18. (a) The R. V. X follows Poisson distribution with parameter $\lambda = 1.6 \times 3 = 4.8$.

(b) $E(X) = \text{Var}(X) = \lambda = 4.8$.

(c) The R command *ppois(9, 4.8)-ppois(4, 4.8)* gives us 0.4986.

(d) $Y = 5000X$, thus $E(Y) = 5000E(X) = 24000$ and $\text{Var}(Y) = 5000^2\text{Var}(X) = 1.2 \times 10^8$.

19. (a) $\text{Var}(X_1) = 2.6$ and $\text{Var}(X_2) = 3.8$.

(b) Let T_1 and T_2 be the event that the article is handled by typesetter 1 and 2, respectively. Then

$$P(\text{No error}|T_1) = e^{-\lambda_1}\frac{\lambda_1^0}{0!} = e^{-2.6} \text{ and } P(\text{No error}|T_2) = e^{-\lambda_2}\frac{\lambda_2^0}{0!} = e^{-3.8}.$$

Thus

$$\begin{aligned} P(\text{No error}) &= P(\text{No error}|T_1)P(T_1) + P(\text{No error}|T_2)P(T_2) \\ &= e^{-2.6} \times 0.6 + e^{-3.8} \times 0.4 = 0.0535. \end{aligned}$$

(c) By Bayes' Rule

$$P(T_2|\text{No error}) = \frac{P(\text{No error}|T_2)P(T_2)}{P(\text{No error})} = \frac{e^{-3.8} \times 0.4}{0.0535} = 0.1672.$$

22. (a) Each fish has a small probability of being caught. It makes sense to assume that each fish behaves independently, thus we have a large number of Bernoulli trials with a small probability of success. As a consequence of Proposition 3.4-1, the number of fish caught by an angler is modeled as a Poisson random variable.

(b) The probability of each disabled vehicle being abandoned on I95 is small and it makes sense to assume that each vehicle owner behaves independently. Thus we have a large number of Bernoulli trials with a small probability of success. As a consequence of Proposition 3.4-1, the number of disabled vehicle being abandoned on I95 in one year is modeled as a Poisson random variable.

(c) Each person has a small probability of dialing a wrong telephone number and it makes sense to assume that each person behaves independently. Thus, we have a large number of Bernoulli trials with a small probability of success. As a consequence of Proposition 3.4-1, the number of wrongly dialed number in a city in one hour is modeled as a Poisson random variable.

(d) Each person has a small probability of living 100 years and it makes sense to assume that each person behaves independently. Thus, we have a large number of Bernoulli trials with a small probability of success. As a consequence of Proposition 3.4-1, the number of people who reach age 100 in a city is modeled as a Poisson random variable.

23. (a) Both of the two events mean that there is one event happened in $[0, t]$ and there is no event happened in $(t, 1]$.

(b) From Proposition 3.4-2, $X(0.6)$ has a Poisson distribution with $\lambda_1 = 2 \times 0.6 = 1.2$ and $X(1) - X(0.6)$ has a Poisson distribution with $\lambda_2 = 2 \times 0.4 = 0.8$. Furthermore, $X(0.6)$ and $X(1) - X(0.6)$ are independent, thus

$$\begin{aligned} P([X(t) = 1] \cap [X(1) - X(t) = 0]) &= P([X(t) = 1])\, P([X(1) - X(t) = 0]) \\ &= e^{-1.2}\frac{1.2^1}{1!}e^{-0.8} = 0.1624. \end{aligned}$$

(c)

(i) Both $T \le t$ and $X(t) = 1$ say that the event happened before or at time t.

(ii) The proof uses Proposition 3.4-2 and the results in (i) and Part (a):

$$\begin{aligned} P(T \le t | X(1) = 1) &= \frac{P([T \le t] \cap [X(1) = 1])}{P(X(1) = 1)} = \frac{P([X(t) = 1] \cap [X(1) = 1])}{P(X(1) = 1)} \\ &= \frac{P([X(t) = 1] \cap [X(1) - X(t) = 0])}{P(X(1) = 1)} \\ &= \frac{P([X(t) = 1])P([X(1) - X(t) = 0])}{P(X(1) = 1)} \\ &= \frac{e^{-\lambda t}\frac{(\lambda t)^1}{1!}e^{-\lambda(1-t)}\frac{(\lambda(1-t))^0}{0!}}{e^{-\lambda \times 1}\frac{(\lambda \times 1)^1}{1!}} = t. \end{aligned}$$

1. Let T be the life time. From the problem, we know that T has a exponential distribution with parameter $\lambda = 1/6$.

(a) $P(T > 4) = 1 - P(T \le 4) = 1 - F(4) = 1 - (1 - e^{-\lambda 4}) = e^{-\lambda 4} = 0.513$.

(b) $Var(T) = 1/\lambda^2 = 36$. To find the 95th percentile, we solve the equation $F(t_{0.95}) = 0.95$, or, $1 - e^{-\lambda t_{0.95}} = 0.95$. $t_{0.95} = 17.9744$.

(c) Let T_r be the remaining life time. By the memoryless property of exponential distribution, T_r still has exponential distribution with $\lambda = 1/6$. Thus,

(i) $P(T_r > 5) = e^{-\lambda 5} = 0.4346$.

(ii) $E(T_r) = 1/\lambda = 6$.

2. Let T_1 be the time, in month, of the first arrival, then $T_1 \sim \text{Exp}(1)$.

   (a) We are looking for $P(7/30 \le T_1 \le 14/30)$, by the CDF of exponential distribution, we have

   $$P\left(\frac{7}{30} \le T_1 \le \frac{14}{30}\right) = \exp\left(-1 \times \frac{7}{30}\right) - \exp\left(-1 \times \frac{14}{30}\right) = 0.1648.$$

   (b) Let T_2 be the remaining waiting time. By the memoryless property of exponential distribution, $T_2 \sim \text{Exp}(1)$. Thus, $E(T_2) = 1$, and $Var(T_2) = 1$.

3. From (3.5.3), $P(X > s+t|X > s) = P(X > t)$, we must have $1 - P(X > s+t|X > s) = 1 - P(X > t)$, or, $P(X \le s+t|X > s) = P(X \le t) = F(t)$. Using the expression for $F(T)$ given in (3.5.1), we have $P(X \le s+t|X \ge s) = 1 - e^{-\lambda t}$.

5. (a) We could calculate this value by R command *qnorm(0.25, 43, 4.5)*, which gives 39.9648.

   (b) We could calculate this value by R command *qnorm(0.9, 43, 4.5)*, which gives 48.76698.

   (c) According to the requirement, $43 + c$ must be the 99.5 percentile, which can be calculated using the command *qnorm(0.995, 43, 4.5)*, resulting in 54.59123. Thus, $c = 11.59123$.

   (d) The probability of a randomly selected A36 steel having strength less than 43 is 0.5. Let Y be the number of A36 steels having strength less than 43 among 15 randomly selected steels, then we are calculating $P(Y \le 3)$. This can be calculated using the R command *sum(dbinom(0:3, 15, 0.5))* and the result is 0.01757812.

6. (a) We could calculate this value by R command *1-pnorm(8.64, 8, 0.5)*, which gives 0.100.

   (b) The probability is $0.1^3 = 0.001$.

7. (a) We could calculate this value by R command *pnorm(9.8, 9, 0.4)-pnorm(8.6, 9, 0.4)*, which gives 0.8185946.

   (b) Let Y be the number of acceptable resistors, then Y has the binomial distribution with $n = 4$ and probability of success as calculated in part (a). By the binomial probability formula, $P(Y = 2) = 0.1323$. We could also get this result by using R command.

8. (a) We could calculate this value by R command *1-pnorm(600, 500, 80)*, which gives 0.1056498.

   (b) We could calculate this value by R command *qnorm(0.99, 500, 80)*, which gives 686.1078.

9. (a) We could calculate this value by R command *qnorm(0.1492, 10, 0.03)*, which gives 9.968804.

   (b) We could calculate this value by R command *pnorm(10.06, 10, 0.03)*, which gives 0.9772499.

10. (a) We could calculate this value by R command *pnorm(7.9, 10, 2)*, which gives 0.1468591.

    (b) We could calculate this value by R command *qnorm(0.3, 10, 2)*, which gives 8.951199.