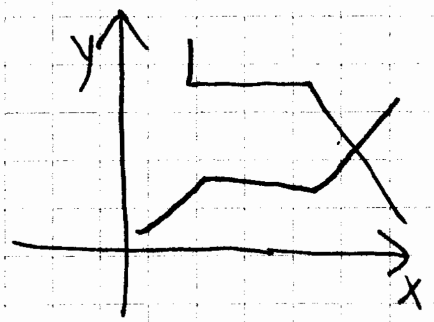
2-21



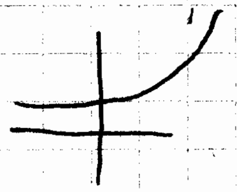
$f(x) = x, \; x \in \mathbb{R}^1$

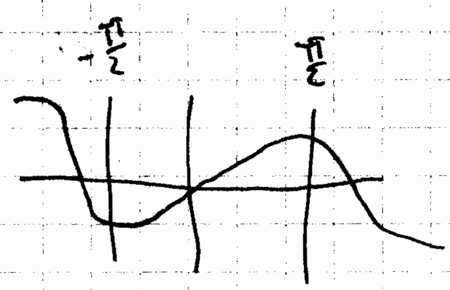
$f(x) = x^2, \; x \geq 0$ monotone increasing

$x \leq 0$ monotone decreasing

There are functions for which no matter how small you take the region, it is not monotone in it

$f(x) = e^x, \; x \in \mathbb{R}^1$





$f(x) = \sin x, \; x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f(x) = \mathbb{I}_{[0,+\infty)}(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$

Indicator function

$(f+g)'(x) = f'(x) + g'(x)$

$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - g(x)f(x)}{g^2(x)}$

$(x^a)' = a x^{a-1}, \; a \in \mathbb{R}^1$

$(e^x)' = e^x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$(\log x)' = \frac{1}{x}$

$(\log_e x)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

Chain Rule

$h(t) = g(f(t)), \; a \leq t \leq b$

$h'(t) = g'(f(x)) f'(x), \; x \in [a,b]$

Suppose $a > 0$

$h(x) = a^x = \exp\{x \log a\}$

$g(s) = \exp(s)$

$s = x \log a$

$g'(s)\frac{ds}{dx} = \exp(s) \log a = \exp(x \log a) \log a$
$= a^x \log a$

Suppose $a > 0$

$h(x) = \log_a x = \frac{\log x}{\log a}$

$((cf)'(x) = c(f(x))$

$= \frac{1}{x \log a}$

Anti-derivatives

$\int x^a \, dx = \frac{1}{a+1} x^{a+1} \quad a \neq -1$

$\int x^{-1} \, dx = \int \frac{1}{x} dx = \log x$

$\int e^x \, dx = e^x$

$\int \sin x \, dx = -\cos x$

$\int \cos x \, dx = \sin x$

f & g integratable on $[a,b]$

α & β real constant

$\int_a^b [\alpha f(x) + \beta g(x)] \, dx$

$= \alpha \int_a^b f(x) \, dx + \beta \int_a^b g(x) \, dx$

f is integrable on $[a,b]$

take c s.t. $a < c < b$

$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$

The Fundamental Theorem of Calculus

if f is integrable on $[a,b]$ and has its anti-derivative F, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Integration by parts

Suppose f + g are differentiable functions on $[a,b]$

$$\int_a^b f(x)\, g'(x)\,dx = f(b)g(b) - f(a)g(a) - \int_a^b g(x)\, f'(x)\,dx$$

$$\int_a^b (fg)'\,dx = \int_a^b (f'g + g'f)\,dx$$

$$\Big[f(x)\, g(x)\Big]_a^b = \int_a^b f'(x)\, g(x)\,dx + \int_a^b f(x)\, g'(x)\,dx$$

Newtons Binomial Theorem

$n \in \mathbb{Z}$, $a \in \mathbb{R}$, $b \in \mathbb{R}$

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

For any $n \in \mathbb{N}$

if $x \neq 1$

$$\sum_{i=0}^{n} x = 1 + x + x^2 + \dots + x^n = \frac{1-x^{n+1}}{1-x}$$