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# HOMEWORK 2

**Exercise 1.** Show that, if $Z \sim \mathcal{N}_n(0, \Sigma)$, then $Z^T \Sigma^{-1} Z \sim \chi_n^2$.

**Exercise 2.** Let

$$Y_{ij} = \mu_i + \epsilon_{ij} \qquad j = 1, ..., n_i \qquad i = 1, ..., a$$

with $\epsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$ being iid random variables. We would like to test, as in the Example 1.17,

$$H_0 : \mu_1 = ... = \mu_a$$
$$H_1 : \exists\, \mu_i \neq \mu_j \quad \text{for some } i, j.$$

Compute the $F$-statistics, justifying each step in detail.

**Exercise 3.** Assume that $X \in \mathbb{R}^{n \times n}$ and that $n^{-1} X^T X = I_n$. Show that the Lasso estimator $\widehat{b}(\lambda)$ is such that

$$\widehat{b}_j(\lambda) = \text{sign}(Z_j) \left( |Z_j| - \frac{\lambda}{2} \right)_+ \qquad Z_j := \frac{(X^T Y)_j}{n} \quad j = 1, ..., n,$$

where $x_+ := \max(x, 0)$.

**Exercise 4.** (Numerical exercise to solve with R or Python).
The goal of an experimental research is to understand whether or not some hormonal treatments have effects on the hormonal concentrations of 12 adult female dogs.
No treatment has been given to 4 dogs. Their hormonal concentrations are 117, 124, 40 and 88. 4 dogs have been treated with oestrogens. Their hormonal concentrations after the treatment are 440, 264, 221, 136. 4 dogs have been treated with progesterone. Their hormonal concentrations after the treatment are 605, 626, 385, 475.

1. Calculate a table that shows the mean and standard deviation of hormonal concentrations by treatment.
2. Use ANOVA to test for a difference between treatments in the hormonal concentrations of the dogs, with a significance level $\alpha = 0.05$. What is your conclusion about the effect of the treatments ?