Notes

2/19/18

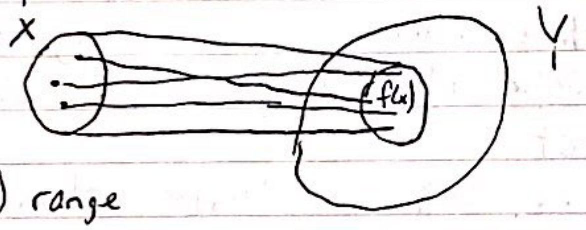
(R.V.)
Random Variables + Their Distributions

- Review calculus
  - R.V.'s, probability distributions
  - parameters of probability distributions

- Functions
  - 2 sets X and Y
    - → elements can be numbers, alphabet, animals, buildings ...
  - Suppose for $\forall x \in X$ $\quad \forall$ = any
    there is an association to an element in Y which we denote $f(x)$.
    This f is said to be a function from X to Y

X: <u>domain</u> of f (f is defined in X)
Y: <u>codomain</u> of f
- elements of $f(x)$ are called values of f. The set of all values of f is called the <u>range</u>

$f: X \to Y$



range $\subset$ codomain

$f(x)$ range

- if $X = \mathbb{R}^1$ then f is called univariate function
- if $X = \mathbb{R}^k$ and $k \geq 2$ then f is called multivariate function
- if $Y = \mathbb{R}^1$ then f is called real valued function
- if $Y = \mathbb{R}^k$ and $k \geq 2$ then f is called vector valued function

$X = \mathbb{R}^1, \quad Y = \mathbb{R}^1, \quad f(x) = x \quad (f(x) = x, \ x \in \mathbb{R}^1)$

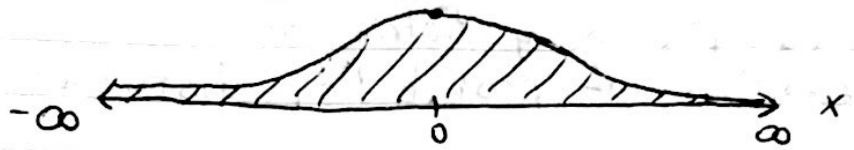
$f(x) = x, \quad x \in [0, 1]$

range $= [0, 1]$

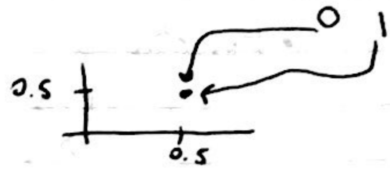
domain $= [0, 1]$

to make area under curve = 1

- $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \quad x \in \mathbb{R}^1$



- $X = \{0, 1\} \quad Y = \mathbb{R}^2$
  $f(0) = f(1) = (0.5, 0.5)$



- probability as a function
  $f = P \quad X = \mathcal{F}$ (set of subsets of the sample space)
  $Y = \mathbb{R}^1$
  - for each event $E \in \mathcal{F}$, we associate a number $P(E)$

- range $[0, 1]$
  $X = \{$ all 78 students in the class $\}$
  $Y = \{$ all majors in Penn State $\}$
  $\forall x \in X$, Let $f(x)$ be his/her major
  - f(Alice Smith) = Civil Engineering
  - f(John Mason) = Biological Engineering

→ "Size" of a set
$\{1, 2, \ldots, 100\}$ is "bigger" than $\{1000, 2000\}$

- <u>Size</u> - how many elements there are in a set
  → cardinality of a set
  → cardinal number of a set

- <u>finite</u> - a set is finite if all its elements can be traversed eventually
- <u>infinite</u> - a set is infinite if it is not finite

- two types of infinity
  - countably infinite (countable) $\mathbb{Z}$
  - uncountably infinite (uncountable) $\mathbb{R}$

$$\begin{array}{ccccccc} -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ 7 & 5 & 3 & 1 & 2 & 4 & 6 \end{array}$$

- Finite sets:

  $\{$All students in class$\}$

  $\{1, 2, \ldots 100^{1000}\}$

  - countable set: $\mathbb{N} = \{0, 1, 2, \ldots\}$

    $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$

- $\mathbb{Q} = \{$All rational numbers$\}$ a/b
- uncountable sets:

  $\mathbb{R}^1 = \{x: -\infty$

  $[a, b] = \{x: a \le x \le b\}$ w/ $a < b$

  $\mathbb{R} \setminus \mathbb{Q} = \{$irrational numbers$\}$

- Monotone Function (univariate function)
  - $f: \mathbb{R}^1 \to \mathbb{R}^1$ is called monotonically
  - non decreasing if for any $x_1 < x_2$ ~~we have~~

we have $f(x_1) \le f(x_2)$ ~~similarly we can define non decreasing~~

$f\uparrow$

- similarly we can define non-increasing $f(x_1) \ge f(x_2)$