Stat 401 note 2/5/2018

multiplication rule

$$P(A\cap B\cap C) = P(A)\,P(B|A)\,P(C|A\cap B)$$

$$\cancel{P(A)}\ \frac{P(\cancel{B\cap A})}{\cancel{P(A)}}\ \frac{P(C\cap A\cap B)}{\cancel{P(A\cap B)}}$$

generally

$$P(A_1)\,P(A_2|A_1)\,P(A_3|A_1\cap A_2)$$
$$P(A_4|A_1\cap A_2\cap A_3)\ldots P(A_n|A_1\cap A_2\cap A_3\cap\ldots\cap A_{n-1})$$
$$= P(A_1\cap A_2\cap\ldots\cap A_n)$$

e.g. Pick 3 cards from deck of 52, find prob. of $\underbrace{\text{first is ace}}_{A}$, $\underbrace{\text{second is king}}_{B}$ and $\underbrace{\text{third is queen}}_{C}$

$P(A)=\frac{1}{13}$ $P(B|A)=\frac{4}{51}$ $P(C|A\cap B)=\frac{4}{50}=\frac{2}{25}$

$\therefore\ P(A\cap B\cap C) = \frac{1}{13}\cdot\frac{4}{51}\cdot\frac{2}{25}\approx 0.000483$

Law of total probability

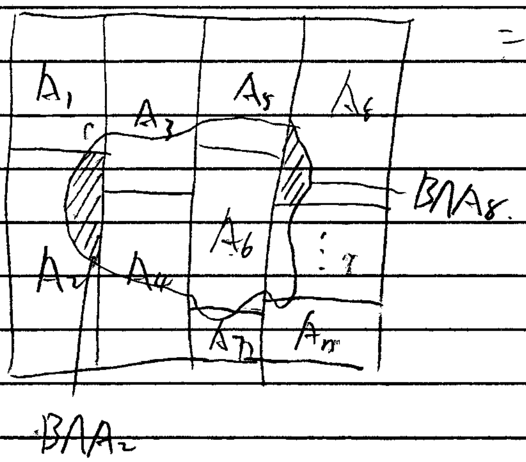
if events $A_1,\ldots A_n$ form a partition of S,

that is $A_1\cup\ldots\cup A_n = S$ and $A_i\cap A_j=\phi$ (events are mutually disjoint from each other) $\forall$ i and j, $1\le i\le n$, $1\le j\le n$

Then, for any event B,

$$P(B) = \sum_{i=1}^{n} P(B\cap A_i)$$

$$= \sum_{i=1}^{n} P(A_i)\,P(B|A_i)$$



$S = A\cup A^c$ where $A\cap A^c \neq \phi$

$P(B) = P(A)P(B|A) + P(A^c)P(B|A^c)$

e.x Two friends dealt card each from 52 cards, random and without replacement. if neither gets an ace, the two cards are put back, reshuffled, and the two cards get dealt again. The game ends when at least one person gets ace. Is the game fair?

$A = \{\text{first draw is ace}\}$ $\quad B = \{\text{second draw is ace}\}$

$P(A) = \frac{1}{13}$ $\quad P(B) = P(A)P(B|A) + P(A^c)P(B|A^c)$

$$= \frac{1}{13} \cdot \frac{3}{51} + \frac{12}{13} \cdot \frac{4}{51}$$

$$= \frac{3+48}{13 \times 51} = \frac{1}{13}$$

$\therefore P(A) = P(B)$, the game is fair.

Bayes Theorem

given $P(B|A)$ what can we say of $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(B|A)P(A) + P(B|A^c)P(A^c)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(B|A^c)P(A^c)}$$

$$= \frac{P(A \cap B)}{P(A \cap B) + P(A^c \cap B)}$$

For $\{A_1 \ldots A_n\}$ for S, $\quad P(A_j | B)$

$$= \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j)P(A_j|B)}{\sum_{i=1}^{n} P(A_i)P(A_i|B)}$$