k \cup \{x_{k+1}\}) \setminus \{\text{a “bad” point}\}$ .
- Take **geometry-improving steps** if the geometry of  $\mathcal{X}_k$  deteriorates.

# Powell's trust-region DFO algorithms and software

- COBYLA: solving general nonlinearly constrained problems using **linear** models; code released in 1992; paper published in 1994
- UOBYQA: solving unconstrained problems using quadratic models; code released in 2000; paper published in 2002
- NEWUOA: solving unconstrained problems using quadratic models; code released in 2004; paper published in 2006
- BOBYQA: solving bound-constrained problems using quadratic models; code released and paper written in 2009
- LINCOA: solving linearly constrained problems using quadratic models; code released in 2013 but no paper written

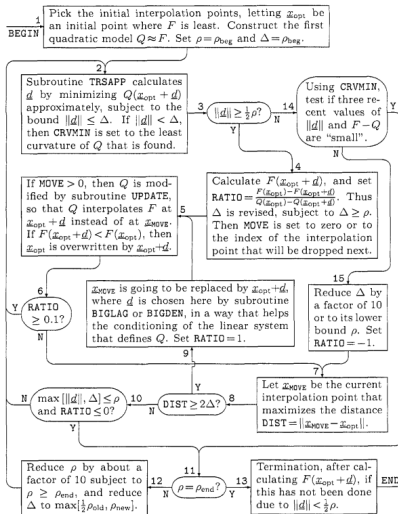


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- What about **COBYQA**?

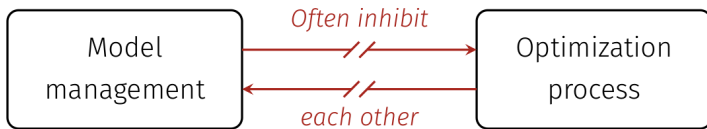
# What do these algorithms look like?

The NEWUOA software 259



NEWUOA

# The central difficulty



# Implementation of these methods is HARD

The development of NEWUOA has taken nearly **three** years. The work was very **frustrating** ...

— M. J. D. Powell

The NEWUOA software for unconstrained optimization without derivatives, 2006

## N.B.

- NEWUOA was Powell's **third** trust-region DFO solver, COBYLA and UOBYQA being the first two.
- Mathematically speaking, NEWUOA and UOBYQA are **essentially the same except for the ways they construct the model**.
- Given the experience with UOBYQA (and COBYLA), Powell still spent **three frustrating years** on the development of NEWUOA.

# Powell's implementation

- Powell implemented these five methods into publicly available solvers.
- The solvers are widely used by scientists and engineers.
- They are often used as benchmarks when designing new algorithms.
- However, the implementation was in Fortran 77, with plenty of GOTOs: in total, 7939 lines of code with 249 GOTOs!

A modernized implementation is greatly needed.

# Why should I work on a modernized implementation

- Professor Powell, April 2015: “It would be a relief to me if you would kindly continue to look after my optimisation software (NEWUOA, BOBYQA and LINCOA). Also I would like you to add COBYLA and TOLMIN if you do not have them already.”
- Stefan Wild, ICCOPT 2016, Tokyo: People do not want interfaces. They want implementations that they can understand and play with.
- Jeff Larson, ISMP 2018, Bordeaux: Numerical linear algebra people have standard implementations for standard algorithms, e.g., LAPACK, whereas we all work on our own implementation of interpolation, model improvement, ...

# Isn't it a perfect project for an engineer or a student?

- Given that Powell spent three frustrating years on the development of his own algorithm NEWUOA despite his abundant experience, where could I find this genius engineer who can learn all the five algorithms from scratch and implement them in a reasonable amount of time?
- Assume that I am lucky to find the abovementioned genius engineer and he/she happens to be my Ph.D. student. How should I persuade him/her to be fully devoted to a project for three years (as I did) without producing a single publication? Am I even allowed to do so?



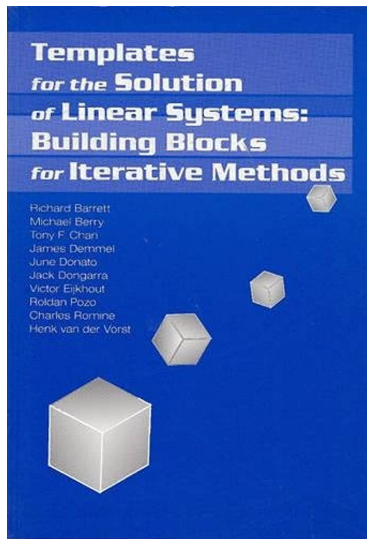
[libprima.net](http://libprima.net)

PRIMA is an acronym for

“Reference Implementation for Powell’s Methods  
with Modernization and Amelioration”,

“P” for Powell.





# An overview of PRIMA

- The solvers are implemented in a structured and modularized way so that they are **understandable**, **maintainable**, **extendable**, **fault-tolerant**, and **future-proof**.
- The code has **no GOTO** and **uses matrix-vector procedures instead of loops** whenever possible.
- The implementation is **mathematically equivalent** to Powell's **except for the bug fixes and improvements we introduce intentionally**.
- The implementation of PRIMA in **modern** Fortran (F2008 or above) has been finished.
- Versions in MATLAB, Python, Julia, R, ... will be implemented using the **modern** Fortran as a reference.
- A MATLAB interface is provided to use the **modern** Fortran version.
- The **inclusion of PRIMA into SciPy** is under discussion, and the major SciPy maintainers are positive about it.

# Why do I start with modern Fortran?



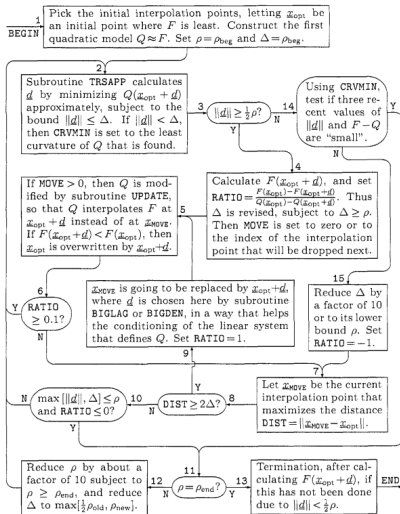
Fortran? Are you a caveman?

- The syntax and style of **modern** Fortran are very similar to MATLAB.
- I **start** with modern Fortran, so that I can **systematically verify** the **bit-to-bit faithfulness** of PRIMA, as the original code is Fortran.
- With other languages, the verification is hard, if not impossible.

Ultimate goal of PRIMA:  
Make Powell's methods **available to everyone in her/his favorite languages**.

# Powell's description of NEWUOA (recapped)

The NEWUOA software 259



NEWUOA

# The original implementation of NEWUOA: a snippet

```
117
118 100 KNEW=0
119 CALL TRSAPP (N,NPT,XOPT,XPT,GQ,HQ,PQ,DELTA,D,W,W(NP),
120 1 W(NP+N),W(NP+2*N),CRVMIN)
121 DSQ=ZERO
122 DO 110 I=1,N
123 110 DSQ=DSQ+D(I)**2
124 DNORM=DMIN1(DELTA,DSQRT(DSQ))
125 IF (DNORM .LT. HALF*RHO) THEN
126 KNEW=-1
127 DELTA=TENTH*DELTA
128 RATIO=-1.0D0
129 IF (DELTA .LE. 1.5D0*RHO) DELTA=RHO
130 IF (NF .LE. NFSAV+2) GOTO 460
131 TEMP=0.125D0*CRVMIN*RHO*RHO
132 IF (TEMP .LE. DMAX1(DIFFA,DIFFB,DIFFC)) GOTO 460
133 GOTO 490
134 END IF
135
136 120 IF (DSQ .LE. 1.0D-3*XOPTSQ) THEN
137 TEMPQ=0.25D0*XOPTSQ
138 DO 140 K=1,NPT
139 SUM=ZERO
140 DO 130 I=1,N
141 130 SUM=SUM+XPT(K,I)*XOPT(I)
142 TEMP=PQ(K)*SUM
143 SUM=SUM-HALF*XOPTSQ
144 W(NPT+K)=SUM
145 DO 140 I=1,N
146 GQ(I)=GQ(I)+TEMP*XPT(K,I)
147 XPT(K,I)=XPT(K,I)-HALF*XOPT(I)
148 VLAG(I)=BMAT(K,I)
149 W(I)=SUM*XPT(K,I)+TEMPQ*XOPT(I)
150 IP=NPT+I
151 DO 140 J=1,I
152 140 BMAT(IP,J)=BMAT(IP,J)+VLAG(I)*W(J)+W(I)*VLAG(J)
153
```

# Faithful pseudocode of NEWUOA in PRIMA

Pick  $\mathcal{X} \subset \mathbb{R}^n$  and  $\rho > 0$ . Let  $M$  interpolate  $f$  on  $\mathcal{X}$ .  $x_o := \arg \min_{x \in \mathcal{X}} f(x)$ .  $\Delta := \rho$ .

- 1: **while** not converged **do**
- 2:   Calculate a **trust-region trial point**  $x_{tr} \approx \arg \min \{M(x) : \|x - x_o\| \leq \Delta\}$
- 3:   **if**  $M(x_o) - M(x_{tr})$  is too small or  $\|x_{tr} - x_o\|$  is too short **then**
- 4:     Reduce  $\Delta$  **subject to**  $\Delta \geq \rho$
- 5:   **else**
- 6:     Evaluate the **reduction ratio** and **update**  $\Delta$  **accordingly** **subject to**  $\Delta \geq \rho$
- 7:     **if** it is proper to replace a point  $x_{drop} \in \mathcal{X}$  with  $x_{tr}$  **then**
- 8:       Set  $\mathcal{X} = (\mathcal{X} \cup \{x_{tr}\}) \setminus \{x_{drop}\}$ , and then **update**  $M$  and  $x_o$
- 9:     **end if**
- 10:   **end if**
- 11:   **improve\_geo** :=  $x_{tr}$  is bad & the geometry of  $\mathcal{X}$  is inadequate
- 12:   **reduce\_rho** :=  $x_{tr}$  is bad & the geometry of  $\mathcal{X}$  is adequate &  $\Delta$  is small
- 13:   **if** **improve\_geo** **then**
- 14:     Decide a point  $x_{drop} \in \mathcal{X}$  to drop and a **geometry-improving point**  $x_{geo}$
- 15:     Set  $\mathcal{X} = (\mathcal{X} \setminus \{x_{drop}\}) \cup \{x_{geo}\}$ , and then **update**  $M$  and  $x_o$
- 16:   **end if**
- 17:   **if** **reduce\_rho** **then** reduce  $\rho$  and reduce  $\Delta$  **subject to**  $\Delta \geq \rho$
- 18: **end while**

**N.B.:** The **updates** keep  $M$  interpolating  $f$  on  $\mathcal{X}$  and  $x_o = \arg \min_{x \in \mathcal{X}} f(x)$ .

# PRIMA NEWUOA: trust-region phase (ln. 2–10)

```
161
162 do tr = 1, maxtr
163   call trsapp(delta, gopt, hq, pq, trtol, xpt, crvmin, d)
164   dnorm = min(delta, norm(d))
165   shortd = (dnorm < HALF * rho)
166   qred = -quadinc(d, xpt, gopt, pq, hq)
167
168   if (shortd .or. .not. qred > 0) then
169     delta = TENTH * delta
170     if (delta <= gamma3 * rho) then
171       delta = rho ! Set DELTA to RHO when it is close to or below.
172     end if
173   else
174     x = xbase + (xpt(:, kopt) + d)
175     call evaluate(calfun, x, f)
176     nf = nf + 1
177
178     dnorm_rec = [dnorm_rec(2:size(dnorm_rec)), dnorm]
179     moderr = f - fval(kopt) + qred
180     moderr_rec = [moderr_rec(2:size(moderr_rec)), moderr]
181
182     ratio = redrat(fval(kopt) - f, qred, etal)
183     delta = trrad(delta, dnorm, etal, eta2, gamma1, gamma2, ratio)
184     if (delta <= gamma3 * rho) then
185       delta = rho ! Set DELTA to RHO when it is close to or below.
186     end if
187
188     ximproved = (f < fval(kopt))
189     knew_tr = setdrop_tr(idz, kopt, ximproved, bmat, d, delta, rho, xpt, zmat)
190     if (knew_tr > 0) then
191       xdrop = xpt(:, knew_tr)
192       xosav = xpt(:, kopt)
193       call updateh(knew_tr, kopt, d, xpt, idz, bmat, zmat)
194       call updatexf(knew_tr, ximproved, f, xosav + d, kopt, fval, xpt)
195       call updateq(idz, knew_tr, ximproved, bmat, d, moderr, xdrop, xosav, xpt, zmat, gopt, hq, pq)
196       call tryqalt(idz, bmat, fval - fval(kopt), ratio, xpt(:, kopt), xpt, zmat, itest, gopt, hq, pq)
197     end if
198   end if ! End of IF (SHORTD .OR. .NOT. QRED > 0). The normal trust-region calculation ends here.
199
```

# PRIMA NEWUOA: improve\_geo, reduce\_rho (ln.11,12)

```
199
200 accurate_mod = all(abs(moderr_rec) <= 0.125 * crvmin * rho**2) .and. all(dnorm_rec <= rho)
201 distsq = sum((xpt - spread(xpt(:, kopt), dim=2, ncopies=npt))**2, dim=1)
202 close_itpset = all(distsq <= 4.0 * delta**2)
203 adequate_geo = (shortd .and. accurate_mod) .or. close_itpset
204 small_trrad = (max(delta, dnorm) <= rho)
205 bad_trstep = (shortd .or. (.not. qred > 0) .or. ratio <= etal .or. knew_tr == 0)
206 improve_geo = bad_trstep .and. .not. adequate_geo
207 bad_trstep = (shortd .or. (.not. qred > 0) .or. ratio <= 0 .or. knew_tr == 0)
208 reduce_rho = bad_trstep .and. adequate_geo .and. small_trrad
209
```



# PRIMA NEWUOA: post-processing phase (ln. 13–17)

```
209
210 if (improve_geo) then
211   knew_geo = int(maxloc(distsq, dim=1), kind(knew_geo))
212   delbar = max(min(TENTH * sqrt(maxval(distsq)), HALF * delta), rho)
213   d = geostep(idz, knew_geo, kopt, bmat, delbar, xpt, zmat)
214   x = xbase + (xpt(:, kopt) + d)
215   call evaluate(calfun, x, f)
216   nf = nf + 1
217
218   dnorm = min(delbar, norm(d))
219   dnorm_rec = [dnorm_rec(2:size(dnorm_rec)), dnorm]
220   moderr = f - fval(kopt) - quadinc(d, xpt, gopt, pq, hq)
221   moderr_rec = [moderr_rec(2:size(moderr_rec)), moderr]
222
223   ximproved = (f < fval(kopt))
224   xdrop = xpt(:, knew_geo)
225   xosav = xpt(:, kopt)
226   call updateh(knew_geo, kopt, d, xpt, idz, bmat, zmat)
227   call updatexf(knew_geo, ximproved, f, xosav + d, kopt, fval, xpt)
228   call updateq(idz, knew_geo, ximproved, bmat, d, moderr, xdrop, xosav, xpt, zmat, gopt, hq, pq)
229 end if ! End of IF (IMPROVE_GEO). The procedure of improving geometry ends.
230
231 if (reduce_rho) then
232   if (rho <= rhoend) then
233     info = SMALL_TR_RADIUS
234     exit
235   end if
236   delta = HALF * rho
237   rho = redrho(rho, rhoend)
238   delta = max(delta, rho)
239   dnorm_rec = REALMAX
240   moderr_rec = REALMAX
241 end if ! End of IF (REDUCE_RHO). The procedure of reducing RHO ends.
242
243 if (sum(xpt(:, kopt)**2) >= 1.0E2 * delta**2) then ! 1.0E2 works better than 1.0E3 on 20230227.
244   call shiftbase(kopt, xbase, xpt, zmat, bmat, pq, hq, idz)
245 end if
246 end do ! End of DO TR = 1, MAXTR. The iterative procedure ends.
247
```

# Issues in the Fortran 77 implementation: an example

```
72 C
73 C   If KNEW is zero initially, then pick the index of the interpolation
74 C   point to be deleted, by maximizing the absolute value of the
75 C   denominator of the updating formula times a weighting factor.
76 C
77 C
78   IF (KNEW .EQ. 0) THEN
79     DENMAX=ZERO
80     DO 100 K=1,NPT
81       HDIAG=ZERO
82       DO 80 J=1,NPTM
83         TEMP=ONE
84         IF (J .LT. IDZ) TEMP=-ONE
85 80     HDIAG=HDIAG+TEMP*ZMAT(K,J)**2
86         DENABS=DABS(BETA*HDIAG+VLAG(K)**2)
87         DISTSQ=ZERO
88         DO 90 J=1,N
89 90     DISTSQ=DISTSQ+(XPT(K,J)-XPT(KOPT,J))**2
90     TEMP=DENABS*DISTSQ*DISTSQ
91     IF (TEMP .GT. DENMAX) THEN
92       DENMAX=TEMP
93       KNEW=K
94     END IF
95 100   CONTINUE
96   END IF
97 C
98 C   Apply the rotations that put zeros in the KNEW-th row of ZMAT.
99 C
100   JL=1
101   IF (NPTM .GE. 2) THEN
102     DO 120 J=2,NPTM
103       IF (J .EQ. IDZ) THEN
104         JL=IDZ
105       ELSE IF (ZMAT(KNEW,J) .NE. ZERO) THEN
```

The above code may crash, as KNEW may be used uninitialized.

# Issues in the Fortran 77 implementation

- The Fortran 77 solvers may **crash with memory violations** (segfaults).  
Reason: Some indices are only initialized under conditions that can never be met because of NaN resulted from floating point exceptions.
- The Fortran 77 solvers may **get stuck in infinite loops**.  
Reason: Some loops are only terminated under conditions that can never be met because of NaN resulted from floating point exceptions.

## N.B.:

- The problems are due to floating point exceptions in the Fortran 77 code rather than flaws in the algorithms.
- The problems affect all implementations or wrappers of these solvers based on the Fortran 77 code, including SciPy (COBYLA), NLopt, ...

# How to ensure PRIMA does not have similar issues?

## Strategy 1. Programming by contract

```
120
121 ! Preconditions
122 if (DEBUGGING) then
123   call assert(n >= 1 .and. npt >= n + 2, 'N >= 1, NPT >= N + 2', srname)
124   call assert(delta > 0, 'DELTA > 0', srname)
125   call assert(size(gopt_in) == n, 'SIZE(GOPT) = N', srname)
126   call assert(size(hq_in, 1) == n .and. issymmetric(hq_in), 'HQ is an NxN symmetric matrix', srname)
127   call assert(size(pq_in) == npt, 'SIZE(PQ) = NPT', srname)
128   call assert(all(is_finite(xpt)), 'XPT is finite', srname)
129   call assert(size(s) == n, 'SIZE(S) = N', srname)
130 end if
131
```

```
431
432 ! Postconditions
433 if (DEBUGGING) then
434   call assert(size(s) == n .and. all(is_finite(s)), 'SIZE(S) = N, S is finite', srname)
435   ! Due to rounding, it may happen that ||S|| > DELTA, but ||S|| > 2*DELTA is highly improbable.
436   call assert(norm(s) <= TWO * delta, '||S|| <= 2*DELTA', srname)
437   call assert(crvmin >= 0, 'CRVMIN >= 0', srname)
438 end if
439
```

- The preconditions and postconditions are **checked** only in the **debug mode**. In the code that users receive, they are disabled by default.
- In the debug mode, if some subroutine receives strange inputs or produces strange outputs, the program will **raise an error** so that the developer (i.e., Zaikun Zhang) can **check the issue and fix it**.

# How to ensure PRIMA does not have similar issues?

## Strategy 2. TOUGH (Tolerance Of Untamed and Genuine Hazards) tests

```
650
651 function f = noisy(f, x, noise_level)
652 if nargin < 3
653     noise_level = 2e-1;
654 end
655 r = cos(1.006 * sin(1.006 * (abs(f) + 1.000) * cos(1.006 * sum(abs(x)))));
656 f = f*(1+noise_level*r);
657 if (r > 0.9)
658     error('Function evaluation fails!');
659 elseif (r > 0.75)
660     f = inf;
661 elseif (r > 0.5)
662     f = NaN;
663 elseif (r < -0.999)
664     f = -1e30;
665 end
666 return
667
```

- In TOUGH tests, objective functions are corrupted as above and then fed to the solvers.
- We make sure that PRIMA solvers work properly even if the objective functions are corrupted in this severe way. ([What about your solvers?](#))

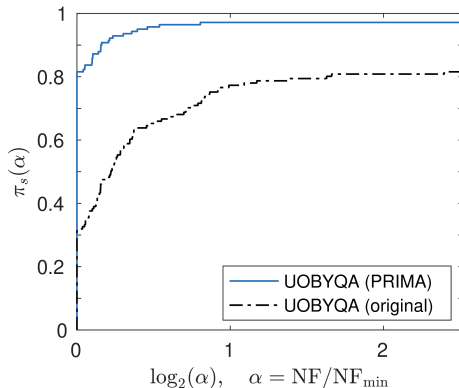
# How to ensure PRIMA does not have similar issues?

## Strategy 3. Automated and randomized tests using GitHub Actions

- Every day, extensive TOUGH tests and other tests are conducted **automatically** on **randomized** variants of CUTEst problems.
- The longer time passes, the more reliable PRIMA is, **automatically**.
- As of June 2023,  $> 42,000$  workflows have been successfully run.
- Each workflow consists of  $\sim 5$  (sometimes more than 150) randomized tests, each test taking from tens of minutes to several hours.
- In other words, PRIMA has been verified by more than 200,000 hours (or **more than 20 years**) of randomized tests.

**Code must be battle-tested before it becomes software.**

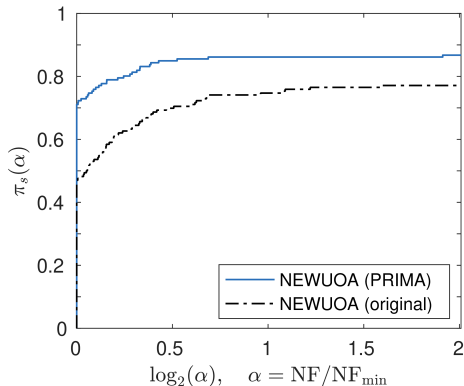
# The performance of PRIMA



UOBYQA

(unconstrained problems, at most 100 variables)

# The performance of PRIMA

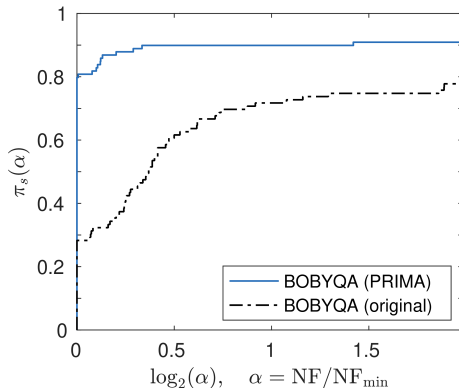


NEWUOA

(unconstrained problems, at most 200 variables)



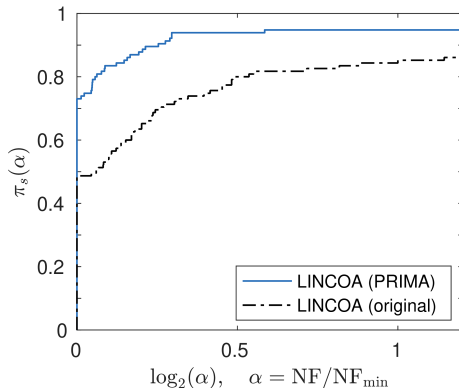
# The performance of PRIMA



BOBYQA

(bound-constrained problems, at most 200 variables)

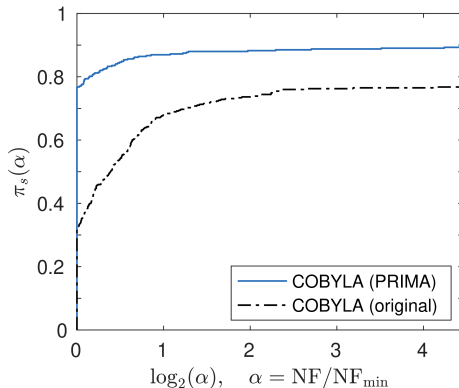
# The performance of PRIMA



LINCOA

(linearly constrained problems, at most 200 variables, 20,000 constraints)

# The performance of PRIMA



COBYLA

(nonlinearly constrained problems, at most 100 variables, 10,000 constraints)

# How to ensure the improvements are not by luck?

Take COBYLA as an example.

The PRIMA implementation of COBYLA is tested on 359 nonlinearly constrained CUTEst problems with at most 100 variables and 10,000 constraints. Seven tests are made.

- ➊ A plain test
- ➋ A test that permutes the variables randomly
- ➌ A test that perturbs the starting point randomly
- ➍ A test based on single-precision objective & constraint values
- ➎ A test using only 5 significant digits of the objective & constraints
- ➏ A test contaminating the objective & constraints by deterministic noise
- ➐ A test contaminating the objective & constraints by random noise

# How to ensure the improvements are not by luck?



168 performance profiles of the new and old implementations of COBYLA

## A “fun” fact ...

- Working on PRIMA, I have spotted a dozen of bugs in reputable Fortran compilers and two bugs in MATLAB.
- Each of them represents days of bitter debugging.
- From an unusual angle, they reflect how intensive the coding is.
- The bitterness behind this fun fact is exactly why I work on PRIMA:
  - I hope all the frustrations that I have experienced will not happen to any user of Powell's methods anymore.
  - I hope I am the last one in the world to decode a maze of 244 GOTOs in 7939 lines of Fortran 77 code — I did this for three years and I do not want anyone else to do it again.

## But it is not quite rewarding in terms of career and life ...

- You may write 3 good papers in 1 year, but not 1 good package in 3 years, especially if you start with a nontrivial Fortran 77 codebase.
- Internet: “Writing software is a low-status academic activity.”
- Internet: “A general problem is that ... professors are usually rewarded for publications, not their software.”
- Comments on my grant proposal: The PI’s expertise seems in software development, but he may not be a good mathematician.
- As a “not-so-good mathematician”, I much prefer spending my time on proofs, which are a lot easier and much more enjoyable for me.
- Sometimes we do things that are not enjoyable but have to be done.
- Teaser: Who translated Euclid’s *Elements* from Ancient Greek to modern languages? You probably do not know (and do not care).

# Concluding remarks

- Implementation of model-based DFO solvers is intrinsically hard
- PRIMA provides the reference implementation of Powell's DFO solvers
- The modern Fortran version and a MATLAB interface is finished
- PRIMA will also be implemented in MATLAB, Python, Julia, R, C++
- PRIMA fixes issues in the original Fortran 77 code
- PRIMA is tested extensively to ensure its correctness & robustness
- PRIMA outperforms the original implementation of the solvers



[libprima.net](http://libprima.net)

Thank you!