Zaitsev_5

October 20, 2024

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1.1 General Information

In this exercise we will further analyze the Facebook network (**Group 8**). Namely, we will: 1. Explore the types of subgraphs in the network. 2. Define 3 community detection algorithms. 3. Detect communities in the network and plot them. 4. Identify the differences between community detection results.

[1]:
```
# need to install leidenalg package which implements Leiden algorithm
# leidenalg only supports igraph graphs, so we also install igraph package
!pip install leidenalg
!pip install igraph
```

```
Requirement already satisfied: leidenalg in d:\progs\python\lib\site-packages
(0.10.2)
Requirement already satisfied: igraph<0.12,>=0.10.0 in d:\progs\python\lib\site-
packages (from leidenalg) (0.11.6)
Requirement already satisfied: texttable>=1.6.2 in d:\progs\python\lib\site-
packages (from igraph<0.12,>=0.10.0->leidenalg) (1.7.0)


[notice] A new release of pip is available: 24.0 -> 24.2
[notice] To update, run: python.exe -m pip install --upgrade pip
```

```
Requirement already satisfied: igraph in d:\progs\python\lib\site-packages
(0.11.6)
Requirement already satisfied: texttable>=1.6.2 in d:\progs\python\lib\site-
packages (from igraph) (1.7.0)


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```

[2]:
```
# imports
import networkx as nx
import leidenalg
import igraph as ig
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.patches as mpatches
from scipy.spatial import ConvexHull
```

[3]:
```
# global variables
DATA_FOLDER = "../data/"
```

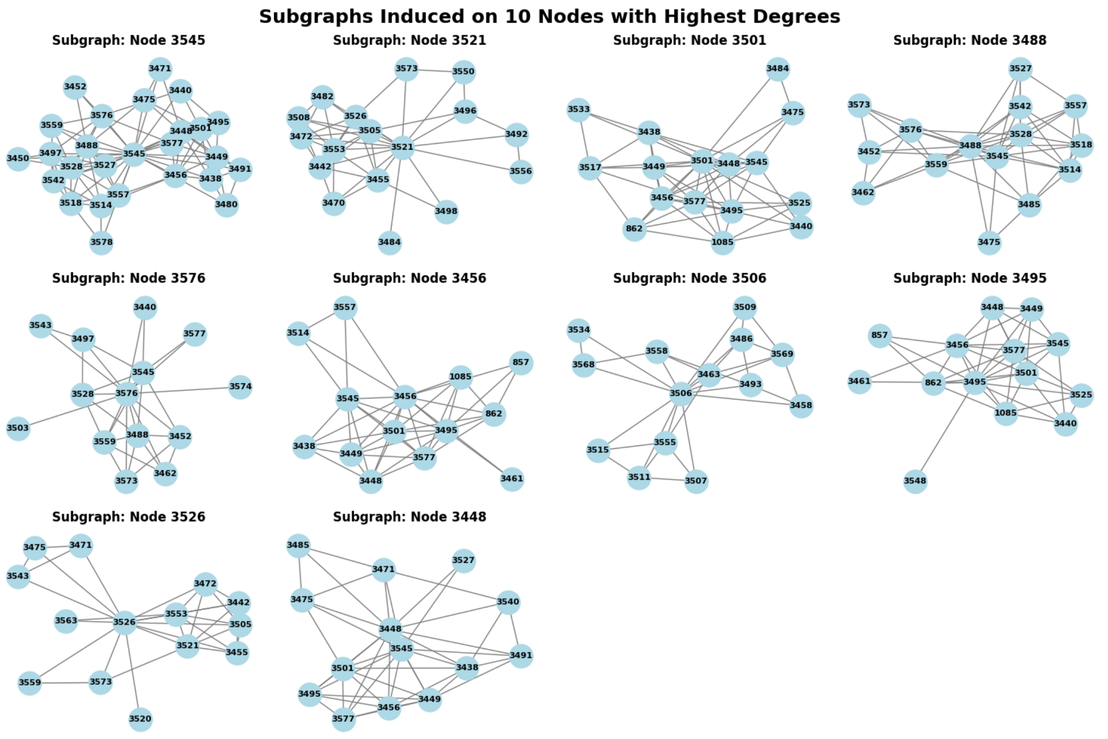
[4]:
```
# utilities
def construct_graph(edges_path: str, directed: bool = False) -> nx.Graph:
    """
    Construct a graph from an edge list.

    Parameters:
        - edges_path (str): The path to the .edges file, which contains the␣
↪edges of the graph.
        - directed (bool): If True, the graph will be treated as directed,␣
↪creating a NetworkX DiGraph.

    Returns:
        - g (nx.Graph or nx.DiGraph): A NetworkX graph containing all the nodes␣
↪and edges from the .edges file.
                                      Additionally, nodes present in the .feat␣
↪file but missing in the .edges
                                      file will be added to the graph as␣
↪disconnected nodes (without any edges).
    """
    # read graph from .edges file
    if directed:
        g = nx.read_edgelist(edges_path, create_using=nx.DiGraph(),␣
↪nodetype=int)
    else:
        g = nx.read_edgelist(edges_path, create_using=nx.Graph(), nodetype=int)
    return g
```

[5]:
```
# load facebook network, indirected graph
G_fb = construct_graph(edges_path=f"{DATA_FOLDER}3437.edges", directed=False)
```

1.2 Question a

[6]:
```
# step 1: choose 10 nodes in the graph that have the highest degrees
# get degree dictionary
degree_dict = dict(G_fb.degree())
# get top-10 nodes in terms of degrees
top_10_nodes_degrees = sorted(degree_dict, key=degree_dict.get, reverse=True)[:
↪10]

# step 2: for each node induce a subgraph of that node and its neighbors
subgraphs = {}
```

```
for node in top_10_nodes_degrees:
    # induce a subgraph on the node and its neighbors
    neighbors = list(G_fb.neighbors(node)) + [node]
    subgraphs[node] = G_fb.subgraph(neighbors)

# step 3: plot all subgraphs
plt.figure(figsize=(15, 10))
for i, (node, subgraph) in enumerate(subgraphs.items(), 1):
    plt.subplot(3, 4, i)
    nx.draw(subgraph, with_labels=True, node_color="lightblue", node_size=500,␣
 ↪font_size=8, font_weight="bold", edge_color="gray")
    plt.title(f"Subgraph: Node {node}", fontsize=12, fontweight="bold")
plt.suptitle(f"Subgraphs Induced on 10 Nodes with Highest Degrees",␣
 ↪fontsize=18, fontweight="bold")
plt.tight_layout()
plt.show()
```



1.3 Question b

There are many algorithms used for community/clusters detection. Some of them are:

1.3.1 Divisive Hierarchical Clustering

Divisive hierarchical clustering: based on edge betweenness. 1. Compute betweenness for all edges in the network. - **Edge Betweenness**: The betweenness of an edge is the number of shortest-paths in the network that pass through that edge. - *Bridges* between communities have high edge betweenness. 2. Remove the edge with highest betweenness. 3. Go to step 1 until no edges left.

1.3.2 Louvain Algorithm

Louvain algorithm: greedily maximises modularity.

1. **Phase 1**:
   1. Assign a different community to each node.
   2. For each node i:
      1. Compute the modularity gain (∇Q) when putting node i into the community of some neighbour j.
      2. Move i to a community of node j that yields the largest modularity gain ∇Q
      3. If no increase is possible, i remains in its original community.
   3. Repeat until no movement yields a modularity gain.
2. **Phase 2**:
   1. Each partition c_i obtained in **Phase 1** forms a new super-node i (a self-loop with weight corresponds to the intra-community weight).
   2. Super-nodes i and j are connected if there is at least one edge betwee nodes of their corresponding partitions (c_i and c_j)
   3. The weight of the edge between new super-nodes i and j is the sum of weights from all edges between their corresponding partitions (c_i and c_j).
3. **Phase 3**:
   1. Apply the Leiden algorithm to the new graph of super-nodes.
   2. Repeat **Phase 1** and **Phase 2** for the new graph until the communities stabilize, and no further modularity gains are possible.

1.3.3 Leiden Algorithm

Leiden algorithm: improvement on the **Louvain algorithm**, addressing some of Louvain's issues with disconnected communities.

1. **Phase 1**: same as Louvain algorithm, but after moving all nodes, **refine** each community to ensure it's connected:
   - If a community splits into multiple disconnected parts, treat each disconnected sub-community as a new community.
   - This guarantees that all resulting communities are internally connected.
2. **Phase 2**:
   1. After Phase 1, examine the connectivity within each community.
      - If a community consists of multiple disconnected sub-communities, split the community into those sub-communities.
   2. Like in the Louvain algorithm, after refining the communities, aggregate nodes that belong to the same community into a **super-node**.
3. **Phase 3**: same as in Louvain algorithm.

1.4 Question c

Here we will detect the communities in the *Facebook* network using **Divisive Hierarchical**, **Louvain** and **Leiden** algorithms and plot the communities.

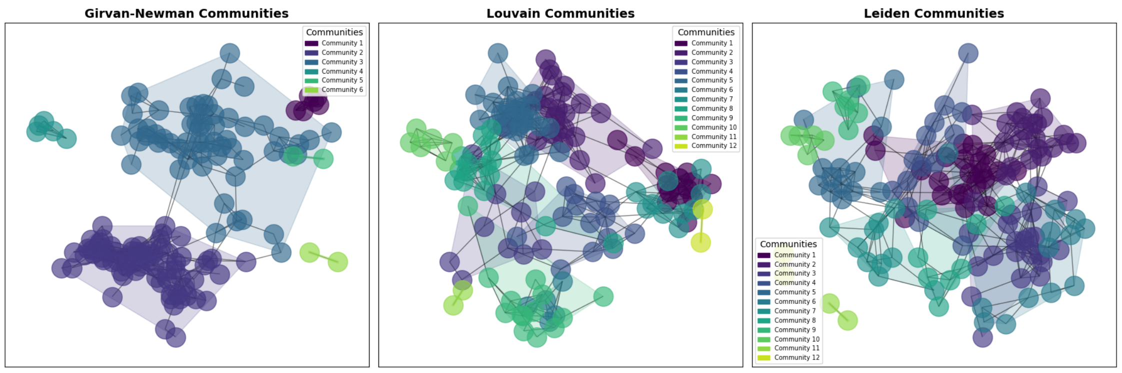
[7]:
```python
def draw_convex_hull(points, ax, color):
    # only draw convex hull if there are 3 or more points
    if len(points) >= 3:
        hull = ConvexHull(points)
        hull_points = points[hull.vertices, :]
        poly = plt.Polygon(hull_points, alpha=0.2, edgecolor=color,␣
 ↪facecolor=color)
        ax.add_patch(poly)
    # for 2 nodes, draw a simple line or ellipse
    elif len(points) == 2:
        ax.plot(points[:, 0], points[:, 1], color=color, alpha=0.5, lw=3)

def plot_communities_in_subplot(ax, graph, communities, title, k_value,␣
 ↪cmap_name="viridis"):
    pos = nx.spring_layout(graph, k=k_value)
    legend_handles = []
    cmap = plt.get_cmap(cmap_name)
    for i, community in enumerate(communities):
        # color and label for each community
        color = cmap(i / len(communities))
        label = f"Community {i + 1}"
        # draw convex hull around the community nodes
        community_pos = np.array([pos[node] for node in community])
        draw_convex_hull(community_pos, ax, color=color)
        # draw the community nodes
        nx.draw_networkx_nodes(
            graph, pos, nodelist=community,
            node_color=[color],
            node_size=500, alpha=0.65, ax=ax
        )
        legend_handles.append(mpatches.Patch(color=color, label=label))
    nx.draw_networkx_edges(graph, pos, alpha=0.5, ax=ax)
    ax.legend(handles=legend_handles, loc="best", title="Communities",␣
 ↪fontsize=7)
    ax.set_title(title, fontsize=14, fontweight="bold")
```

[8]:
```python
def find_girvan_newman_communities(graph: nx.Graph):
    """
    Divisive Hierarchical (Girvan-Newman) algorithm for finding communities in␣
 ↪the graph.
    """
    communities = nx.community.girvan_newman(graph)
    return list(sorted(c) for c in next(communities))
```

```
def find_louvain_communities(graph: nx.Graph):
    """
    Louvain algorithm for finding communities in the graph.
    """
    communities = nx.community.louvain_communities(graph)
    return communities

def find_leiden_communities(graph):
    """
    Leiden algorithm for finding communities in the graph.
    """
    # step 1: convert the NetworkX graph to an iGraph graph

    # NOTE: ig.Graph.from_networkx() uses sequential integer indices for nodes,␣
  ↪starting from 0.
    #       when we convert a NetworkX graph to iGraph, it internally reassigns␣
  ↪the node labels to 0-based indices.
    #       thus, we need to create a map from actual node numbers in G_fb to␣
  ↪reassigned node numbers

    # create a mapping from node labels to node indices
    node_mapping = {node: i for i, node in enumerate(graph.nodes())}
    # reverse mapping for later
    reverse_mapping = {i: node for node, i in node_mapping.items()}
    # convert the NetworkX graph to an iGraph graph
    g_igraph = ig.Graph.from_networkx(graph)
    # step 2: run the Leiden algorithm to detect communities
    partition = leidenalg.find_partition(g_igraph, leidenalg.
  ↪ModularityVertexPartition)
    # step 3: convert iGraph community nodes back to the original NetworkX␣
  ↪labels
    communities = [[reverse_mapping[node] for node in community] for community␣
  ↪in partition]
    return communities
```

[9]:
```
# compute the communities using specified algorithms
gn_communities = find_girvan_newman_communities(graph=G_fb)
lou_communities = find_louvain_communities(graph=G_fb)
lei_communities = find_leiden_communities(graph=G_fb)
# plot all the communities results in a single plot with 1 row and 3 columns
fig, axes = plt.subplots(1, 3, figsize=(18, 6))
plot_communities_in_subplot(axes[0], G_fb, gn_communities, "Girvan-Newman␣
  ↪Communities", k_value=0.2)
plot_communities_in_subplot(axes[1], G_fb, lou_communities, "Louvain␣
  ↪Communities", k_value=0.3)
```

```
plot_communities_in_subplot(axes[2], G_fb, lei_communities, "Leiden␣
 ↪Communities", k_value=0.3)
plt.tight_layout()
plt.show()
```



[14]:
```
def printCommunitySizes(algos_communities: list, titles: list) -> None:
    for algo_communities, title in zip(algos_communities, titles):
        print(f"{title} community sizes:")
        sorted_algo_communities = sorted(algo_communities, key=len,␣
 ↪reverse=True)
        for i, community in enumerate(sorted_algo_communities):
            print(f"    Community {i}, size: {len(community)}")
printCommunitySizes([gn_communities, lou_communities, lei_communities],␣
 ↪["Givan-Newman", "Louvain", "Leiden"])
```

```
Givan-Newman community sizes:
    Community 0, size: 67
    Community 1, size: 58
    Community 2, size: 6
    Community 3, size: 5
    Community 4, size: 2
    Community 5, size: 2
Louvain community sizes:
    Community 0, size: 25
    Community 1, size: 23
    Community 2, size: 15
    Community 3, size: 14
    Community 4, size: 14
    Community 5, size: 14
    Community 6, size: 10
    Community 7, size: 10
    Community 8, size: 6
    Community 9, size: 5
    Community 10, size: 2
    Community 11, size: 2
Leiden community sizes:
    Community 0, size: 22
    Community 1, size: 22
    Community 2, size: 21
    Community 3, size: 14
    Community 4, size: 14
    Community 5, size: 14
    Community 6, size: 10
    Community 7, size: 8
    Community 8, size: 6
    Community 9, size: 5
    Community 10, size: 2
    Community 11, size: 2
```

1.5 Question d

The differences between the community detection results we are going to discuss are the number of communities detected and the community sizes.

We can see a clear difference in how communities are detected in the Girvan-Newman algorithm compared to the Louvain and Leiden algorithms. The Girvan-Newman algorithm detects only 6 communities, with 2 significantly larger communities (sizes 67 and 58) compared to the other 4 smaller ones (sizes $6, 5, 2, 2$). The reason for this difference lies in the splitting criteria between the algorithms:

1. **Girvan-Newman** is an edge-betweenness-based algorithm, which gradually removes the edges that are most "between" communities, typically leading to the detection of a **few larger communities** and several much smaller ones. This algorithm focuses on topological "bottlenecks" within the network, which leads to a more hierarchical and less granular community structure.
2. On the other hand, **Louvain and Leiden** algorithms focus on maximizing modularity, a measure that favors denser intra-community links while minimizing inter-community links. Both algorithms begin by partitioning the network into small communities and then iteratively refining them to optimize modularity. This results in a **larger number of smaller, more balanced community sizes** compared to Girvan-Newman.

Finally, the Leiden algorithm produces results that are similar to the Louvain algorithm but with a slight improvement. It detects 12 communities compared to Louvain's 11. This is expected because **Leiden** improves on Louvain by allowing communities to **split into disconnected sub-communities**, addressing one of the limitations of Louvain. As a result, Leiden identifies slightly finer-grained community structures, which explains the additional community it detects in this case.