STAT 401 Lecture Notes 3/19

Bookstore: Purchased 3 books @ \$6 each

- sells them at \$12 each

X : # of copies sold

X	0	1	2	3
$P(X)$	.1	.2	.2	.5

- Y is net revenue (unsold copies are returned for \$2 each)
- we need $E[Y]$

$$Y = 12X + 2(3-X) - 18$$
$$= 12X + 6 - 2X - 18$$
$$= 10X - 12$$

- $E[X] = .2 + .4 + 1.5 = 2.1$

$$E[Y] = E[10X - 12] = 10E[X] - 12 \Rightarrow 10(2.1) - 12 = 9$$

Ex.) Time T in days for completion of a project is r.v. with pdf $f_T(t) = .1e^{(-.1t)}$

- If $T > 15$ then cost $= \$10(T-15)$
- If $T < 15$ then cost $= \$5(15-T)$

need to find expected cost

$$\text{Cost}(T) = h(T) = \begin{cases} 5(15-T) & T \le 15 \\ 10(T-15) & T \ge 15 \end{cases}$$

$$E[C] = \int_{-\infty}^{\infty} h(t) f_T(t)\,dt = \int_0^{\infty} h(t) f_T(t)\,dt$$

$$= \int_0^{15} h(t) f_T(t)\,dt + \int_{15}^{\infty} h(t) f_T(t)\,dt$$

$$= \int_0^{15} 5(15-T)(.1e^{-.1t})\,dt + \int_{15}^{\infty} 10(T-15)(.1e^{-.1t})\,dt$$

$$= 75\int_0^{15} .1e^{-.1t}\,dt - 5\int_0^{15} t\,.1e^{-.1t}\,dt + 10\int_{15}^{\infty} t\,.1e^{(-.1t)}\,dt - 150\int_{15}^{\infty} .1e^{(-.1t)}\,dt$$

$$\int_0^{15} .1e^{-.1t}\,dt = \left[-e^{-.1t}\right]_0^{15} = 1 - e^{-1.5} = .777$$

$$\int_{15}^{\infty} .1e^{-.1t}\,dt = \left[-e^{-.1t}\right]_{15}^{\infty} = e^{-1.5} = .223$$

$$\left[te^{-.t}\right]' = e^{-.1t} - t\,.1e^{-.1t} \Rightarrow te^{-.1t}\Big]_a^b = \int_a^b e^{-.1t}\,dt - \int_a^b t\,.1e^{-.1t}\,dt$$

$$= \int_a^b .1e^{-.1t}\,dt = \int_a^b e^{-.1t}\,dt - \left(te^{-.1t}\right)_a^b$$

$$\int_0^{15} e^{-.1t}\,dt - \left[te^{-.1t}\right]\Big|_0^{15} = 10 - 25e^{-1.5} = 4.42$$

$$\int_{15}^{\infty} e^{-.1t}\,dt - \left[te^{-.1t}\right]\Big|_{15}^{\infty} = \frac{e^{-.1t}}{-.1}\Bigg]_{15}^{\infty} + 15e^{-1.5} = 25e^{-1.5} = 5.578$$

$$\Rightarrow 75(.777) - 5(4.42) + 10(.223) - 150(5.578) = -798 \; ??$$

<u>Variance</u> of a r.v. X denoted by $V(X)$ or σ^2_X is defined to be

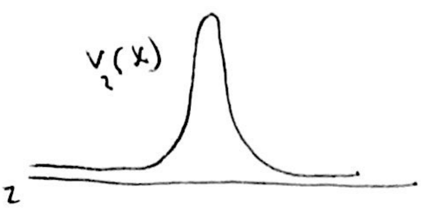
$$Var(X) = E\left[\left(X - E(X)\right)^2\right]$$

<u>Standard Deviation</u> donated by $sd(X)$ or σ_X is defined to be the square root of the variance: $sd(X) = \sqrt{Var(X)}$

$$h(x) = \left(X - E(X)\right)^2 = E\left[x^2 - 2xE(x) + E[x]^2\right] = E(x^2) - 2E[x]^2 + E(x)^2$$

$$= E[x^2] - E[x]^2$$ (variance must be non-negotive)

$>$



Variance Example:

X : outcomes of fair die

find $V[X]$

$= E[X^2] - E[X]^2$

$$\frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$

$$\frac{1+2+3+4+5+6}{6} = \left(\frac{21}{6}\right)^2 = \frac{49}{4}$$

$$V[X] = \frac{91}{6} - \frac{49}{4} = 2.917$$