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$$\sum_{i=0}^{n} x^i = 1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1-x}$$

$$\sum_{i=0}^{\infty} x^i \text{ for } |x| < 1 = \frac{1}{1-x}$$

since $0.9^{\infty} \approx 0$

A random variable X is a function that associates a number with each outcome of the sample space of a random experiment

$S = \{H, T\}$, $S = \{\text{Red}, \text{Blue}, \text{Green}\}$

A random variable (r.v.) X is a function

$$\underbrace{X}_{\text{function}} : \underbrace{S}_{\text{domain}} \to \underbrace{\mathbb{R}}_{\text{codomain}}$$

range of X denoted by S_X (also denoted by $\underline{x}$) small x

$(\underbrace{S_X}_{\text{range}} = \{0, 1\})$

- sometimes called the sample space

r.v.s are typically denoted by capital letters

$X, Y, Z, X_1, Y_1, Z_1, \dots$

an element of S we denote by ω (omega not w)

$\{\omega : \text{'some condition in } \omega\text{'}\}$

is a subset of S

$\{X(\omega) = x : \text{`some condition on } x\text{'}\}$

- still a subset of S

Ex. $S = \{(a_1, a_2, a_3) : a_i \in \{H, T\}, i = 1, 2, 3\}$

- for each ω in S define:

3 H,T values

$X(\omega)$ = Number of H in ω

$X((T,T,H)) = 1$

$X((H,H,H)) = 3$

$X(\omega)$ = Number of T in ω

$X((T,T,H)) = 2$

$X((H,H,H)) = 0$

$S_x = \{0, 1, 2, 3\}$

Ex. $S = [0, 1]$

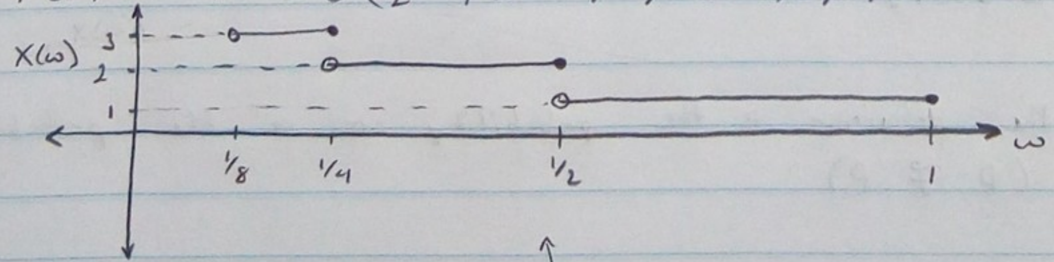
$X(\omega) = \omega$

- randomly picks a number from $[0,1]$, records its coordinate

$S_x = [0, 1]$

$S = (0, 1]$

$X(\omega) = n$ if $\omega \in \left(\frac{1}{2^n}, \frac{1}{2^{n-1}}\right)$, $n = 1, 2, 3, \ldots$



continues to grow smaller as $n \to \infty$

$S_x = \{1, 2, 3, 4, \ldots\}$

Ex. $S = \{$All students in class$\}$

$$X(\omega) = \begin{cases} 1 & \text{if hometown of } \omega \text{ is in PA} \\ 0 & \text{otherwise} \end{cases}$$

$S_X = \{0, 1\}$

Ex. $S = \{$All pairs of students in class$\}$

$X(\omega)$ = Number of students in ω majoring in civil engineering

$S_X = \{0, 1, 2\}$

The (cummulative) distribution function, cdf of X denoted by $F_X(x)$ is defined as $F_X(x) = P(X \leq x), x = \mathbb{R}^1$

$P(\{\omega : X(\omega) \leq x\})$

- probability maps a subset of the sample space S to a number from 0 to 1

F_X defined on $\mathbb{R}^1 = (-\infty, \infty)$

range is $[0, 1]$

P in the definition is the probability are in the probability space $(P, \mathcal{F}, P)$

Ex. $X(\omega) = \#$ of H's

$S = \{(a_1, a_2, a_3) : a_i = \{H, T\}, i = 1, 2, 3\}$

$P(X \leq 1) = P(\{(T, T, T), (H, T, T), (T, H, T), (T, T, H)\}) = \frac{4}{8} = \frac{1}{2}$