Math 584, Fall 2020 Syllabus

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1 Course Information

Course Number: MATH 584

Course Title: Selected Topics: Computational Algebra

Course Description: Introductory algebraic geometry based on computational methods in polynomial rings, particularly methods based on Gröbner bases.

1.1 Instructor

Instructor: Zach Teitler

Email: zteitler@boisestate.edu

Website: https://sites.google.com/site/zteitler/home

Office: MB 233A

Office Phone: 208-426-1086

1.2 Section

Section Number: 001

Meeting Times: MoWeFr 12:00-1:15

Meeting Remotely: We will meet remotely using Zoom. Zoom sessions may be recorded for students who are not able to attend.

Zoom Room: `984 1606 8308`

Zoom Link: https://boisestate.zoom.us/j/98416068308

2 Course Learning Outcomes

By the end of this course, students will be able to:

1. Demonstrate familiarity with introductory concepts, definitions, and theorems in algebraic geometry, including:
   - Affine varieties and ideals
   - Regular and rational maps
   - Elimination and projection
   - Hilbert's Nullstellensatz
2. Demonstrate familiarity with introductory concepts, definitions, and theorems in computational algebra, including:
   - Computations with multivariable polynomials, including division with remainder
   - Monomial orderings
   - Gröbner bases and Buchberger's algorithm
3. Read and write rigorous proofs.
4. Demonstrate good mathematical writing skills and style.
5. Demonstrate familiarity with research and conjectures in algebraic geometry and computational algebra.

3 Textbook

3.1 Required Textbook

1. Brendan Hassett, *Introduction to Algebraic Geometry*. Cambridge University Press, 2007.

   ebook: 978-0-511-28529-5

   hardback: 978-0-521-87094-8

   paperback: 978-0-521-69141-3

We will cover material from the chapters of the textbook listed above the dividing line in the following table; additional material from chapters listed after the dividing line may be covered if time permits:

Table 3.1

Chapter	Title
1	Guiding Problems
2	Division Algorithm and Gröbner Bases
3	Affine Varieties
4	Elimination
6	Irreducible Varieties
7	Nullstellensatz
5	Resultants
8	Primary Decomposition
9	Projective Geometry

3.2 Recommended Optional Textbooks

1. Mateusz Michałek and Bernd Sturmfels, *Invitation to Nonlinear Algebra*. https://personal-homepages.mis.mpg.de/michalek/NonLinearAlgebra.pdf (pdf).

2. Cox, Little, O'Shea, *Ideals, Varieties, and Algorithms*

4 Grading

4.1 Components of course grade

Graded student work will consist of a short paper and a term paper, midterm and final exams, and written homework. Course grades will be based primarily on term papers and exams, and to a lesser extent on homework.

4.2 Papers

Each student will write two papers for this class. Each paper is intended to provide:

- an opportunity to explore the mathematical research literature and seek a personally appealing research topic
- an authentic experience of working with the published mathematical literature
- an authentic experience of writing a substantial mathematical text, well beyond typical homework solutions
- preparation for writing even an larger text, such as a thesis
- an authentic experience of receiving feedback on a draft manuscript and responding to that feedback through multiple rounds of revision
- an authentic experience of writing mathematical exposition at a high level

The details of the two papers are as follows.

1. A short paper about an open question, unsolved problem, or currently studied research area within algebra, algebraic geometry, or computational algebra, or an application of one of those areas.

2. A term paper about a topic of the student's choice within algebra, algebraic geometry, or computational algebra, or an application of one of those areas (but emphasizing the algebraic aspect).

Lengths and deadlines of the papers are as follows:

1. The short paper will be 2–3 pages long, with the following deadlines:
   - A topic and source are due by the end of week 3.
   - A first draft is due by the end of week 4.
   - The short paper is due by the end of week 5.

2. The term paper will be 6–15 pages long (it can be shorter or longer if needed; please discuss with me), with the following deadlines:

- A topic (1–2 pages) is due by the end of week 7.
- An outline plus one section are due by the end of week 10.
- A first draft is due by the end of week 13.
- The term paper is due by the end of week 15.

For both papers, highly recommended sources are the recommended additional texts listed above. They contain numerous examples, applications of algebraic geometry, and links to other books and articles. If you find a topic that interests you in one of those texts, you can use it as a starting point for your paper.

Additional recommended sources include:

- MAA Writing Awards
- American Mathematical Monthly
- College Mathematics Journal
- Mathematics Magazine
- Math Horizons
- What's Happening in the Mathematical Sciences

You should use **high-quality published sources**. For simplicity, this means the recommended sources listed above, or any publication listed in MathSciNet. If you have questions or wish to use other sources, talk to me.

Papers for this class may not be about your own research.

4.3 Midterm and final exams

A midterm exam in the 8th week of the semester, and the final exam, will be individual oral exams. The exams are intended to gauge mastery of course material, but also reflection on the course and its place in a larger context.

4.4 Written homework

You are encouraged to work collaboratively on homework but you must turn in your own solutions.

Homework will contribute to course grades, but it is intended primarily to give feedback and guidance to students throughout the semester.

4.4.1 Turning in homework

Homework submissions and grading will be paperless. You will turn in your homework by uploading PDFs to BlackBoard. PDFs should have filenames in the following format:

`584-Homework-`⟨number⟩`-`⟨your last name⟩`.pdf`

For example: `584-Homework-01-Teitler.pdf`.

4.4.2 Homework formatting

Homework must be typed in LaTeX. LaTeX tutorials are available online, e.g., https://www.latex-tutorial.com and https://www.gnu.org/software/teximpatient/. You may wish to use a free online LaTeX system such as https://overleaf.com. (Overleaf includes a LaTeX tutorial.)

Use a new page (`\newpage`) for each problem. State which question you are answering (textbook section and exercise number) and the actual question. Then, start your answer in a new paragraph. Use environments such as `proof` and `theorem` (via `\begin{proof}...\end{proof}`) to organize your work and display it clearly.

For legibility, use the `12pt` option (`\documentclass[12pt]{amsart}`) and `\linespread{2.4}`. If you use figures, I recommend learning to use `TikZ` to generate high-quality figures within LaTeX. Alternatively you may use figures/plots generated in other programs such as Sage, Mathematica, Maple, or Inkscape, saved to PDF, and included in your document with commands like `\includegraphics`. It's also fine to include hand-drawn figures that you scanned or photoed.

5 Help

5.1 Allowed resources

In this 500 level class, I expect you to use the full array of resources at your disposal to learn the material "by any means possible." For homework assignments, you are encouraged (in fact, expected) to collaborate with your classmates and to ask me questions. Other resources (books, online sources, people outside the class) are highly recommended for clarifying class topics or for enrichment (but *not* for getting solutions to problems).

You are allowed to use things that you learn from a book, online source, or person outside the class that *help you and your classmates to find your own solution* for a problem. However, if you read a full solution, so that there's little or nothing left for you and your classmates to figure out, then you *may not turn in that solution for credit.*

The Mathematics Stack Exchange is a very useful question-and-answer site for undergraduate/graduate level mathematics. You are welcome to browse the Mathematics Stack Exchange and even post questions there. Hopefully it will help you learn and understand the material! However please remember that if you use that site to get a solution to a problem, then you can't turn that solution in for credit. Other helpful websites for basic information include Wikipedia and MathWorld.

5.2 A note on collaboration

Solving mathematical problems has three parts:

The discovery phase: This is the time you spent trying to figure out how to solve the problems, and it often takes most of the time. You are welcome and encouraged to collaborate with other students in this phase. Collaboration is a healthy practice, and this is how mathematics is done in real life.

This phase starts with working to understand what a problem or question is asking for. That might include reviewing material from previous textbook sections.

The write-up phase: This consists of writing your solutions once you have an idea of how the problem can be solved. You should do this entirely by yourself. Be alone when you write your solutions. If you collaborate on this part, or you copy part of your solutions from somebody else, or you have notes written by somebody else in front of you when you write your solutions, you are hurting yourself by depriving yourself of an opportunity to learn and practice.

If you need help in the write-up phase, talk to me! I can help you.

The editing phase: This consists of editing and revising your write-up for clarity, organization, and presentation. At this stage it can be very helpful to get feedback and suggestions from other students.

5.3 Expectations of lectures

Please read "Mathematics Professors and Mathematics Majors' Expectations of Lectures in Advanced Mathematics".

5.4 How to read mathematics

Please read the following, especially the first:

1. https://www.lboro.ac.uk/media/media/schoolanddepartments/mathematics-education-centre/downloads/research/SE-booklet.pdf
2. https://personal.utdallas.edu/~zweck/SEbooklet.pdf
3. http://web.stonehill.edu/compsci/History_Math/math-read.htm
4. https://brownmath.com/stfa/read.htm

6 Important Dates

Table 6.1

Monday	8/24	First day of classes.
Friday	9/4	Last day to register/add or to drop without a W.
Monday	9/7	Labor Day. No classes.
Friday	9/11	Short paper topic proposal due.
Friday	9/18	Short paper first draft due.
Friday	9/25	Short paper final version due.
Friday	10/9	Term paper topic proposal due.
	10/12-16	Midterm exam.
Friday	10/30	Term paper outline plus one section due
Friday	10/30	Last day to drop with a W or completely withdraw.
Friday	11/20	Term paper first draft due.
	11/23-11/29	Thanksgiving Holiday. No classes.
Friday	12/11	Term paper final version due.
Friday	12/11	Last day of instruction for regular classes.
Monday	12/14	Final Exam (scheduled), 12:00-2:00.
Tuesday	12/22	Grades due. (You will be able to see your grade by this date.)

7 Other

Respect for Diversity: Students from all backgrounds and with all perspectives are welcome in this course. It is my intent that all students be well served by this course, that students's learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength, and benefit. It is my intent to maintain a classroom atmosphere that is welcoming and respectful of diversity: gender, sexuality, disability, age, socioeconomic status, ethnicity, race, and culture. Your suggestions are encouraged and appreciated. Please let me know ways to improve the effectiveness of the course for you personally or for other students or student groups.

ADA Policy Statement: Students with disabilities needing accommodations to fully participate in this class should contact the EAC. All accommodations must be approved through the EAC prior to being implemented. To learn more about the accommodation process, visit the EAC's website at `https://www.boisestate.edu/eac/new-students/`.

Email: In accordance with Boise State University Policy #2280, it is expected that you will receive and read emails sent to your `boisestate.edu` email address.

Communication: Additional information and updates may be announced in class, sent by email, and/or posted on BlackBoard (`http://blackboard.boisestate.edu/`).

Academic Integrity: Getting answers to homework or exam problems from unauthorized sources is a very serious form of academic misconduct. For this class, *all online sources* are unauthorized for this purpose. You are allowed to *learn and increase your understanding* from online sources or other textbooks; you are *not* allowed to use those sources to find answers to homework or exam problems.

Behavioral Expectations: Every student has the right to a respectful learning environment. In order to provide this right to all students, students must take individual responsibility to conduct themselves in a mature and appropriate manner and will be held accountable for their behavior in accordance with Boise State University Policy #2050.