4/18/18

Sample version of correlation
$(X_1, Y_1), \dots (X_n, Y_n)$
sample

Sample variance: $\sigma_X = S^2_X \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$

Sample Covariance

$$\hat{\nabla}^2_{X,Y} = S_{X,Y} = \frac{1}{n-1} \sum_{i=1} (X_i - \hat{X})(Y_i - \hat{Y})$$

Sample correlation

$$\hat{\rho}_{X,Y} = r_{X,Y} = \frac{\hat{\nabla}_{X,Y}}{\sigma_X \cdot \sigma_Y}$$

ex: X force applied to beam for 150 hours
Y 1 or 0 depending on whether it fails or not

$$X = \begin{cases} 4 & \text{w.p } 0.3 \\ 5 & \text{w.p } 0.5 \\ 6 & \text{w.p } 0.2 \end{cases} \quad (\text{in } 100\text{-lb units})$$

Suppose $P(Y=1 \mid X=x) = \dfrac{(-0.8+0.04x)^4}{1+(-0.8+0.04x)^4}$

Joint pmf $\underbrace{\quad}_{1 \quad\quad 0}$

$X \begin{cases} 4 \\ 5 \\ 6 \end{cases}$ Try to tabulate

X and Y independent

$X \not\perp Y$

$P(Y=1 \mid X=4) \neq P(Y=4 \mid X=5)$

Find average force applied to beams that fail

$$P(X=x \mid Y=1) = \frac{P(X=x, Y=1)}{P(Y=1)}$$

$$= \frac{P(Y=1 \mid X=x)\, P(X=i)}{P(Y=1)}$$

$$P(Y=1) = \sum_{x=4}^{6} P(Y=1, X=x)$$

$$= \sum_{x=4}^{6} P(Y=1 \mid X=x)\, P(X=x)$$

reduce to mean calculation for invariant discrete random variable

Average force applied to beams that do not fail

$$P(X=x \mid Y=0) = \frac{P(X=x \mid Y=0)}{P(Y=0)} = \frac{P(Y=0 \mid X=x)\, P(X=x)}{P(Y=0)}$$

$$= \frac{[1 - P(Y=1 \mid X=x)]\, P(X=x)}{1 - P(Y=1)}$$ => use this to calculate the mean

(X, Y)

$$f(x,y) = \begin{cases} Ae^{-2(x+y)} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Scanned with CamScanner

find A.

$$\int_0^\infty \int_0^\infty A e^{-(2x+y)}\, dx\, dy = A\int_0^\infty e^{-y} dy \int_0^\infty e^{-2x} dx$$

$$= A\left[e^{-y}\right]_0^\infty \left[\frac{-e}{2}^{-2x}\right]_0^\infty = A\,{}^1/_2 \Rightarrow A = 2$$

$$P(X<2, Y<1) = \int_0^2 \int_0^1 2e^{-2x} e^{-y}\, dy\, dx = 2\int_0^2 e^{-2x} \int_0^1 e^{-y} dy$$

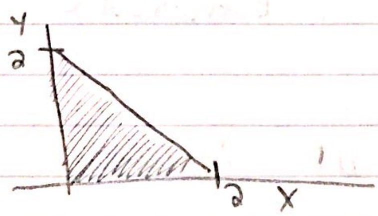
$$= 2\left[-\frac{e^{2x}}{2}\right]_0^2 \left[-e^{y}\right]_0^1 = 2\left[\frac{1}{2} - \frac{e}{2}^{-4}\right]\left[1-e^{-1}\right]$$

$$= \left[1-e^{-4}\right]\left[1-e^{-1}\right]$$

marginal pdf of X

$$\int_0^\infty 2e^{-2x} e^{-y}\, dy = 2e^{-2x}\int_0^\infty e^{-y} dy = 2e^{-2x}\left[-e^{-y}\right]_0^\infty$$
$$= 2e^{-2x}$$

$P(X+Y<2)$



$$\int_0^2 \int_0^{2-x} 2e^{-2x} e^{-y}\, dy\, dx$$

$$= \int_0^2 2e^{-2x} \int_0^{2-x} e^{-y} dy\, dx = \int_0^2 2e^{-2x} \left(-e^{-y}\right]_0^{2-x} dx$$

$$= \int_0^2 2e^{-2x}\left[1-e^{-2+x}\right] dx = \int_0^2 2e^{-2x} - 2e^{-2-x}\, dx$$

$$= \left[-e^{-2x}\right]_0^2 - 2e^{-2}\left[-e^{-x}\right]_0^2 = \left[1-e^{-4}\right] - 2e^{-2}\left[1-e^{-2}\right] \Rightarrow$$

$= [1-e^{-4}] - 2[e^{-2} - e^{-4}] = 1 - e^{-4} - 2e^{-2} + 2e^{-4}$

$= 1 + e^{-4} - 2e^{-2}$

Conditional density

$F_{X|Y=y}(x) = \frac{F(x,y)}{f_y(y)} = \frac{2e^{-2x}e^{-y}}{e^{-y}} = 2e^{-2x} = f_x(x)$

$P(X < 2 | Y < 1) = P(X < 2) = [-e^{-2x}]_0^2 = 1 - e^{-4}$

J distribution of
X = height
Y = radius of cylinder

$$F(x,y) = \begin{cases} \frac{3x}{8y^2} & 1 \le x \le 3,\ 0.5 \le y \le 0.75 \\ 0 & \text{otherwise} \end{cases}$$

Find variance of value of randomly selected cylinder

$V = \pi y^2 x$

$$F(V) = \int_1^3 \int_{0.5}^{0.75} \pi y^2 x \frac{3x}{8y^2}\, dy\, dx$$

$$E(V^2) = \int_1^3 \int_{0.5}^{0.75} \pi^2 y^4 x \frac{3x}{8y^2} \, dy\, dx$$

$$Var(V) = E(V^2) - E(V)^2$$

X, Y, Z independent
U(0,1) r.v.
$X_1 = X + Z$
$Y_1 = Y + 2Z$

Find $V(X_1 + Y_1)$ and $V(X_1 - Y_1)$

$V(X_1 + Y_1) = V(X_1) + V(Y_1) + 2\,Cov(X_1, Y_1)$

$V(X_1) = V(X + 2) = V(X) + V(Z) = 1/12 + 1/12 = 1/6$

$V(Y_1) + V(Y + 2Z) = V(Y) + V(2Z) = 1/12 + 4V(2) =$

$= 1/12 + 4/12 = 5/12$

$Cov(X + Z, Y + 2Z) = Cov(X, Y) + 2\,Cov(X, Z) + Cov(Z, Y) + 2\,Cov(Z$

$= 2\,var(Z) = 1/6$

$cov(aX + bY + cZ, dX + eY + fZ)$

$= E[(aX + bY + cZ - aE(X) - bE(Y) - cE(Z)]$
$[aX + eY + fZ + aE(X) + eE(Y) + FE(Z)]$

$var(X_1, Y_1) = 1/6 + 2/12 + 2/6 = 6/12 + 5/12 = 11/12$