



Dr. Mark C. Paulk
SE 4367 – Software Testing, Verification, Validation, and Quality Assurance

Topics: Software Testing

Part II: Test Generation

- 3. Domain Partitioning
 - Equivalence Partitioning
 - Boundary Value Analysis
 - Category-Partition Method
 - 4. Predicate Analysis
 - 5. Test Generation from Finite State Models
 - 6. Test Generation from Combinatorial Designs

Requirements Are the Starting Point

Requirements specifications may be informal, rigorous, and/or formal.

The input domain is derived from the informal and rigorous specifications.

Black-box testing...

Test Selection Problem

Select a subset T of test such that execution of program p against each element of T will reveal all errors in p.

In general, there does not exist an algorithm to construct such a test set.

The problem of test selection is primarily because of the size and complexity of the input domain of p.

Mathur, Example 3.1

P sorts a sequence of integers into ascending order.

- input domain of integers [-32768, 32767]
- limit N_{max} > 1
- then the size of the input domain depends on the value of N

S, the size of the input domain, is given by the formula where *v* is the number of possible values each element of the input sequence may assume, i.e., 65,536.

$$S = \sum_{i=1}^{N} v^{i}$$

Equivalence Partitioning

Subdivide the input domain into a relatively small number of subdomains.

- subdomains are disjoint
- subdomains cover the entire input domain
 - both legal inputs and illegal (unexpected) inputs

Assumes program P exhibits the same behavior for every element within the class.

Each subdomain is known as an equivalence class.

Mathur, Example 3.5

wordCount

- takes a word w and a filename f as input
- returns the number of occurrences of w in the text contained in the file named f
- an exception is raised if there is no file with name f

Using the legal / illegal (E and U) partitioning method, there are two equivalence classes.

black-box

E1: (w,f) where w is a string and f is an existing file

E2: (w,f) where w is a string and f is a file that does not exist

Program P2.1

```
1) begin
2) string w, f;
3)
  input (w, f);
   if (not exists(f))
5) {
6)
       raise exception;
7)
      return(0);
8) }
9) if (length(w)==0)
10) return(0);
11) if (empty(f))
12) return(0);
13) return(getCount(w,f));
14) end
```

How many feasible paths?

- 1-2-3-4-6-7
 1-2-3-4-9-10
 1-2-3-4-9-11-12
 1-2-3-4-9-11-13
- 4 (Mathur says 6, 2³ is 8)
- each if-then terminates (return) the program
- white-box analysis!

How many equivalence classes?

- 4 (Mathur says 6)
- depending on which of the four feasible paths is covered by a test case

Equivalence class	W	<u>f</u>
E1	non-null	exists, nonempty
E2	non-null	does not exist
E 3	non-null	exists, empty
E4	null	exists, nonempty
E5 - does not matter whether	null rwis null if file	does not exist

E6 null exists, empty

- does not matter whether file is empty if word is null

Equivalence Classes for Variables

Range (integers and floating point)

- one class with values inside the range
 - there may be multiple legal ranges, which may or may not be adjacent
- two classes with values outside the range
 - less than
 - greater than

Strings

- at least one class containing legal strings
- at least one class containing illegal strings
- the empty string ε is an (illegal) equivalence class
- legality is determined based on constraints on the length and other semantic features of the string

Enumerations

- each value in a separate class
- may combine values if program behavior is the same

Arrays

- one class containing all legal arrays
 - may be additional legal equivalence classes depending on semantics, e.g., values must be in [-3,3]
- one class containing only the empty array
- one class containing arrays larger than the expected size

Compound data types (e.g., structures in C++)

legal and illegal values for each component

String ECP Example

The input is a string that is 8 alphabetic characters long.

- The first character is either "R" or "W".
- Characters 2-8 are in {A-Z}.

```
E<sub>1</sub>: "Rxxxxxxx" – legal input
```

E₂: "Wxxxxxxx" – legal input

- where x is in {A-Z}

E₃: "Rxxxx" – illegal input, short R string

E₄: "Wxxxx" – illegal input, short W string

E₅: "Xxxxxxxx" – illegal input, not R/W

- where X is not R or W

E₆: "Rxxxxxxxxxx" – illegal input, long R string

E₇: "Wxxxxxxxxx" – illegal input, long W string

E₈: "Rxxx&xxx" – illegal input, non-A-Z R string

E₉: "Wxxx&xxx" – illegal input, non-A-Z W string

- where & is not an alphabetic character

 E_{10} : ϵ – illegal input, null string

Questions: String ECP Example Applying Judgment and Knowledge

Should you have equivalence classes with more than one mistake?

Do you need "short" strings for both R and W?

- how long should a short string be?
- 1 character (R/W)?
- 2 characters?
- 7 characters?

Do you need "long" strings for both R and W?

- how long should a long string be?
- 9 characters?

Does it matter what position the non-alphabetic characters are in?

Unidimensional Partitioning

One input variable at a time

simple and scalable

Multidimensional partitioning can become too large.

Combining illegal inputs does not add much value.

- Addressing one illegal input frequently masks subsequent illegal inputs.
- Combinations of errors can cause unexpected results, so it's a judgment call.

Mathur, Example 3.7

Application with integer inputs x and y

- $3 \le x \le 7$
- $5 \le y \le 9$

Unidimensional partitioning

 E_1 : x < 3 E_2 : 3 \le x \le 7 E_3 : x > 7

 E_4 : y < 5 E_5 : 5 \le y \le 9 E_6 : y > 9

Multidimensional partitioning

 E_1 : x < 3, y < 5 E_2 : x < 3, 5 \le y \le 9 E_3 : x < 3, y > 9

 E_4 : $3 \le x \le 7$, y < 5 E_5 : $3 \le x \le 7$, $5 \le y \le 9$ E_6 : $3 \le x \le 7$, y > 9

 E_7 : x > 7, y < 5 E_8 : x > 7, 5 \le y \le 9 E_9 : x > 7, y > 9

Mathur, Example 3.11

findPrice application

- code in [99, 999]
- quantity in [1, 100]

Equivalence classes for code

• E1: code < 99

• E2: $99 \le code \le 999$

• E3: code > 999

Equivalence classes for quantity

• E4: quantity < 1

• E5: 1 ≤ *quantity* ≤ 100

• E6: *quantity* > 100

Equivalence Class Test Set

<i>a</i> [99, 999], <i>b</i> [1, 100]	<u>t_n: <a, b=""></a,></u>
E ₁ : <i>code</i> < 99	t ₁ : <50, 50>
E_2 : 99 \leq code \leq 999	t ₂ : <500, 50>
E_3^- : <i>code</i> > 999	t_3^- : <2000,50>
E ₄ : quantity < 1	t ₄ : <300, 0>
E_5 : $1 \le quantity \le 100$	t ₅ : <300, 60>
E ₆ : <i>quantity</i> > 100	t ₆ : <300, 200>

When creating a test case in unidimensional partitioning, combine legal values for the variables not being varied with the legal and illegal inputs for the varying variable.

Income Tax Example

Given a program to calculate income tax based on the following marginal tax rates.

Income	<u>Tax</u>
Income < \$10K	no tax
\$10K ≤ Income < \$20K	10%
\$20K ≤ Income < \$30K	12%
\$30K ≤ Income < \$40K	15%
Income ≥ \$40K	20%

An example of using marginal tax rates: on an income of \$25K, you pay \$0 on the first \$10K, 10% for income from \$10-20K, and 12% for income from \$20-25K, i.e., 0 + 1K + 0.6K = \$1.6K.

Income Tax Equivalence Examples

```
Income < $10K no tax

$10K ≤ Income < $20K 10%

$20K ≤ Income < $30K 12%

$30K ≤ Income < $40K 15%

Income ≥ $40K 20%
```

What are the equivalence classes for this problem?

Either notation is acceptable. Note that (,) are the same as <,> and [,] are the same as ≤,≥ in standard mathematical notation.

Deriving an Equivalence Class

The equivalence class (and associated BVA) for income < 0 is based on an understanding of the application domain.

For taxes, income cannot be less than 0 (you round negative numbers to 0).

The equivalence classes bounded by 0 are based on knowledge of the application domain.

An Income Tax Test Set

Create a test set generated from your equivalence classes.

Note that this test set is simply an example of the very large number of possible test sets that could be generated.

```
\begin{array}{lll} T_e = \{ & -10,000, & E_1: Income < 0 \\ & 5,000, & E_2: 0 \le Income < 10,000 \\ & 15,000, & E_3: 10,000 \le Income < 20,000 \\ & 25,000, & E_4: 20,000 \le Income < 30,000 \\ & 35,000, & E_5: 30,000 \le Income < 40,000 \\ & 100,000 & E_6: 40,000 \le Income \\ \} \end{array}
```

Procedure for Equivalence Partitioning

Identify the input domain

Equivalence classing

- partition the input domain
- tester defines same way of program behavior
- consider output-driven equivalence classes

Combine equivalence classes

- usually omitted (e.g., enumerated variables)
- multidimensional partitioning

Identify infeasible equivalence classes

GUI interface may only allow valid inputs

GUI Design

GUI may offer only correct choices via menu.

GUI may ask the user to fill in a box.

illegal values are possible

Test design must take into account GUI design.

Makes the assumption the GUI has been correctly implemented.

Boundary Value Analysis

Programmers make mistakes in process values at or near the boundaries of equivalence classes.

The "requirement" is if (x ≤ 0) then return f1; else return f2;

But the program executes if (x < 0) then return f1; else return f2;

x=0 lies at the boundary between the equivalence classes...

Two-Point and Three-Point BVA (IEEE 29119:4 section 5.2.3)

Three-point boundary value analysis

- three test cases derived from each boundary
- based on values on the boundary and an incremental distance on <u>each side</u> of the boundary

"Incremental distance" is defined as the smallest significant value for the data type.

- integer → ± 1
- floating point with granularity of 0.01 \rightarrow \pm 0.01

Two-point boundary value analysis

- a value on the boundary
 - no need to test a value inside the boundary if this works
- a value an incremental distance <u>outside</u> the boundary

Boundary Value Analysis Procedure

Partition the input domain using unidimensional partitioning.

- as many partitions as there are input variables
 - a single partition of an input domain can also be created using multidimensional partitioning

Identify the boundaries for each partition.

- also use any special relationships among inputs

Select test data such that each boundary value occurs in at least one test input.

- near-boundary values: just inside, just outside
 - 3-point BVA

Mathur, Example 3.11

findPrice application

- code in [99, 999]
- quantity in [1, 100]

Equivalence classes for code

• E1: code < 99

• E2: 99 ≤ *code* ≤ 999

• E3: code > 999

Equivalence classes for quantity

• E4: quantity < 1

• E5: 1 ≤ *quantity* ≤ 100

• E6: *quantity* > 100

Boundaries

Boundaries for code: 99, 999

code values near the boundaries: 98, 100, 998, 1000

Boundaries for *quantity*: 1, 100

quantity values near the boundaries: 0, 2, 99, 101

A Boundary Value Test Set

Would the following test T' set cover the boundary values?

Would it be a good test set? Why or why not?

```
[99, 999], [1, 100] \underline{t_n}: <a, b>

E<sub>1</sub>: code: 98, quantity: 0 \underline{t_1}: <98, 0>

E<sub>2</sub>: code: 99, quantity: 1 \underline{t_2}: <99, 1>

E<sub>3</sub>: code: 100, quantity: 2 \underline{t_3}: <100, 2>

E<sub>4</sub>: code: 998, quantity: 99 \underline{t_4}: <998, 99>

E<sub>5</sub>: code: 999, quantity: 100 \underline{t_5}: <999, 100>

E<sub>6</sub>: code: 1000, quantity: 101 \underline{t_6}: <1000, 101>
```

Interactions Between Inputs – Masking Defect Interactions

T' covers the boundary values.

There is no test case for a legal value of *code* and an illegal value of *quantity*...

... or a illegal value of code and a legal value of quantity

A Better Boundary Value Test Set

[99, 999], [1, 100 <u>]</u>	<u>t_n: <a, b=""></a,></u>
E ₁ : <i>code</i> : 98	t ₁ : <98,50>
E ₂ : <i>code</i> : 99	t ₂ : <99, 50>
E ₃ : <i>code</i> : 100	t_3^- : <100,50>
E ₄ : <i>code</i> : 998	t ₄ : <998, 50>
E ₅ : <i>code</i> : 999	t ₅ : <999, 50>
E ₆ : <i>code</i> : 1000	t ₆ : <1000, 50>
E ₇ : quantity: 0	t ₇ : <300, <mark>0</mark> >
E ₈ : quantity: 1	t ₈ : <300, 1>
E ₉ : quantity: 2	t ₉ : <300, <mark>2</mark> >
E ₁₀ : <i>quantity</i> : 99	t ₁₀ : <300, <mark>99</mark> >
E ₁₁ : <i>quantity</i> : 100	t ₁₁ : <300, 100>
E ₁₂ : <i>quantity</i> : 101	t ₁₂ : <300, 101>

Income Tax Example

What are the boundaries for this problem?

0	E ₁ : Income < 0
10,000	E ₂ : 0 ≤ Income < 10,000
20,000 30,000 40,000	E_3 : 10,000 \leq Income \leq 20,000
	E₄: 20,000 ≤ Income < 30,000
	E_5 : 30,000 \leq Income $<$ 40,000
	E ₆ : 40,000 ≤ Income

- Questions: Would it make sense to have an lower or upper bound for income?
- If so, what might those bounds be?
- How would you discover them? (Talk to the customer!)
- Should it be driven by the problem (tax on income) or technology (how small/big can a number be on the computer)?

BVA Income Tax Test Set

What is your test set based on boundary value analysis?

```
T<sub>b</sub> = { -1, 0, 1,
9,999, 10,000, 10,001,
19,999, 20,000, 20,001,
29,999, 30,000, 30,001,
39,999, 40,000, 40,001
}
```

Question: Does boundary value analysis "subsume" equivalence class partitioning?

Question: Is it worthwhile to generate test cases based on both criteria? That is, $T = T_e + T_b$

Boundaries at "Infinity"

You could add a large positive income to test what happens when an input exceeds the "practical" maximum income that we might expect.

 You should always talk to the customer about this kind of boundary.

You could add an income near the max (or min) value that will fit into a variable.

- This boundary is set by the technology, not the customer or the problem.
- What happens when an integer value overflows? A floating point value?

Incremental Distance

What is the "incremental distance"?

What is 10% for \$9,999? \$10,000? \$10,001?

- marginal tax rate
- \$0 \$0 \$0.10
- Are outputs rounded or truncated by your programming language?
- Should just inside/outside be 9990, 10000, 10010?

If the answer for an "off-by-one" calculation is the same as the correct calculation, is that a "good" test case?

Closed Boundaries

Would this set of equivalence classes be preferable to the set we identified before?

No open boundaries...

E₁: Income ≤ -1

 E_2 : $0 \le Income \le 9,999$

 E_3 : 10,000 \leq Income \leq 19,999

 E_{4} : 20,000 \leq Income \leq 29,999

 E_5 : 30,000 \leq Income \leq 39,999

E₆: 40,000 ≤ Income

```
Three-point boundary values are (-2, -1, 0), (-1, 0, 1), (9998, 9999, 10000), (9999, 10000, 10001), (19998, 19999, 20000), (19999, 20000, 20001), (29998, 29999, 30000), (29999, 30000, 30001), (39998, 39999, 40000), (39999, 40000, 40001)
```

After removing duplicates, boundary values are

```
{-2, -1, 0, 1, 9998, 9999, 10000, 10001, 19998, 19999, 20000, 20001, 29998, 29999, 30000, 30001, 39998, 39999, 40000, 40001}
```

Boundary values considering incremental distance might be better at

```
{-5, 0, 5, 9995, 10000, 10005, 19995, 20000, 20005, 29995, 30000, 30005, 39995, 40000, 40005}
```

Two-Point BVA

Two-point BVA for the income tax problem:

```
{-1, 0, -1, 0, 9999, 10000, 19999, 20000, 19999, 20000, 19999, 20000, 29999, 30000, 39999, 40000}
```

After removing duplicates

{-1, 0, 9999, 10000, 19999, 20000, 29999, 30000, 39999, 40000}

E₁: Income \leq -1 E₂: $0 \leq$ Income \leq 9,999 E₃: $10,000 \leq$ Income \leq 19,999 E₄: $20,000 \leq$ Income \leq 29,999 E₅: $30,000 \leq$ Income \leq 39,999 E₆: $40,000 \leq$ Income

SE 4367 BVA

We will do three-point boundary value analysis and not worry about whether bounds are closed or open when we have adjacent valid ranges.

- remove duplicates!

For boundaries between valid and invalid inputs, the valid range shall be closed (inclusive), e.g., [0,100] or 0≤x≤100.

Don't worry about the incremental distance unless asked to in the problem.

We won't worry about boundaries at "infinity."

Category-Partition Methods

A systematic approach to generation of tests from requirements

 a systematization of equivalence partitioning and boundary value techniques

Mix of manual and automated steps

Transforms requirements into test specifications

 categories corresponding to program inputs and environment objects

Test specs are input to test-frame generator

generate test scripts

Test Frames

A collection of choices, corresponding to each category

A template for one or more test cases that are combined into one or more test scripts

Steps in the Category-Partition Method

- Functional specification
- 1) Analyze specification
 - Functional units
- 2) Identify categories
 - Categories
- 3) Partition categories
 - Choices
- 4) Identify constraints
 - Constraints
- 5) (Re)write test specification
 - Test specification (TS)
- 6) Process specification
 - Test frames
- 7) Evaluate generator output: revise TS, goto step 6
 - Test frames
- 8) Generate test scripts
 - Test scripts

Mathur Running Example for Category-Partition Method

findPrice application

- 8-digit code
- qty
- weight

code leftmost digitInterpretation

0	ordinary grocery items
2	variable-weight items
3	health-related items
5	coupon
1, 6-9	unused

fP finds unit price, description, total price of item

displays error message if any of three is incorrect

Running Example Keying on "code"

Use of *qty* and *weight* depends on leftmost digit of *code*

- second digit is \$ amount third and fourth digits are ¢ amount
- 1, 6-9 ignored
 - what about 4?

Running Example 1) Analyze Specification

Identify each functional unit to test separately

findPrice → fP

Running Example 2) Identify Categories

Isolate inputs, identify objects in environment

Determine characteristics (categories)

- qty and weight are related to code
- no bounds on qty and weight
- fP accesses database → environment object

code: length, leftmost digit, remaining digits

qty: integer

weight: float

database: contents???

Running Example 3) Partition Categories

What are the different cases (choices) against which the functional unit must be tested?

Partition each category into at least two subsets: correct values + erroneous values

 in a networked environment, this might include events like network failure

Running Example Inputs and Environment Objects

code

- length
 - valid (8 digits)
 - invalid (<,> 8 digits)
- leftmost digit
 - 0
 - 2
 - 3
 - 5
 - others
- remaining digits
 - valid string
 - invalid string???

qty

- integer
 - valid quantity
 - invalid quantity (0)

weight

- float
 - valid weight
 - invalid weight (0)

Is there an upper bound to qty or weight?

Environment: database

- item exists
- item does not exist

Running Example 4) Identify Constraints

A test for a functional unit consists of a combination of choices for each parameter and environment object.

- Certain combinations might not be possible.
- Some combinations must satisfy specific relationships.

A constraint is specified using a property list and selector expression.

[property P1, P2, ...]

- property is a key word
- P1, P2, ... are the names of individual properties

Running Example Selector Expressions

[if P]

[if P1 and P2 and ...]

[error]

- can be assigned to error conditions

[single]

 specifies that the associated choice is not to be combined with choices of other parameters or environment objects

Running Example Sampled Choices

Comment lines start with

```
# Leftmost digit of code
0 [property ordinary-grocery]
2 [property variable-weight]
# Remaining digits of code
valid string [single]
# Valid value of qty
valid quantity [if ordinary-grocery]
# Incorrect value of qty
invalid quantity [error]
```

Running Example 5) (Re)write Test Specification

Write test spec in a test specification language (TSL)

```
Parameters
code
length
valid
invalid [error]
leftmost digit
0 [property Ordinary-grocery]
2 [property Variable-weight]
3 [property Health-related]
5 [property Coupon]
```

remaining digits

valid string [single]

invalid string [error]

qty

valid quantity [if Ordinary-grocery]

invalid quantity [error]

weight

valid weight [if Variable-weight]

invalid weight [error]

Environments: database

item exists

item does not exist [error]

Running Example 6) Process Specification

TSL specification is processed by an automatic test-frame generator

Analyze for redundancy

Sample test frame:

Test number identifies the test

Test case 2: (Key=1.2.1.0.1.1) Key identifies the choices

0 indicates no choice

Length: valid

Leftmost digit: 2

Remaining digits: valid string

qty: ignored

weight: 3.19

database: item exists

Running Example Test Frames

A test frame is not a test case.

test cases are derived from test frames

Test frames are generated from all possible combinations of choices while satisfying the constraints.

Choices marked [error] or [single] are not combined with others and only produce one test case.

Running Example 7) Evaluate Generator Output

Examine test frames for redundancy or missing cases

Think about a code 4 input...

Running Example 8) Generate Test Scripts

Combine test cases generated from test frames into test scripts

A test script is a grouping of test cases

 group test cases that do not require any changes in settings of the environment objects

Summary – Things to Remember

Equivalence class partitioning

Boundary value analysis

three point BVA

Questions and Answers

