

TASK 1

1. What is the theoretical maximum value of f_{u_max} and f_{v_max} if error-free image reconstruction from the digital image should be possible (not using any compressive-sensing techniques)?

The theoretical maximum spatial frequencies f_{u_max} and f_{v_max} that can be represented without aliasing, based on the sampling rates provided. This can be solved using the **Nyquist-Shannon sampling theorem**, which states that the maximum representable frequency is half the sampling frequency.

So:

The sampling interval in the x-direction (Δx) and the y-direction (Δy) respectively:

$$\Delta x = 1/20 \text{ samples/mm} \ \& \ \Delta y = 1/10 \text{ samples/mm}$$

Based on the Nyquist-Shannon theorem states the maximum frequency that can be represented without aliasing is:

$$f_{_max} = 1/2\Delta$$

Thus we get the theoretical maximum values:

$$f_{_umax} = 1/2\Delta x = 10 \text{ cycles/mm} \ \& \ f_{_vmax} = 1/2\Delta y = 5 \text{ cycles/mm}$$

2. What is the minimum memory requirement for the color image $f_F(x, y)$ when stored in a conventional computer system, if 1024 values are to be distinguished per color channel. Describe the image format to be used.

Sampling Rate: 20 samples/mm in the x-direction and 10 samples/mm in the y-direction.

Image Dimensions: Assume the image dimensions are $W \times H$ mm.

Color Channels: A color image typically has three channels: Red, Green, and Blue (RGB).

Bit Depth: Each channel can distinguish 1024 values. This implies a bit depth of:

$$\text{Bit depth per channel} = \log_2(1024) = 10 \text{ bits.}$$

So,

$$\text{Number of sampling points} = (20 \text{ samples/mm}) \cdot (10 \text{ samples/mm}) \cdot W \cdot H = 200 \cdot W \cdot H \text{ samples}$$

$$\text{Memory per sample} = 3 \cdot 10 = 30 \text{ bits} = 3.75 \text{ bytes}$$

$$\text{So: Total memory} = (\text{Number of sampling points}) \cdot (\text{Memory per sample in bytes}) = 750 \cdot W \cdot H \text{ bytes}$$

Since we are at a 10 bits RGB image case the only known format that easily deals with this in a lossless manner is TIFF (can go to 3×16 bits).

3. How many colors could be represented with the quantization chosen in sub-task 2?

Since there are 1024 potential values per color channel, there are as many color representations as the number of value combinations.

So, # of combinations = $1024^3 = 1,073,741,824$ colors represented with the quantization we chose.