2.1.12

$$P(X \geq 4|X \geq 1) = \frac{P(X \geq 4)}{P(X \geq 1)} = \frac{1 - P(X \leq 3)}{1 - P(X = 0)}$$
$$= \frac{1 - [1 - 1/2 + 1/2 - 1/3 + 1/3 - 1/4 + 1/4 - 1/5]}{1 - [1 - 1/2]} = \frac{2}{5}$$

2.2.2

$$E(X) = (-1)(\frac{4}{9}) + (0)(\frac{1}{9}) + (1)(\frac{4}{9}) = 0;$$
$$E(X^2) = (-1)^2(\frac{4}{9}) + (0)(\frac{1}{9}) + (1)^2(\frac{4}{9}) = \frac{8}{9};$$
$$E(3X^2 - 2X + 4) = 3(\frac{8}{9}) - 2(0) + 4 = \frac{20}{3}$$

2.3.18

$$P(X > k + j|X > k) = \frac{P(X > k + j)}{P(X > k)}$$
$$= \frac{q^{k+j}}{q^k} = q^j = P(X > j).$$

2.4.20abd

a (i)Binomial, $b(5, 0.7)$; (ii) $\mu = 3.5$,$\sigma^2 = 1.05$; (iii) 0.1607;

b (i)Geometric, p =0.3; (ii) $\mu = 10/3$, $\sigma^2 = 70/9$; (iii) 0.51;

d (i)Discrete distribution; (ii) $\mu = 2.1$, $\sigma^2 = 0.89$; (iii) 0.7

2.6.10 $\sigma = \sqrt{9} = 3$

$$P(3 < X < 15) = P(X \leq 14) - P(X \leq 3) = \Sigma_{x=4}^{14} e^{-9} \frac{9^x}{x!} = 0.959 - 0.021 = 0.938$$

3.1.4 $X \sim U(4, 5)$; **(a)** $\mu = 9/2$ **(b)** $\sigma^2 = 1/12$ **(c)** 0.5

3.1.10

(a)

$$\int_1^\infty \frac{c}{x^2} dx = 1$$
$$[\frac{-c}{x}]|_1^\infty = 1$$
$$c = 1$$

(b) $E(X) = \int_1^\infty \frac{x}{x^2} dx = [lnx]|_1^\infty$, which is unbounded.

3.2.8

1. Using integration by part

$$P(X \leq 5) = \int_0^5 \frac{x^{2-1} e^{-x/4}}{\Gamma(2) 4^2} dx$$
$$= -\frac{1}{4}[xe^{-x/4} + 4e^{-x/4}]|_0^5$$
$$= 0.35536$$

2. Using Equation 3.2-1

$$\begin{aligned} F(x) &= P(X \leq x) \\ &= 1 - \Sigma_{k=0}^{\alpha-1} \frac{(\lambda x)^k e^{-\lambda x}}{k!}. \end{aligned}$$

Thus, with $\lambda = 1/\theta = 1/4$ and $\alpha = 2$,

$$\begin{aligned} P(X < 5) &= 1 - e^{5/4} - (\frac{5}{4})e^{-5/4} \\ &= 0.35536 \end{aligned}$$

4.1.1ad

a

$$\begin{aligned} 1 &= \Sigma_{x=1}^{2}\Sigma_{y=1}^{3} c(x+2y) \\ 1 &= c(3+4+5+6+7+8) \\ 1 &= 33c \\ c &= \frac{1}{33} \end{aligned}$$

d

$$\begin{aligned} \Sigma_{x=1}^{\infty}\Sigma_{y=1}^{\infty} c(\frac{1}{4})^x(\frac{1}{3})^y &= 1 \\ \Sigma_{x=1}^{\infty} c(\frac{1}{4})^x \Sigma_{y=1}^{\infty}(\frac{1}{3})^y &= 1 \\ \Sigma_{x=1}^{\infty} c(\frac{1}{4})^x \frac{1}{2} &= 1 \\ c\Sigma_{x=1}^{\infty}(\frac{1}{4})^x &= 2 \\ \frac{1}{3}c &= 2 \\ c &= 6 \end{aligned}$$

4.2.10

$$\begin{aligned} h(v) &= E|(X - \mu_X) + v(Y - \mu_Y)|^2 \\ &= E[|X - \mu_X|^2] + 2vE[(X - \mu_X)(Y - \mu_Y)] + E[v^2(Y - \mu_Y)^2] \\ &= \sigma_X^2 + 2Cov(X,Y)v + \sigma_Y^2 v^2 \geq 0. \end{aligned}$$

Thus the discriminant of this quadratic must be less than or equal to 0. So we have

$$\begin{aligned} [Cov(X,Y)]^2 - \sigma_X^2\sigma_Y^2 &\leq 0 \\ \rho^2 &\leq 1 \\ -1 \leq \rho &\leq 1 \end{aligned}$$

4.3.10

(a) $f(x,y) = 1/[10(10-x)]$, where $x = 0, 1, 2, \cdots, 9$, $y = x, x+1, \cdots, 9$;

(b) $f_Y(y) = \Sigma_{x=0}^{y} \frac{1}{10(10-x)}$, $y = 0, 1, \cdots, 9$;

(c) $E[Y|x] = (x+9)/2$

4.4.10 The area of the space is

$$\int_2^6 \int_1^{14-2t_2} dt_1 dt_2 = \int_2^6 (13 - 2t_2) dt_2 = 20;$$

Thus

$$\begin{aligned} P(T_1 + T_2 > 10) &= \int_2^4 \int_{10-t_2}^{14-2t_2} \frac{1}{20} dt_1 dt_2 \\ &= \int_2^4 \frac{4 - t_2}{20} dt_2 \\ &= [-\frac{(4 - t_2)^2}{40}]|_2^4 = \frac{1}{10}. \end{aligned}$$