

# Regression

Prototyping with Deep Learning

# Learning outcomes

After this lesson you will be able to:

- Understand regression principles
- Identify appropriate evaluation metrics for regression tasks
- Recognize applications of regression models in DL

# What is regression?

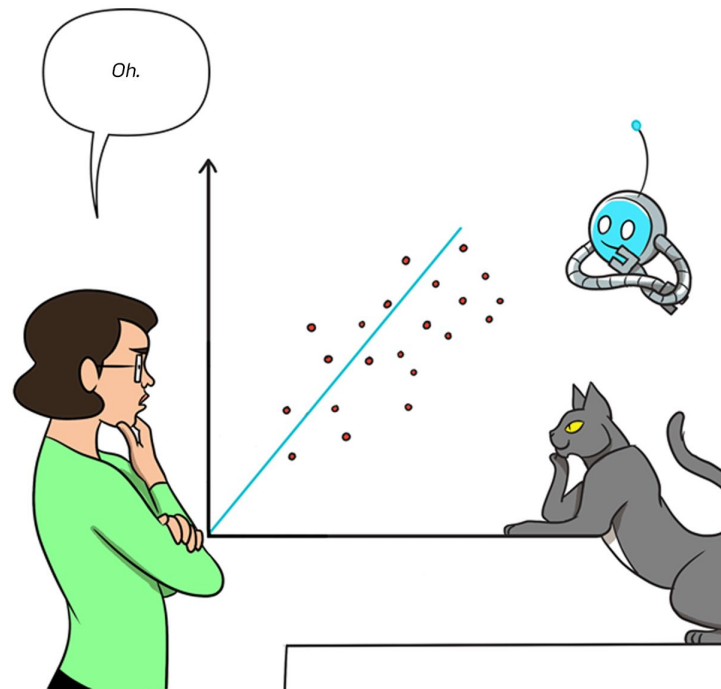
Predict a **continuous** value associated with a feature vector

Examples:

$f(\text{room}) = \text{temperature}$

$f(\text{trajectory}) = \text{time}$

...



<https://cloud.google.com/products/ai/ml-comic-1/>

# Linear regression

Linear Regression: Single Variable

$$\boxed{\hat{y}} = \underbrace{\beta_0 + \beta_1}_{\text{Coefficients}} \underbrace{x}_{\text{Input}} + \underbrace{\epsilon}_{\text{Error}}$$

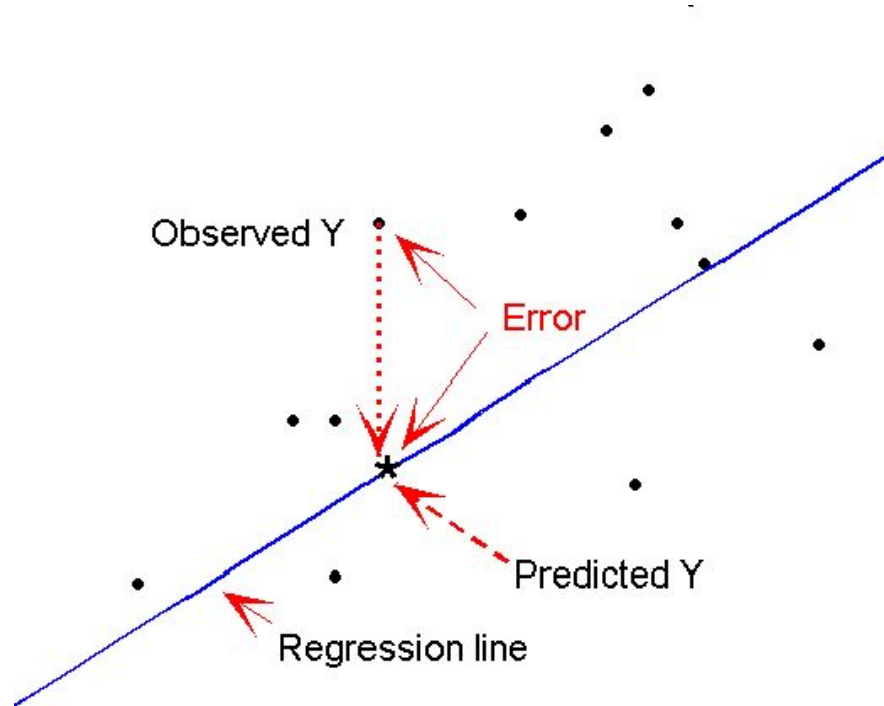
Predicted output

Linear Regression: Multiple Variables

$$\boxed{\hat{y}} = \underbrace{\beta_0 + \beta_1 x_1}_{\text{Coefficients}} + \dots + \underbrace{\beta_p x_p}_{\text{Input}} + \underbrace{\epsilon}_{\text{Error}}$$

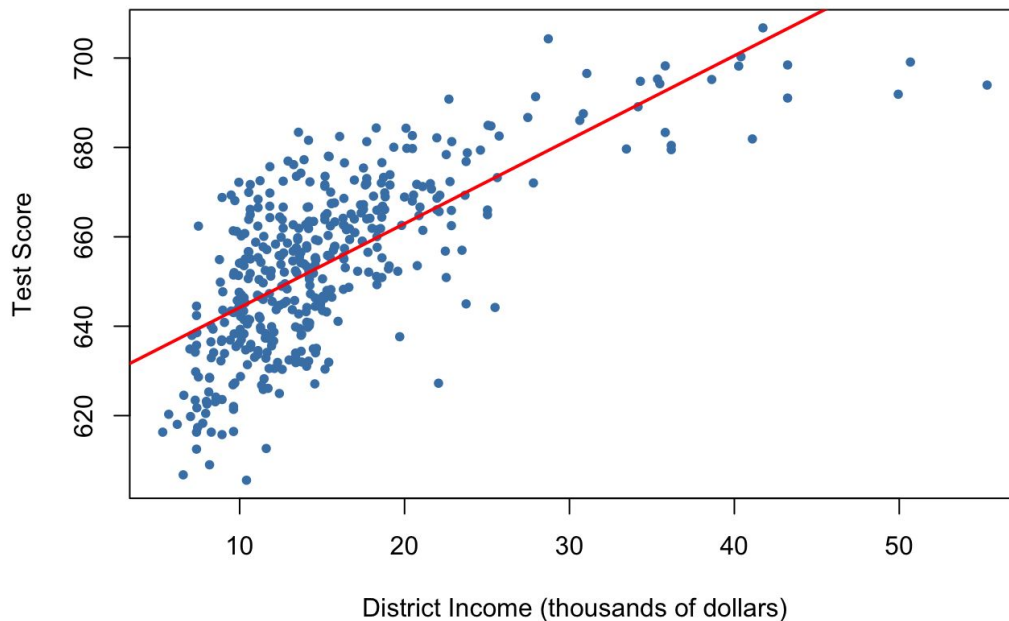
<https://www.dataquest.io/blog/understanding-regression-error-metrics/>

# Residuals

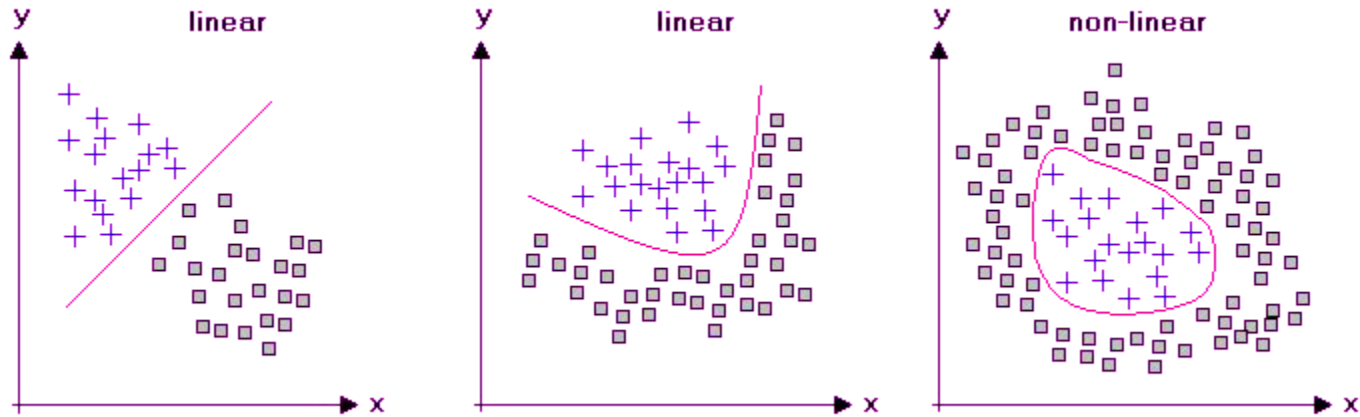


# Non-linear regression

Test Score vs. District Income and a Linear OLS Regression Function



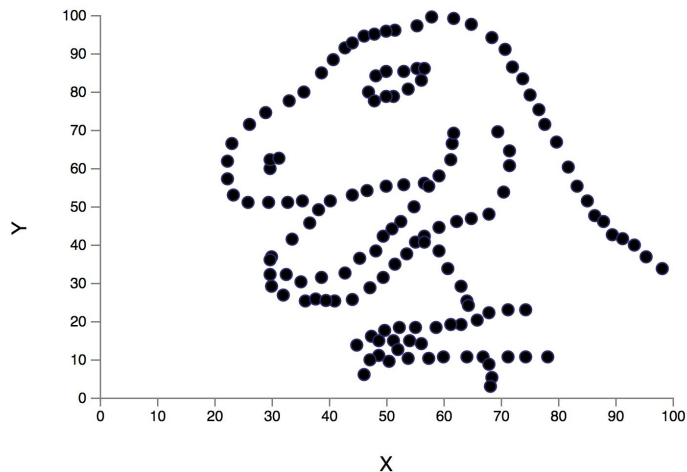
# Non-linear regression with linear models



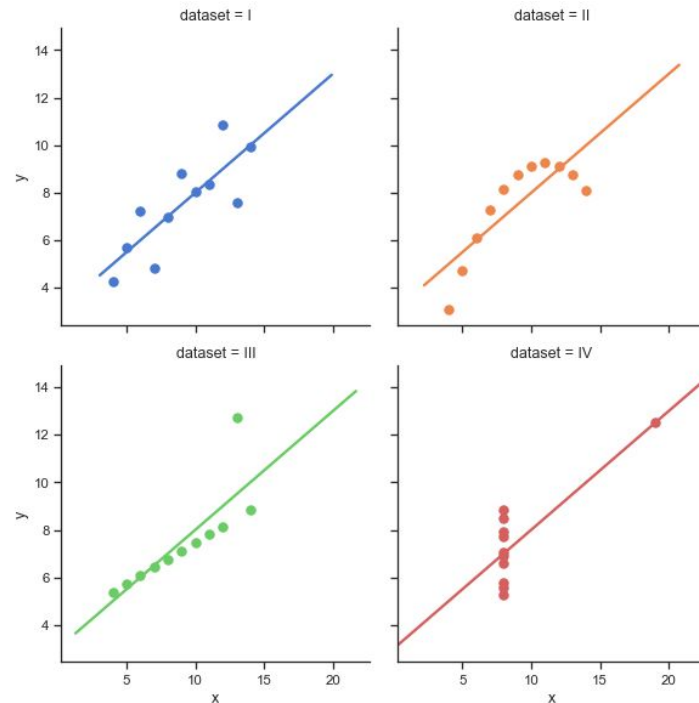
[http://www.statistics4u.info/fundstat\\_eng/cc\\_linvsnonlin.html](http://www.statistics4u.info/fundstat_eng/cc_linvsnonlin.html)

<https://blog.minitab.com/en/adventures-in-statistics-2/what-is-the-difference-between-linear-and-nonlinear-equations-in-regression-analysis>

# Anscombe's quartet



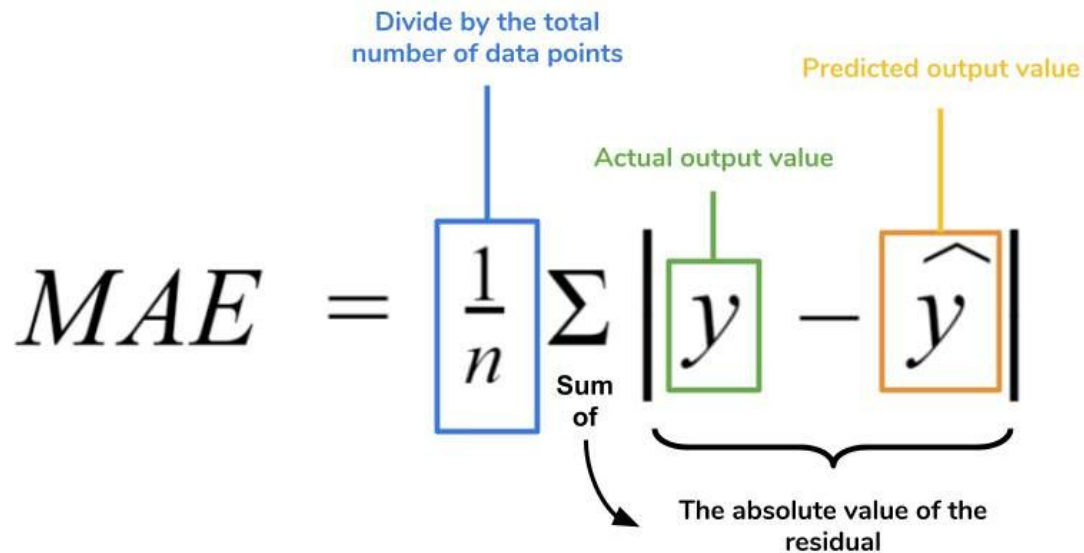
<https://www.autodeskresearch.com/publications/samestats>



<https://www.heap.io/blog/anscombes-quartet-and-why-summary-statistics-dont-tell-the-whole-story>



## Mean Average Error



The diagram illustrates the Mean Absolute Error (MAE) formula with the following components and annotations:

- Divide by the total number of data points:** Points to the fraction  $\frac{1}{n}$ .
- Sum of:** Points to the summation symbol  $\Sigma$ .
- Actual output value:** Points to the variable  $y$  inside a green box.
- Predicted output value:** Points to the variable  $\hat{y}$  inside an orange box.
- The absolute value of the residual:** Points to the absolute value bars  $| \dots |$  surrounding the difference  $y - \hat{y}$ .

$$MAE = \frac{1}{n} \sum |y - \hat{y}|$$

## Mean Square Error

$$MSE = \frac{1}{n} \sum \underbrace{\left( y - \hat{y} \right)^2}_{\substack{\text{The square of the difference} \\ \text{between actual and} \\ \text{predicted}}}$$

## Mean Absolute Percentage Error

$$MAPE = \frac{100\%}{n} \sum \left| \frac{\overbrace{y - \hat{y}}^{\text{The residual}}}{\underbrace{y}_{\text{Each residual is scaled against the actual value}}} \right|$$

Multiplying by 100% converts to percentage

## Mean Percentage Error

$$MPE = \frac{100\%}{n} \sum \left( \frac{y - \hat{y}}{y} \right)$$

# Evaluation metrics

**CASE 1: Evenly distributed errors**

ID	Error	Error	Error^2
1	2	2	4
2	2	2	4
3	2	2	4
4	2	2	4
5	2	2	4
6	2	2	4
7	2	2	4
8	2	2	4
9	2	2	4
10	2	2	4

<b>MAE</b>	<b>RMSE</b>
2.000	2.000

**CASE 2: Small variance in errors**

ID	Error	Error	Error^2
1	1	1	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	3	3	9
7	3	3	9
8	3	3	9
9	3	3	9
10	3	3	9

<b>MAE</b>	<b>RMSE</b>
2.000	2.236

**CASE 3: Large error outlier**

ID	Error	Error	Error^2
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0	0
10	20	20	400

<b>MAE</b>	<b>RMSE</b>
2.000	6.325

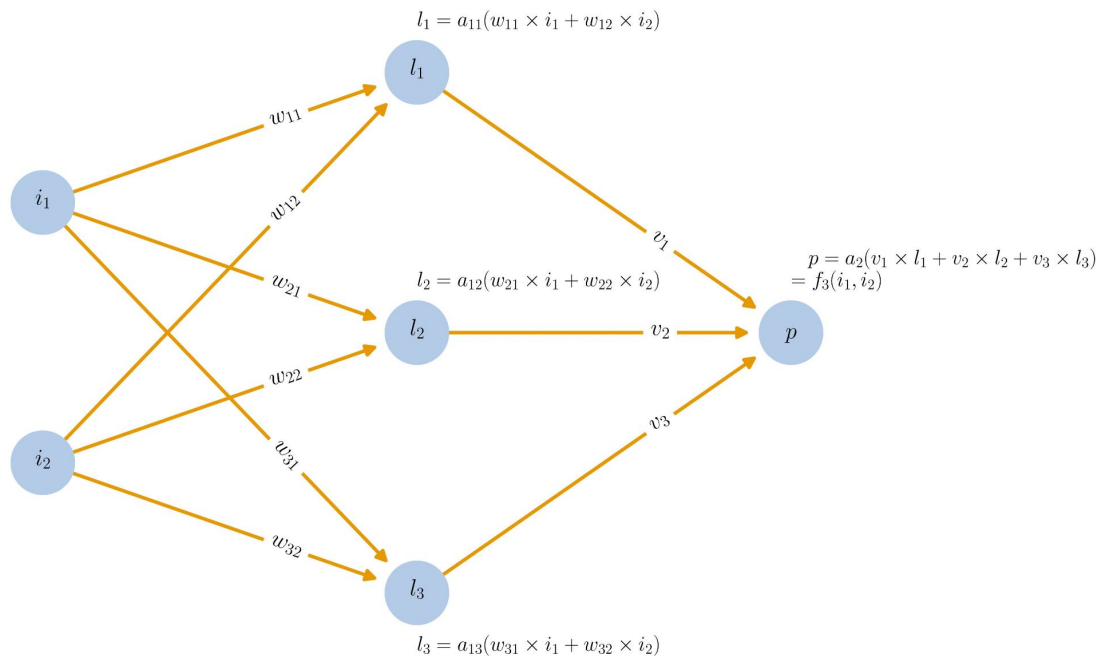
<https://medium.com/human-in-a-machine-world/e60ac3bde13d>

# Evaluation metrics

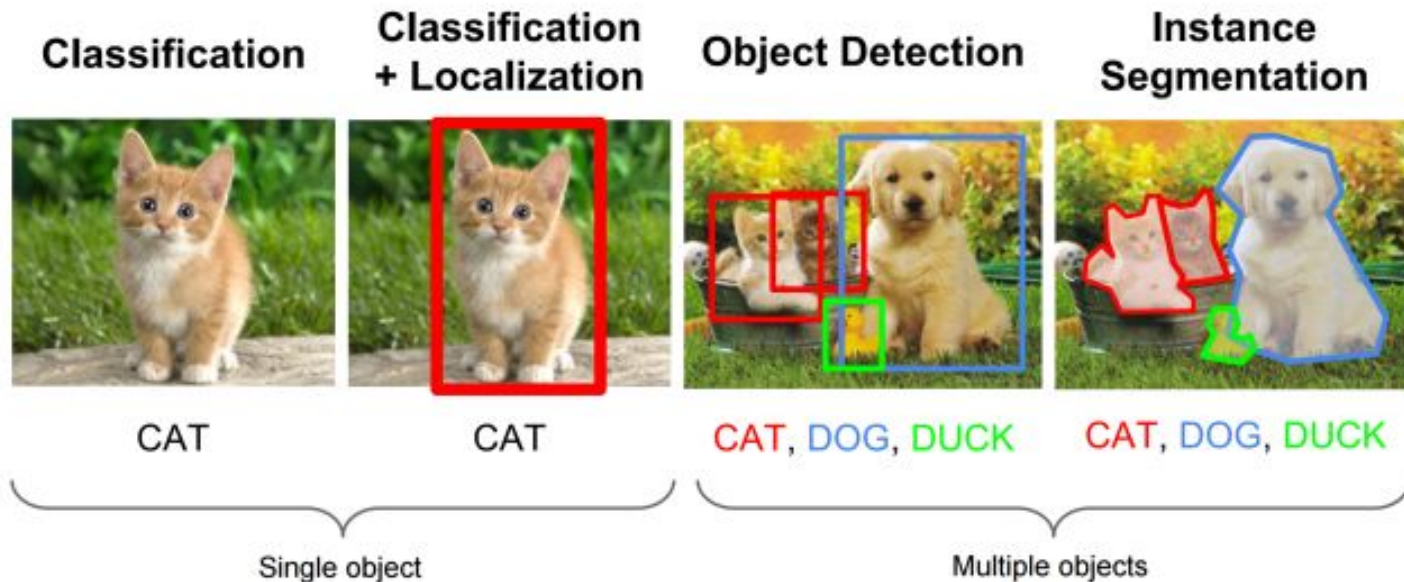
Acronym	Name	Residual Operation	Robust To Outliers
MAE	Mean Absolute Error	Abs. diff	yes
MSE	Mean Squared Error	Squared diff	no
RMSE	Root Mean Squared Error	Squared diff	no
MAPE	Mean Absolute Percentage Error	Abs. diff	yes
MPE	Mean Percentage Error	Raw diff	yes

<https://towardsdatascience.com/cdc5703d242d>

# A simple regression model architecture



# There is more to regression!



[https://leonardoaraujosantos.gitbooks.io/artificial-intelligence/content/object\\_localization\\_and\\_detection.html](https://leonardoaraujosantos.gitbooks.io/artificial-intelligence/content/object_localization_and_detection.html)



# Practical use cases

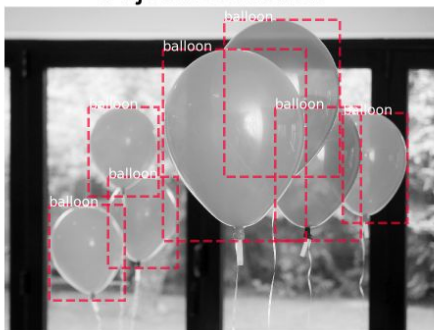
Classification



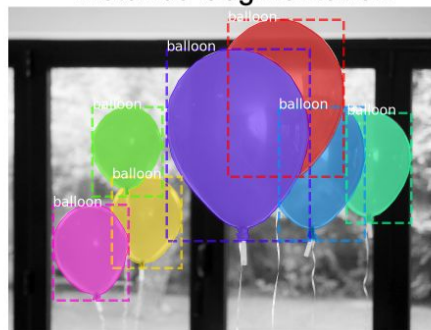
Semantic Segmentation



Object Detection

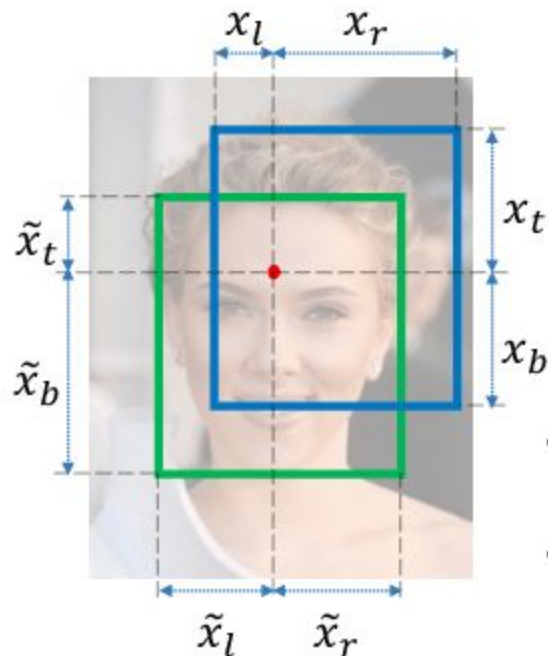



Instance Segmentation




<https://engineering.matterport.com/7c761e238b46>

# Evaluation metric: Intersection over Union



 Ground truth:  $\tilde{x} = (\tilde{x}_t, \tilde{x}_b, \tilde{x}_l, \tilde{x}_r)$

 Prediction:  $x = (x_t, x_b, x_l, x_r)$

- $\ell_2 \text{ loss} = ||\square - \square||_2^2$

- $\text{IoU loss} = -\ln \frac{\text{Intersection}(\square, \square)}{\text{Union}(\square, \square)}$

# Classic architectures

FCN (2014)

DeconvNet (2015)

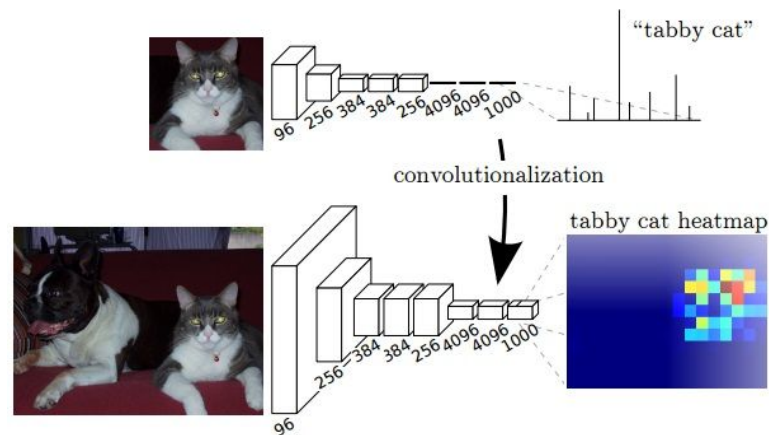
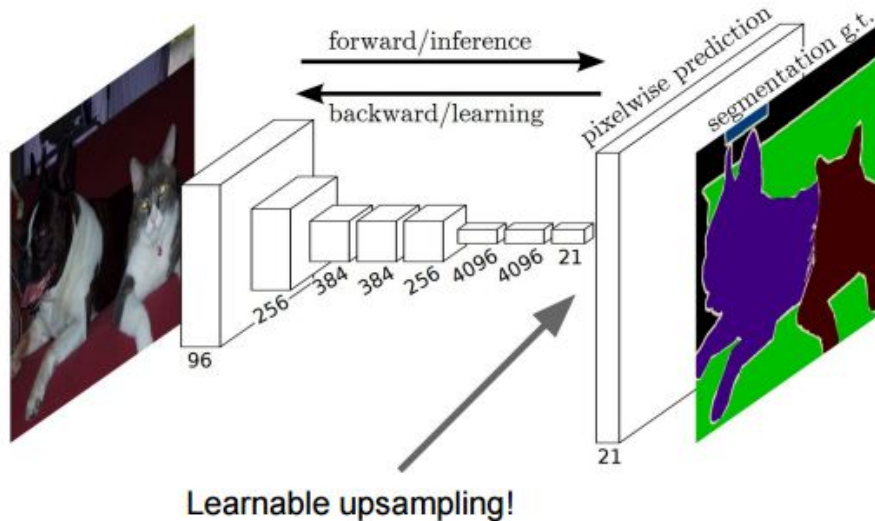
U-Net (2015)

Fast R-CNN (2014) and Faster R-CNN (2016)

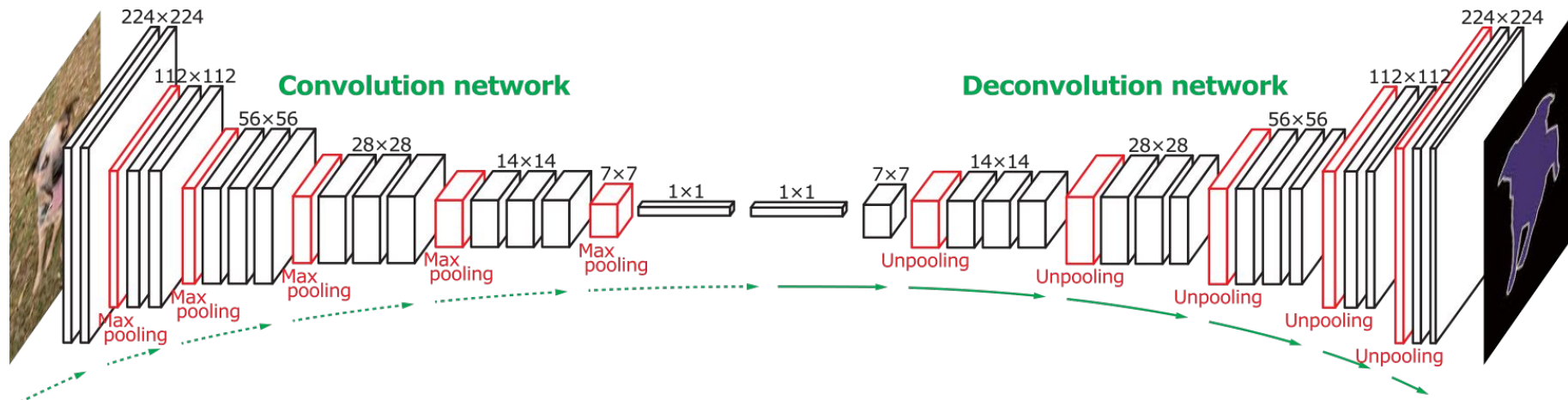
Mask R-CNN (2017)

YOLO (2016)

# Classic architecture: FCN

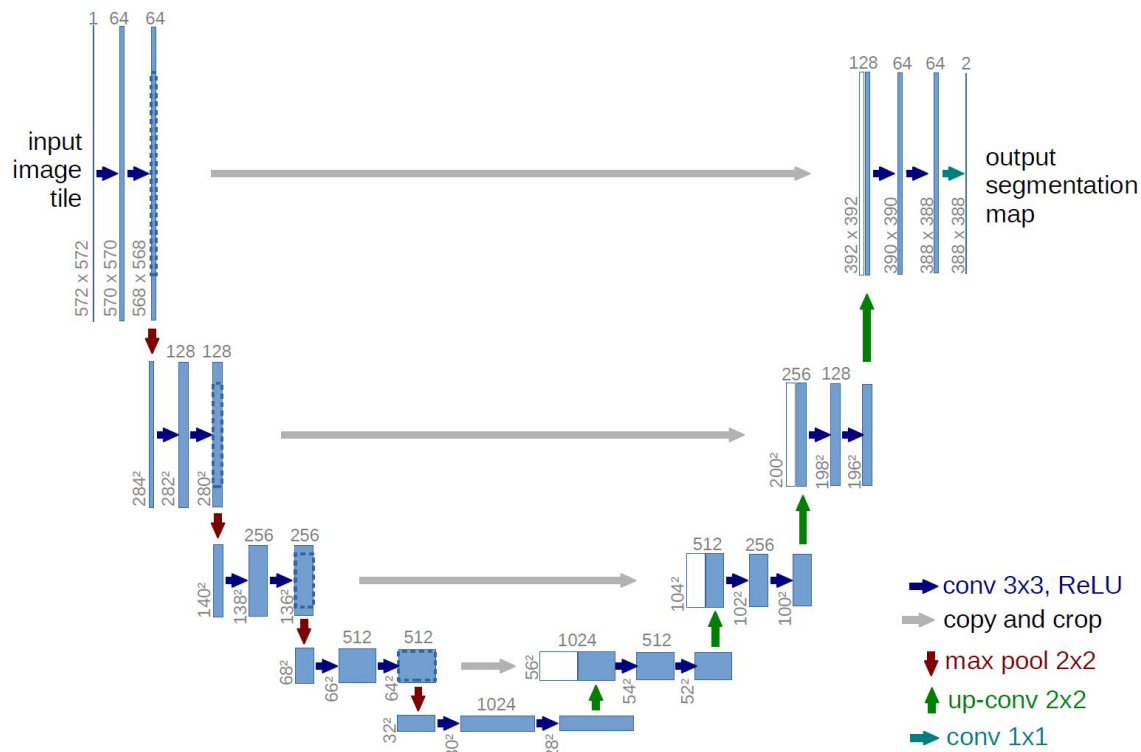


# Classic architecture: DeconvNet



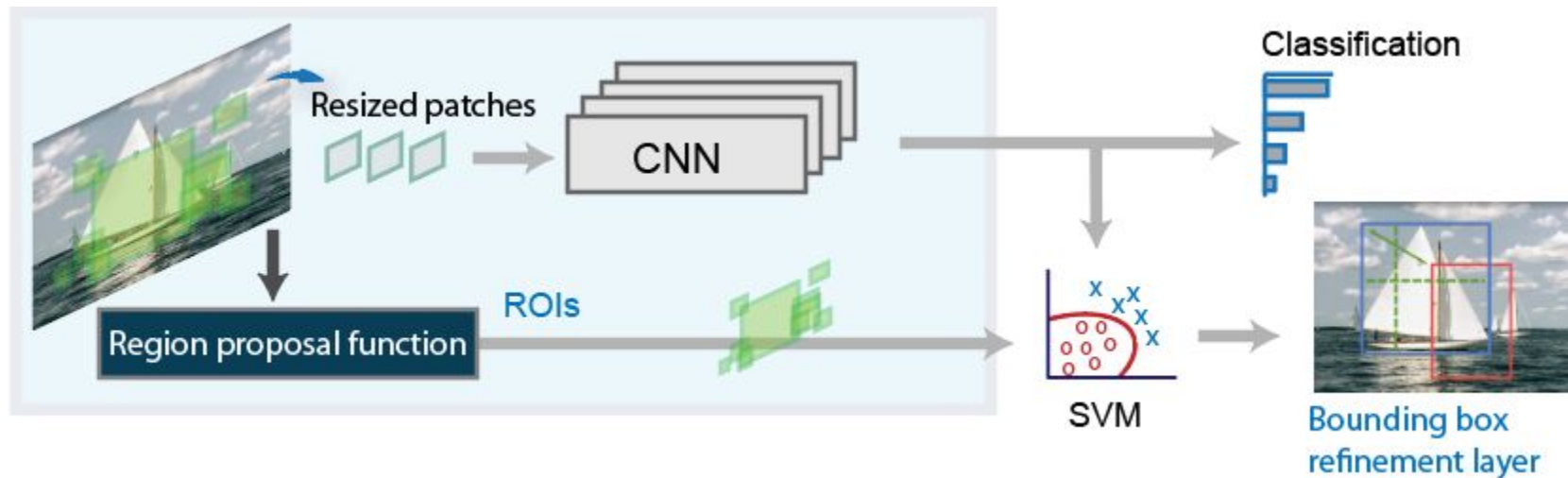
<https://towardsdatascience.com/55cf8a6e380e>

# Classic architecture: U-Net



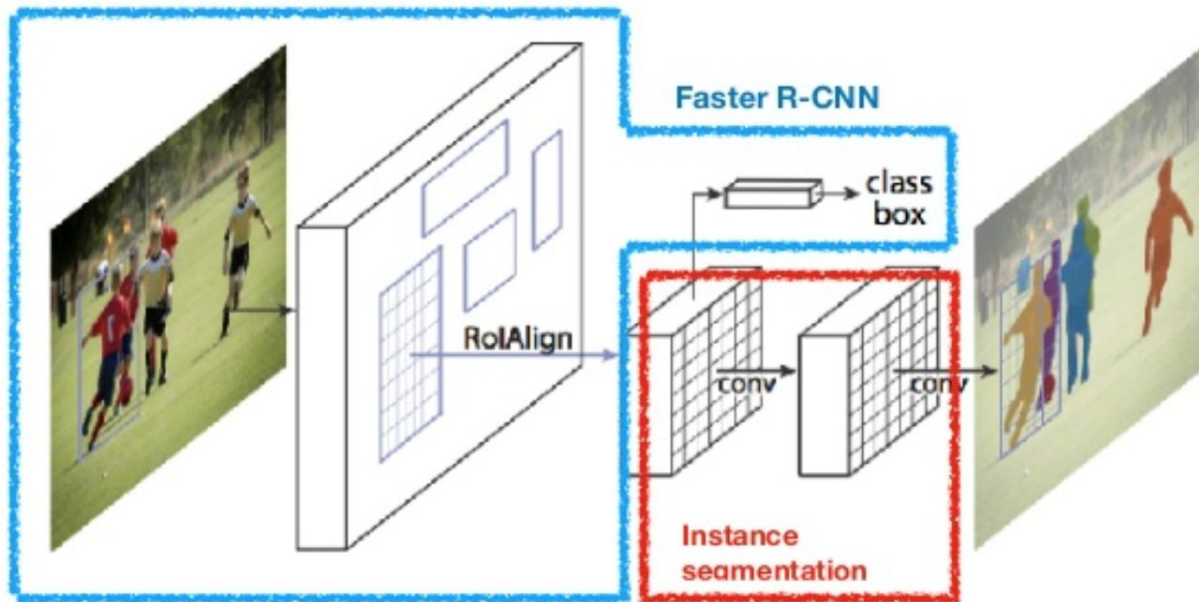
<https://heartbeat.fritz.ai/ff17f6e4c1cf>

# Classic architecture: Fast(er) R-CNN



<https://www.mathworks.com/help/vision/ug/getting-started-with-r-cnn-fast-r-cnn-and-faster-r-cnn.html>

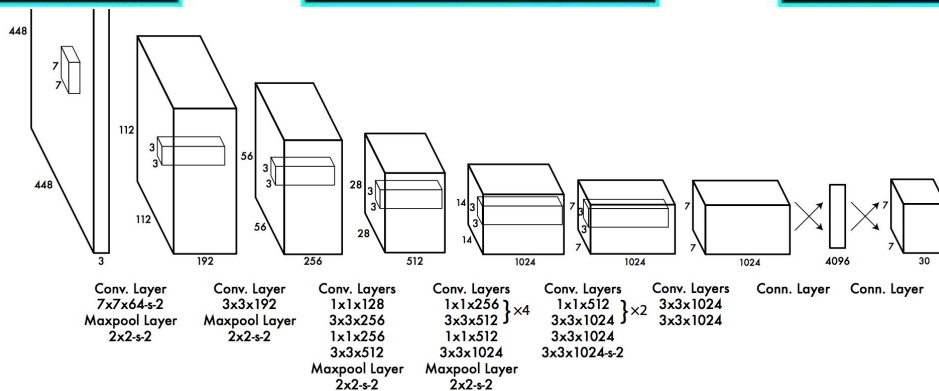
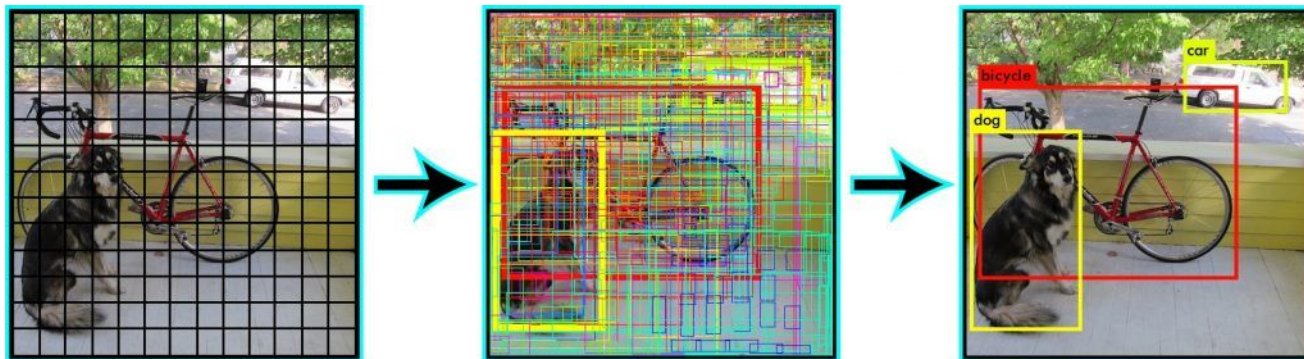
# Classic architecture: Mask R-CNN



<https://lilianweng.github.io/lil-log/2017/12/31/object-recognition-for-dummies-part-3.html>



# Classic architecture: YOLO



<https://dzone.com/articles/understanding-object-detection-using-yolo>

# Faster R-CNN vs YOLO



<https://www.youtube.com/watch?v=Vrx2rKt1xSc>



<https://www.youtube.com/watch?v=Vrx2rKt1xSc>