

William Bathe's Table of Canons: 16th Century Algorithmic Music

Canon #1 (at the third above)



Using The Table

In addition to instruction in reading music notation and some information about basic counterpoint, "A Briefe Introduction to the Skill of Song" contains a table intended to help the reader compose two-voice canons over a cantus firmus. My work with the table is closely guided by the fantastic presentation by Elam Rotem and Alon Schab⁵. In this process, we will write a cantus firmus, select a pitch interval for the canon, use the table to find a melodic canon part in whole notes satisfying our restrictions, and finally add ornamentation.

I have chosen to write my compositions in MuseScore, a free and open-source music composition software. It allows me to easily copy my canon melody and transpose it into the correct pitch and time, and the lyrics feature allows me to easily label the cantus firmus notes with potential intervals.

Rules of Song.

The obser- uations of the places vp are fixe	6 5 4 3 2 1	8 5 1 7 2 1	11 7 6 1 6 2	10 6 5 7 7 3	9 5 4 6 1 4	8 4 3 5 2 5	7 3 2 4 3 6	6 2 1 3 4 7
Places vp.	1	7	6	5	4	3	2	
Cantus vp	1	1356	6	135	16	35	136	5
Cantus down	2	7	6	135	16	35	136	5
	3	6	135	16	35	136	5	1356
	4	5	16	35	136	5	1356	6
	5	4	35	136	5	1356	6	135
	6	3	136	5	1356	6	135	36
	7	2	5	1356	6	135	16	35
8 vt fu: I	1356	6	135	16	35	136	5	
Places down	1	2	3	4	5	6	7	
The obser- uations of the places down are fixe.	1 2 3 4 5 6	1 5 2 7 1 5	2 6 3 1 2 6	3 7 4 2 3 7	4 1 5 3 4 8	5 2 6 4 5 9	6 3 7 5 6 10	7 4 1 6 7 11

⁵ Rotem and Schab, "The Magic Table That Lets You Compose Canons Over A Cantus Firmus (1596)."

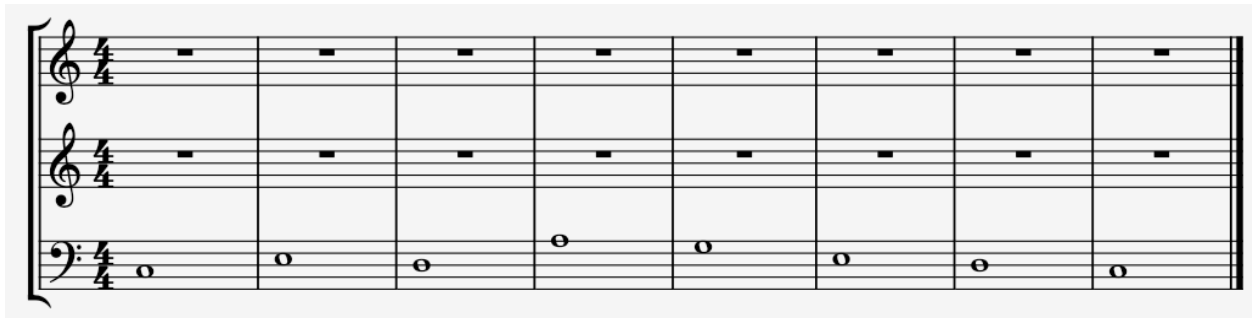
Prerequisites

Here are some concepts you should be familiar with in order to use the table to write a canon over a cantus firmus.

1. Reading modern music notation. The examples below are presented with the cantus firmus in bass clef, and the two canon voices in treble clef.
2. Musical intervals. You should be able to determine the interval between two given pitches - unisons, seconds, thirds, etc. Bathe's table uses decimal numbers to denote intervals, e.g. 6 represents a sixth and 1 represents a unison.
3. Basic counterpoint. While thorough knowledge of late 16th century counterpoint practice is not needed here (and my example canons do not conform strictly to it!), it is useful to understand a couple of basic principles.
 - a. Consonance and dissonance. Intervals of unisons, thirds, sixths, and perfect fourths and fifths are considered consonant. Other intervals (seconds, sevenths, and diminished or augmented fourths and fifths) are considered dissonant⁶. Dissonances should be avoided, and used only on weak beats if necessary.
 - b. Parallel perfect intervals. Perfect fifths and unisons/octaves are consonant and allowed in this counterpoint system. However, if two voices form a perfect interval, they must not move in the same direction and to reach the same perfect interval. For example, if two voices have a C and G in one measure, they may not move to D and A in the next measure. This prohibition is intended to help maintain the independence of voices and avoid them sounding like a single melodic line⁷.

Getting Started

To use the table, you will first need to write a cantus firmus and choose the pitch interval for your canon. The cantus firmus should be entirely in whole notes. For example, in Canon #1 I chose to have a canon at the third above and wrote this cantus firmus.



⁶ Palisca and Moore, "Consonance."

⁷ Drabkin, "Consecutive Fifths, Consecutive Octaves."

Finding Candidate Pitches - Rows and Columns

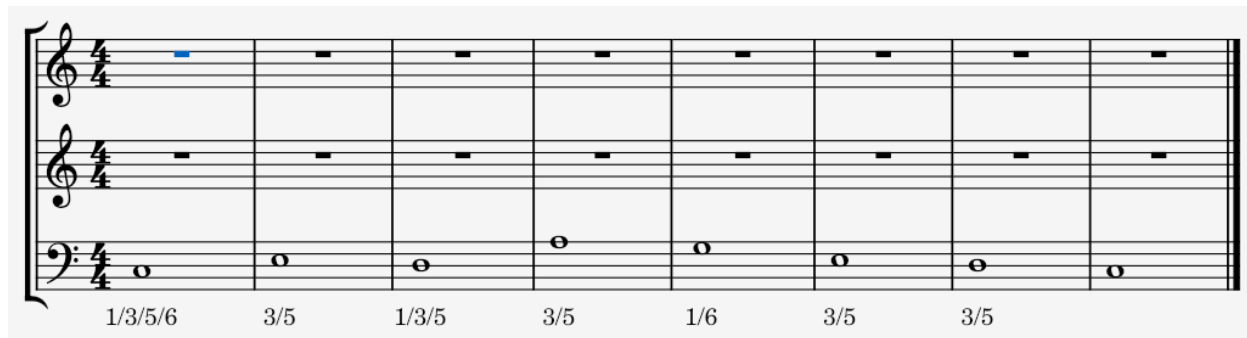
This directs you to a column of the table that will be used for the entire canon. The correct column is indicated by the rows reading "Places up" and "Places down". For a third above, I will use the column "3 places up".

Rules of Song.

	6	8	5	11	10	9	8	7	6
The obser	5	1	7	6	5	4	3	2	1
uations of	4	7	6	5	4	3	2	1	1
the places	3	2	1	7	6	5	4	3	2
vp are fixe	2	5	6	7	1	2	3	4	5
	1	1	2	3	4	5	6	7	8
Places vp.	1	7	6	5	4	3	2	1	1
Courses	1	1356	6	135	16	35	136	5	1356
Courses	2	7	6	135	16	35	136	5	1356
Courses	3	6	135	16	35	136	5	1356	6
Courses	4	5	16	35	136	5	1356	6	135
Courses	5	4	35	136	5	1356	6	135	36
Courses	6	3	136	5	1356	6	135	16	35
Courses	7	2	5	1356	6	135	16	35	136
8 vt fu: I	1	1356	6	135	16	35	136	5	1356
Places down	1	2	3	4	5	6	7	8	9
The obser	1	1	2	3	4	5	6	7	8
uations of	2	5	6	7	1	2	3	4	5
the places	3	2	3	4	5	6	7	8	9
down are	4	7	1	2	3	4	5	6	7
fixe.	5	1	2	3	4	5	6	7	8
	6	5	6	7	8	9	10	11	12

Having found our column, we now look at the melodic motion of the cantus firmus to find the relevant rows of the table. Looking at each successive melodic interval in the cantus firmus, we find the corresponding row of the table and its intersection with our column. Bathe calls these melodic intervals "courses up" and "courses down" and labels them on the left side of the table. The numbers we find in the selected cell give the melodic intervals above the current cantus firmus note that are guaranteed to be consonant both with the current cantus firmus note and the next one - i.e. both the leader and follower part will work with the cantus firmus if we choose one of these notes.

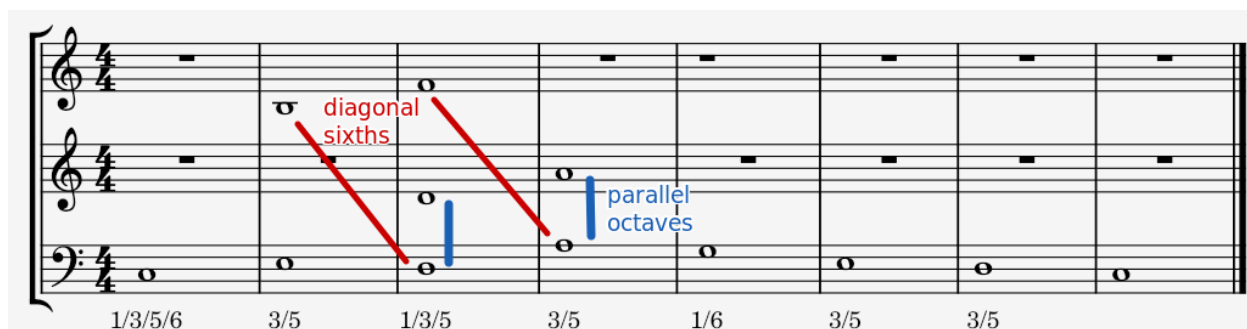
For example, in Canon #1, the first interval is a third up from C to E. I therefore look at the row for 3 courses up and see the numbers 1356, meaning I may choose a unison, third, fifth, or sixth above the first cantus firmus note as the first note of the melody. I thus enter "1/3/5/6" as a lyric under the first note of the cantus firmus.



Each cantus firmus note can be labeled in this way (other than the final note, which has no successor and is therefore free). Once this is done, we can start writing in a melody conforming to these intervals. However, there may still be problems with dissonance between the two parts or with parallel perfect intervals (fifths and octaves), which are forbidden in the counterpoint style Bathe is using. These problems can be avoided by using the “Observations” in the top and bottom parts of the table.

Further Restrictions: Observations 1-6

Observations 1 and 2 relate to the intervals found in the “diagonal” relationships between the leader melody and the cantus firmus. They indicate intervals that should not be used twice in a row in this diagonal location. Otherwise, parallel perfect fifths or octaves will occur. For example, since Canon #1 is a third above, observations 1 and 2 indicate of a third and a sixth. If we choose a B in the second measure (a fifth above the E in the cantus firmus) and an F in the third (a third above the D in the cantus firmus), we have created two “diagonal sixths” between the leader melody and our cantus firmus. When we then fill in the follower melody (one measure later, transposed up a third), we see that we have parallel octaves between the follower voice and the cantus firmus. If we want to abide by the rules of counterpoint, we should avoid this.



Observations 3 and 4 relate to melodic intervals within the leader melody. Those on the upper part of the table represent ascending intervals, and those on the lower represent descending intervals. These intervals should always be avoided within the melody or a dissonance will occur between the leader and follower. For our example Canon #1, which is a third above, our melody should avoid ascending seconds and fourths and descending fifths and sevenths. If we use an A in the first measure and a B in the second, we will have an ascending melodic second. This creates a dissonance in the second bar between the leader and follower voices.

1/3/5/6 3/5 1/3/5 3/5 1/6 3/5 3/5

ascending second dissonance

Observations 5 and 6 are less clearly defined. Rotem and Schab observe that if they are treated as relating to melodic intervals of the leader melody like Observations 3 and 4, they will cause the leader and follower to have an octave or a fifth between them. This would cause parallel fifths or octaves if these intervals are used in succession. For our example, these intervals are ascending thirds and sevenths and descending sixths and tenths. If we use a G in the first measure, a B in the second, and a D in the third, we have two ascending major thirds in a row, violating Observation 5. Thus, we have parallel unisons in measures 3 and 4 between the leader and follower melodies.

1/3/5/6 3/5 1/3/5 3/5 1/6 3/5 3/5

ascending thirds parallel unisons

Once we have filled in the notes as best we can, attempting to adhere to the intervals from the center of the table, the Observations from the edges of the table, and the general rules of counterpoint, we have a basis for our canon. The base structure for Canon #1 is shown below.

1/3/5/6 3/5 1/3/5 3/5 1/6 3/5 3/5

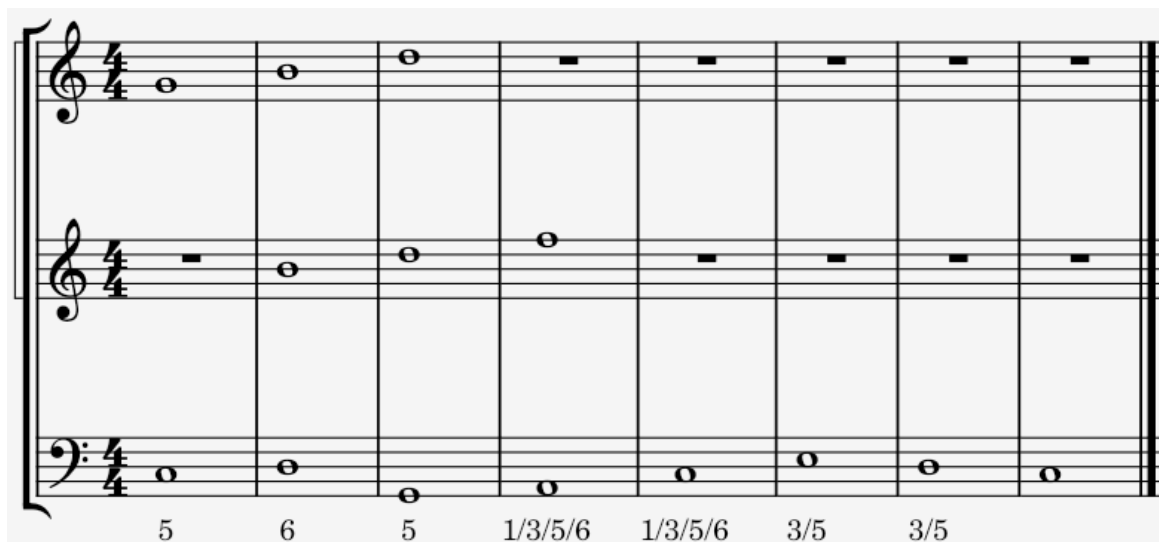
After this, we can ornament the canon. In particular, this is useful for places in the canon where the counterpoint is not perfect. By adding rests, passing notes, suspensions, and other ornaments, we can decorate the canon and draw attention away from problematic areas. For instance, in measure 5 of Canon #1, the two voices converge to a unison, which is not ideal for maintaining their independence. To deal with this, I added a quarter rest in this measure as well

as a tied note from the preceding measure to reduce the time the two voices spend on the same pitch. The complete ornamented canon is shown below.



Lessons Learned and Future Work

In using the table, it became apparent that it is quite restrictive. Even before applying the Observation rules, there are often no good melodic solutions for a given cantus firmus and pitch interval. For example, I started working on a canon at the unison using the cantus firmus below. Using only the information from the center of the table, the first three notes of the melody have only a single option, producing parallel unisons in measures 2 and 3. This guided me to consider the table more thoroughly when composing a cantus firmus and choosing a pitch interval.



In order to produce stylistically correct music of this era, I need to learn more about counterpoint. I plan to read Thomas Morley's "A Plain and Easy Introduction to Practical Music", published in England in 1597, to better understand the "rules" of the music practice of this time and place.

The table might also be useful for slightly different kinds of compositions. I am interested in applying the idea to create rounds, which are essentially canons at the unison. Compositions like "Sumer is icumen in" demonstrate precedent for rounds over a ground bass, which is essentially a cantus firmus.

Works Cited

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