Poisson Limit Theorem

n is large, p is small
then $Bin(n,p)$ will be
distributed simarlarily to $Pois(np)$

Geometric

$p>0$ $\qquad p(x) = (1-p)^{x-1}p$

$x = 1, 2, 3 \ldots \qquad E[x] = 1/p$

$$E[x(x-1)] = \sum_{x=1}^{\infty} x(x-1)(1-p)^{x-1}p$$

$$= p(1-p)\sum_{x=1}^{\infty} x(x-1)(1-p)^{x-2} = p(1-p)\sum_{x=1}^{\infty} \frac{d^2[(1-p)^x]}{dp^2}$$

$$-x(1-p)^{x-1} \rightarrow x(x-1)(1-p)^{x-2}$$

$$= p(1-p)\frac{d^2\left[\sum_{x=1}^{\infty}(1-p)^x\right]}{dp^2} = p(1-p)\frac{d^2\left[\frac{(1-p)}{1-(1-p)}\right]}{dp^2}$$

$$= p(1-p)\frac{d^2\left(\frac{1-p}{p}\right)}{dp^2} = \boxed{\frac{1-p}{p^2}}$$

as $p \rightarrow 0$, $V[x] \rightarrow \infty$

as $p \rightarrow 1$, $V[x] \rightarrow 0$

Negative Binomial

$p>0 \quad r \in N_+ \qquad N_+ = \{1, 2, 3 \ldots\}$

$$p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

How many trials before you succeed r times?

$r=1$

$\binom{x-1}{0} p^1 (1-p)^{x-1} = p(1-p)^{x-1} \rightarrow$ geometric

$\binom{x-1}{r-1} p^{r-1} (1-p)^{x-r} p$

$$\sum_{x=r}^{\infty} x \binom{x-1}{r-1} p^r (1-p)^{x-r} = \sum_{x=r}^{\infty} \frac{x!}{(r-1)!(x-r)!} p^r (1-p)^{x-r}$$

$$= \frac{r}{p} \sum_{x=r}^{\infty} \binom{x}{r} p^{r+1} (1-p)^{x-1-(r-1)}$$

$\frac{r}{p} = E[x]$ $\qquad E[x(x+1)]$

$$= \frac{r(1-p)}{p^2}$$

<u>Hypergeometric</u>

$M_1, M_2, n \in \mathbb{N}_+$

$$p(x) = \frac{\binom{M_1}{x}\binom{M_2}{n-x}}{\binom{M_1+M_2}{n}}$$

A bag has M_1 red, M_2 blue, you choose n out of the bag. x is the number of set.

if only 5 red balls, you cannot have $x>5$

if only 5 blue balls and you choose 10 balls, $x \geq 5$

$x = 0 \vee (n-M_2), n \wedge M_1$

$a \vee b$ = larger of a and b

$a \wedge b$ = smaller of a and b

$$E[X] = \sum_{x=0 \vee (n-M_2)}^{n \wedge M} \frac{x \binom{M_1}{x} \binom{M_2}{n-x}}{\binom{N}{n}}$$

Example

suppose 70% of all purchases in a certain store are made with credit card. Let x be the number of credit card uses in the next 10 purchases.

$X \sim \text{Binomial}(10, 0.7)$

$E[X] = 7$

$$P(5 \le X \le 8) = \sum_{x=5}^{8} \binom{10}{x} p^x (1-p)^{10-x} = 0.803$$

Standard Normal Distribution

r.v. Z is said to be standard normal if its pdf is $p(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), \; x \in \mathbb{R}^1$

$Z \sim N(0,1)$