

PROJECT 0: INAUGURAL PROJECT

Vision: The inaugural project teaches you to solve a simple economic model and present the results.

- **Objectives:** In your inaugural project, you should show that you can:

1. Apply simple numerical solution and simulation methods
2. Structure a code project
3. Document code
4. Present results in text form and in figures

- **Content:** In your inaugural project, you should:

1. Solve and simulate a pre-specified economic model (see next page)
2. Visualize results

Example of structure: [See this repository](#).

- **Structure:** Your inaugural project should consist of:

1. A README.md with a short introduction to your project
2. A single self-contained notebook (.ipynb) presenting the analysis
3. Fully documented Python files (.py)

- **Hand-in:** On GitHub by uploading it to the subfolder *inauguralproject*, which is located in:

github.com/NumEconCopenhagen/projects-YEAR-YOURGROUENAME

- **Deadline:** See [Calendar](#).

- **Peer feedback:** After handing in, you will be asked to give peer feedback on the projects of two other groups.

- **Exam:** Your inaugural project will be a part of your exam portfolio. You can incorporate feedback before handing in the final version.

Exchange Economy

We consider an exchange economy with two consumers, A and B , and two goods, x_1 and x_2 . The initial endowments are $\omega_1^A \geq 0$ and $\omega_2^A \geq 0$. The total endowment of each good is always one, such that

$$\begin{aligned}\omega_1^B &= 1 - \omega_1^A \\ \omega_2^B &= 1 - \omega_2^A.\end{aligned}$$

We define the vectors $\mathbf{p} = (p_1, p_2)$, $\boldsymbol{\omega}^A = (\omega_1^A, \omega_2^A)$, and $\boldsymbol{\omega}^B = (\omega_1^B, \omega_2^B)$.

Utility and demand functions with prices $p_1 > 0$ and $p_2 > 0$ are

$$\begin{aligned}u^A(x_1, x_2) &= x_1^\alpha x_2^{1-\alpha}, \quad \alpha \in (0, 1) \\ x_1^{A*}(\mathbf{p}, \boldsymbol{\omega}^A) &= \alpha \frac{p_1 \omega_1^A + p_2 \omega_2^A}{p_1} \\ x_2^{A*}(\mathbf{p}, \boldsymbol{\omega}^A) &= (1 - \alpha) \frac{p_1 \omega_1^A + p_2 \omega_2^A}{p_2} \\ u^B(x_1, x_2) &= x_1^\beta x_2^{1-\beta}, \quad \beta \in (0, 1) \\ x_1^{B*}(\mathbf{p}, \boldsymbol{\omega}^B) &= \beta \frac{p_1 \omega_1^B + p_2 \omega_2^B}{p_1} \\ x_2^{B*}(\mathbf{p}, \boldsymbol{\omega}^B) &= (1 - \beta) \frac{p_1 \omega_1^B + p_2 \omega_2^B}{p_2}.\end{aligned}$$

The (Walras) market equilibrium requires market clearing for both goods,

$$\begin{aligned}x_1^{A*}(\mathbf{p}, \boldsymbol{\omega}^A) + x_1^{B*}(\mathbf{p}, \boldsymbol{\omega}^B) &= \omega_1^A + \omega_1^B \\ x_2^{A*}(\mathbf{p}, \boldsymbol{\omega}^A) + x_2^{B*}(\mathbf{p}, \boldsymbol{\omega}^B) &= \omega_2^A + \omega_2^B.\end{aligned}$$

Walras' law apply, so if one market clears, the other one does as well.

Calibration We use the following parameter values

$$\begin{aligned}\alpha &= \frac{1}{3} \\ \beta &= \frac{2}{3}.\end{aligned}$$

Numeraire The numeraire is $p_2 = 1$.

Questions

Code to start from is provided in *IntroProg-lectures/projects/InauguralProject2024.ipynb*

The initial endowment is

$$\begin{aligned}\omega_1^A &= 0.8 \\ \omega_2^A &= 0.3.\end{aligned}$$

1. Illustrate the following set in the Edgeworth box

$$\mathcal{C} = \left\{ (x_1^A, x_2^A) \mid \begin{aligned} &u^A(x_1^A, x_2^A) \geq u^A(\omega_1^A, \omega_2^A) \\ &u^B(x_1^B, x_2^B) \geq u^B(\omega_1^B, \omega_2^B) \\ &x_1^B = 1 - x_1^A, x_2^B = 1 - x_2^A \\ &x_1^A, x_2^A \in \{0, \frac{1}{N}, \frac{2}{N}, \dots, 1\}, N = 75 \end{aligned} \right\}$$

That is, find the pairs of combinations of x_1^A and x_2^A that leave both players as least as well off as they were when consuming their endowments. This is thus Pareto improvements relative to the endowment.

2. For $p_1 \in \mathcal{P}_1 = \{0.5, 0.5 + 2\frac{1}{N}, 0.5 + 2\frac{2}{N}, \dots, 2.5\}$ calculate the error in the market clearing condition s

$$\begin{aligned}\epsilon_1(\mathbf{p}, \boldsymbol{\omega}) &= x_1^{A*}(\mathbf{p}, \boldsymbol{\omega}^A) - \omega_1^A + x_1^{B*}(\mathbf{p}, \boldsymbol{\omega}^B) - \omega_1^B \\ \epsilon_2(\mathbf{p}, \boldsymbol{\omega}) &= x_2^{A*}(\mathbf{p}, \boldsymbol{\omega}^A) - \omega_2^A + x_2^{B*}(\mathbf{p}, \boldsymbol{\omega}^B) - \omega_2^B\end{aligned}$$

3. What is market clearing price?

Assume that A chooses the price to maximize her own utility.

- 4a. Find the allocation if only prices in \mathcal{P}_1 can be chosen, i.e.

$$\max_{p_1 \in \mathcal{P}_1} u^A(1 - x_1^B(\mathbf{p}, \boldsymbol{\omega}^B), 1 - x_2^B(\mathbf{p}, \boldsymbol{\omega}^B))$$

- 4b. Find the allocation if any positive price can be chosen, i.e.

$$\max_{p_1 > 0} u^A(1 - x_1^B(\mathbf{p}, \boldsymbol{\omega}^B), 1 - x_2^B(\mathbf{p}, \boldsymbol{\omega}^B))$$

Assume that A chooses B 's consumption, but such that B is not worse off than in the initial endowment. A is thus the market maker.

- 5a. Find the allocation if the choice set is restricted to \mathcal{C} , i.e.

$$\max_{(x_1^A, x_2^A) \in \mathcal{C}} u^A(x_1^A, x_2^A)$$

- 5b. Find the allocation if no further restrictions are imposed, i.e.

$$\begin{aligned} &\max_{(x_1^A, x_2^A) \in [0,1] \times [0,1]} u^A(x_1^A, x_2^A) \\ \text{s.t. } &u^B(1 - x_1^A, 1 - x_2^A) \geq u^B(\omega_1^B, \omega_2^B)\end{aligned}$$

Assume A 's and B 's consumption are chosen by a utilitarian social planner to maximize aggregate utility

6a. Find the resulting allocation

$$\max_{(x_1^A, x_2^A) \in [0,1] \times [0,1]} u^A(x_1^A, x_2^A) + u^B(1 - x_1^A, 1 - x_2^A)$$

6b. Illustrate and compare with your results in questions 3)-5).
Discuss the pros and cons of the various allocations.

Consider the random set

$$\mathcal{W} = \left\{ \left(\omega_1^A, \omega_2^A \right) \mid \omega_1^A \sim \mathcal{U}(0, 1), \omega_2^A \sim \mathcal{U}(0, 1) \right\}$$

7. Draw a set \mathcal{W} with 50 elements

8. Find the market equilibrium allocation for each $\omega^A \in \mathcal{C}$ and plot them in the Edgeworth box