

# Graph Theory Homework 1

November 2023

## 1. Exercise 1:

Let  $G = (V, E)$  a simple undirected graph. Let  $G' = (V', E')$  the simple undirected graph satisfying  $V' = E$  and  $E' = \{\{e_1, e_2\} \mid e_1, e_2 \in E, e_1 \neq e_2, e_1 \cap e_2 \neq \emptyset\}$ . Prove or disprove the following assertions.

- (i)  $G'$  is hamiltonian implies that  $G$  is eulerian,
- (ii)  $G$  is eulerian implies that  $G'$  is hamiltonian.

## 2. Exercise 2:

Let  $K_n = (V, E)$  the complete graph. Let  $e \in E$ . Determine the number of spanning trees of  $K_n - e$  and prove that your statement holds.

## 3. Exercise 3:

Let  $G = (V, E, w)$  a simple undirected weighted graph with  $w(v) \neq w(u)$  for all  $u, v \in V$  such that  $u \neq v$ . Prove that if  $T$  is a minimum spanning tree of  $G$ , then it is unique.

## 4. Exercise 4:

Let  $G = (V, E)$  a simple undirected graph such that  $G$  contains no cycle and  $G + e$  contains a cycle for any  $e \in \{\{v, u\} \mid v, u \in V, v \neq u \text{ and } e \notin E\}$ . Let  $\varphi : G \rightarrow G$  an isomorphism (for graphs). Show that the following holds

$$\exists v \in V \text{ with } \varphi(v) = v \quad \text{or} \quad \exists e \in E \text{ with } \varphi(e) = e.$$