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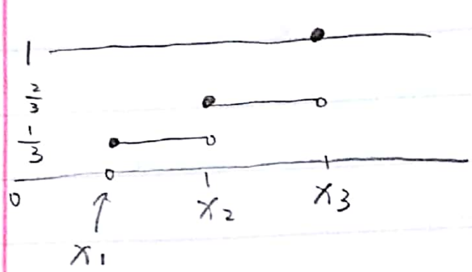
$F_X(x) = P(X \le x)$ not $<$

$P(\{w : X(w) \le x\})$ not x



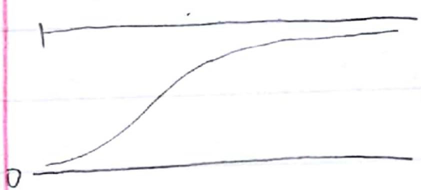
$F_X(x_1) = \frac{1}{3} = P(X \le x_1) = \frac{1}{3}$

but $P(X < x_1) = 0$

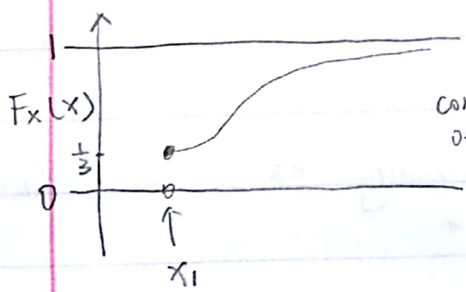
because for $F_X(x)$ when $x < x_1$, $F_X(x) = 0$

corresponds to a discrete random variable

$$X = \begin{cases} x_1 & \text{w.p. } \frac{1}{3} \\ x_2 & \text{w.p. } \frac{1}{3} \\ x_3 & \text{w.p. } \frac{1}{3} \end{cases}$$



continuous



combination of continuous & discrete

$$X = \begin{cases} x_1 & \text{w.p. } \frac{1}{3} \\ Y & \text{w.p. } \frac{2}{3} \end{cases}$$

Y are exponential with mean plus X_1

Properties of cdf

The cdf F of any r.v. X satisfies the following properties

① if $a < b$ $F(a) \le F(b)$

$P(X \le a) \le P(X \le b)$

$\{X \le a\} \subseteq \{X \le b\}$

Scanned by CamScanner

② $F(-\infty) = P(X \le -\infty) = 0$

$F(\infty) = P(X \le \infty) = 1$

③ if $a < b$

$P(a < X \le b) = P(X \le b) - P(X \le a)$

$= F(b) - F(a)$

$\{x \le a\}$

$\{w : X(w) \le a\}$

$\{x \le a\}$ & $\{a < x \le b\}$ then disjoint & the union makes $\{x \le b\}$

look at cdf

Types random variable

step wise function

$\Rightarrow$ discrete r.v.

continuous function

$\Rightarrow$ continuous r.v.

X is discrete

S_x continous at most countably many elements

↑

range

$S_x = \{x_1, x_2, x_3 \cdots, x_n\}$

X is continuous if S_x contains an interval

$[a, b] \subseteq S_x$, $a < b$, $a, b \in \mathbb{R}$

then

X is continuous

exercise 2.3.15

we select 5 cards from 52

(a) 5 cards contain all form 2's

$\binom{52}{5}$ $\boxed{2}$ $\boxed{2}$ $\boxed{2}$ $\boxed{2}$ $\boxed{}$

$\uparrow$

denominator

$$\frac{48}{\binom{52}{5}}$$

(b) 5 cards contain two aces and two 7's

$\binom{52}{5}$ $\boxed{A}$ $\boxed{A}$ two suits from 4

$\uparrow$

denominator

$\binom{4}{2}\binom{4}{2}$

$\boxed{7}$ $\boxed{7}$

$$\frac{\binom{4}{2}\binom{4}{2}44}{\binom{52}{5}}$$

(c) 5 cards contain 3 K's and the other two are of different denomination

$\binom{52}{5}$ $\binom{4}{1}$ because 4 ways to choose a suit not to include in the triple

$\binom{n}{1} = n$

$\frac{4!}{3!1!} = 4$ $\binom{n}{n-1} = n$

$$\frac{4\binom{48}{2}}{\binom{52}{5}}$$

in choosing two cards from 48 with denomination 1 to 2

there are R that result in a pair

AA, 22, 33, ... QQ

sample space

$$\frac{4\left[\binom{48}{2}-12\right]}{\binom{52}{5}}$$ $$\frac{\binom{48}{1}\binom{46}{1}}{2}$$

$$\frac{\binom{4}{1}\left(\binom{48}{2}-12\times\binom{4}{2}\right)}{\binom{52}{2}}$$

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cdf. $F_X(X)$ of r.v. X, summarizes all useful informations about the r.v.

$F(X)$, $F(X^2)$, $F(X^3)$ ....

and every r.v. has a cdf F_X.

$P(X \le x)$

discrete r.v. probability mass function (pmf).

$S_X = \{x, \dots x_u\}$ it's continuous r.v.

then probability density function (pdf)