PMF/CDF/PDF

Monday, March 12, 2018 10:11 AM

X is a discrete random variable with $S_X = \{x_1, \ldots\}$ where $x_1 < x_2 < \ldots$ Then the probability mass function (pmf) of X is a function p from S_X to $[0,1]$

$p(x_i) = P(X = x_i), \quad i = 1, 2, \ldots$

Suppose S_X is finite

Say with cardinality N

x	x_1	$x_2 \ldots$	x_n
$p(x)$	$p(x_1)$	$p(x_2) \ldots$	$p(x_n)$

$p(x_1) + \ldots + p(x_n) = 1 = P(X \in S_X)$

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pmf is easier to work with than cdf for discrete random variable.

<u>Example</u>

We flip 3 coins and count # of heads

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

From this pdf, find the cdf

$$F(x) = P(X \le x) = \begin{cases} 0, & x < 0 \\ 1/8, & 0 \le x < 1 \\ 4/8, & 1 \le x < 2 \\ 7/8, & 2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

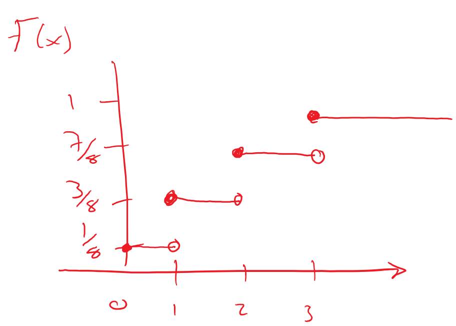
$\uparrow$ small x

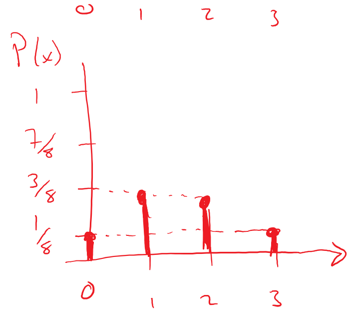
$1/8 + 3/8 = 4/8$

$1/8 + 3/8 + 3/8 = 7/8$

$1/8 + 3/8 + 3/8 + 1/8 = \frac{8}{8} = 1$

- cdf is defined on the whole real line
- pmf is only defined on its range (finite, $x_1, \dots, x_n$)





General procedure for moving from pdf to cdf.

General procedure for moving from pdf to cdf.

1.) Enumerate the set of jump points

$S_X = \{x_1, \ldots, x_n\}$

$x_1 < x_2 < \ldots < x_n$

$F(x_i) = \sum_{k=1}^{i} p(x_k)$ $, i = 1, \ldots N$ $\quad F(x_i) = p(x_i)$

Now going from cdf to pmf

↳ Find values of cdf at jump points.

$\{x_1, \ldots, x_N\}$

$$p(x_i) = \begin{cases} F(x_1) - 0, & i = 1 \\ F(x_i) - F(x_{i-1}), & i = 2 \end{cases}$$

$$\sum_{k=1}^{i} p(x_k) - \sum_{k=1}^{i-1} p(x_k)$$

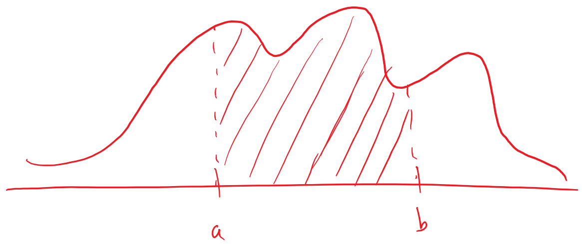
$$= p(x_i) + \sum_{k=1}^{i-1} p(x_k) - \sum_{k=1}^{i-1} p(x_k)$$

$$= \underline{\underline{p(x_i)}}$$

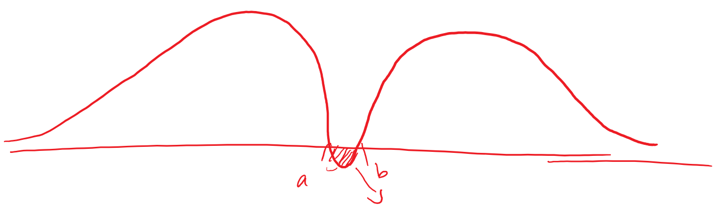
Suppose X is a continuous random variable.

Then the probability density function (pdf) of X is a non-negative function

f from $\mathbb{R}^1$ to $\mathbb{R}^1$ such that for any $a < b$, $P(a < X \le b) = \int_a^b f(x)\,dx$



$\underline{P(a < X \le b)}$

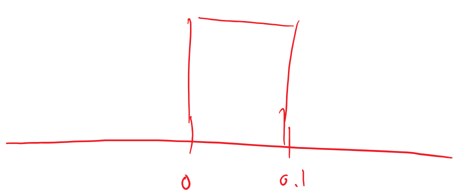


Negative # $\Rightarrow$ $\underline{P(a < X \le b) < 0}$

Can't happen

pdf sometimes takes a value larger than 1

↳



$P(0 < X \le 0.1) = 1$

$\int_0^{0.1} f(x)\,dx = 1$