LECTURE 28

04-11-18

$$E[Y|X=x] = \begin{cases} \sum_{y \in S_Y} y\, P_{Y|X=x}(y) & (X,Y) \text{ DISCRETE} \\ \int_{-\infty}^{\infty} y\, f_{Y|X=x}(y) & (X,Y) \text{ CONTINUOUS} \end{cases}$$

$$VAR(Y|X=x) = E[Y^2/X=x) - (E[Y/X=x])^2$$

$E[X|Y=y]$, $VAR[X|Y=y]$

		0	1	2	3
	0	0.84	0.03	0.02	0.01
X	1	0.06	0.01	0.008	0.002
	2	0.01	0.005	0.004	0.001

$V[Y|X=0]$ $\quad E[Y|X=0] = 0\left(\frac{0.84}{0.9}\right) + 1\left(\frac{0.03}{0.9}\right) + 2\left(\frac{0.02}{0.9}\right) + 3\left(\frac{0.01}{0.9}\right)$

$$= \frac{3}{90} + \frac{4}{90} + \frac{3}{90} = \frac{1}{9}$$

$$E[Y^2/X=0] = 1\left(\frac{0.003}{0.9}\right) + 4\left(\frac{0.02}{0.9}\right) + 9\left(\frac{0.01}{0.9}\right)$$

$$= \frac{3+8+9}{90}$$

$$VAR[Y|X=0] = \frac{2}{9} - \frac{1}{81} = \frac{18}{81} - \frac{1}{81} = \frac{17}{81}$$

$$f(x,y) = \begin{cases} \frac{e^{-x/y}\, e^{-y}}{y} & (x>0, y>0) \\ 0 & \text{OTHERWISE} \end{cases}$$

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$E[Y|X=3]$ $V[Y|X=3]$

$$f_{X|Y}(y) = \frac{f(x,y)}{f_X(x)}$$

$$f_Y(y) = \int_0^\infty \frac{e^{-x/y}\, e^{-y}}{y}\, dx = \frac{e^{-y}}{y}\left[-y e^{-x/y}\right]_0^\infty$$

$$= \frac{e^{-y}}{y}[y] = e^{-y} \qquad f_{X|Y}(y) = \frac{e^{-x/y}\, \cancel{e^{-y}}}{y\, \cancel{e^{-y}}} = \frac{e^{-x/y}}{y}$$

$$f_{X|Y=3}(y) = \frac{e^{-x/3}}{3}$$

$$f_{X|Y=3}(y) = \frac{e^{-x/3}}{3}$$ EXPONENTIAL $\lambda = \frac{1}{3}$

$E_c(X|Y=3) = 3$

$V(X|Y=3) = 9$ CALCULATE & UNDERSTAND THIS.

THE REGRESSION FUNCTION

(SYNONYM FOR CONDITIONAL EXPECTATION)

$E[Y/X=x))$ REGRESSION FUNCTION OF Y ON X DENOTED BY $Y|X$

SO YOU CAN USE THE SAME FORMULA TO CALL. 04-11-18
THEM

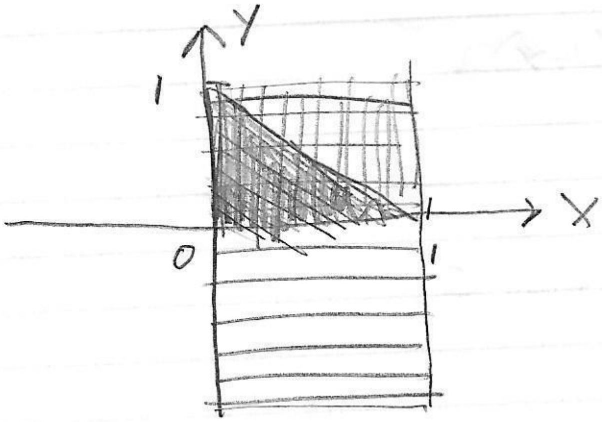
BECAUSE THE JOINT DISTRIBUTION OF X, Y IS UNKNOWN
CLASSICAL LINEAR REGRESSION MODEL EXPLORES
ONLY THE RELATIONSHIP OF THE FORM

$$\mu_{Y|X}(x) = \alpha_1 + \beta_1 X$$

α_1 & β_1 ARE PARAMETERS TO BE ESTIMATES FROM
THE DATA.

(X, Y) HAS JOINT PDF

$$f(x,y) = \begin{cases} 24xy & 0 \le x \\ & 0 \le y \\ & x + y \le \\ 0 & \leftarrow \text{OTHERWI} \end{cases}$$



$$f_{Y|X}(y) = \frac{f(x,y)}{f_Y(y)}$$

$$f_Y(y) = \int_0^{1-y} 24xy\,dx$$

$$= 24y \frac{x^2}{2} \Big|_0^{1-y}$$

$$= 12\,y\,(1-y)^2$$

$$= 12y(1-y)^2 \quad 0 \le y \le 1$$

$$E[Y|X] =$$

$$f_{X|Y}(y) = \frac{f(x,y)}{f_Y(y)}$$

$$= \frac{24xy}{12y(1-y)^2} = \frac{2x}{(1-y)^2} \quad \cancel{0 \le x \le 1}$$

THE RANGE OF X DEPENDS ON Y.

IN FACT $0 \le x \le 1-y$ 04-11-18

$E[Y|X] = \int_0^{1-y}$

BY SYMMETRY $f_{Y}(x) = \int_0^{1-x} 24xy\,dy$

$= 12x(1-x)^2 \quad 0 \le y \le 1$

$f_{Y|X}(y) \; \frac{24xy}{12x(1-x)^2} = \frac{2y}{(1-x)^2} \quad 0 \le y \le 1-x$

$E[Y|X] = \int_0^{1-x} \frac{y\,2y}{(1-x)^2}\,dy \quad \frac{2y^3}{3(1-x)^2}\Big|_0^{1-x}$

$= \frac{2(1-x)^3}{3(1-x)^2} = \frac{2}{3}(1-x)$

$\mu_{Y|X} = \underbrace{\frac{2}{3}}_{\text{INTERCEPT}} \underbrace{- \frac{2}{3}}_{\text{SLOPE}} X$

FOR A LARGER X, EXPECT A SMALLER Y

INDEPENDENCE OF RANDOM VARIABLES

(CH 2 INDEPENDENCE OF EVENTS)

A, B EVENTS $P(A \cap B) = P(A)\,P(B|A)$

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TWO RANDOM VARIABLES ARE INDEPENDENT IF FOR ANY $x \in \mathbb{R}^1$ & $y \in \mathbb{R}^1$

$$P(X \leq x, Y \leq y) = P(X \leq x) P(Y \leq y)$$

IN TERMS OF DISTRIBUTION FUNCTIONS,

$$F_{X,Y}(x,y) = F_X(x) F_Y(y) \quad \forall \; x,y \in \mathbb{R}^1$$

CHARACTERIZED INDEPENDENCE WITH PMF/PDF.

(X,Y) DISCRETE

$$X \perp\!\!\!\perp Y \iff p(x,y) = p_X(x) p_Y(y) \quad \forall \; (x,y) \in S_{XY}$$