

8. (a) Prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$:

$$\begin{aligned}
 x \in (A - B) \cup (B - A) &\Leftrightarrow x \in A - B \text{ or } x \in B - A \\
 &\Leftrightarrow x \in A \text{ but } x \notin B \text{ or } x \in B \text{ but } x \notin A \\
 &\Leftrightarrow x \in A \text{ or } x \in B \text{ but not in both} \\
 &\Leftrightarrow x \in A \cup B \text{ and } x \notin A \cap B \\
 &\Leftrightarrow x \in (A \cup B) - (A \cap B).
 \end{aligned}$$

- (b) Prove that $(A \cap B)^c = A^c \cup B^c$:

$$\begin{aligned}
 x \in (A \cap B)^c &\Leftrightarrow x \notin A \cap B \\
 &\Leftrightarrow x \in A - B \text{ or } x \in B - A \text{ or } x \in (A \cup B)^c \\
 &\Leftrightarrow [x \in A - B \text{ or } x \in (A \cup B)^c] \text{ or } [x \in B - A \text{ or } x \in (A \cup B)^c] \\
 &\Leftrightarrow x \in B^c \text{ or } x \in A^c \Leftrightarrow x \in A^c \cup B^c.
 \end{aligned}$$

- (c) Prove that $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$:

$$\begin{aligned}
 x \in (A \cap B) \cup C &\Leftrightarrow x \in A \cap B \text{ or } x \in C \\
 &\Leftrightarrow x \in A \text{ and } x \in B \text{ or } x \in C \\
 &\Leftrightarrow [x \in A \text{ or } x \in C] \text{ and } [x \in B \text{ or } x \in C] \\
 &\Leftrightarrow x \in A \cup C \text{ and } x \in B \cup C \\
 &\Leftrightarrow x \in (A \cup C) \cap (B \cup C).
 \end{aligned}$$

9. (a) The sample space is $S = \{(x_1, x_2, x_3, x_4, x_5) | x_i = 5.3, 5.4, 5.5, 5.6, 5.7, i = 1, 2, 3, 4, 5\}$. The size of the sample space is $5^5 = 3125$.

1. $P(E_1) = 0.5, P(E_2) = 0.5, P(E_1 \cap E_2) = 0.3, P(E_1 \cup E_2) = 0.7, P(E_1 - E_2) = 0.2,$
 $P((E_1 - E_2) \cup (E_2 - E_1)) = 0.4.$

3. $E_1 = \{6.8, 6.9, 7.0, 7.1\}$ and $E_2 = \{6.9, 7.0, 7.1, 7.2\}$. Thus, $P(E_1) = P(E_2) = 4/5$.
 $E_1 \cap E_2 = \{6.9, 7.0, 7.1\}$ and $P(E_1 \cap E_2) = 3/5$. $E_1 \cup E_2 = S$ and $P(E_1 \cup E_2) = 1$.
 $E_1 - E_2 = \{6.8\}$ and $P(E_1 - E_2) = 1/5$. Finally, $(E_1 - E_2) \cup (E_2 - E_1) = \{6.8, 7.2\}$,
so $P(E_1 - E_2) \cup (E_2 - E_1) = 2/5$.

6. (a) The number of ways to finish the test is $2^5 = 32$.
- (b) The sample space for the experiment that records the test score is $S = \{0, 1, 2, 3, 4, 5\}$.
- (c) The PMF of X is given by

X	0	1	2	3	4	5
$p(x)$	0.03125	0.15625	0.31250	0.31250	0.15625	0.03125

7. The number of assignments is

$$\binom{4}{1, 1, 1, 1} = 24.$$