Friday 4/6

$f_1 = x+y$ on $0 < x, y < 1$

$f_2 = (x+\frac{1}{2})(y+\frac{1}{2})$ on $0 < x, y < 1$

$$\int_0 \int_0 (x+y)\,dx\,dy = 1$$

$$\int_0^1 \int_0^1 (x+\tfrac{1}{2})(y+\tfrac{1}{2})\,dx\,dy$$

marginals

$$f_{1,x}(x) = \int_0^1 x+y\,dy$$
$$= \left[xy + \frac{y^2}{2}\right]_0^1$$
$$= x + \frac{1}{2}$$

$f_{1,y}(y) = y + \frac{1}{2}$ (symmetric distribution)

$$f_{2x}(x) = \int_0^1 xy + \frac{x}{2} + \frac{y}{2} + \frac{1}{4}\,dy$$
$$= \left[\frac{xy^2}{2} + \frac{x}{2}y + \frac{y^2}{4} + \frac{1}{4}y\right]_0^1$$
$$= x + 1/2$$

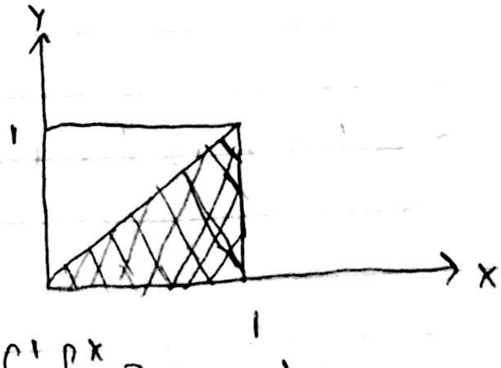
$f_{2y}(y) = y + 1/2$ (also symmetric)

Example:
$$f(x,y) = \begin{cases} \frac{12}{7}(x^2+xy) & 0 \le x,y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{12}{7}\int_0^1\int_0^1 (x^2+xy)\,dx\,dy$$
$$= \frac{12}{7}\int_0^1 \left[\frac{x^3}{3} + \frac{x^2y}{2}\right]_0^1 dy$$
$$= \frac{12}{7}\left[\frac{1}{3}y + \frac{y^2}{4}\right]_0^1 = \boxed{\frac{12}{7}\left(\frac{1}{3}+\frac{1}{4}\right)}$$

$$\frac{12}{7}\left(\frac{4}{12}+\frac{3}{12}\right) = 1 \checkmark$$

Find Probability of $x > y$



$$\int_0^1 \int_0^x \frac{12}{7}(x^2+xy)\,dy\,dx$$

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or $\frac{12}{7}\int_0^1\int_y^1 (x^2+xy)\,dx\,dy$

$\frac{12}{7}\int_0^1 \left[\frac{x^3}{3}+\frac{x^2y}{2}\right]_y^1$

$\frac{12}{7}\int_0^1\left[\left(\frac{1}{3}+\frac{y}{2}\right)-\left(\frac{y^3}{3}+\frac{y^3}{2}\right)\right]dy$

$\frac{12}{7}\left[\left(\frac{1}{3}y+\frac{y^2}{4}\right)-\left(\frac{y^4}{12}+\frac{y^4}{8}\right)\right]_0^1$

$\frac{12}{7}\left[\frac{1}{3}+\frac{1}{4}-\frac{1}{12}-\frac{1}{8}\right]$

$\frac{12}{7}\left[\frac{8}{24}+\frac{6}{24}-\frac{2}{24}-\frac{3}{24}\right] = \frac{\cancel{12}}{7}\,\frac{9}{\cancel{24}_{\,2}} = \frac{9}{14}$

$\frac{12}{7}\int_0^1\int_0^x (x^2+xy)\,dy\,dx$

$\frac{12}{7}\int_0^1\left[\underbrace{x^2y}_{x^3}+\underbrace{\frac{xy^2}{2}}_{\frac{x^3}{2}}\right]_0^x$

$\frac{12}{7}\left[\frac{x^4}{4}+\frac{x^4}{8}\right]_0^1 = \frac{12}{7}\left(\frac{3}{8}\right) = \frac{9}{14}$

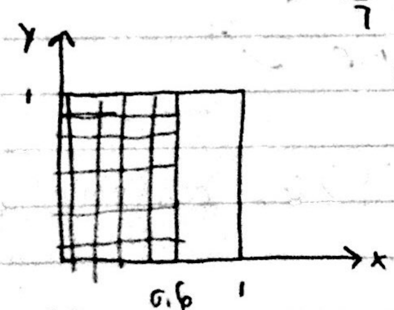
they are the same so it checks out

* obviously there is an easier way to do it be sure to draw the distribution and determine the integrals. You should be able to figure out which one is easier

$p(x \le 0.6)$

① $\frac{12}{7}\int_0^1\int_0^{0.6} (x^2+xy)\,dx\,dy$

② $\frac{12}{7}\int_0^{0.6}\int_0^1 (x^2+xy)\,dy\,dx$



#2 is better because 0.6 is annoying to integrate with, even though you can treat 0.6 like a variable until the end.

① $\frac{12}{7}\int_0^1\int_0^{0.6} (x^2+xy)\,dx\,dy$

$\frac{12}{7}\int_0^1 \frac{0.6^3}{3}+\frac{0.6^2y}{2}\,dy$

$\frac{12}{7}\left[\frac{0.6^3}{3}+\frac{0.6^2}{4}\right]$

even before simplifying they are the same

$\frac{1}{7}\left[4\cdot 0.6^3+3\cdot 0.6^2\right]$

≈ 0.2777

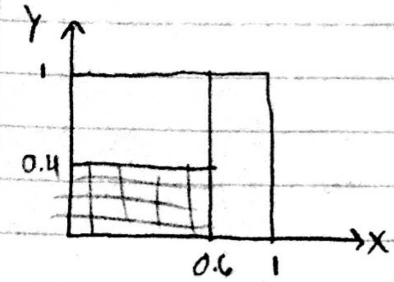
② $\frac{12}{7}\int_0^{0.6}\int_0^1 (x^2+xy)\,dy\,dx$

$\frac{12}{7}\int_0^{0.6} x^2+\frac{x}{2}\,dx$

$\frac{12}{7}\left[\frac{0.6^3}{3}+\frac{0.6^2}{4}\right]$

"Harder way"

$x \leq 0.6$ and $y \leq 0.4$



$$\frac{12}{7}\int_0^{0.4}\int_0^{0.6}(x^2+xy)\,dx\,dy$$

$$\frac{12}{7}\int_0^{0.4}\left[\frac{0.6^3}{3}+\frac{0.6^2 y}{2}\right]dy$$

$$\frac{12}{7}\left[\frac{0.6^3\cdot 0.4}{3}+\frac{0.6^2\cdot 0.4^2}{4}\right] = 0.0741$$

* Here with two conditions look at the original function $\frac{12}{7}(x^2+xy)$, pick the variable with less terms to integrate first to make it easier.

* Note that area was <10% of the distribution, this is because the joint pdf $\frac{12}{7}(x^2+xy)$ grows linearly with y and quadractically with X.

Marginals

$$f_X(x) = \frac{12}{7}\int_0^1 (x^2+xy)\,dy = \frac{12}{7}\left[x^2 y + \frac{xy^2}{2}\right]_0^1 = \frac{12}{7}\left(x^2+\frac{x}{2}\right) \quad 0 \leq x \leq 1$$

Quadratic

$$f_Y(y) = \frac{12}{7}\int_0^1 (x^2+xy)\,dx = \frac{12}{7}\left[\frac{x^3}{3}+\frac{x^2 y}{2}\right]_0^1 = \frac{12}{7}\left(\frac{1}{3}+\frac{y}{2}\right) = \frac{1}{7}(4+6y) \quad 0 \leq y \leq 1$$

linear

There are two events $A + B$ with $P(A) > 0$, conditional probability of B given A is $P(B|A) = \frac{P(A \cap B)}{P(A)}$

- events A, B can be induced by two variables X and Y or the random vector (X, Y)
- we want to study the distribution of X given the value of Y.
- The conditional distribution of X given Y (Y given X) we need to treat discrete case as continuous