MS_DS-19 - Introduction to Deep Learning for Image Analysis and Computer Vision
with Examples in Medical Applications

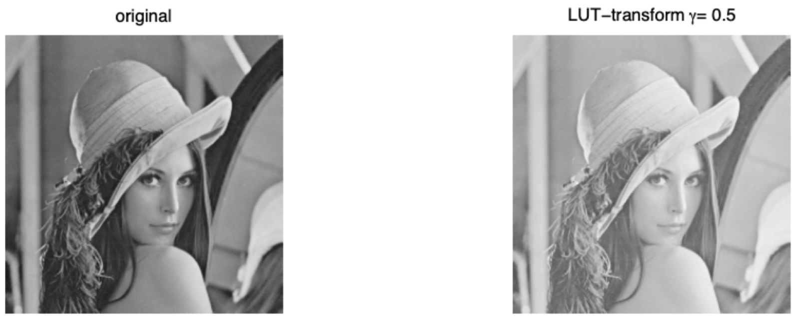
Self study: Geometric Image Transformations

Andreas Husch, PhD
Luxembourg Centre for Systems Biomedicine
Interventional Neuroscience Group

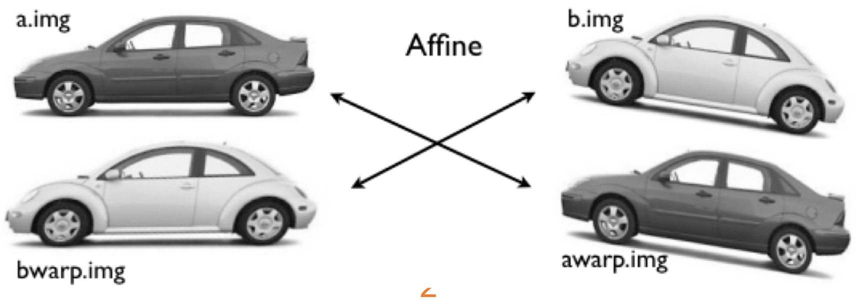
andreas.husch@uni.lu

Geometric transformations

- So far we talked about operations **changing** the image *content*, i.e. the intensity values (the *range* of the image function)



- Morphological operators might alter the *shape* of some image *contents*
- However the *geometry* of the *image space* remained untouched. Geometric transformations change the geometry trying to **preserve** the content.



Geometric transformations

- Geometric transformations $\mathrm{T_G}$ transform the ***coordinates*** $(\mathrm{x,y})$:

$$T_G : (x, y)^t \mapsto (x', y')^t$$

where $(x, y)^t$ and $(x', y')^t$ are column vectors of *coordinate* pairs.

- Basic Examples: Translation, Rotation, Scaling.
- Coordinates $(\mathrm{x',y'})$ are *real-valued (!)*: grid coordinates result from rounding (nearest neighbor)
- For the determination of the pixel values, => **interpolations** (e.g. bi-linear, tri-linear, nearest neighbor).
- We **don't** transform image intensities! We "just" **interpolate** new intensity values on the new grid to **obtain the new image**!

Affine transformations

- Affine ("linear+translation") mappings can be formulated for all **coordinates** in matrix form

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R \cdot \begin{pmatrix} x \\ y \end{pmatrix} + V$$

- Rotation matrix R for rotation by angle ϕ and translation vector V for displacement by (Δx,Δy):

$$\begin{aligned} R(\varphi) &= \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \\ V &= \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \end{aligned}$$

Examples

- Rotation around ϕ = -90°

$$R(-\pi/2) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} 0 \\ M-1 \end{pmatrix}$$

Scaling: x-axis with $s_x = 2$
and y-axis with $s_y=1/2$

$$R = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$V = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Examples

- **Rotations** can be considered as a **subset** of the **affine** transformations $\mathrm{f(x)} = \mathrm{Ax}$ where the matrix A is constraint to have $\mathrm{determinant}=1$
- In software implementation, affine transform $\mathrm{C_2} = \mathrm{TC_1} + \mathrm{t}$ often represented using **homogenous coordinates,** $\mathrm{C_2} = \mathrm{T_H C_1}$

2D Example (translation only):

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Idea: augmenting the matrix to express the translation ($+\ \mathrm{t}$) directly in the linear matrix
- (more in lecture on image registration)

Geometric Transforms – key point

Remember: Geometric transform **don't** transform image *intensities*!

Geometric transforms, transform the (x,y) pairs, i.e. **the grid!**

We "just" **interpolate** new intensity values on the new grid to **obtain the new image**!

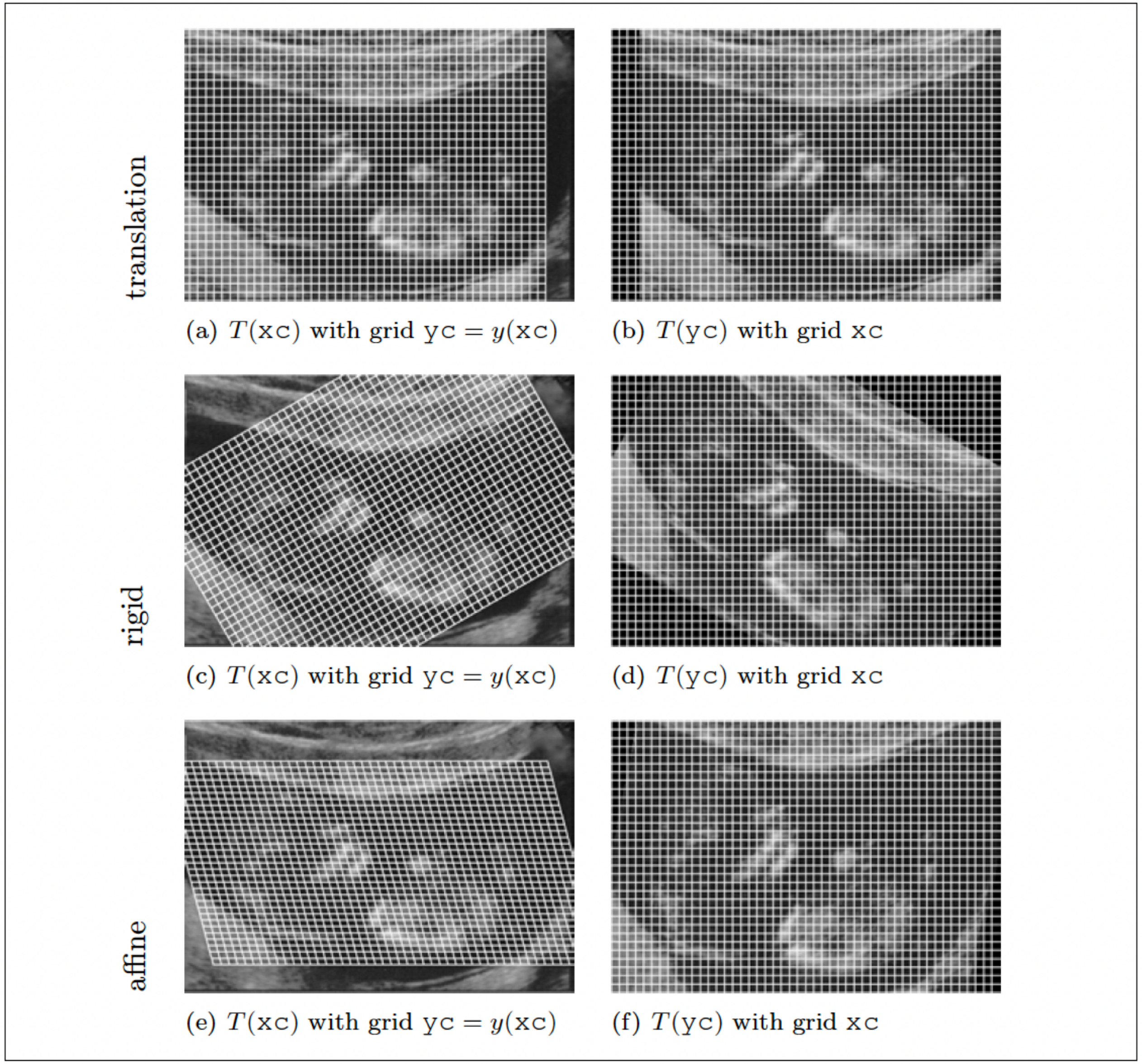


Figure 4.1: *Translation of a US image, rigid, and affine linear transformations.*

Figure from Modersitzky, 2009.