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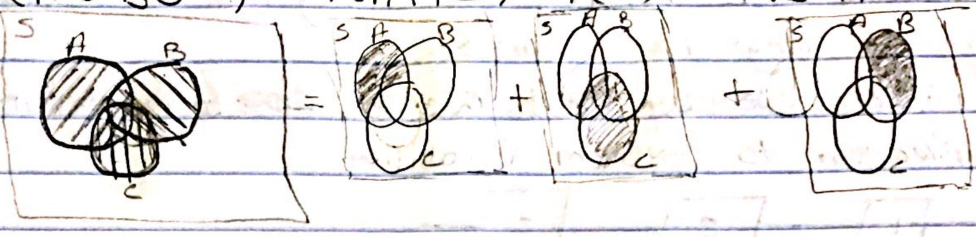
STAT 401 lecture #6 02/14/18

① $P(A \cup B \cup C) = P(A \cap C^c) + P(C) + P(B \cap A^c \cap C^c)$



② Woman has two kids, given that one of them is a girl, what is the prob. of the other child also being a girl?

$S = \{BB, GG, GB, BG\}$

$P(\text{at least one girl}) = 3/4$

$P(\text{Both girls}) = 1/4$

$P(A|B) = 1/4 / 3/4 = \frac{1}{4} \cdot \frac{4}{3} = 1/3$

③ Throw two fair die

$A = \{\text{sum is } 7\}; A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$B = \{\text{first die is } 3\}; B = \{(3,x) : 1 \le x \le 6\}$

$C = \{\text{Second die is } 4\}\ C = \{(x,4) : x \le x \le 6\}$

$P(A) = P(B) = P(C) = 1/6$

$P(A \cap B) = P(A \cap C) = P(B \cap C) = 1/36$

$P(A \cap B \cap C) = 1/36 \neq P(A)P(B)P(C)$?

ex. roll one die

$E_1 = \{1, 2, 3\}$

$E_2 = \{3, 4, 5\}$

$E_3 = \{1, 2, 3, 4\}$

$P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3)$

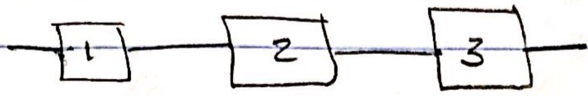
$P(E_1 \cap E_2) \neq P(E_1)P(E_2)$

$E_1 \cap E_2 \cap E_3 = \{3\}$ $P(E_1 \cap E_2 \cap E_3) = 1/6$ $(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{6})$

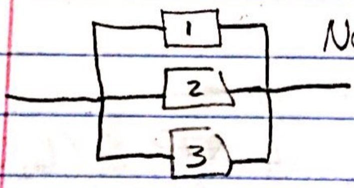
$E_1 \cap E_2 = \{3\}$ $P(E_1 \cap E_2) = 1/6 \neq \frac{1}{2} \cdot \frac{1}{2}$ $(P(E_1) \cdot P(E_2) = \frac{1}{2} \cdot \frac{1}{2})$

↳ <u>NOT Independent</u>

- Independence is important b/c using this assumption makes things much simpler.
  - many everyday events are very close to independent
- application to system reliability



  - failure Probabilities: $P_1 = .1$, $P_2 = .15$, $P_3 = .2$
    assume failure probabilities are independent.
  - What is probability that the system will fail (only 1 needs to fail)
  - What is probability that system will survive? (look at compliments)
    $(1-P_1)\cdot(1-P_2)\cdot(1-P_3)$; This is prob. of sys. surviving
  - let A denote sys. fail : $P(A^c) = P(A_1^c \cap A_2^c \cap A_3^c)$
    $= P(A_1^c)P(A_2^c)P(A_3^c) = (1-P_1)\cdot(1-P_2)\cdot(1-P_3) = \boxed{0.612}$
- Next sys. reliability example:



Now all 3 must fail in order for sys. to fail

using same notation as above: $P(A) = P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$
$= (0.1)\cdot(0.15)\cdot(0.2) = 0.003$
- Chapter Conclusion:
  - Sample Space
  - events (subsets of sample space)
  - axioms of probability
  - Classical prob. model (each outcome has same prob.)
  - Basic set operations: $\cap$, $\cup$, $-$, compliments
  - Basic counting techniques: $n!$, $P_{nk} = \frac{n!}{(n-k)!}$, $C_{nk} = \frac{n!}{(n-k)!\,k!}$
  - law of total probability: Partition sample space
  - Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$