Question 1

Consider a deep neural net applied to decide between the following four categories:

    cat, tiger, human face, lion

The neural net uses a softmax unit at the output layer. Consider the case where the values fed into the output layer are:

cat	0.5
tiger	0.8
human face	-3
lion	0.6

The softmax converts these values into a probability vector.

1. Compute the probabiity vector.

Answer:

$$e^{0.5} = 1.64872, \quad e^{0.8} = 2.22554, \quad e^{-3} = 0.0497871, \quad e^{0.6} = 1.82212, \quad e^{0.5} + e^{0.8} + e^{-3} + e^{0.6} = 5.74617$$

$$p = (0.286925, 0.387309, 0.00866439, 0.317102)$$

2. Which outcome is the most likely?

Answer: tiger

3. Which outcome is the least likely?

Answer: human face

4. What is the result of cross-entropy cost function if the target output is lion?

Answer:
$$E = \ln(1/0.317102) = 1.14853$$

Question 2

In the table below cases 3,4 are distributions, and cases 1, 2 can be converted into distributions.

case	A	B	C	D
1	1	-2	3	-4
2	1	2	-3	0
3	1	0	0	0
4	1/4	1/4	1/4	1/4

Converting 1 into a probability distribution using softmax:

$$V = \{1, -2, 3, -4\}$$
$$q = \{2.71828, 0.135335, 20.0855, 0.0183156\}$$
$$Z = 22.9575$$
$$p = \{0.118405, 0.00589504, 0.874902, 0.000797807\}$$

Converting 2 into a probability distribution using softmax:

$$V = \{1, 2, -3, 0\}$$
$$q = \{2.71828, 7.38906, 0.0497871, 1\}$$
$$Z = 11.1571$$
$$p = \{0.243636, 0.662272, 0.00446236, 0.0896288\}$$

1. Use cross entropy to determine which distribution among 1,2,3 is most similar to 4. **Show your computations.**

case	A	B	C	D	cross entropy of p_4 with candidate:
1	0.118405	0.00589504	0.874902	0.000797807	5.24224
2	0.243636	0.662272	0.00446236	0.0896288	3.47989
3	1	0	0	0	infinity
4	1/4	1/4	1/4	1/4	2

Answer: 1 / $\boxed{2}$ / 3

2. Use cross entropy to determine which distribution among 1,2,4 is most similar to 3. **Show your computations.**

case	A	B	C	D	cross entropy of p_3 with candidate:
1	0.118405	0.00589504	0.874902	0.000797807	3.0782
2	0.243636	0.662272	0.00446236	0.0896288	2.0372
3	1	0	0	0	0
4	1/4	1/4	1/4	1/4	2

Answer: 1 / 2 / $\boxed{4}$