LECTURE 17

03-16-18

- EXPECTED VALUE (MEAN, EXPECTATION) OF A R.V. X:

$E[X]\ (=\mu)\ (=\mu_X)$

$$= \begin{cases} \sum_{x \in S_X} x\, p(x) & \text{IF } X \text{ IS DISCRETE} \\ \int_{-\infty}^{\infty} f(x)\, dx & \text{IF } X \text{ IS CONTINUOUS} \end{cases}$$

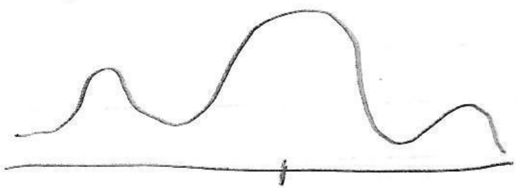
X	X_1	X_2	... X_n
P(x)	$1/N$	$1/N$	$1/N$

$$E(X) = \sum_{i=1}^{N} x \frac{1}{N} = \frac{X_1 + \ldots + X_N}{N}$$

ALGEBRAIC MEAN OF $\{X_1, \ldots X_n\}$

SUPPOSE CONTINUOUS R.V. X HAS PDF f S.T. ("SUCH THAT")
f IS SYMMETRIC ABOUT 0

$$f(x) = f(-x),\ \forall x \in \mathbb{R}^1$$



0

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{\infty} x f(x)\, dx$$

$$-t = x$$

$$-\infty \leq x \leq 0$$
$$\downarrow$$
$$0 \leq t \leq \infty \qquad \frac{dx}{dt} = -1 \qquad dx = -dt$$

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$$\int_{\infty}^{0} -t f(t)(-dt) = \int_{\infty}^{0} t f(t)\,dt = \int_{\infty}^{0} t f(t)\,dt$$

$$= -\int_{0}^{\infty} t f(t)\,dt$$

$$\int_{b}^{a} f(t)\,dt = F(a) - F(b)$$

$$\int_{a}^{b} f(t)\,dt = F(b) - F(a) = -[F(a) - F(b)]$$

IF f IS SYMMETRIC ABOUT A VALUE c, WHAT IS $E(X)$ WHERE R.V. X HAS PDF f

ANS: c

$$\int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} (x-c) f(x)\,dx + \int_{-\infty}^{\infty} c f(x)\,dx$$

$$x - c = t \rightarrow x = t + c$$

$$\int_{-\infty}^{\infty} (t+c) f(t+c)\,d(t+c)$$

R.V. X HAS PDF $p(x) = (1-p)^{x-1} p \quad x = 1, 2, \ldots$

WHERE $0 \le p \le 1$

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$$E(X) = \sum_{x=1}^{\infty} x(1-p)^{x-1}p$$

USE $\dfrac{d[(1-p)^x]}{dp} = -x(1-p)^x$

$$= p\sum_{x=1}^{\infty} x(1-p)^{x-1}$$

$$= p\sum_{x=1}^{\infty} \frac{d[-(1-p)^x]}{dp} = p\,\frac{d\left(-\sum_{x=1}^{\infty}(1-p)^x\right)}{dp}$$

$$\sum_{x=1}^{\infty}(1-p)^x = \frac{(1-p)-0}{1-(1-p)} = \frac{1-p}{p}$$

$$= \frac{p\,d\left[-\frac{1-p}{p}\right]}{dp} = \frac{p\,d\left(-\frac{1}{p}+1\right]}{dp}$$

$$= p\left(+\frac{1}{p^2}\right) = \frac{1}{p}$$

$$[(t)(0.1e^{-0.1t})]'$$

$$= 1\cdot 0.1e^{-0.1t} - t\,0.1^2e^{-0.1t}$$

$$t^2 0.1^2 e^{-0.1t} = 0.1e^{-0.1t} - [t\,0.1e^{-0.1t}]'$$

$$0.1te^{-0.1t} = e^{-0.1t} - [te^{-0.1t}]'$$

$$= \int_0^{\infty} e^{-0.1t}\,dt - [te^{-0.1t}]_0^{\infty}$$

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$\lim_{t \to \infty} t e^{-0.1t} = 0$ 03-16-18

$\lim_{t \to 0} \frac{t}{e^{0.1t}} = \lim_{t \to \infty} \frac{1}{0.1 e^{0.1t}} = 0$

$$\int_0^\infty e^{-0.1t}\,dt = \left[\frac{-e^{-0.1t}}{0.1}\right]_0^\infty = 0 + \frac{1}{0.1} = 10$$

X IS CONTINUOUS

$h(x)$ DEFINED ON $\mathbb{R}^1$

$$E[h(x)] = \int_{-\infty}^{\infty} h(x) f(x)\,dx$$

IF $h(x) = ax + b$ (LINEAR FUNCTION)

~~LINEARITY OF EXPECTATION~~

$$E[h(x)] = E[aX + b] = aE[X] + b$$

$$\int_{-\infty}^{\infty} (ax+b) f(x)\,dx = \int_{-\infty}^{\infty} ax f(x) + b f(x)\,dx$$

IF X IS A R.V. WHAT IS $h(x)$? ANS → R.V.

NO NEED TO DETERMINE DISTRIBUTION OF $h(x)$ TO DETERMINE $E[h(x)]$

→ MORE GENERALLY, FOR THIS FUNCTION: h, hx

DOES NOT HAVE TO BE LINEAR.

BOOKSTORE PURCHASES 3 BOOKS, \$6 EACH. AND SELL THEM FOR \$12 EACH. UNSOLD BOOKS WILL BE RETURNED TO PUBLISHER FOR \$2 EACH

X : RV OF # OF BOOKS SOLD 03-16-18
Y : R.V. OF h A REVENUE

X	0	1	2	3
p(x)				