Calculating Var

Monday, March 26, 2018 10:10 AM

$$E(y(y-1))$$

$$= \sum_{y=0}^{n} y(y-1)\binom{n}{y} p^{y}(1-p)^{n-y}$$

$$= \sum_{y=2}^{n} y(y-1)\frac{n!}{y!(n-y)!} p^{y}(1-p)^{n-y}$$

$$= \sum_{y=2}^{n} \frac{n!}{(y-2)!(n-y)!} p^{y}(1-p)^{n-y} = n(n-1)p^2 \sum_{y=2}^{n} \frac{(n-2)!}{(y-2)!(n-y)!} p^{y-2}(1-p)^{n-y}$$

$$\begin{pmatrix} y-2=u \\ y=y+2 \end{pmatrix} \to n(n-1)p^2 \sum_{u=0}^{n-2} \frac{(n-2)!}{u!(n-2-u)!} p^{u}(1-p)^{n-2-u} = n(n-1)p^2$$

$$\Rightarrow E[y]^2 - E[y] = n(n-1)p^2$$

$$E[y] = np$$

$$E[y]^2 = n(n-1)p^2 + np$$

$$= (n^2-n)p^2 + np$$

$$V(y) = E[y^2] - E[y]^2 = n^2p^2 - np^2 - np - n^2p^2$$

$$= np(1-p)$$

By Newtons Theorem, $P(Y=y)$ exactly equal

to the $y+1$ st term of the binomial expansion

of $(p+1-p)^n = 1^n = 1$

$$= \binom{n}{0} p^0 (1-p)^{n-0} + \binom{n}{1} p^1 (1-p)^{n-1}$$

$$\binom{n}{n} p^n (1-p)^{n-n} + \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

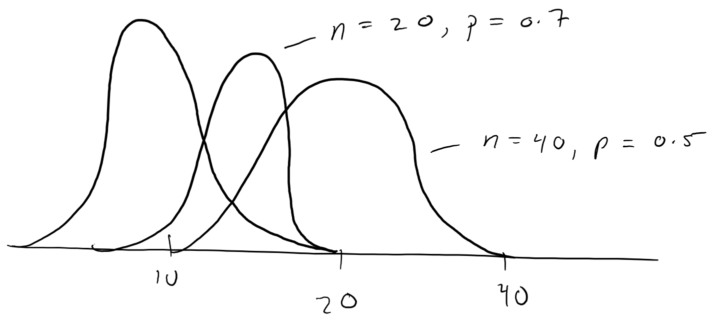
$$P(y=k) = \binom{1}{k} p^k (1-p)^{1-k} = p^k (1-p)^{1-k}$$

for $n = 1$

k is either 0 or 1

$$\binom{1}{0} - \binom{1}{1} = 1$$

Bernoulli



Poisson Distribution

- Counts # of times an event happens.
- Measures how many times an event occurs over a period of time
- No bound on how many times an event can happen.

Parameter

$\lambda > 0$

$X \sim \text{poisson}(\lambda)$

Poisson can be approximated to the binomial when "n" is large and "p" is small "np" is medium.

Ex! Poisson with np approximates to

$\underline{\underline{Bin(n,p)}}$

Parameter $\lambda = 0$

Range $S_x = \{0, 1, 2, \dots, n\}$

pmf $P(X=x) - e^{-\lambda} \dfrac{\lambda^x}{x!}$

$x = 0, 1, 2, \dots$

$x!$ grows faster than λ^x

But for small x, λ^x may be larger than $x!$

$$\sum_{x=0}^{\infty} e^{-\lambda} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^x}{x!}}$$

$$= e^{-\lambda} \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^x}{x!}}_{\text{Taylor expansion}}$$

$$= e^{-\lambda} e^{\lambda} = 1$$

$$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{3} + \dots + \frac{\lambda^k}{k}$$

$$E[x] = \sum_{x=0}^{\infty} x\, e^{-\lambda} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$((x-1) = u)$$

$$\Rightarrow \lambda e^{-\lambda} \underbrace{\sum_{x=1}^{\infty} \frac{\lambda^u}{u!}}_{} \rightarrow e^{\lambda}$$

$$= \lambda e^{-\lambda} e^{\lambda} = \underline{\lambda}$$

$$E[x(x-1)] = \sum_{x=0}^{\infty} x(x-1)\, e^{-\lambda} \frac{\lambda^x}{x!}$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-2)!}$$

$$= e^{-\lambda} \lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}$$

$$(x - 2 = u)$$

$$(x-2=u)$$

$$\Rightarrow e^{-\lambda}\lambda^2 \sum_{x=0}^{\infty} \frac{\lambda^u}{u!}$$

$$= e^{-\lambda}\lambda^2 e^{\lambda} = \lambda^2$$

$$E[x^2] - E[x] = \lambda^2$$

$$E[x^2] = \lambda^2 + \lambda$$

$$\Rightarrow Var[x] = E[x^2] - E[x]^2$$

$$= \lambda^2 + \lambda - \lambda^2 = \underline{\lambda}$$

For $n \to$ large
$p \to$ small
$np \to$ med

$$\rightarrow \binom{n}{p} p^k (1-p)^{n-k} \quad — ①$$

$$\text{poisson}(np) = e^{-np}\frac{(np)!}{k!} \quad — ②$$

$$\frac{①}{②} = \frac{\frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}}{e^{-np}\frac{(np)!}{k!}}$$

$$= \underbrace{\frac{e^{np} n!}{(n-k)(np)!} p^k (1-p)^{n-k}}$$

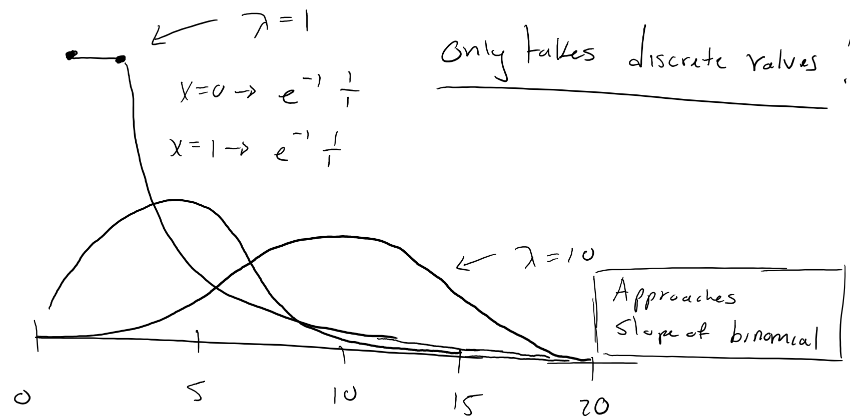
$\rightarrow$ Not easy to Calculate

$(n-k)(np)!$

Calculate

Will re-do this next class

Only takes discrete values!



Other Common discrete distributions

- Geometric

↳ $0 < p \leq 1$

↳ $p(x) = (1-p)^{x-1}\, p$

$$X = 1, 2, \ldots$$

$$E[x] = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$$

$$= \underline{\underline{1/p}}$$