PRIMA: What, Why, How, and So What?

Zaikun Zhang

The Hong Kong Polytechnic University

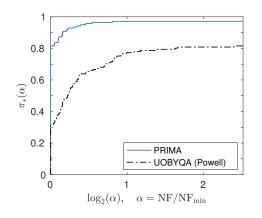
Optimization 2023, Aveiro, Portugal

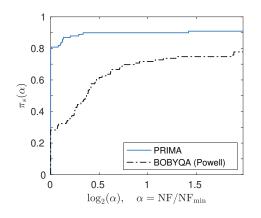
What is this talk about?

Topics of this talk:

- A new DFO solver named PRIMA
- 2 The mathematics behind it

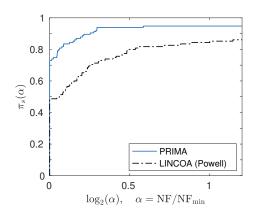
Let us start with some tests of PRIMA based on the CUTEst problems.





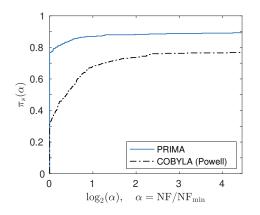
PRIMA v.s. BOBYQA

(bound-constrained problems, at most 200 variables)



PRIMA v.s. LINCOA

(linearly constrained problems, at most 200 variables, 20,000 constraints)



PRIMA v.s. COBYLA

(nonlinearly constrained problems, at most 100 variables, $10,\!000$ constraints)

Sorry, I lied ...

Topics of this talk:

- A new DFO solver named PRIMA
 PRIMA is not a new solver but a re-implementation of Powell's solvers.
- The mathematics behind it
 There is no mathematics in this talk.

PRIMA: Reference Implementation for Powell's Methods with Modernization and Amelioration

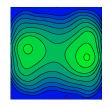
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Dedicated to the late Professor M. J. D. Powell FRS (1936–2015)

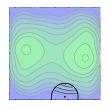
Funding: Hong Kong RGC grants 253012/17P, 153054/20P, and 153066/21P.



$$x_{k+1} \approx x_k + \underset{\|d\| \le \Delta_k}{\operatorname{arg \, min}} M_k(x_k + d)$$

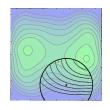
- M_k is the trust-region model (surrogate)
 - $M_k(x) \approx f(x)$ around x_k
 - ullet M_k interpolates f on a set \mathcal{X}_k consisting of previous iterates
- $||d|| \leq \Delta_k$ is the trust-region constraint
 - ullet If "things work well", increase Δ_k
 - Otherwise, decrease Δ_k

Images by Dr. F. V. Berghen from http://www.applied-mathematics.net.



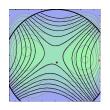
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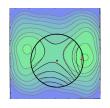
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Maintenance of the interpolation set

The interpolation set \mathcal{X}_k must be updated with care.

- ullet \mathcal{X}_k must reuse previous iterates as much as possible.
- ullet The geometry of \mathcal{X}_k must ensure the well-conditioning of the problem

$$M_k(x) = f(x), \quad x \in \mathcal{X}_k.$$

- Normally, $\mathcal{X}_{k+1} = (\mathcal{X}_k \cup \{x_{k+1}\}) \setminus \{a \text{ "bad" point}\}.$
- ullet Take geometry-improving steps if the geometry of \mathcal{X}_k deteriorates.

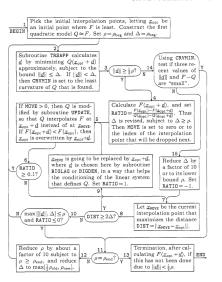
Powell's trust-region DFO algorithms and software

- COBYLA: solving general nonlinearly constrained problems using linear models; code released in 1992; paper published in 1994
- UOBYQA: solving unconstrained problems using quadratic models; code released in 2000; paper published in 2002
- NEWUOA: solving unconstrained problems using quadratic models;
 code released in 2004; paper published in 2006
- BOBYQA: solving bound-constrained problems using quadratic models; code released and paper written in 2009
- LINCOA: solving linearly constrained problems using quadratic models; code released in 2013 but no paper written
- COBYQA is the successor of COBYLA using quadratic models. Visit:



What do these algorithms look like?

The NEWUOA software



NEWUOA

Implementation of these methods is HARD

The development of NEWUOA has taken nearly three years. The work was very frustrating ...

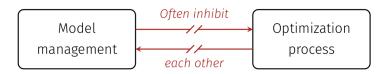
— M. J. D. Powell

The NEWUOA software for unconstrained optimization without derivatives, 2006

N.B.

- NEWUOA was Powell's third trust-region DFO solver, COBYLA and UOBYQA being the first two.
- Mathematically speaking, NEWUOA and UOBYQA are essentially the same except for the ways they construct the model.
- Given the experience with UOBYQA (and COBYLA), Powell still spent three frustrating years on the development of NEWUOA.

The central difficulty



Powell's implementation

- Powell implemented these five methods into publicly available solvers.
- The solvers are widely used by scientists and engineers.
- They are often used as benchmarks when designing new algorithms.
- However, the implementation was in Fortran 77, with plenty of GOTOs: in total, 7939 lines of code with 249 GOTOs!

A modernized implementation is greatly needed.

Why should ${f I}$ work on a modernized implementation

- Professor Powell, April 2015: "It would be a relief to me if you would kindly continue to look after my optimisation software (NEWUOA, BOBYQA and LINCOA). Also I would like you to add COBYLA and TOLMIN if you do not have them already."
- Stefan Wild, ICCOPT 2016, Tokyo: People do not want interfaces.
 They want implementations that they can understand and play with.
- Jeff Larson, ISMP 2018, Bordeaux: Numerical linear algebra people have standard implementations for standard algorithms, e.g., LAPACK, whereas we all work on our own implementation of interpolation, model improvement, ...

Isn't it a perfect project for an engineer or a student?

- Given that Powell spent three frustrating years on the development of his own algorithm NEWUOA despite his abundant experience, where could I find this genius engineer who can learn all the five algorithms from scratch and implement them in a reasonable amount of time?
- Assume that I am lucky to find the abovementioned genius engineer and he/she happens to be my Ph.D. student. How should I persuade him/her to be fully devoted to a project for three years (as I did) without producing a single publication? Am I even allowed to do so?

PRIMA



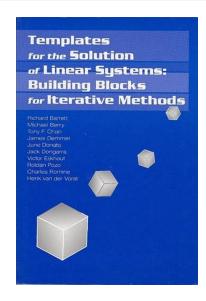
libprima.net

PRIMA is an acronym for

"Reference Implementation for Powell's Methods with Modernization and Amelioration",

"P" for Powell.

An inspiration



An overview of PRIMA

- The solvers are implemented in a structured and modularized way so that they are understandable, maintainable, extendable, fault-tolerant, and future-proof.
- The code has no GOTO and uses matrix-vector procedures instead of loops whenever possible.
- The implementation is mathematically equivalent to Powell's except for the bug fixes and improvements we introduce intentionally.
- The implementation of PRIMA in modern Fortran (F2008 or above) has been finished.
- Versions in MATLAB, Python, Julia, R, ... will be implemented using the modern Fortran as a reference.
- A MATLAB interface is provided to use the modern Fortran version.
- The inclusion of PRIMA into SciPy is under discussion, and the major SciPy maintainers are positive about it.

Why do I start with modern Fortran?



Fortran? Are you a caveman?

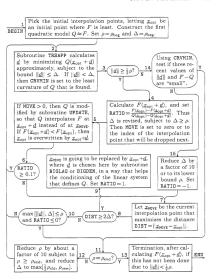
- The syntax and style of modern Fortran are very similar to MATLAB.
- I start with modern Fortran, so that I can systematically verify the bit-to-bit faithfulness of PRIMA, as the original code is Fortran.
- With other languages, the verification is hard, if not impossible.

Ultimate goal of PRIMA:

Make Powell's methods available to everyone in her/his favorite languages.

Powell's description of NEWUOA (recapped)

The NEWUOA software



NEWUOA

The original implementation of NEWUOA: a snippet

```
118
      100 KNEW=0
119
          CALL TRSAPP (N, NPT, XOPT, XPT, GQ, HQ, PQ, DELTA, D, W, W(NP),
         1 W(NP+N), W(NP+2*N), CRVMIN)
          DS0=ZERO
          DO 110 I=1,N
      110 DSO=DSO+D(I)**2
124
          DNORM=DMIN1 (DELTA, DSQRT(DSQ))
          IF (DNORM .LT. HALF*RHO) THEN
              KNEW=-1
              DELTA=TENTH*DELTA
              RATIO=-1.0D0
129
              IF (DELTA .LE. 1.5D0*RHO) DELTA=RHO
              IF (NF .LE. NFSAV+2) GOTO 460
              TEMP=0.125D0*CRVMIN*RHO*RHO
              IF (TEMP .LE. DMAX1(DIFFA, DIFFB, DIFFC)) GOTO 460
              GOTO 490
134
          END IF
      120 IF (DSO .LE. 1.0D-3*XOPTSO) THEN
              TEMPO=0.25D0*XOPTSO
              DO 140 K=1,NPT
139
              SUM=ZERO
140
              DO 130 I=1,N
141
              SUM=SUM+XPT(K,I)*XOPT(I)
142
              TEMP=PO(K)*SUM
              SUM=SUM-HALF*XOPTSQ
144
              W(NPT+K)=SUM
145
              DO 140 I=1,N
146
              GO(I)=GO(I)+TEMP*XPT(K,I)
147
              XPT(K,I)=XPT(K,I)-HALF*XOPT(I)
148
              VLAG(I)=BMAT(K,I)
149
              W(I)=SUM*XPT(K,I)+TEMPO*XOPT(I)
150
              IP=NPT+I
              DO 140 J=1,I
              BMAT(IP.J)=BMAT(IP.J)+VLAG(I)*W(J)+W(I)*VLAG(J)
```

Faithful pseudocode of NEWUOA in PRIMA

```
Pick \mathcal{X} \subset \mathbb{R}^n and \rho > 0. Let M interpolate f on \mathcal{X}. x_0 := \arg\min_{x \in \mathcal{X}} f(x). \Delta := \rho.
 1: while not converged do
 2:
         Calculate a trust-region trial point x_{\mathsf{tr}} \approx \arg\min\{M(x) : ||x - x_{\mathsf{o}}|| \leq \Delta\}
         if M(x_0) - M(x_{tr}) is too small or ||x_{tr} - x_0|| is too short then
 3:
             Reduce \Delta subject to \Delta \geq \rho
 4:
         else
 5:
             Evaluate the reduction ratio and update \Delta accordingly subject to \Delta \geq \rho
 6:
 7:
             if it is proper to replace a point x_{\sf drop} \in \mathcal{X} with x_{\sf tr} then
 8:
                Set \mathcal{X} = (\mathcal{X} \cup \{x_{\mathsf{tr}}\}) \setminus \{x_{\mathsf{drop}}\}, and then update M and x_{\mathsf{o}}
            end if
 9:
         end if
10:
         improve_geo := x_{tr} is bad & the geometry of \mathcal{X} is inadequate
11:
         reduce rho := x_{tr} is bad & the geometry of \mathcal{X} is adequate & \Delta is small
12:
13:
         if improve geo then
14:
             Decide a point x_{drop} \in \mathcal{X} to drop and a geometry-improving point x_{geo}
            Set \mathcal{X} = (\mathcal{X} \setminus \{x_{\mathsf{drop}}\}) \cup \{x_{\mathsf{geo}}\}, and then update M and x_{\mathsf{o}}
15:
         end if
16:
17:
         if reduce_rho then reduce \rho and reduce \Delta subject to \Delta \geq \rho
18: end while
```

N.B.: The updates keep M interpolating f on \mathcal{X} and $x_0 = \arg\min_{x \in \mathcal{X}} f(x)$.

16/34

PRIMA NEWUOA: trust-region phase (ln. 2–10)

```
161
162 do tr = 1, maxtr
        call trsapp(delta, gopt, hq, pq, trtol, xpt, crvmin, d)
        dnorm = min(delta, norm(d))
        shortd = (dnorm < HALF * rho)
166
        gred = -quadinc(d, xpt, gopt, pq, hq)
168
        if (shortd .or. .not. gred > 0) then
            delta = TENTH * delta
170
            if (delta <= gamma3 * rho) then
                delta = rho ! Set DELTA to RHO when it is close to or below.
           end if
        else
            x = xbase + (xpt(:, kopt) + d)
           call evaluate(calfun, x, f)
            nf = nf + 1
178
            dnorm rec = [dnorm rec(2:size(dnorm rec)), dnorm]
179
            moderr = f - fval(kopt) + gred
180
            moderr_rec = [moderr_rec(2:size(moderr_rec)), moderr]
            ratio = redrat(fval(kopt) - f, gred, etal)
            delta = trrad(delta, dnorm, eta1, eta2, gamma1, gamma2, ratio)
184
            if (delta <= gamma3 * rho) then
                delta = rho ! Set DELTA to RHO when it is close to or below.
186
            end if
188
            ximproved = (f < fval(kopt))</pre>
189
            knew_tr = setdrop_tr(idz, kopt, ximproved, bmat, d, delta, rho, xpt, zmat)
190
            if (knew_tr > 0) then
                xdrop = xpt(:, knew_tr)
                xosav = xpt(:, kopt)
                call updateh(knew_tr, kopt, d, xpt, idz, bmat, zmat)
194
                call updatexf(knew_tr, ximproved, f, xosav + d, kopt, fval, xpt)
                call updateq(idz, knew_tr, ximproved, bmat, d, moderr, xdrop, xosav, xpt, zmat, gopt, hq, pq)
196
                call tryqalt(idz, bmat, fval - fval(kopt), ratio, xpt(:, kopt), xpt, zmat, itest, gopt, hq, pq)
            end if
198
        end if ! End of IF (SHORTD .OR. .NOT. ORED > 0). The normal trust-region calculation ends here.
199
```

PRIMA NEWUOA: improve_geo, reduce_rho (ln.11,12)

```
199
200
        accurate mod = all(abs(moderr rec) <= 0.125 * crymin * rho**2) .and. all(dnorm rec <= rho)
201
        distsq = sum((xpt - spread(xpt(:, kopt), dim=2, ncopies=npt))**2, dim=1)
202
        close_itpset = all(distsq <= 4.0 * delta**2)</pre>
        adequate geo = (shortd .and. accurate mod) .or. close itpset
204
        small_trrad = (max(delta, dnorm) <= rho)</pre>
205
        bad_trstep = (shortd .or. (.not. gred > 0) .or. ratio <= eta1 .or. knew_tr == 0)
206
207
        improve_geo = bad_trstep .and. .not. adequate_geo
        bad_trstep = (shortd .or. (.not. gred > 0) .or. ratio <= 0 .or. knew_tr == 0)
208
        reduce_rho = bad_trstep .and. adequate_geo .and. small_trrad
```

PRIMA NEWUOA: post-processing phase (ln. 13–17)

```
209
210
        if (improve_geo) then
211
            knew_geo = int(maxloc(distsq, dim=1), kind(knew_geo))
212
            delbar = max(min(TENTH * sqrt(maxval(distsq)), HALF * delta), rho)
213
            d = geostep(idz, knew geo, kopt, bmat, delbar, xpt, zmat)
214
            x = xbase + (xpt(:, kopt) + d)
215
           call evaluate(calfun, x, f)
216
            nf = nf + 1
217
218
           dnorm = min(delbar. norm(d))
219
            dnorm_rec = [dnorm_rec(2:size(dnorm_rec)), dnorm]
220
            moderr = f - fval(kopt) - quadinc(d, xpt, gopt, pq, hq)
221
            moderr_rec = [moderr_rec(2:size(moderr_rec)), moderr]
222
223
            ximproved = (f < fval(kopt))
224
            xdrop = xpt(:, knew_geo)
225
            xosav = xpt(:, kopt)
226
            call updateh(knew_geo, kopt, d, xpt, idz, bmat, zmat)
227
            call updatexf(knew_geo, ximproved, f, xosav + d, kopt, fval, xpt)
228
            call updateg(idz, knew geo, ximproved, bmat, d, moderr, xdrop, xosay, xpt, zmat, gopt, hg, pg)
229
        end if ! End of IF (IMPROVE GEO). The procedure of improving geometry ends.
230
231
        if (reduce rho) then
232
            if (rho <= rhoend) then
233
                info = SMALL_TR_RADIUS
234
                exit
            end if
235
236
            delta = HALF * rho
237
           rho = redrho(rho, rhoend)
238
            delta = max(delta, rho)
239
           dnorm rec = REALMAX
240
            moderr_rec = REALMAX
241
        end if ! End of IF (REDUCE RHO). The procedure of reducing RHO ends.
242
243
        if (sum(xpt(:, kopt)**2) >= 1.0E2 * delta**2) then ! 1.0E2 works better than 1.0E3 on 20230227.
244
            call shiftbase(kopt, xbase, xpt, zmat, bmat, pq, hq, idz)
245
        end if
246 end do ! End of DO TR = 1. MAXTR. The iterative procedure ends.
247
```

Issues in the Fortran 77 implementation: an example

```
72 C
         If KNEW is zero initially, then pick the index of the interpolation
74 C
           point to be deleted, by maximizing the absolute value of the
           denominator of the updating formula times a weighting factor.
         IF (KNEW .EQ. 0) THEN
             DENMAX=ZERO
80
             DO 100 K=1,NPT
             HDIAG=ZERO
             DO 80 J=1.NPTM
             TEMP=ONE
             IF (J .LT. IDZ) TEMP=-ONE
          HDIAG=HDIAG+TEMP*ZMAT(K, J)**2
86
             DENABS=DABS(BETA*HDIAG+VLAG(K)**2)
             DISTSO=ZERO
88
            DO 90 J=1.N
            DISTSO=DISTSO+(XPT(K,J)-XPT(KOPT,J))**2
90
             TEMP=DENABS*DISTSQ*DISTSQ
             IF (TEMP .GT. DENMAX) THEN
                 DENMAX=TEMP
                 KNEW=K
94
             END IF
     100
             CONTINUE
         END IF
98 C
         Apply the rotations that put zeros in the KNEW-th row of ZMAT.
99 C
100
         IF (NPTM .GE. 2) THEN
             DO 120 J=2,NPTM
             IF (J .EQ. IDZ) THEN
                 JL=IDZ
             ELSE IF (ZMAT(KNEW.J) .NE. ZERO) THEN
```

The above code may crash, as KNEW may be used uninitialized.

Issues in the Fortran 77 implementation

- The Fortran 77 solvers may crash with memory violations (segfaults).
 Reason: Some indices are only initialized under conditions that can never be met because of NaN resulted from floating point exceptions.
- The Fortran 77 solvers may get stuck in infinite loops.
 Reason: Some loops are only terminated under conditions that can never be met because of NaN resulted from floating point exceptions.

N.B.:

- The problems are due to floating point exceptions in the Fortran 77 code rather than flaws in the algorithms.
- The problems affect all implementations or wrappers of these solvers based on the Fortran 77 code, including SciPy (COBYLA), NLopt, ...

How to ensure PRIMA does not have similar issues?

Strategy 1. Programming by contract

```
Preconditions
122 if (DEBUGGING) then
       call assert(n >= 1 .and. npt >= n + 2, 'N >= 1, NPT >= N + 2', srname)
       call assert(delta > 0. 'DELTA > 0', srname)
       call assert(size(gopt_in) == n, 'SIZE(GOPT) = N', srname)
       call assert(size(hq_in, 1) == n .and. issymmetric(hq_in), 'HO is an NxN symmetric matrix', srname)
       call assert(size(pq_in) == npt, 'SIZE(PQ) = NPT', srname)
       call assert(all(is_finite(xpt)), 'XPT is finite', srname)
       call assert(size(s) == n, 'SIZE(S) == N', srname)
432 ! Postconditions
433 if (DEBUGGING) then
       call assert(size(s) == n .and. all(is_finite(s)), 'SIZE(S) == N, S is finite', srname)
       ! Due to rounding, it may happen that ||S|| > DELTA, but ||S|| > 2*DELTA is highly improbable.
       call assert(norm(s) <= TWO * delta, '||S|| <= 2*DELTA', srname)</pre>
       call assert(crvmin >= 0, 'CRVMIN >= 0', srname)
438 end if
```

- The preconditions and postconditions are checked only in the debug mode. In the code that users receive, they are disabled by default.
- In the debug mode, if some subroutine receives strange inputs or produces strange outputs, the program will raise an error so that the developer (i.e., Zaikun Zhang) can check the issue and fix it.

How to ensure PRIMA does not have similar issues?

Strategy 2. TOUGH (Tolerance Of Untamed and Genuine Hazards) tests

```
651 function f = noisy(f, x, noise_level)
652 if nargin < 3
        noise_level = 2e-1;
654 end
655 \text{ r} = \cos(1.006 * \sin(1.006 * (abs(f) + 1.000) * \cos(1.006 * sum(abs(x)))));
656 f = f*(1+noise_level*r);
657 if (r > 0.9)
        error('Function evaluation fails!');
659 elseif (r > 0.75)
   f = inf:
661 elseif (r > 0.5)
       f = NaN:
663 elseif (r < - 0.999)
664 f = -1e30;
665 end
666 return
```

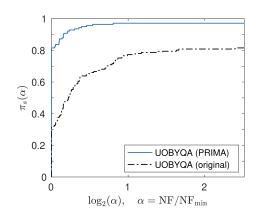
- In TOUGH tests, objective functions are corrupted as above and then fed to the solvers.
- We make sure that PRIMA solvers work properly even if the objective functions are corrupted in this severe way. (What about your solvers?)

How to ensure PRIMA does not have similar issues?

Strategy 3. Automated and randomized tests using GitHub Actions

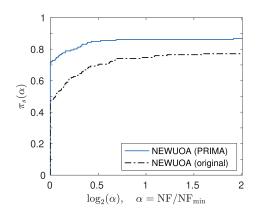
- Every day, extensive TOUGH tests and other tests are conducted automatically on randomized variants of CUTEst problems.
- The longer time passes, the more reliable PRIMA is, automatically.
- ullet As of June 2023, > 42,000 workflows have been successfully run.
- \bullet Each workflow consists of \sim 5 (sometimes more than 150) randomized tests, each test taking from tens of minutes to several hours.
- In other words, PRIMA has been verified by more than 200,000 hours (or more than 20 years) of randomized tests.

Code must be battle-tested before it becomes software.



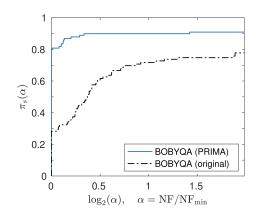
UOBYQA

(unconstrained problems, at most 100 variables)



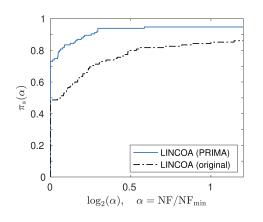
NEWUOA

(unconstrained problems, at most 200 variables)



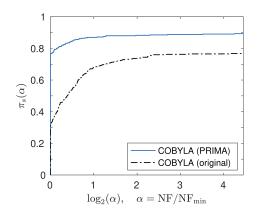
BOBYQA

(bound-constrained problems, at most 200 variables)



LINCOA

(linearly constrained problems, at most 200 variables, 20,000 constraints)



COBYLA

(nonlinearly constrained problems, at most 100 variables, 10,000 constraints)

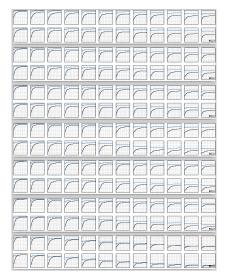
How to ensure the improvements are not by luck?

Take COBYLA as an example.

The PRIMA implementation of COBYLA is tested on 359 nonlinearly constrained CUTEst problems with at most 100 variables and 10,000 constraints. Seven tests are made.

- A plain test
- A test that permutes the variables randomly
- A test that perturbs the starting point randomly
- A test based on single-precision objective & constraint values
- A test using only 5 significant digits of the objective & constraints
- A test contaminating the objective & constraints by deterministic noise
- A test contaminating the objective & constraints by random noise

How to ensure the improvements are not by luck?



168 performance profiles of the new and old implementations of COBYLA

A "fun" fact ...

- Working on PRIMA, I have spotted a dozen of bugs in reputable Fortran compilers and two bugs in MATLAB.
- Each of them represents days of bitter debugging.
- From an unusual angle, they reflect how intensive the coding is.
- The bitterness behind this fun fact is exactly why I work on PRIMA:
 - I hope all the frustrations that I have experienced will not happen to any user of Powell's methods anymore.
 - I hope I am the last one in the world to decode a maze of 244 GOTOs in 7939 lines of Fortran 77 code — I did this for three years and I do not want anyone else to do it again.

But it is not quite rewarding in terms of career and life ...

- You may write 3 good papers in 1 year, but not 1 good package in 3 years, especially if you start with a nontrivial Fortran 77 codebase.
- Internet: "Writing software is a low-status academic activity."
- Internet: "A general problem is that ... professors are usually rewarded for publications, not their software."
- Comments on my grant proposal: The PI's expertise seems in software development, but he may not be a good mathematician.
- As a "not-so-good mathematician", I much prefer spending my time on proofs, which are a lot easier and much more enjoyable for me.
- Sometimes we do things that are not enjoyable but have to be done.
- Teaser: Who translated Euclid's *Elements* from Ancient Greek to modern languages? You probably do not know (and do not care).

Concluding remarks

- Implementation of model-based DFO solvers is intrinsically hard
- PRIMA provides the reference implementation of Powell's DFO solvers
- The modern Fortran version and a MATLAB interface is finished
- PRIMA will also be implemented in MATLAB, Python, Julia, R, C++
- PRIMA fixes issues in the original Fortran 77 code
- PRIMA is tested extensively to ensure its correctness & robustness
- PRIMA outperforms the original implementation of the solvers



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Muito obrigado!