given simple random sample

$\{X_1, \dots, X_n\}$ from F_θ

We learned how to construct point estimator $\hat{\theta}$ of θ

interval estimator (or confidence interval(CI)) of θ

$[\hat{\theta}_L, \hat{\theta}_U]$

Symmetric $[\hat{\theta} - M \times SE_{\hat{\theta}}, \hat{\theta} + M \times SE_{\hat{\theta}}]$

need to look at sampling distribution of $\hat{\theta}$

(distribution of $\hat{\theta}$) not easy

Normally you can simplify

Simulation can always be done

3 Fundamental sampling distributions

$Z_1, \dots, Z_\nu$ iid $\sim N(0,1)$

$X \equiv Z_1^2 + \dots + Z_\nu^2$ is χ^2 with df ν $(X \sim \chi^2_\nu)$

$Z \sim N(0,1)$, $X \sim \chi^2_\nu$

$Z \perp\!\!\!\perp X$

Then $T \equiv \frac{Z}{\sqrt{X/\nu}}$ is called t distribution with df ν $(T \sim t_\nu)$

if $X_1 \sim \chi^2_{\nu_1}$, $X_2 \sim \chi^2_{\nu_2}$

$X_1 \perp\!\!\!\perp X_2$

$F \equiv \frac{X_1/\nu_1}{X_2/\nu_2}$ is called F distribution with df ν_1 and ν_2 $(F \sim F_{\nu_1, \nu_2})$

$X_1, \dots, X_n$ iid $\sim N(\mu, \sigma^2)$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$S^2 = \frac{1}{n-1} \sum_{i=0}^{n} (X_i - \bar{X})$

$\bar{X} \perp\!\!\!\perp S^2$

$\frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \sim N(0,1)$

$\frac{\sqrt{n}(\bar{X} - \mu)}{S} \sim t_{n-1}$

$Y_1, \dots Y_n$ iid $N(\mu, \sigma^2)$

$\{X_i\}_{i=1}^n \perp\!\!\!\perp \{Y_i\}_{i=1}^n$

then $\frac{S_X^2}{S_Y^2} \sim F_{n,w}$

CLT

$X_1, \dots, X_n$ iid $\sim \underbrace{(\mu, \sigma^2)}_{\text{not necessarily normal}}$

$\frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \to N(0,1)$ in distribution as $n \to \infty$

$X_1, \dots X_n$ iid Bern(p)

$\hat{p} = \bar{x}$ p(of interest)

$\frac{\sqrt{n}(\hat{p} - p)}{p(1-p)} \sim N(0,1)$

$\frac{\sqrt{n}(\hat{p} - p)}{\hat{p}(1-\hat{p})} \sim N(0,1)$

$1-\alpha$ is confidence level

$(100(1-\alpha)\%$ CI)

$X_1, \dots X_n$ iid $N(\mu, \sigma_0^2)$

σ_0^2 is known, μ is unknown

find $100(1-\alpha)\%$ CI for μ

$\frac{\sqrt{n}(\bar{X} - \mu)}{\sigma_0} \sim N(0,1)$ pivot

$P(-z_{\alpha/2} \le \frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \le z_{\alpha/2}) = 1-\alpha$

$\bar{X} + z_{\frac{\alpha}{2}} \frac{\sigma_0}{\sqrt{n}} \geq \mu$

$\bar{X} - z_{\frac{\alpha}{2}} \frac{\sigma_0}{\sqrt{n}} \le \mu$

multiplier $\swarrow$ $\swarrow$ standard error

$\bar{X} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma_0}{\sqrt{n}} \le \mu \le \bar{X} + z_{\alpha/2} \frac{\sigma_0}{\sqrt{n}}$

t-interval μ $X_1, \dots X_n \sim$ iid $N(\mu, \sigma^2)$

μ, σ^2 unknown

$(\bar{X} - t_{n-1, \frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}, \bar{X} + t_{n-1, \frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$

$X_1, \dots X_n$ Bernoulli(p) CI Bor p

$\hat{p} - z_{\frac{\alpha}{2}} \cdot \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}, \hat{p} + z_{\frac{\alpha}{2}} \cdot \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$

nonsymmetric example

$X_1, \dots X_n$ iid $N(\mu, \sigma^2)$ find $100(1-\alpha)\%$ CI for σ^2

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$P(\chi^2_{n-1,1-\frac{\alpha}{2}} \le \frac{(n-1)S^2}{\sigma^2} \le \chi^2_{n-1,\frac{\alpha}{2}}) = 1-\alpha$

$\left[\frac{(n-1)S^2}{\chi^2_{n-1,\frac{\alpha}{2}}}, \frac{(n-1)S^2}{\chi_{n-1,1-\frac{\alpha}{2}}}\right]$

model (population)

$\{F_\theta : \theta \in \Theta\}$ Θ is the parameter space, $\Theta_0 \in \Theta$

$X_1, \dots, X_n \sim F_\theta$

hypothesis, for example

$\Theta_0 \in \Theta$, $\Theta_0 \in \Theta$

null Hypothesis H_0 alternative hypothesis H_a

Complements of each other

For example

$H_0: \theta = \Theta_0$

$H_a: \theta \neq \Theta_0$

$H_0: \theta \le \theta_0$ versus $H_a: \theta > \theta_0$

$H_0: \theta \ge \theta_0$ versus $H_a: \theta < \theta_0$

$H_0: \bar{X} = 0$ vs $H_a: \bar{X} \neq 0$ ⇐ this is wrong, no hypothesis, just fact

Hypothesis tests are based on samples: So there is uncertainty involved

never say "Ho is proved true", only say "Ho is not rejected at significance level α"

$H_0: \theta = 0$ vs $H_a: \theta = 1$ (1) reject Ho in (1)

$H_0: \theta = 1$ vs $H_a: \theta = 0$ (2) accept Ho in (2)

not the same confidence in these two conclusions are not the same

Ho & Ha not treated equally

Ho is favored unless evidence against it is very strong

Ho is not rejected