Homework B

Zhentao Jiang

Georgetown University

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Question 1

Consider an economy with a constant labor force L. Let E_t and U_t denote the numbers of employed and unemployed individuals at time t, with $E_t + U_t = L$. Assume that each period, a fraction s of employed workers are separated from their jobs (for reasons related to frictional or structural unemployment) while a fraction f of unemployed workers find jobs.

Part a

a. How would you model the dynamics of the unemployment rate over time starting from a given unemployment rate at time t?

$$U_{t+1} = U_t + sE_t - fU_t \tag{1}$$

$$E_t = L_t - U_t \tag{2}$$

$$U_{t+1} = sL_t + (1 - f - s)U_t \tag{3}$$

Dividing both sides by L we can get the unemployment rate:

$$u_{t+1} = s + (1 - f - s)u_t \tag{4}$$

Part b

b. What is the steady-state equilibrium of your dynamic model?

From part (a) we know that:

$$u_{t+1} = s + (1 - f - s)u_t$$

In steady state, $u_{t+1} = u_t$, thus we can have

$$u - (1 - f - s)u = s \tag{5}$$

$$u = \frac{s}{f + s} \tag{6}$$

Part c

c. Show that the steady-state equilibrium is **stable** in the following sense: if we start at a lower (higher) unemployment rate at t, then the unemployment rate rises (falls) over time, converging to the steady-state value as time tends to infinity.

We can rewrite equation (4) as

$$u_{t+1} - u_t = s - (f + s)u_t$$

Where s can be written as $s = (f + s)u$

$$u_{t+1} - u_t = (f + s)[u - u_t] \tag{7}$$

Consider if $u_t > u$? What about $u_t < u$

Question 2

Consider the following 2-period model of saving for retirement. In Period 1, a consumer-worker has one unit of time to allocate to labor (L) or leisure (1-L). The real wage rate is w, and labor income (wL) is taxed at the rate t ($0 < t < 1$). The worker chooses his consumption (C_1) in Period 1 and saving (S) for retirement:

$$S = (1 - t)wL - C_1$$

Savings grow at the real interest rate r, and the worker gets a transfer (pension) T from the government in Period 2. Consumption in the second period (C_2) is

$$C_2 = (1 + r)S + T$$

The consumer-worker maximizes

$$u(C_1) + \beta u(C_2) + v(1 - L), 0 < \beta < 1$$

subject to (1) and (2). The functions $u(.)$ and $v(.)$ are strictly increasing, continuously differentiable, and strictly concave. Assume throughout that leisure and consumption in the two periods are normal goods. Also assume the transfer T is sufficiently small to make saving S positive.

Part a

a. Derive the optimality condition for the consumer's choice over C_1 and L (i.e., the labor-leisure margin), and give a brief intuitive explanation of what this condition implies.

Plug in the C_1 and C_2 we can have the utility function as

$$u((1-t)wL - S) + u((1+r)S + T) + v(1-L) \tag{8}$$

Find FOC w.r.t. L:

$$(1-t)wu'((1-t)wL - S) - v'(1-L) = 0$$

$$(1-t)wu'(C_1) = v'(1-L)$$

Interpretation

The forgone utility of working = The utility gain from consuming the proceeds

Part b

b. Derive the optimality condition for the consumer's choice over C_1 and C_2 (i.e. the Euler equation), and give a brief intuitive explanation of what this condition implies. Briefly indicate how your results reflect (or don't reflect) a consumption smoothing motive.

Lagrangian's method:

$$\mathcal{L} = u(C_1) + \beta u(C_2) + v(1-L) + \lambda[(1-t)wL - C_1 - S] + \mu[(1+r)S + T - C_2] \quad (9)$$

$$u'(C_1) = \lambda$$

$$\beta u'(C_2) = \mu$$

$$-\lambda + \mu(1+r) = 0$$

$$u'(C_1) = \beta(1+r)u'(C_2) \quad (10)$$

Interpretation

Reducing consumption by one unit and saving the proceeds entail a utility loss in Period 1 **equal to** the utility gain from consuming the proceeds in Period 2

Part c

c. Briefly indicate what are the income and substitution effects of an increase in the tax rate t on C_1, C_2, S, and L.

Increase tax rate t reduces real (permanent) income.

- reduces leisure and consumption in both periods $\Rightarrow$ consumption smoothing.
- reduces C_1 and C_2, and increase L
- $C_2 = (1 + r)S + T$, thus S falls

increase in tax rate t is equivalent to a reduce in real wage.

- leisure becomes less expensive relative to consumption.
- increases leisure and reduce consumption in both periods.
- reduces C_1, C_2, and L.
- $C_2 = (1 + r)S + T$, thus S falls.

Part d

d. Briefly indicate what are the income and substitution effects of an increase in the transfer (pension benefit) T on C_1, C_2, S, and L.

increase in transfer T increases real (permanent) income.

- raises leisure and consumption in both periods.
- raises C_1 and C_2, and reduces L.
- $S = (1 - t)wL - C_1$, thus S falls

increase in transfer T does not change relative prices.