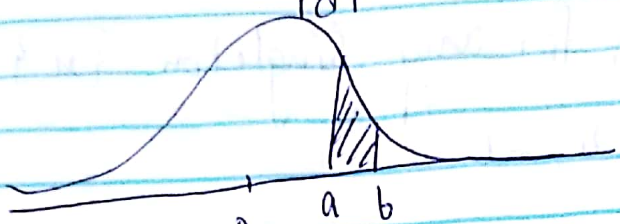
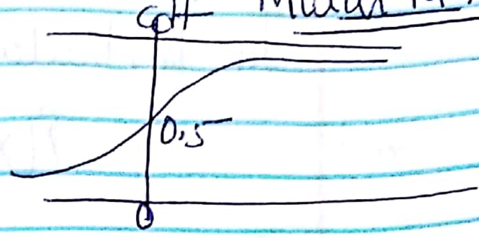
March '14 '18

eg -





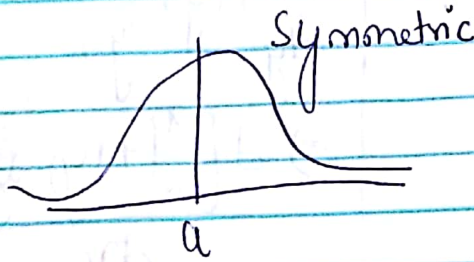
$P(a < X \leq b) = \int_a^b f(x)\,dx$

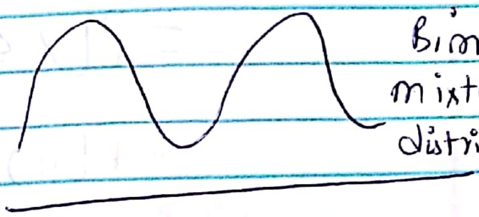
$P(-\infty < X \leq \infty) = 1$

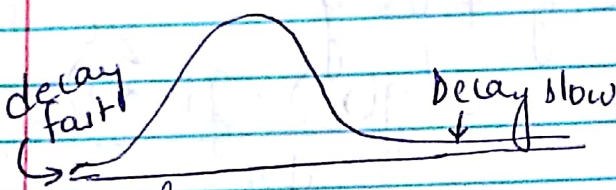
$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$P(a < X \leq b) = F(b) - F(a)$

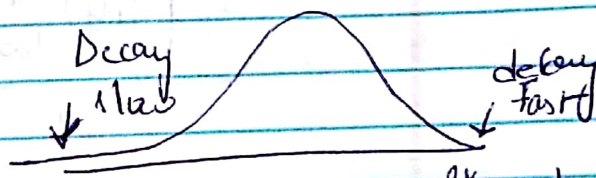
eg -







Positively Skewed



Negatively Skewed.

— Properties of Pdf

Suffice to Check: $f(x) \geq 0 \quad \forall x \in \mathbb{R}^1$

$\int_{-\infty}^{\infty} f(x) = 1$ to determine weather f is a Pdf.

For ~~ ~~ for $a < b$

$P(a < X < b) = P(a < X \leq b) = P(a \leq X \leq b)$

$= P(a \leq X \leq b)$

$= \int_a^b f(x)\,dx = F(b) - F(a)$

In particular, for any singleton $\{x\}$
$$P(X=x)=0$$

- To obtain cdf from pdf ~

$$F(x) = \int_{-\infty}^{x} f(y)\,dy$$
$$= P(-\infty < X \leq x)$$
$$= P(X \leq x)$$
$$= F(x)$$

$f(x) = F'(x)$

$\frac{d}{dy} F(y) \Big|_{y=x}$

$\Rightarrow \frac{d\left[\int_{-\infty}^{x} f(y)\,dy\right]}{dx}$

Call antiderivative of f as g

$$\frac{d\left(\left[g(y)\right]_{-\infty}^{x}\right)}{dx} = \frac{d\left(g(x) - g(-\infty)\right)}{dx}$$

c

$$= \frac{d\,g(x)}{dx} = f(x)$$

X

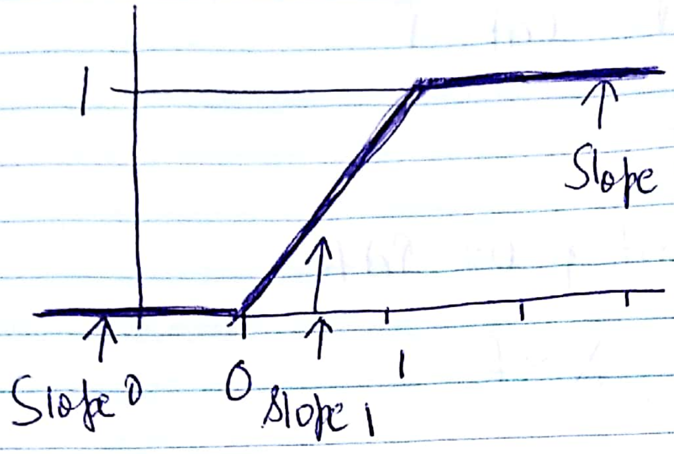
- Uniform $[0,1]$ r.V

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

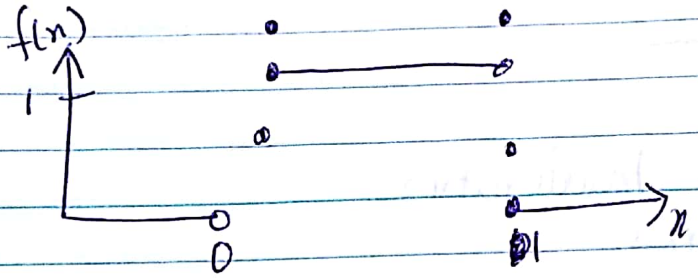
$P(X \leq c) = 0, \quad c < 0$ ✓

$P(X \leq c) = c, \quad 0 \leq c < 1$ ✓

$P(X \leq c) = 1, \quad c \geq 1$ ✓



graphical form



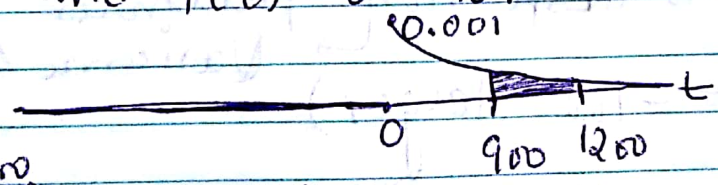
$f(x) = \begin{cases} 1 & 0 \le x \\ 0 & \text{otherwise} \end{cases}$

eg – Life time 'T' of an electrical component has pdf $f(t) = 0$ for $t < 0$.

Exponential Distribution $f(t) = 0.001 \exp(-0.001t)$ for $t \geq 0$

Find Prob. that component last between 900 & 1200 hours.

and $f(t) = 0$ for $t < 0$



Shaded Area $= \int_{900}^{1200} 0.001 \exp(-0.001t)\, dt$

$= \Big[-\exp(-0.001t)\Big]^1 \quad = 0.001 \exp(-0.001t)$

$= \exp(-0.001t)\Big]_{900}^{1200}$

$= \exp(-1.2) + \exp(-0.9) = 0.1054$

r.v X with cdf F

$\Rightarrow X \sim F$

if it has pmf p or pdf f

$\Rightarrow X \sim p$, $X \sim f$

– Parameters

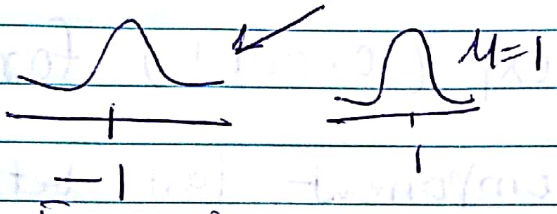
attributes

Numerical characteristics

Summaries

μ and σ for normal

pdf with $\mu = -1$



μ is a location parameter

(μ)

Note ~ Even if we change $\sigma \Rightarrow$ Location don't change (expected value percentiles)

~ σ is a dispersion parameter

↳ Variance Standard deviation

$\sigma = 100$ (larger)