8. The probability is
$$\frac{26^2 \times 10^3}{26^3 \times 10^4} = 0.0038.$$

9. (a) The number of possible committees is
$$\binom{12}{4} = 495.$$

   (b) The number of committees consisting of 2 biologists, 1 chemist, and 1 physicist is
$$\binom{5}{2}\binom{4}{1}\binom{3}{1} = 120.$$

   (c) The probability is $120/495 = 0.2424$.

10. (a) The number of possible selections is
$$\binom{10}{5} = 252.$$

   (b) The number of divisions of the 10 players into two teams of 5 is $252/2 = 126$.

   (c) The number of handshakes is
$$\binom{12}{2} = 66.$$

11. (a) In order to go from the lower left corner to the upper right corner, we need to totally move 8 steps, with 4 steps to the right and 4 steps upwards. Thus, the total number of paths is

$$\binom{8}{4} = 70.$$

(b) We decompose the move as two stages: stage 1 is from lower left corner to circled point, which needs 5 steps with 3 steps to the right and 2 steps upwards; stage 2 is from the circled point to the upper right corner, which needs 3 steps with 1 step to the right and 2 steps upwards. Thus, the total number of paths passing the circled point is

$$\binom{5}{3}\binom{3}{1} = 30.$$

(c) The probability is $30/70 = 3/7$.

12. (a) In order to keep the system working, the nonfunctioning antennas cannot be next to each other. There are 8 antennas functioning; thus, the 5 nonfunctioning antennas must be in the 9 spaces created by the 8 functioning antennas. The number of arrangements is

$$\binom{9}{5} = 126.$$

(b) The total number of the 5 nonfunctioning antennas is $\binom{13}{5} = 1287$. Thus, the required probability is $126/1287 = 0.0979$.

13. (a) The total number of selections is

$$\binom{15}{5} = 3003.$$

(b) The number of selections containing three defective buses is

$$\binom{4}{3}\binom{11}{2} = 220.$$

(c) The asked probability is $220/3003 = 0.07326$.

(d) The probability all five buses are free of the defect is calculated as

$$\frac{\binom{11}{5}}{\binom{15}{5}} = 0.1538.$$

14. (a) The number of samples of size five is $\binom{30}{5} = 142506$.

(b) The number of samples that include two of the six tagged moose is $\binom{6}{2}\binom{24}{3} = 30360$.

(c)

(i) The probability is

$$\frac{\binom{6}{2}\binom{24}{3}}{\binom{30}{5}} = \frac{30360}{142506} = 0.213.$$

(ii) The probability is

$$\frac{\binom{24}{5}}{\binom{30}{5}} = \frac{30360}{142506} = 0.298.$$

18. (a)

$$2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^k 1^{n-k} = \sum_{k=0}^{n} \binom{n}{k}.$$

(b)

$$\begin{aligned}(a^2+b)^4 &= \binom{4}{0}(a^2)^0 b^{4-0} + \binom{4}{1}(a^2)^1 b^{4-1} + \binom{4}{2}(a^2)^2 b^{4-2} + \binom{4}{3}(a^2)^3 b^{4-3} \\ &\quad + \binom{4}{4}(a^2)^4 b^{4-4} \\ &= b^4 + 4a^2b^3 + 6a^4b^2 + 4a^6b + a^8\end{aligned}$$