SE 4367 Homework #8, BOR-MI

Use the BOR-MI-CSET procedure to derive the constraint set for the following predicate

p_r: (a + b)(c + !cd)

where a,b,c,d are Boolean variables.

Show all the steps in generating T_{BOR-MI}.

Draw the abstract syntax tree (AST) and label the nodes N1 to Nm (for the BOR part of the problem).

Explicitly list the true and false constraint sets for each node in the AST for the BOR part of the problem.

Remember to generate a test set T_{BOR-MI} corresponding to the root node in the AST.

Grading Rubric

Setting up the AST wrong or not having an AST, -10 points

Not generating the T_{BOR-MI} test set, -5 points

- **don’t care whether you use true/false or t/f**

For the individual steps in the BOR-MI procedure, if you get the wrong answer, it cascades (usually) through the rest of the problem…

Missing the class, assignment, or your name at the beginning of your submission or in the filename, -5 points each

Grading the BOR-MI-CSET problem

- **not breaking the equation into the correct mutually singular components, -35 points**
- **getting the wrong t/f sets for e_1, -5 points**
- **getting the wrong T sets for e_2, -5 points**
- **getting the wrong TS sets for e_2, -5 points**
- **getting the wrong S^t sets for e_2, -5 points**
- **getting the wrong F sets for e_2, -5 points**
- **getting the wrong FS sets for e_2, -5 points**
- **getting the wrong S^f sets for e_2, -5 points**

BOR-MI-CSET Step 1

Consider the nonsingular Boolean expression

$E = (a + b)(c + !cd)$

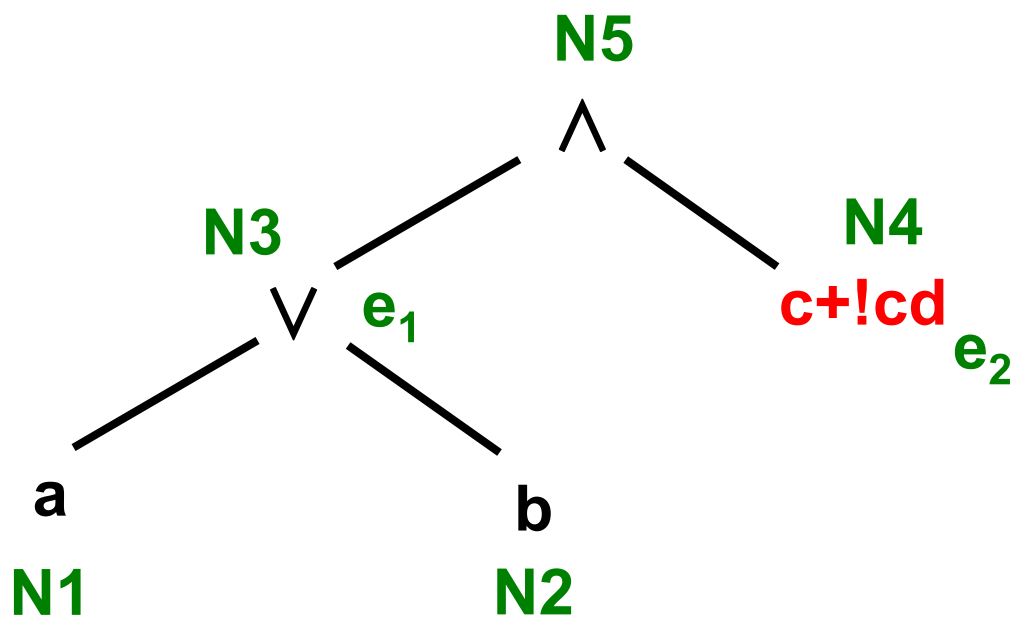
Step 1. Partition E into a set of n mutually singular components $E = \{E_1, E_2, \dots E_n\}$

Mutually singular components of E

$e_1 = a + b$ (singular)

$e_2 = c + !cd$ (nonsingular and DNF)

$AST\ for\ (a + b)(c + !cd)$



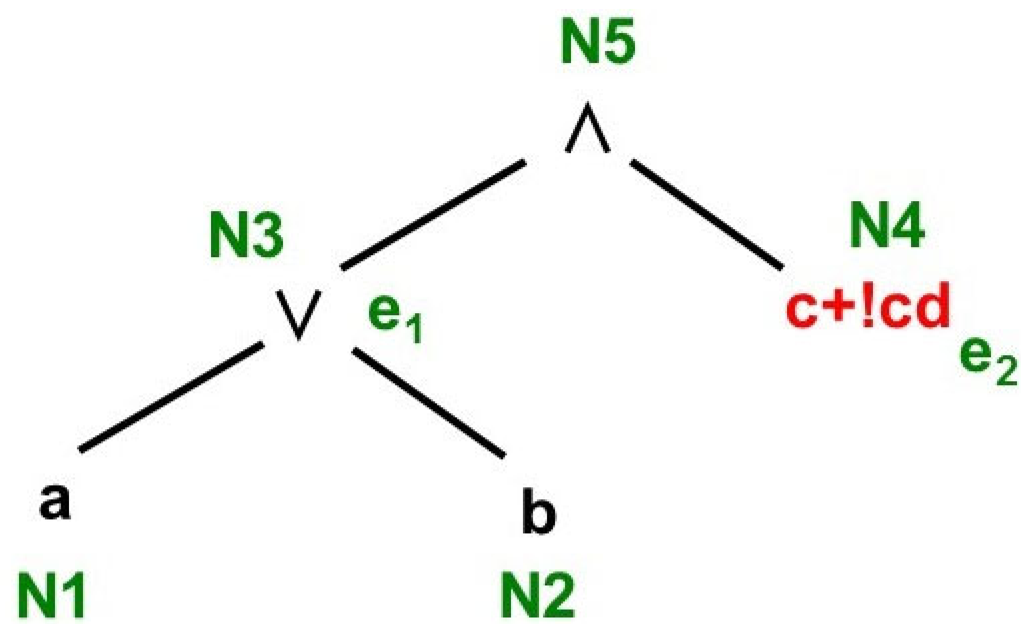
BOR-MI-CSET Step 2

Step 2. Generate the BOR-constraint set for each singular component in E using the BOR-CSET procedure

Use the BOR-CSET procedure to generate the constraint set for the singular component e_1 = a + b

$S_{N1}^{t} = S_{N2}^{t} = \{(t)\}$

$S_{N1}^{f} = S_{N2}^{f} = \{(f)\}$



N3 is an OR node for f(a,b).

$S_{N3}^{t} = (S_{N1}^{t} \times \{f_{N2}\}) \cup (\{f_{N1}\} \times S_{N2}^{t})$

$= (\{(t)\} \times \{(f)\}) \cup (\{(f)\} \times \{(t)\})$

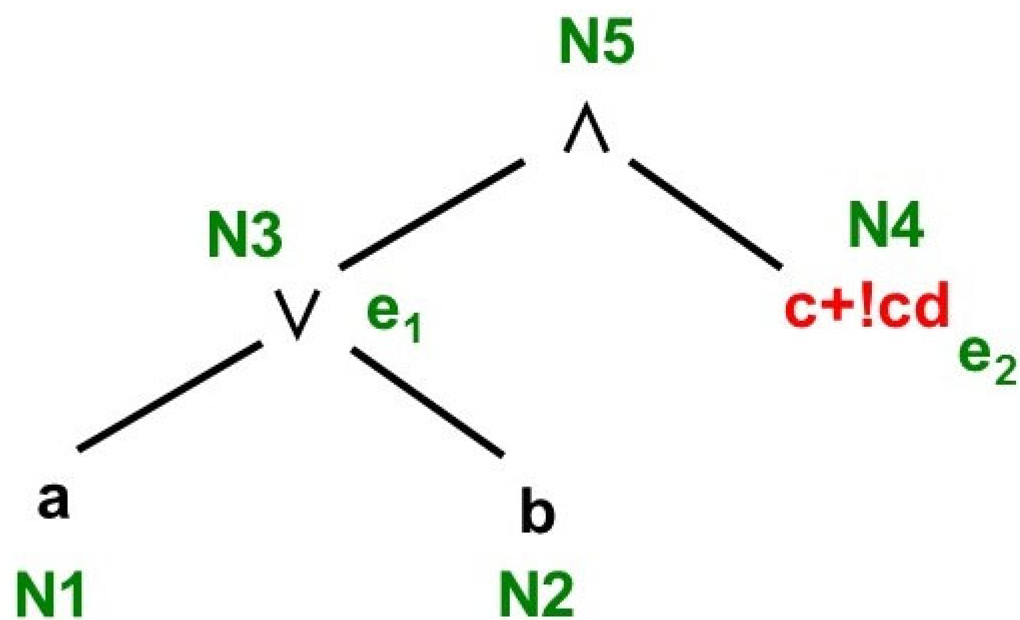
$= \{(t,f)\} \cup \{(f,t)\}$

$= \{(t,f), (f,t)\}$

$S_{N3}^{f} = S_{N1}^{f} \otimes S_{N2}^{f}$

$= \{(f)\} \otimes \{(f)\}$

$= \{(f,f)\}$



For component $e_1 = a + b$ we get

$S_{e1}^t = \{(t,f), (f,t)\}$ $S_{e1}^f = \{(f,f)\}$

S_{e1}^t is the true constraint set for e_1

S_{e1}^f is the false constraint set for e_1

BOR-MI-CSET Step 3

Step 3. Generate the MI-constraint set for each nonsingular component in E using the MI-CSET procedure

Use the MI-CSET procedure for the DNF nonsingular component e_2 = c + !cd

e_2 can be written as e_2 = u + v
where u = c, v = !cd

Apply the MI-CSET procedure to obtain the BOR constraint set for e_2

T_u = {(t,t), (t,f)} T_v = {(f,t)}

MI-CSET Step 2

MI-CSET Step 2. Let $TS_{ei} = T_{ei} - \cup_{j=1,i\neq j}^{n} T_{ej}$

- **$T_u = \{(t,t), (t,f)\}$**
- **$T_v = \{(f,t)\}$**

There are no duplicate Ts.

$TS_u = T_u = \{(t,t), (t,f)\}$

$TS_v = T_v = \{(f,t)\}$

MI-CSET Step 3

MI-CSET Step 3. Construct S_E^t by including one constraint from each TS_e

- **$TS_u = \{(t,t), (t,f)\}$**
- **$TS_v = \{(f,t)\}$**

$S_{e2}^t = \{(t,t), (f,t)\}$

MI-CSET Step 4

MI-CSET Step 4. Let e_i^j denote the term obtained by complementing the j^{th} literal in term e_i, for $1 \le i \le n$ and $1 \le j \le l_i$

- **u = c, v = !cd**

u_1 = !c

v_1 = cd **v_2 = !c!d**

Construct F_{eij} as the set of constraints that make e_i^j false

F_{u1}= {(f,t), (f,f)}

F_{v1}= {(t,t)} **F_{v2} = {(f,f)}**

MI-CSET Step 5

MI-CSET Step 5. Let $FS_{eij} = F_{eij} - \cup_{k=1}^{n} T_e$

- **T_u = {(t,t), (t,f)}**
- **T_v = {(f,t)}**

Eliminate candidate Fs that are really true.

$FS_{u1} = F_{u1} =$ {~~(f,t),~~ (f,f)}

$FS_{v1} = F_{v1} =$ {~~(t,t)~~}

$FS_{v2} = F_{v2} =$ {(f,f)}

MI-CSET Step 6

MI-CSET Step 6. Construct S_E^f that is minimal and covers each FS_{eij} at least once

- $FS_{u1} = \{(f,f)\}$
- $FS_{v1} = ø$
- $FS_{v2} = \{(f,f)\}$

$S_{e2}^f = \{(f,f)\}$

MI-CSET Step 7

MI-CSET Step 7. Construct the desired constraint set for E as $S_E = S_E^t \cup S_E^f$

- $S_{e2}^t = \{(t,t), (f,t)\}$
- $S_{e2}^f = \{(f,f)\}$

$$S_{e2} = \{(t,t), (f,t), (f,f)\}$$

BOR-CSET and MI-CSET

Recap

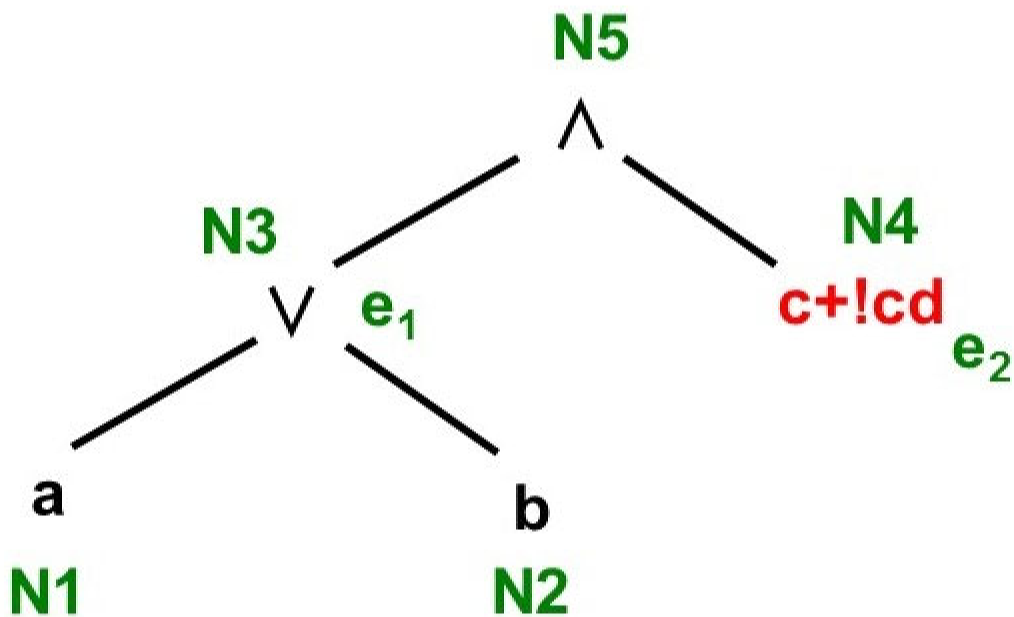
From Step 2 of BOR-MI-CSET, for component $e_1 = a + b$

$S_{e1}^{t} = \{(t,f), (f,t)\}$ $S_{e1}^{f} = \{(f,f)\}$

From Step 3 of BOR-MI-CSET, for component $e_2 = c + !cd$

$S_{e2}^{t} = \{(t,t), (f,t)\}$ $S_{e2}^{f} = \{(f,f)\}$

AST for (a + b)(c + !cd)



BOR-MI-CSET Step 4

Step 4. Combine the constraints generated in steps 2 and 3 using Step 2 from the BOR-CSET procedure to obtain the constraint set for E

N5 is an AND Node

$$
\begin{aligned}
S_{N5}^{t} &= S_{e1}^{t} \otimes S_{e2}^{t} \\
&= \{(t,f), (f,t)\} \otimes \{(t,t), (f,t)\} \\
&= \{(t,f,t,t), (f,t,f,t)\}
\end{aligned}
$$

$$
\begin{aligned}
S_{N5}^{f} &= (S_{e1}^{f} \times \{t_{e2}\}) \cup (\{t_{e1}\} \times S_{e2}^{f}) \\
&= (\{(f,f)\} \times \{(t,t)\}) \cup (\{(t,f)\} \times \{(f,f)\}) \\
&= \{(f,f,t,t)\} \cup \{(t,f,f,f)\} \\
&= \{(f,f,t,t), (t,f,f,f)\}
\end{aligned}
$$

Test Set for BOR-MI

S_{N5} = {(t,f,t,t), (f,t,f,t), (f,f,t,t), (t,f,f,f)}

$T_{BOR\text{-}MI}$ = {t_1: <a=true, b=false, c=true, d=true>,

t_2: <a=false, b=true, c=false, d=true>,

t_3: <a=false, b=false, c=true, d=true>,

t_4: <a=true, b=false, c=false, d=false>}