

## William Bathe's Table of Canons: 16th Century Algorithmic Music

Canon #1 (at the third above)



Canon #2 (at the unison)



### What is a “Canon over a Cantus Firmus”?

In music, a “canon” originally meant a set of instructions from which musical parts may be realized<sup>1</sup>. In these examples, we are concerned with a two voice canon. The second voice repeats the exact melody of the first voice after a one-measure delay, possibly with some interval transposition. The second voice in Canon #1 is a third above the first, so this is called a “canon at the third above”. The second voice in Canon #2 is in unison with the first, so it is a “canon at the unison”. The interval between the two voices is called the canon’s “pitch interval”. The first voice of these canons is called the “leader” and the second the “follower”.

“Cantus firmus”, Latin for “fixed melody”, denotes a melody used as the basis for another composition<sup>2</sup>. In this period of music, these are often written as a series of plain whole notes. Each canon here has an original cantus firmus in the lowest voice.

Thus, a canon over a cantus firmus must follow a strict set of conditions. The notes of the leading voice must be chosen so they do not create dissonances with the following voice, and both voices must also remain consonant with the cantus firmus.

### The Book

William Bathe’s “A Briefe Introduction to the Skill of Song” was published in 1596, but probably written in the mid-1580s<sup>3</sup>. At the time of the book’s writing, Bathe was one of a number of English authors competing to produce popular music textbooks<sup>4</sup>. Bathe likely produced this work during employment as a tutor teaching basic musical skills to wealthy children. Musical instruction books like this were targeted toward socially aspiring families who may not be able to afford a tutor, but wanted to learn skills associated with the “gentle” class.

<sup>1</sup> Mann, Wilson, and Urquhart, “Canon (i).”

<sup>2</sup> Bloxam, “Cantus Firmus.”

<sup>3</sup> Bathe, *A Briefe Introduction to the Skill of Song*, 4.

<sup>4</sup> Bathe, 3.

## Using The Table

In addition to instruction in reading music notation and some information about basic counterpoint, "A Brief Introduction to the Skill of Song" contains a table intended to help the reader compose two-voice canons over a cantus firmus. My work with the table is closely guided by the fantastic presentation by Elam Rotem and Alon Schab<sup>5</sup>. In this process, we will write a cantus firmus, select a pitch interval for the canon, use the table to find a melodic canon part in whole notes satisfying our restrictions, and finally add ornamentation.

I have chosen to write my compositions in MuseScore, a free and open-source music composition software. It allows me to easily copy my canon melody and transpose it into the correct pitch and time, and the lyrics feature allows me to easily label the cantus firmus notes with potential intervals.

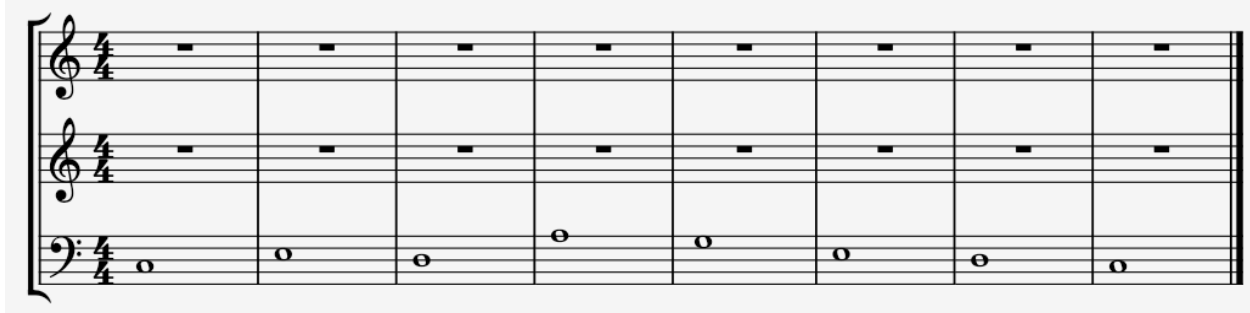
## Rules of Song.

The obser- uations of the places vp are fixe	6 5 4 3 2 1	8 5 1 7 2 5 1	11 7 6 1 6 2	10 6 5 7 7 3	9 5 4 6 1 4	8 4 3 5 2 5	7 3 2 4 3 6	6 2 1 3 4 7
Places vp.	1	7	6	5	4	3	2	
Cantus vp	1	1356	6	135	16	35	136	5
Cantus down	2	7	6	135	16	35	136	5
	3	6	135	16	35	136	5	1356
	4	5	16	35	136	5	1356	6
	5	4	35	136	5	1356	6	135
	6	3	136	5	1356	6	135	36
	7	2	5	1356	6	135	16	35
8 vt fu: I	1356	6	135	16	35	136	5	
Places down	1	2	3	4	5	6	7	
The obser- uations of the places down are fixe.	1 2 3 4 5 6	1 5 2 7 1 5	2 6 3 1 2 6	3 7 4 2 3 7	4 1 5 3 4 8	5 2 6 4 5 9	6 3 7 5 6 10	7 4 1 6 7 11

<sup>5</sup> Rotem and Schab, "The Magic Table That Lets You Compose Canons Over A Cantus Firmus (1596)."

### Getting Started

To use the table, you will first need to write a cantus firmus and choose the pitch interval for your canon. The cantus firmus should be entirely in whole notes. For example, in Canon #1 I chose to have a canon at the third above and wrote this cantus firmus.



### Finding Candidate Pitches - Rows and Columns

This directs you to a column of the table that will be used for the entire canon. The correct column is indicated by the rows reading "Places up" and "Places down". For a third above, I will use the column "3 places up".

**Rules of Song.**

	6	8	5	11	10	9	8	7	6
The obser	5	1	7	6	5	4	3	2	1
uations of	4	7	6	5	4	3	2	1	1
the places	3	2	1	7	6	5	4	3	2
vp are fixe	2	5	6	7	1	2	3	4	5
	1	1	2	3	4	5	6	7	8
Places vp.	1	7	6	5	4	3	2	1	1
Courses	1	3	5	6	1	3	5	6	1
Courses	2	7	6	1	3	5	6	1	3
	3	6	1	3	5	6	1	3	5
	4	5	1	6	3	5	6	1	3
	5	4	3	5	1	3	5	6	1
	6	3	1	6	5	1	3	5	6
	7	2	5	1	3	5	6	1	3
8 vt fu:1	1	3	5	6	1	3	5	6	1
Places down	1	2	3	4	5	6	7	8	9
The obser	1	2	3	4	5	6	7	8	9
uations of	2	5	6	7	1	2	3	4	5
the places	3	2	3	4	5	6	7	8	9
down are	4	7	1	2	3	4	5	6	7
fixe.	5	1	2	3	4	5	6	7	8
	6	5	6	7	8	9	10	11	12

Having found our column, we now look at the melodic motion of the cantus firmus to find the relevant rows of the table. Looking at each successive melodic interval in the cantus firmus, we find the corresponding row of the table and its intersection with our column. Bathe calls these melodic intervals "courses up" and "courses down" and labels them on the left side of the

table. The numbers we find in the selected cell give the melodic intervals above the current cantus firmus note that are guaranteed to be consonant both with the current cantus firmus note and the next one - i.e. both the leader and follower part will work with the cantus firmus if we choose one of these notes.

For example, in Canon #1, the first interval is a third up from C to E. I therefore look at the row for 3 courses up and see the numbers 1356, meaning I may choose a unison, third, fifth, or sixth above the first cantus firmus note as the first note of the melody. I thus enter "1/3/5/6" as a lyric under the first note of the cantus firmus.

1/3/5/6    3/5    1/3/5    3/5    1/6    3/5    3/5

Each cantus firmus note can be labeled in this way (other than the final note, which has no successor and is therefore free). Once this is done, we can start writing in a melody conforming to these intervals. However, there may still be problems with dissonance between the two parts or with parallel perfect intervals (fifths and octaves), which are forbidden in the counterpoint style Bathe is using. These problems can be avoided by using the "Observations" in the top and bottom parts of the table.

#### **Further Restrictions: Observations 1-6**

Observations 1 and 2 relate to the intervals found in the "diagonal" relationships between the leader melody and the cantus firmus. They indicate intervals that should not be used twice in a row in this diagonal location. Otherwise, parallel perfect fifths or octaves will occur. For example, since Canon #1 is a third above, observations 1 and 2 indicate of a third and a sixth. If we choose a B in the second measure (a fifth above the E in the cantus firmus) and an F in the third (a third above the D in the cantus firmus), we have created two "diagonal sixths" between the leader melody and our cantus firmus. When we then fill in the follower melody (one measure later, transposed up a third), we see that we have parallel octaves between the follower voice and the cantus firmus. If we want to abide by the rules of counterpoint, we should avoid this.

diagonal sixths    parallel octaves

1/3/5/6    3/5    1/3/5    3/5    1/6    3/5    3/5

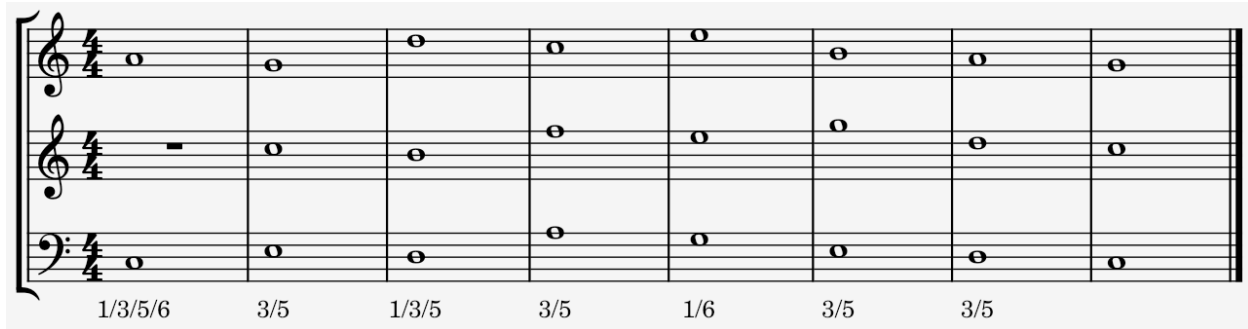
Observations 3 and 4 relate to melodic intervals within the leader melody. Those on the upper part of the table represent ascending intervals, and those on the lower represent descending intervals. These intervals should always be avoided within the melody or a dissonance will occur between the leader and follower. For our example Canon #1, which is a third above, our melody should avoid ascending seconds and fourths and descending fifths and sevenths. If we use an A in the first measure and a B in the second, we will have an ascending melodic second. This creates a dissonance in the second bar between the leader and follower voices.

A musical score for Canon #1 in 4/4 time, showing the first three measures. The leader melody is in the treble clef, and the follower melody is in the bass clef. The intervals between the two voices are indicated by red and blue lines and labels. The first measure has an interval of 1/3/5/6. The second measure has an interval of 3/5, with a red line labeled 'ascending second' and a blue line labeled 'dissonance'. The third measure has an interval of 1/3/5. The fourth measure has an interval of 3/5. The fifth measure has an interval of 1/6. The sixth measure has an interval of 3/5. The seventh measure has an interval of 3/5.

Observations 5 and 6 are less clearly defined. Rotem and Schab observe that if they are treated as relating to melodic intervals of the leader melody like Observations 3 and 4, they will cause the leader and follower to have an octave or a fifth between them. This would cause parallel fifths or octaves if these intervals are used in succession. For our example, these intervals are ascending thirds and sevenths and descending sixths and tenths. If we use a G in the first measure, a B in the second, and a D in the third, we have two ascending major thirds in a row, violating Observation 5. Thus, we have parallel unisons in measures 3 and 4 between the leader and follower melodies.

A musical score for Canon #1 in 4/4 time, showing the first three measures. The leader melody is in the treble clef, and the follower melody is in the bass clef. The intervals between the two voices are indicated by red and blue lines and labels. The first measure has an interval of 1/3/5/6. The second measure has an interval of 3/5, with a red line labeled 'ascending thirds' and a blue line labeled 'parallel unisons'. The third measure has an interval of 1/3/5. The fourth measure has an interval of 3/5. The fifth measure has an interval of 1/6. The sixth measure has an interval of 3/5. The seventh measure has an interval of 3/5.

Once we have filled in the notes as best we can, attempting to adhere to the intervals from the center of the table, the Observations from the edges of the table, and the general rules of counterpoint, we have a basis for our canon. The base structure for Canon #1 is shown below.



After this, we can ornament the canon. In particular, this is useful for places in the canon where the counterpoint is not perfect. By adding rests, passing notes, suspensions, and other ornaments, we can decorate the canon and draw attention away from problematic areas. For instance, in measure 5 of Canon #1, the two voices converge to a unison, which is not ideal for maintaining their independence. To deal with this, I added a quarter rest in this measure as well as a tied note from the preceding measure to reduce the time the two voices spend on the same pitch. The complete ornamented canon is shown below.



### Lessons Learned and Future Work

In using the table, it became apparent that it is quite restrictive. Even before applying the Observation rules, there are often no good melodic solutions for a given cantus firmus and pitch interval. For example, I started working on a canon at the unison using the cantus firmus below. Using only the information from the center of the table, the first three notes of the melody have only a single option, producing parallel unisons in measures 2 and 3. This guided me to consider the table more thoroughly when composing a cantus firmus and choosing a pitch interval.

5 6 5 1/3/5/6 1/3/5/6 3/5 3/5

In order to produce stylistically correct music of this era, I need to learn more about counterpoint. I plan to read Thomas Morley's "A Plain and Easy Introduction to Practical Music", published in England in 1597, to better understand the "rules" of the music practice of this time and place.

The table might also be useful for slightly different kinds of compositions. I am interested in applying the idea to create rounds, which are essentially canons at the unison. Compositions like "Sumer is icumen in" demonstrate precedent for rounds over a ground bass, which is essentially a cantus firmus.

### Works Cited

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