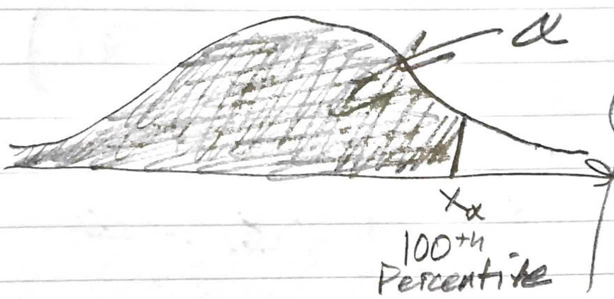
The issue can be solved easily

$$\max x \quad s.t. \quad F(x) = 0$$

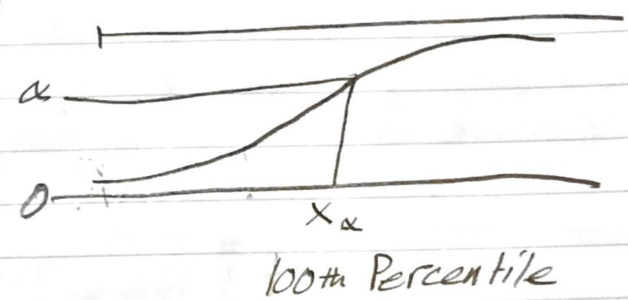
or

$$\min x \quad s.t. \quad F(x) = 1$$

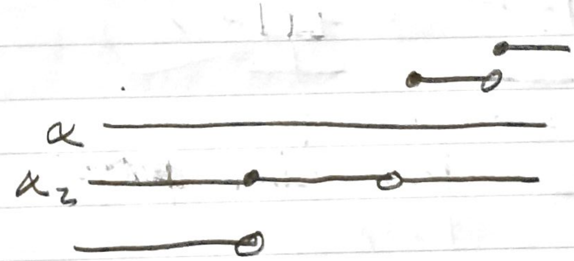
PDF Percentiles



CDF Percentiles

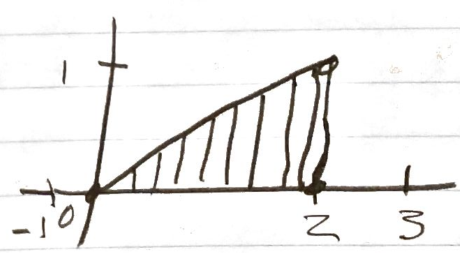


VS Discrete CDF



Ex) X Continuous r.v.

$$pdf = \begin{cases} x/2, & x \in (0,2) \\ 0, & other \end{cases}$$



$$\boxed{A = \frac{1}{2}(2 \cdot 1) = 1}$$

*Find Quantile

$$\int_0^x \left(\frac{x}{2}\right) dt = \frac{x^2}{4}$$

$\Rightarrow Q_1$ 25% $\Rightarrow x = 1$

Q_2 50% $\Rightarrow x = \sqrt{2}$

Q_3 75% $\Rightarrow x = \sqrt{3}$

Interquantile Range (IQR)

- Distance between 25th and 75th percentiles

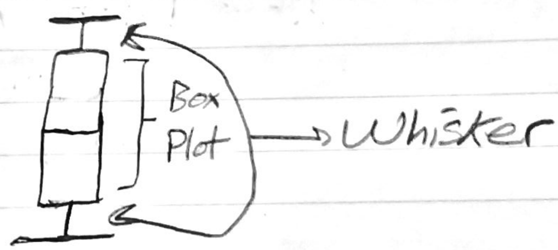
$$IQR = Q_3 - Q_1$$

(Measure of dispersion)

Population of IQR (True IQR) Uses WHOLE Sample

Empirical IQR (From Sample)

Upper Extreme
Upper Quartile Q_3
Median Q_2
Lower Quantile Q_1
Lower Extreme



Note: Parameter
PMF
Expected Value
Variance
Interpretation

Bernoulli Trials (Bernoulli r.v.)

- Bernoulli Trial/Experiment is one whose outcome can be classified as either <u>success</u> or <u>failure</u>

① Succeed with probability

$$p \in (0,1)$$

→ X (Bernoulli r.v.) takes 1 if Success

→ X (Bern r.v) takes 0 if Failure

$X \sim$ Bernoulli(p) — Defining Parameter p

$S_X = \{0,1\}$

X	0	1
$p(x)$	$1-p$	p

⇒ $p(x) = P(X=x)$

$= p^x(1-p)^{1-x}$

$x = 0, 1$

$= (1-p)$ $\qquad$ $= p$

$$E(x) = 0 \cdot (1-p) + (1 \cdot p) = p$$

$$E(x^2) = 0^2 \cdot (1-p) + (1^2 \cdot p) = p$$

$$V(x) = p - p^2 = p(1-p)$$

Binomial r.V.

- Do $\boxed{n}$ Bernoulli Trials

$$X_1, X_2, X_3, \ldots X_n$$

$$X_1 = \begin{cases} 1, & \text{If Success} \\ 0, & \text{If Failure} \end{cases}$$

$$Y = \sum_{i=1}^{n} X_i \qquad Y \sim Bin(n, p)$$

Y is counting total number of success in $\underline{n}$ Bernoulli Trials

Repeat ⇒ Identical
Independent ⇒ Unique

⇒ Independent + Identical Distributed (i,i,d)

X_i is (i,i,d)

Defining Parameter: n, p

$$S_Y = \{0, 1, \ldots n\}$$

PMF of Y : $P(Y=y) = \binom{n}{y} p^y (1-p)^{n-y}$

$$y = 0, 1, 2, \ldots, n$$

For Example

$\binom{n}{y}$ Ways to place y success in n trials

$$E[Y] = \sum_{y=0}^{n} y \left[\binom{n}{y}\right] p^y (1-p)^{n-y}$$

$$= \sum_{y=1}^{n} y \frac{n!}{y!(n-y)!} p^y (1-p)^{n-y}$$

$$= \sum_{y=1}^{n} \frac{n!}{(y-1)!(n-y)!} p^y (1-p)^{n-y}$$

$$= n \sum_{y=1}^{n} \frac{(n-1)!}{(y-1)!(n-y)!} p^y (1-p)^{n-y}$$

$$= np \sum_{y=1}^{n} \binom{n-1}{y-1} p^{y-1} (1-p)^{n-y}$$

$\Rightarrow$ $y-1 = s$, $y = s+1$ $\Rightarrow$ $s = 0$ (since $y=1$ in Σ)

$$= np \underbrace{\sum_{s=0}^{n-1} \binom{n-1}{s} p^s (1-p)^{n-1-s}}$$

$$= np\left(\quad = 1 \quad \right)$$

$$\Rightarrow \binom{n-1}{s} p^s (1-p)^{n-1-s}$$

So PMF of Bin $\boxed{(n-1, p)}$ ✓

$$E[Y(Y-1)] = \sum_{y=0}^{n} y(y-1) \frac{n!}{y!(n-y)!} p^y (1-p)^{n-y}$$

$$= \sum_{y=2}^{n} \frac{n!}{(y-2)!(n-y)!} p^y (1-p)^{n-y}$$

$$= n(n-1)p^2 \cdot \sum_{y=2}^{n} \frac{(n-2)!}{(y-2)!(n-y)!} \cdot p^{y-2}(1-p)^{n-y}$$

$$= n(n-1)p^2 \cdot \sum_{y=2}^{n} \binom{n-2}{y-2} p^{y-2} (1-p)^{n-y}$$

$$y-2 = t \qquad t = y+2$$

$$\downarrow$$

$$E[Y^2] = E[Y(Y-1)]$$