1. $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.37 + 0.23 - 0.47 = 0.13$.

2. (a) $P(A_1) = \cdots = P(A_m) = 1/m$.

   (b) If $m = 8$, $P(A_1 \cup A_2 \cup A_3 \cup A_4) = P(A_1) + P(A_2) + P(A_3) + P(A_4) = 4 \times 1/m = 1/2$.

4. (a)

   (i) $E_1 = \{5, 6, 7, 8, 9, 10, 11, 12\}$. $P(E_1) = 4/36 + 5/36 + 6/36 + 5/36 + 4/36 + 3/36 + 2/36 + 1/36 = 5/6$.

   (ii) $E_2 = \{2, 3, 4, 5, 6, 7, 8\}$. $P(E_2) = 1/36 + 2/36 + 3/36 + 4/36 + 5/36 + 6/36 + 5/36 = 13/18$.

   (iii) $E_3 = E_1 \cup E_2 = \{2, \cdots, 12\}$, $P(E_3) = 1$. $E_4 = E_1 - E_2 = \{9, 10, 11, 12\}$, $P(E_4) = 4/36 + 3/36 + 2/36 + 1/36 = 5/18$. $E_5 = E_1^c \cap E_2^c = \emptyset$, $P(E_5) = 0$.

   (b) $P(E_3) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = 30/36 + 26/36 - (4/36 + 5/36 + 6/36 + 5/36) = 1$.

   (c) $P(E_5) = P(E_1^c \cap E_2^c) = P((E_1 \cup E_2)^c) = P(E_3^c) = 1 - P(E_3) = 1 - 1 = 0$.

5. (a)

   (i) $E_1 = \{(>3, V), (<3, V)\}$, $P(E_1) = 0.25 + 0.3 = 0.55$.

   (ii) $E_2 = \{(<3, V), (<3, D), (<3, F)\}$, $P(E_2) = 0.3 + 0.15 + 0.13 = 0.58$.

   (iii) $E_3 = \{(>3, D), (<3, D)\}$, $P(E_3) = 0.1 + 0.15 = 0.25$.

   (iv) $E_4 = \{(>3, V), (<3, V), (<3, D), (<3, F)\}$, $P(E_4) = 0.25 + 0.3 + 0.15 + 0.13 = 0.83$. $E_5 = \{(>3, V), (<3, V), (<3, D), (<3, F), (>3, D)\}$, $P(E_5) = 0.25 + 0.3 + 0.15 + 0.13 + 0.1 = 0.93$.

   (b) $P(E_4) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.55 + 0.58 - 0.3 = 0.83$.

   (c) $P(E_5) = P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) = 0.55 + 0.58 + 0.25 - 0.3 - 0 - 0.15 + 0 = 0.93$.

6. (a) The probability that, in any given hour, only machine A produces a batch with no defects is

$$P(E_1 \cap E_2^c) = P(E_1) - P(E_1 \cap E_2) = 0.95 - 0.88 = 0.07.$$

   (b) The probability, in that any given hour, only machine B produces a batch with no defects is

$$P(E_2 \cap E_1^c) = P(E_2) - P(E_1 \cap E_2) = 0.92 - 0.88 = 0.04.$$

(c) The probability that exactly one machine produces a batch with no defects is

$$P((E_1 \cap E_2^c) \cup (E_2 \cap E_1^c)) = P(E_1 \cap E_2^c) + P(E_2 \cap E_1^c) = 0.07 + 0.04 = 0.11.$$

(d) The probability that at least one machine produces a batch with no defects is

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.95 + 0.92 - 0.88 = 0.99.$$

7. The probability that at least one of the machines will produce a batch with no defectives is

$$\begin{aligned} P(E_1 \cup E_2 \cup E_3) &= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) \\ &\quad - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) \\ &= 0.95 + 0.92 + 0.9 - 0.88 - 0.87 - 0.85 + 0.82 = 0.99. \end{aligned}$$

8. (a)

(i) $P(E_1) = 0.10 + 0.04 + 0.02 + 0.08 + 0.30 + 0.06 = 0.6.$
(ii) $P(E_2) = 0.10 + 0.08 + 0.06 + 0.04 + 0.30 + 0.14 = 0.72.$
(iii) $P(E_1 \cap E_2) = 0.1 + 0.04 + 0.08 + 0.3 = 0.52.$

(b) The probability mass function for the experiment that records only the online monthly volume of sales category is given as

Online Sales	0	1	2
Probability	0.16	0.44	0.4

10. (a) If two dice are rolled, there are a total of 36 possibilities, among which 6 are tied. Hence, the probability of tie is $6/36 = 1/6$.

(b) By symmetry of the game $P(A \text{ wins}) = P(B \text{ wins})$ and $P(A \text{ wins}) + P(B \text{ wins}) + P(\text{tie}) = 1$. Using the result of (a), we can solve that $P(A \text{ wins}) = P(B \text{ wins}) = 5/12$.