LECTURE 35

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		NULL HYPOTHESIS TRUE	NULL HYPOTHESIS FALSE
JUDGEMENT OF THE TEST	FAIL TO REJECT	$1-\alpha$ CORRECT INFERENCE TRUE NEGATIVE	TYPE II ERROR FALSE NEGATIVE β
	REJECT	TYPE I ERROR FALSE POSITIVE α	CORRECT INFERENCE TRUE POSITIVE $1-\beta$

TEST HAS SIGNIFICANCE LEVEL α IF $P(\text{TYPE I ERROR}) = P_{H_0}(\text{REJECTING } H_0) \leq \alpha$

LEVEL-α TEST

GIVEN THAT IT IS LEVEL-α, WE WANT THE POWER $1-\beta$ TO BE AS LARGE AS POSSIBLE

HYPOTHESIS THAT INVESTIGATOR WANTS TO CLAIM TRUE SHOULD BE TAKEN AS THE ALTERNATIVE.

IF DATA INDICATES REJECTING H_0, THEN THE MOST POSSIBLE REASON IS THAT H_0 IS UNTRUE

$H_0^c = H_1 \Rightarrow$ YOU CHOOSE H_1

BY CONTRAST, WE DO NOT HAVE SAME LEVEL OF CONFIDENCE BY CLAIMING H_0 IS TRUE WHEN WE FAIL TO REJECT H_0.

WE DIDN'T REALLY IMPOSE ANYTHING ON:

$P_{H_0}(\text{FAIL TO REJECT } H_0)$

$H_a: \mu < 28,000$
$H_0: \mu \geq 28,000$] – FIRM SUSPECTING MEAN TIRE LIFE 04-27-18

$H_a: \mu > 28,000$
$H_0: \mu \leq 28,000$] – MANUFACTURER TESTING TIRE LIFE

$H_0: \theta = \theta_0$

$H_a: \theta < \theta_0 \Rightarrow H_0: \theta \geq \theta_0$
$H_a: \theta > \theta_0 \Rightarrow H_0: \theta \leq \theta_0$
$H_a: \theta \neq \theta_0 \Rightarrow H_0: \theta = \theta_0$

HYPOTHESIS THAT INVESTIGATOR WANTS TO CLAIM TRUE SHOULD BE TAKEN AS THE ALTERNATIVE.

$X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$ σ^2 IS KNOWN

$H_0: \mu = \mu_0$ $H_a: \mu > \mu_0$

$P(\bar{X} > \mu_0 + c)$

$$P_{H_0}\left(\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > \sqrt{n}\,\frac{c}{\sigma}\right) = P_{H_0}\left(Z > \sqrt{n}\,\frac{c}{\sigma}\right) = \alpha$$

$$= P(Z > \underbrace{Z_\alpha}_{Z_{1-\alpha}}) = \alpha$$

$$Z_\alpha = \frac{\sqrt{n}\,c}{\sigma} \Rightarrow c = \frac{Z_\alpha \sigma}{\sqrt{n}}$$

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$$\left\{ \bar{X} : \bar{X} > \mu_0 + \frac{\sigma z_\alpha}{\sqrt{n}} \right\}$$

REJECTION REGION

$$\left\{ \bar{x} : \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > z_\alpha \right\}$$

WHEN σ^2 IS UNKNOWN

$$T_{H_0} = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

$t_{n-1,1-\alpha}$

IF $H_a : \mu > \mu_0$

$T_{H_0} > t_{n-1,\alpha}$ IF $H_a : \mu < \mu_0$

$T_{H_0} < -t_{n-1,\alpha}$

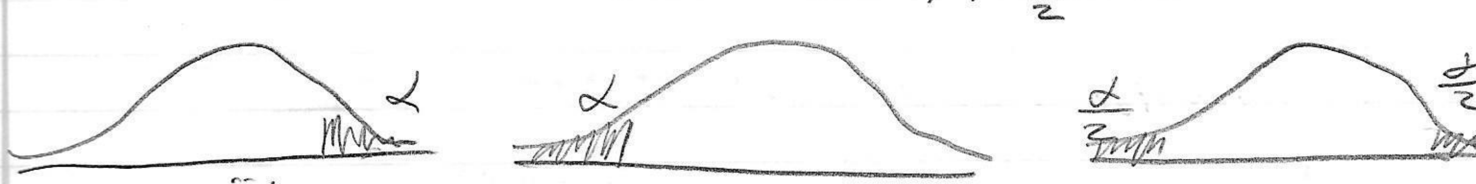
$-t_{n-1,1-\alpha}$

IF $H_a : \mu \neq \mu_0$

$T_{H_0} > t_{n-1,\frac{\alpha}{2}}$ OR $T_{H_0} < -t_{n-1,\frac{\alpha}{2}}$

$t_{n-1,1-\frac{\alpha}{2}}$

$-t_{n-1,1-\frac{\alpha}{2}}$



$X_1, \dots, X_n \overset{iid}{\sim} BERN(p)$

$H_0 : p = p_0$ WE USE CLT $(n > 30)$ $\hat{p} = \frac{X_1 + \dots + X_n}{n}$

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$$Z_{H_0} = \frac{\hat{p} - p_0}{\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}}$$

RR

$Z_{H_0} > Z_{\alpha}$ ($Z_{1-\alpha}$) : $H_a : p > p_0$

$Z_{H_0} < -Z_{\alpha}$ ($Z_{1-\alpha}$) ; $H_a : p < p_0$

~~EX~~ $|Z_{H_0}| > Z_{\frac{\alpha}{2}}$ $\quad H_a : p \neq p_0$

$\downarrow$

$Z_{1-\frac{\alpha}{2}}$

EX: $n = 15$, $\bar{x} = 16.0367$ $\quad s = 0.0551$

LEVEL 0.05

TEST HYPOTHESIS THAT

$H_0 : \mu = 16$ vs $H_a \; \mu \neq 16$

$$|T_{H_0}| = \left| \frac{\bar{x} - 16}{s/\sqrt{n}} \right| = \left| \frac{0.0367}{\frac{0.0551}{\sqrt{15}}} \right| = 2.58$$

$t_{14, 0.025} = 2.145$ $\quad t_{14, 1-0.025}$

$|t_{H_0}| > 2.145$ WE REJECT AT $\underline{\alpha = 0.05}$

H_0 IS REJECTED AT LEVEL $\alpha = 0.05$

DO NOT KNOW ANYTHING ABOUT WHETHER H_0 IS REJECTED AT LEVEL $\alpha = 0.01$

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H_0 NOT REJECTED AT $\alpha = 0.05$

DO NOT KNOW WHETHER H_0 WILL BE NOT REJECTED AT $\alpha = 0.1$

P-VALUE IS THE PROBABILITY OF OBTAINING A RESULT EQUAL TO OR MORE THAN WHAT WAS ACTUALLY OBSERVED, WHEN THE NULL IS TRUE.

$T_{H_0} = 2.58$ $\quad P(|t_{n=14}| > 2.58)$

$$= [1 - P(t_{14} \le 2.58)]\,2$$

$$= 0.0216$$

↳ P-VALUE

$$P(t_{14} \le 2.58) = F_{t_{14}}(2.58)$$

$H_a: \mu > \mu_0$

$$P(t_{14} > T_{H_0}) = 1 - P(t_{14} < T_{H_0})$$
$$= 1 - F_{t_{14}}(T_{H_0})$$

$H_a: \mu < \mu_0$

$$= P(t_{14} < T_{H_0})$$
$$= P(t_{14} \le T_{H_0}) = F_{t_{14}}(T_{H_0})$$

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$P(|t_{n-1}| > |T_{H_0}|)$

$= 2P(t_{n-1} > |T_{H_0}|) = 2[1 - P(t_{n-1} \leq |T_{H_0}|)$

$= 2(1 - F_{t_{n-1}}(|T_{H_0}|)$