4. (a) X is Binomial R.V. with $n = 20$ and $p = 0.5$. Thus, $E(X) = 10 \times 0.5 = 5$ and $\text{Var}(X) = 10 \times 0.5 \times 0.5 = 2.5$.

   (b) $P(X = 5)$ can be calculated using R command *dbinom(5, 10, 0.5)*, which gives 0.2461.

   (c) This is $P(X \leq 5) = F(5)$, which can be calculated by the command *pbinom(5, 10, 0.5)* and the result is 0.6230.

   (d) Y is the total number of incorrectly answered questions.

   (e) $P(2 \leq Y \leq 5) = P(2 \leq 10 - X \leq 5) = P(5 \leq X \leq 8) = P(X \leq 8) - P(X \leq 4) = F(8) - F(4)$. This can be calculated by the command *pbinom(8, 10, 0.5)-pbinom(4, 10, 0.5)* and the result is 0.6123.

5. (a) X is Binomial R.V. with $n = 10$ and $p = 0.9$.

   (b) $E(X) = 10 \times 0.9 = 9$ and $\text{Var}(X) = 10 \times 0.9 \times 0.1 = 0.9$.

   (c) $P(X \geq 7) = 1 - P(X \leq 6)$, the R command is *1-pbinom(6, 10, 0.9)*, and the result is 0.9872.

   (d) Let Y be the catering cost, then $Y = 100+10X$, thus $E(Y) = 100+10E(X) = 190$, and $\text{Var}(Y) = 100\text{Var}(X) = 90$.

6. (a) Let X be the number of guilty votes when the defendant is guilty, then X has a Binomial distribution with $n = 9$ and $p = 0.9$. In order to convict a guilty defendant, we must have $X > 4$. Thus, the probability of convicting is $P(X > 4) = 1 - P(X \leq 4)$, which could be calculated by *1- pbinom(4,9,0.9)* resulting in 0.9991. Similarly, the probability of convicting an innocent defendant is *1 - pbinom(4,10,0.1) = 0.00089*. Thus, the proportion of all defendants convicted is $0.4 \times 0.00089 + 0.6 \times 0.9991 = 0.5998$.

   (b) Let G and I be the defendant is guilty and innocent, respectively and let VG and VI be the events that the defendant is voted as guilty and innocent, respectively. Then, $P(G) = 0.6$, $P(I) = 0.4$ and $P(VG|G) = 0.9991$ from part (a). Let Y be the number of guilty votes when the defendant is innocent. Then Y has a Binomial distribution with $n = 9$ and $p = 0.1$. Thus, $P(VI|I) = P(Y \leq 4)$, which could be calculated by *pbinom(4,9,0.1)* resulting in 0.9991. Thus,

$$\begin{aligned} P(\text{Correct}) &= P((VG \cap G) \cup (VI \cap I)) = P(VG|G)P(G) + P(VI|I)P(I) \\ &= 0.9991 \times 0.6 + 0.9991 \times 0.4 = 0.9991. \end{aligned}$$

7. (a) The R. V. X follows Negative Binomial distribution with parameter $r = 1$ and $p = 0.3$.

   (b) The sample space is $S = \{1, 2, \cdots\}$, and $p(x) = P(X = x) = p(1 - p)^{x-1}$.

   (c) $E(X) = 1/p = 3.33$, and $\text{Var}(X) = (1 - p)/p^2 = 7.778$.

9. (a) Let X be the number of games needed for team A to win twice, then X has the negative binomial distribution with $r = 3$ and $p = 0.6$. Team A will win the series if $X = 3$ or $X = 4$ or $X = 5$. Thus, the probability can be calculated using the command *sum(dnbinom(0:2, 3, 0.6))*, which gives 0.6826.

   (b) The probability for a better team to win a best-of-five series is larger. With more games played, the better team will win more games.

10. (a) The R. V. X follows Negative Binomial distribution with parameter $r = 1$ and $p = 0.01$.

    (b) The sample space is $S = \{1, 2, \cdots\}$ and $p(x) = P(X = x) = p(1-p)^{x-1}$.

    (c) $E(X) = 1/p = 100$.

11. (a) The R. V. Y follows Negative Binomial distribution with parameter $r = 5$ and $p = 0.01$.

    (b) $E(Y) = r/p = 500$ and $\mathrm{Var}(X) = r(1-p)/p^2 = 49500$.

13. (a) The R. V. X follows Hypergeometric distribution with parameter $n = 5$, $M_1 = 3$ and $M_2 = 17$. $N = M_1 + M_2 = 20$.

    (b) The sample space is $S_X = \{0, 1, 2, 3\}$ and the PMF is

$$p(x) = P(X = x) = \frac{\binom{3}{x}\binom{17}{5-x}}{\binom{20}{5}}.$$

    (c) The R command is *dhyper(1, 3, 17, 5)* and it gives 0.4605.

    (d) $E(X) = nM_1/N = 5 \times 3/20 = 0.75$, and

$$\mathrm{Var}(X) = n\frac{M_1}{N}\left(1 - \frac{M_1}{N}\right)\frac{N-n}{N-1} = 5\frac{3}{20}\left(1 - \frac{3}{20}\right)\frac{20-5}{20-1} = 0.5033.$$

16. (a) Poisson.

    (b) $E(Y) = \mathrm{Var}(Y) = \lambda = 1800 \times 0.6 = 1080$.

    (c) The R command is *1-ppois(1100, 1080)* and it gives 0.2654.

17. Let X be the number of loads during the next quarter, then X has a Poisson distribution with $\lambda = 0.5$. We are looking for $P(X > 2)$, the R command *1-ppois(2, 0.5)* gives us 0.0144.