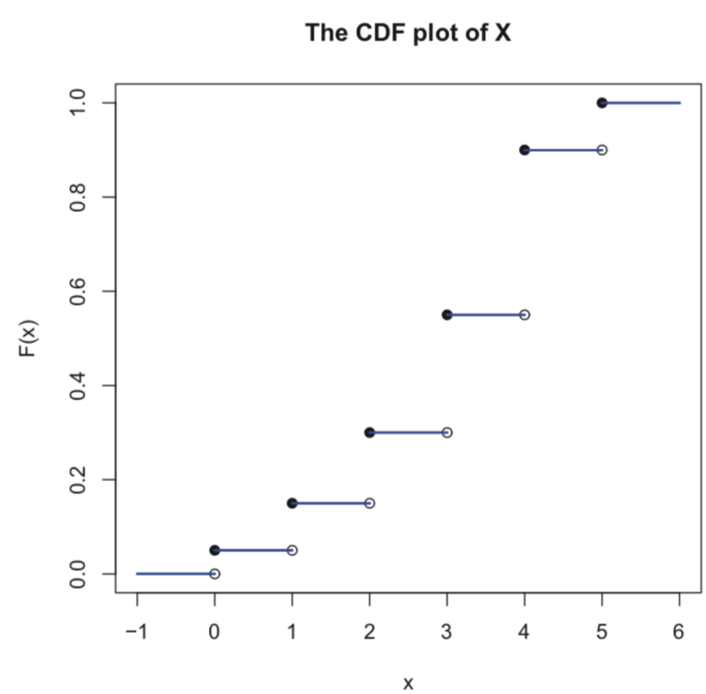
1. (a) $p_1(x)$ is not a valid probability mass function but $p_2(x)$ is.

   (b) To find k, solve the equation $0.2k + 0.3k + 0.4k + 0.2k = 1$. The solution is $k = 1/1.1$.

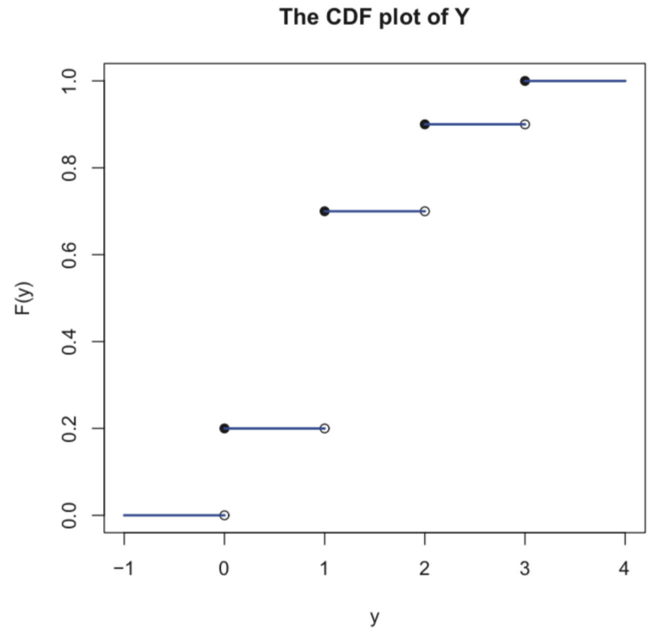
2. (a) The CDF of X is

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 0.05, & \text{if } 0 \leq x < 1 \\ 0.15, & \text{if } 1 \leq x < 2 \\ 0.3, & \text{if } 2 \leq x < 3 \\ 0.55, & \text{if } 3 \leq x < 4 \\ 0.9, & \text{if } 4 \leq x < 5 \\ 1, & \text{if } x \geq 5 \end{cases}$$



(b) $P(1 \leq X \leq 4) = P(X \leq 4) - P(X < 1) = F(4) - F(0) = 0.9 - 0.05 = 0.85$.

3. (a) $P(Y \geq 2) = 1 - P(Y < 2) = 1 - 0.7 = 0.3$. The plot is given below.



(b) The possible values of Y are the jumping points 0, 1, 2, 3, and the probability at the jumping point is the jumping size. Thus $p(0) = 0.2 - 0 = 0.2$, $p(1) = 0.7 - 0.2 = 0.5$, $p(2) = 0.9 - 0.7 = 0.2$, and $p(3) = 1 - 0.9 = 0.1$.

4. X can assume values 0, 1, 2, and 3. We have the PMF of X as

$$p(0) = P(X = 0) = \frac{\binom{7}{3}}{\binom{10}{3}} = 0.292, \qquad p(1) = P(X = 1) = \frac{\binom{3}{1}\binom{7}{2}}{\binom{10}{3}} = 0.525,$$

$$p(2) = P(X = 2) = \frac{\binom{3}{2}\binom{7}{1}}{\binom{10}{3}} = 0.175, \qquad p(3) = P(X = 3) = \frac{\binom{3}{3}}{\binom{10}{3}} = 0.008.$$

The CDF $F(x) = 0$ if $x < 0$; if $0 \leq x < 1$, $F(x) = p(0) = 0.292$; if $1 \leq x < 2$, $F(x) = p(0)+p(1) = 0.292+0.525 = 0.817$; if $2 \leq x < 3$, $F(x) = p(0)+p(1)+p(2) = 0.292 + 0.525 + 0.175 = 0.993$; if $3 \leq x$, $F(x) = p(0) + p(1) + p(2) + p(3) = 1$.

5. (a) $f_1(x)$ is not a valid PDF because for some $x \in (0,2)$, $f_x(x) < 0$. For example, $f_1(1.9) = -0.58$. $f_2(x)$ is a valid PDF because it is easy to verify that $f_2(x) \geq 0$ and $\int_0^2 f_2(x)dx = 1$.

(b)

(i) To find k, we must have $\int_{-\infty}^{\infty} f(x)dx = 1$, that is $\int_8^{10} kxdx = 1$. Solving the equation, we have $k = 1/18$.
It is clear that $F_X(x) = 0$ if $x < 8$ and $F_X(x) = 1$ if $x > 10$. For $x \in [8, 10]$,

$$F_X(x) = \int_8^x f(t)dt = \frac{1}{18}\int_8^x tdt = \frac{1}{36}\, t^2\big|_8^x = \frac{x^2 - 64}{36}.$$

Using the CDF,

$$P(8.6 \leq X \leq 9.8) = F_X(9.8) - F_X(8.6) = \frac{9.8^2 - 64}{36} - \frac{8.6^2 - 64}{36} = 0.61333.$$

(ii)

$$\begin{aligned} P(X \leq 9.8|X \geq 8.6) &= \frac{P(8.6 \leq X \leq 9.8)}{P(X \geq 8.6)} = \frac{P(8.6 \leq X \leq 9.8)}{1 - P(X < 8.6)} \\ &= \frac{P(8.6 \leq X \leq 9.8)}{1 - F_X(8.6)} = \frac{0.61333}{1 - \frac{8.6^2 - 64}{36}} = 0.8479. \end{aligned}$$

6. $X \sim U(0, 1)$ and $Y = 3 + 6X$, then clearly the sample space for Y is (3, 9). Thus, the CDF of Y, $F_Y(y) = 0$ if $y < 3$ and $F_Y(y) = 1$ if $y > 9$. For $y \in (3, 9)$, we calculate $F_Y(y)$ as

$$F_Y(y) = P(Y \leq y) = P(3 + 6X \leq y) = P\left(X \leq \frac{y-3}{6}\right) = \frac{y-3}{6} = \frac{y-3}{9-3}.$$

Comparing to the CDF of $U(A, B)$ in Examples 3.2-5, we can see that $F_Y(y)$ is the CDF of $U(3, 9)$, hence $Y \sim U(3, 9)$.

7. The sample space of Y is $(0, \infty)$. If $y < 0$, clearly, the CDF $F_Y(y) = 0$. For $y > 0$,

$$\begin{aligned} F_Y(y) &= P(Y \le y) = P(-\log X \le y) = P(X \ge e^{-y}) \\ &= 1 - P(X < e^{-y}) = 1 - F_X(e^{-y}) = 1 - e^{-y}. \end{aligned}$$

The PDF of Y is $f_Y(y) = 0$ if $y < 0$ and $f_Y(y) = e^{-y}$ for $y > 0$.

8. (a) $P(0.5 \le X \le 2) = F(1) - F(0.5) = 1/4 - 1/16 = 0.1875$.

(b) Taking derivative to $F(x)$, we find the PDF of X is

$$f(x) = \begin{cases} \frac{x}{2}, & \text{if } 0 < x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

(c) Since Y is in seconds, $Y = 60X$. Thus, $F(y) = 0$ for $y \le 0$ and $F(y) = 1$ for $y > 120$, for $0 < y \le 120$,

$$F(y) = P(Y \le y) = P(60X < y) = P\left(x < \frac{y}{60}\right) = \frac{1}{4}\left(\frac{y}{60}\right)^2 = \frac{y^2}{14400}.$$

Taking derivative to $F(y)$, we find the PDF of Y is

$$f(y) = \begin{cases} \frac{y}{7200}, & \text{if } 0 < x \le 120 \\ 0, & \text{otherwise.} \end{cases}$$

9. (a)

$$P(X > 10) = P(30/D > 10) = P(D < 3) = \frac{\pi 3^2}{\pi 9^3} = 1/9.$$

(b) Since D must between 0 and 9, then the sample space of $X = 30/D$ is $(30/9, \infty)$. We first calculate the CDF of X. Clearly, $F_X(x) = 0$ if $x < 30/9$. For $x \ge 30/9$, we have

$$\begin{aligned} F_X(x) &= P(X \le x) = 1 - P(X > x) = 1 - P\left(\frac{30}{D} > x\right) \\ &= 1 - P\left(D < \frac{30}{x}\right) = 1 - \frac{\pi(\frac{30}{x})^2}{\pi 9^2} = 1 - \frac{100}{9x^2}. \end{aligned}$$

Differentiating $F_X(x)$, we have $f_X(x) = 0$ for $x < 10/3$, otherwise, $f_X(x) = 200/(9x^3)$.

10. (a) The event "no cost" means that $X < 24 \times 3 = 72$. Thus, the probability is

$$\begin{aligned} P(X < 72) &= \int_{48}^{72} f(x)dx = \int_{48}^{72} 0.02e^{-0.02(x-48)}dx = [-e^{-0.02(x-48)}]\Big|_{48}^{72} \\ &= 1 - e^{-0.02\times 24} = 0.3812. \end{aligned}$$

(b) The additional cost is between \$400 and \$800. This means that it takes more than 4 days but less than 7 days for the fixture to arrive. That is, $4 \times 24 < X < 7 \times 24$, or $96 < X < 168$. Thus, the corresponding probability is