1. (a) The random variable X can take 0, 1, 2, 3. The PMF can be calculated as

$$p(0) = P(X=0) = \frac{\binom{16}{3}}{\binom{20}{3}} = 0.4912, \qquad p(1) = P(X=1) = \frac{\binom{4}{1}\binom{16}{2}}{\binom{20}{3}} = 0.4211,$$

$$p(2) = P(X=2) = \frac{\binom{4}{2}\binom{16}{1}}{\binom{20}{3}} = 0.0842, \qquad p(3) = P(X=3) = \frac{\binom{4}{3}}{\binom{20}{3}} = 0.0035.$$

(b) $E(X) = \sum_0^3 xp(x) = 0 \times 0.491 + 1 \times 0.4211 + 2 \times 0.0842 + 3 \times 0.0035 = 0.6$, $Var(X) = E(X^2) - E(X)^2 = 0 \times 0.491 + 1 \times 0.4211 + 4 \times 0.0842 + 9 \times 0.0035 - 0.6^2 = 0.429$.

2. (a) $E(X) = 1 \times 0.4 + 2 \times 0.3 + 3 \times 0.1 + 4 \times 0.2 = 2.1$, and $E(1/X) = 1/1 \times 0.4 + 1/2 \times 0.3 + 1/3 \times 0.1 + 1/4 \times 0.2 = 0.63333$.

(b) We need to compare the expectation of $1000/E(X)$ and $1000/X$: $E(1000/E(X)) = 1000/E(X) = 1000/2.1 = 476.19$, while $E(1000/X) = 1000E(1/X) = 1000 \times 0.63333 = 633.33$. Thus, the player should choose $1000/X$.

3. (a) The sample space of X is $S_X = \{0, 400, 750, 800, 1150, 1500\}$. We can find the PMF as

$$\begin{aligned} p(0) &= P(\text{both do not buy TV}) \\ &= 0.7 \times 0.7 = 0.49, \end{aligned}$$

$$\begin{aligned} p(400) &= P(\text{one buys \$400 TV, the other one does not buy TV}) \\ &= 2 \times 0.7 \times 0.3 \times 0.4 = 0.168, \end{aligned}$$

$$\begin{aligned} p(750) &= P(\text{one buys \$400 TV, the other one does not buy TV}) \\ &= 2 \times 0.7 \times 0.3 \times 0.6 = 0.252, \end{aligned}$$

$$\begin{aligned} p(800) &= P(\text{both buy \$400 TV}) \\ &= 0.3 \times 0.4 \times 0.3 \times 0.4 = 0.0144, \end{aligned}$$

$$\begin{aligned} p(1150) &= P(\text{one buys \$400 TV, the other buys \$750 TV}) \\ &= 2 \times 0.3 \times 0.4 \times 0.3 \times 0.6 = 0.0432, \end{aligned}$$

and

$$\begin{aligned} p(1500) &= P(\text{both buy \$750 TV}) \\ &= 0.3 \times 0.6 \times 0.3 \times 0.6 = 0.0324, \end{aligned}$$

(b) $E(X) = \sum xp(x) = 0 \times 0.49 + 400 \times 0.168 + 750 \times 0.252 + 800 \times 0.0144 + 1150 \times 0.0432 + 1500 \times 0.0324 = 366.$

$Var(X) = 0^2 \times 0.49 + 400^2 \times 0.168 + 750^2 \times 0.252 + 800^2 \times 0.0144 + 1150^2 \times 0.0432 + 1500^2 \times 0.0324 - 366^2 = 173,922.$

4. (a) $E(X) = 0 \times 0.05 + 1 \times 0.1 + 2 \times 0.15 + 3 \times 0.25 + 4 \times 0.35 + 5 \times 0.1 = 3.05$, and $E(X^2) = 0^2 \times 0.05 + 1^2 \times 0.1 + 2^2 \times 0.15 + 3^2 \times 0.25 + 4^2 \times 0.35 + 5^2 \times 0.1 = 11.05$, thus, $\text{Var}(X) = E(X^2) - E(X)^2 = 1.7475$.

(b) Let Y be the bonus, then $Y = 15000X$, thus $E(Y) = E(15000X) = 15000 \times 3.05 = 45750$, and $\text{Var}(Y) = \text{Var}(15000X) = 15000^2\text{Var}(X) = 15000^2 \times 1.7475 = 393187500$.

5. Use the commands *g=function(x) {0.01*x*x*exp(-0.1*x)}; integrate(g,lower=0, upper=Inf)* to find $E(X)$, and the result is 20. Similarly, we find $E(X^2) = 600$, thus $\sigma_X^2 = 600 - 20^2 = 200$.

6. (a) Since $\tilde{T} = T + 5$ and $T > 0$. Thus, the sample space of $\tilde{T}$ is $(5, \infty)$. The CDF of $\tilde{T}$ is $F_{\tilde{T}}(\tilde{t}) = 0$ if $\tilde{t} < 5$, for $\tilde{t} > 5$

$$F_{\tilde{T}}(\tilde{t}) = P(\tilde{T} \le \tilde{t}) = P(T + 5 \le \tilde{t}) = P(T \le \tilde{t} - 5) = \int_0^{\tilde{t}-5} f_T(t)dt$$

$$= \int_0^{\tilde{t}-5} 0.1e^{-0.1t}dt.$$

Differentiating the CDF of $\tilde{T}$, we get the PDF of $\tilde{T}$ as

$$f_{\tilde{T}}(\tilde{t}) = \begin{cases} 0.1e^{-0.1(\tilde{t}-5)}, & \text{if } \tilde{t} \ge 5 \\ 0, & \text{otherwise.} \end{cases}$$

(b) The expected cost is

$$E(\tilde{Y}) = E(h(\tilde{T})) = \int_{-\infty}^{\infty} h(\tilde{T}) f_{\tilde{T}}(\tilde{t})d\tilde{t}$$

$$= \int_5^{15} 5(15 - \tilde{t})0.1e^{-0.1(\tilde{t}-5)}d\tilde{t} + \int_{15}^{\infty} 10(\tilde{t} - 15)0.1e^{-0.1(\tilde{t}-5)}d\tilde{t} = 55.1819.$$

The two integrals are calculated by the commands

g=function(x){5(15-x)*0.1*exp(-0.1*x+0.5)}*
integrate(g, lower=5, upper=15)

and

g=function(x) {10(x-15)*0.1*exp(-0.1*x+0.5)}*
integrate(g, lower=15, upper=Inf)

the company's plan to delay the work on the project does reduce the expected cost.

7. (a) To find the median, solve the equation $F(\tilde{\mu}) = 0.5$, which is $\tilde{\mu}^2/4 = 0.5$, and results in $\tilde{\mu} = \sqrt{2}$. The 25th percentile is the solution to $F(x_{0.25}) = 0.25$, which is $x_{0.25}^2/4 = 0.25$, and results in $x_{0.25} = 1$; The 75th percentile is the solution to $F(x_{0.75}) = 0.75$, which is $x_{0.75}^2/4 = 0.75$, and results in $x_{0.75} = \sqrt{3}$. Thus, the IQR is $IQR = x_{0.75} - x_{0.725} = \sqrt{3} - 1 = 0.732$.

   (b) We can get the PDF $f(x) = x/2$ for $0 \le x \le 2$, and otherwise, $f(x) = 0$. So

$$E(X) = \int_0^2 xf(x)dx = \int_0^2 \frac{x^2}{2}dx = \frac{x^3}{6}\bigg|_0^2 = 4/3 = 1.333,$$

   and

$$\sigma_X^2 = \int_0^2 x^2 f(x)dx - E(X)^2 = \int_0^2 \frac{x^3}{2}dx - E(X)^2 = \frac{x^4}{8}\bigg|_0^2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}.$$

   Thus, $\sigma_X = \sqrt{2}/3 = 0.4714$.

8. (a) Since $Y = 60X$ and the sample space for X is $(0, 3)$, the sample space for Y is (0, 180). Thus, the CDF of Y is $F_Y(y) = 0$ if $y < 0$, $F_Y(y) = 1$ if $y \ge 180$, for $y \in (0, 180)$, we have

$$\begin{aligned} F_Y(y) &= P(Y \le y) = P(60X \le y) = P(X \le y/60) = F_X(y/60) \\ &= \frac{\log(1 + y/60)}{\log 4}. \end{aligned}$$

   Thus, by differentiating $F_Y(y)$, we have the pdf of Y is

$$f_Y(y) = \begin{cases} \frac{1}{(60+y)\log 4}, & \text{if } 0 < y < 180 \\ 0, & \text{otherwise.} \end{cases}$$

(b) $V = h(y)$, and $h(y) = 0$ for $0 \le y \le 120$, and $h(y) = 200 + 6(y - 120)$ for $y > 120$. Then we can calculate

$$
\begin{aligned}
E(V) = E(h(Y)) &= \int_{-\infty}^{\infty} h(y) f_Y(y) dy \\
&= \int_{120}^{180} (200 + 6(y - 120)) \frac{1}{(60 + y)\log 4} dy = 77.0686.
\end{aligned}
$$

The integral can be calculated using the following R commands

```
g=function(y)(200+6*(y-120))/((60+y)*log(4))
integrate(g, lower=120, upper=180)
```

$$
\begin{aligned}
E(V^2) = E(h(Y)^2) &= \int_{-\infty}^{\infty} h(y)^2 f_Y(y) dy \\
&= \int_{120}^{180} (200 + 6(y - 120))^2 \frac{1}{(60 + y)\log 4} dy = 30859.97.
\end{aligned}
$$

The integral can be calculated using the following R commands

```
g=function(y)(200+6*(y-120))**2/((60+y)*log(4))
integrate(g, lower=120, upper=180)
```

As a result, $\sigma_V^2 = E(V^2) - E(V)^2 = 30859.97 - 77.0686^2 = 24920.4$.

(c) Let D be the fine expressed in dollars, then $D = Y/100$. Thus, $E(D) = E(Y)/100 = 0.7707$, and $\sigma_D^2 = \sigma_V^2/100^2 = 2.492$.

9. (a)

$$E(P) = \int_0^1 p f_P(p) dp = \int_0^1 \theta p^\theta dp = \theta \frac{1}{\theta+1} p^{\theta+1}\Big|_0^1 = \frac{\theta}{\theta+1},$$

and

$$E(P^2) = \int_0^1 p^2 f_P(p) dp = \int_0^1 \theta p^{\theta+1} dp = \theta \frac{1}{\theta+2} p^{\theta+2}\Big|_0^1 = \frac{\theta}{\theta+2},$$

hence

$$\sigma_P^2 = E(P^2) - E(P)^2 = \frac{\theta}{\theta+2} - \left(\frac{\theta}{\theta+1}\right)^2 = \frac{\theta}{(\theta+2)(\theta+1)^2}.$$

(b) Clearly, if $p \leq 0$, $F_P(p) = 0$, and if $p \geq 1$, $F_P(p) = 1$. If $0 < p < 1$,

$$F_P(p) = \int_0^p f_P(t) dt = \int_0^p \theta t^{\theta-1} dt = t^\theta\Big|_0^p = p^\theta.$$

(c) Denote the 25th percentile and 75th percentile as $p_{0.25}$ and $p_{0.75}$, respectively. Then $F_P(p_{0.25}) = 0.25$ and $F_P(p_{0.75}) = 0.75$, or, $p_{0.25}^\theta = 0.25$ and $p_{0.75}^\theta = 0.75$. Thus, $p_{0.25} = 0.25^{1/\theta}$ and $p_{0.75} = 0.75^{1/\theta}$. So $IQR = p_{0.75} - p_{0.25} = 0.75^{1/\theta} - 0.25^{1/\theta}$.

1. (a) X is Binomial R.V.

   (b) The sample space is $S_X = \{0, 1, \cdots, 5\}$ and the PMF is $p(x) = \binom{5}{x} 0.3^x 0.7^{5-x}$ for $x = 0, 1, 2, 3, 4, 5$.

   (c) $E(X) = 5 \times 0.3 = 1.5$ and $\text{Var}(X) = 5 \times 0.3 \times 0.7 = 1.05$.

   (d)

   (i) The probability that there are more than 2 fails, $P(X > 2)$, can be calculated by the R command *1-pbinom(2, 5, 0.3)*, which gives the result 0.163.

   (ii) Let Y be the cost from failed grafts, then $Y = 9X$. Thus, $E(Y) = E(9X) = 9E(X) = 13.5$, and $\text{Var}(Y) = \text{Var}(9X) = 81\text{Var}(X) = 85.05$.