1. (a) Let μ be the mean soil heat flux, then the null and alternative hypotheses are

$$H_0 : \mu \leq 31 \quad \text{vs.} \quad H_a : \mu > 31.$$

(b) If the null hypothesis is rejected, we should use the coal dust cover.

2. (a) The null and alternative hypotheses are

$$H_0 : p \leq 0.25 \quad \text{vs.} \quad H_a : p > 0.25.$$

(b) If the null hypothesis is rejected, we should adopt the modified bumper design.

3. (a) If the CEO wants to adopt it unless there is evidence that it has a lower protection index, then $H_a : \mu < \mu_0$.

(b) If the CEO does not want to adopt it unless there is evidence that it has a higher protection index, then $H_a : \mu > \mu_0$.

(c) If the null hypothesis is rejected for part (a), the CEO should not adopt the new grille guard and for part (b), the CEO should adopt the new grille guard.

4. (a) If the manufacturer does not want to buy the new machine unless there is evidence it is more productive than the old one, then $H_a : \mu > \mu_0$.

(b) If the manufacturer wants to buy the new machine unless there is evidence it is less productive than the old one, then $H_a : \mu < \mu_0$.

(c) If the null hypothesis is rejected for part (a), the manufacturer should buy the new machine and for part (b) the manufacturer should not buy the new machine.

5. (a) Let p be the proportion of all customers who qualify for membership, then the hypotheses are

$$H_0 : p \geq 0.05 \quad \text{vs.} \quad H_a : p < 0.05.$$

(b) If the null hypothesis is rejected, the airline should not proceed with the establishment of the traveler's club.

6. (a) The statement is true.

(b) We determine C from the requirement that the probability of incorrectly rejecting H_0 is no more than 0.05 or, in mathematical notation,

$$P(\bar{X} \geq C) \leq 0.05 \text{ if } H_0 \text{ is true.}$$

Over the range of μ values specified by H_0 (i.e., $\mu \leq 28,000$), the probability $P(\bar{X} \geq C)$ is largest when $\mu = 28,000$. Thus, the requirement will be satisfied if C is chosen so that when $\mu = 28,000$, $P(\bar{X} \geq C) = 0.05$. This is achieved by choosing C to be the 95th percentile of the distribution of $\bar{X}$ when $\mu = 28,000$. Recall that σ is assumed to be known, this yields $C = 28,000 + z_{0.05}\sigma/\sqrt{n}$.

(c) Let

$$Z_{H_0} = \frac{\bar{X} - 28,000}{\sigma/\sqrt{n}}.$$

Then the standardized version of the rejection region is $Z_{H_0} \geq z_{0.05}$.

8. (a) Let p be the proportion of all detonators that will ignite, then the null and alternative hypotheses are

$$H_0 : p \geq 0.9 \quad \text{vs.} \quad H_a : p < 0.9.$$

(b) The standardized test statistic is

$$Z_{H_0} = \frac{\hat{p} - 0.9}{\sqrt{0.9 \times 0.1/n}}.$$

(c) The statement is false.

10. (a) The value of the test statistic is

$$Z_{H_0} = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{28640 - 28000}{900/\sqrt{25}} = 3.556.$$

Since the RR is $Z_{H_0} \geq z_\alpha$, the smallest level at which H_0 is rejected is found by solving $3.556 = z_\alpha$ for α. Let $\Phi(\cdot)$ be the cumulative distribution function of $N(0,1)$. The solution to this equation, which is also the p-value, is

$$p - \text{value} = 1 - \Phi(3.556) = 0.00019.$$

(b) Since p-value< 0.05, the null hypothesis should be rejected at a 0.05 level of significance.

11. (a) The standardized test statistic is

$$Z_{H_0} = \frac{8/50 - 0.25}{\sqrt{0.25 \times 0.75/50}} = -1.469694.$$

To sketch the figure, draw a $N(0,1)$ PDF and shade the right of -1.469694, which represents the p-value.

(b) The p-value is calculated as 0.9292. Since p-value> 0.05, the null hypothesis should not be rejected at a 0.05 level of significance.

1. (a) The value of test statistic is

$$T_{H_0} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{9.8 - 9.5}{1.095/\sqrt{50}} = 1.94.$$

The RR is $T_{H_0} > t_{n-1,\alpha} = t_{49,0.05} = 1.68$. Since $1.94 > 1.68$, we should reject H_0.

(b) Since the sample size $n = 50 > 30$, no additional assumptions are needed.

2. (a) Let μ be the average permissible exposure, then the null and alternative hypotheses are

$$H_0 : \mu \leq 1 \quad \text{vs.} \quad H_a : \mu > 1.$$

(b) The value of test statistic is

$$T_{H_0} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{2.1 - 1}{4.1/\sqrt{36}} = 1.61.$$

The RR is $T_{H_0} > t_{n-1,\alpha} = t_{35,0.05} = 1.69$. Since $1.69 > 1.61$, we should not reject H_0. Since the sample size $n = 36 > 30$, no additional assumptions are needed.

(c) By using Table A4 the p-value should be between 0.05 and 0.1. Using the R command *1-pt(1.61,35)*, the exact p-value is 0.058.

3. (a) Let μ be the (population) mean penetration, then the null and alternative hypotheses are

$$H_0 : \mu \leq 50 \quad \text{vs.} \quad H_a : \mu > 50.$$

(b) The value of test statistic is

$$T_{H_0} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{52.7 - 50}{4.8/\sqrt{16}} = 2.25.$$

The RR is $T_{H_0} > t_{n-1,\alpha} = t_{15,0.1} = 1.34$. Since $2.25 > 1.34$, we should reject H_0. In order to make the test valid, we need the assumption that the population is normal.

(c) By using Table A4 p-value should be between 0.01 and 0.025. Using R command *1-pt(2.25, 15)*, the exact p-value is 0.02.

4. (a) The value of test statistic is

$$T_{H_0} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{30.79 - 29}{6.53/\sqrt{8}} = 0.775.$$

The RR is $T_{H_0} > t_{n-1,\alpha} = t_{7,0.05} = 1.89$. Since $1.89 > 0.775$, we should not reject H_0. Using R command *1-pt(0.775, 7)*, the exact p-value is 0.2318.

(b) In order to make the test valid, we need the assumption that the population is normal.

5. (a) The standardized test statistic is

$$Z_{H_0} = \frac{40/500 - 0.05}{\sqrt{0.05 \times 0.95/500}} = 3.0779.$$

The RR is $Z_{H_0} < -z_\alpha = -z_{0.01} = -2.33$. Since $3.0779 > -2.33$, we should not reject H_0, and thus the traveler's club should be established.

(b) The p-value is calculated as 0.999 by using R command *pnorm(3.0779)*.

6. (a) Let p be the proportion of all customers in states east of the Mississippi who prefer the bisque color, then the the null and alternative hypotheses are

$$H_0 : p \leq 0.3 \quad \text{vs.} \quad H_a : p > 0.3.$$

(b) The standardized test statistic is

$$Z_{H_0} = \frac{185/500 - 0.3}{\sqrt{0.3 \times 0.7/500}} = 3.416.$$

The RR is $Z_{H_0} > z_\alpha = z_{0.05} = 1.645$. Since $3.416 > 1.645$, we should reject H_0 at level 0.05.

(c) The p-value is calculated as 0.0003 by using R command *1-pnorm(3.416)*. Since p-value is less than the significant level $\alpha = 0.01$, we should reject H_0.

7. (a) Let p be the proportion of all consumers who would be willing to try this new product, then the the null and alternative hypotheses are

$$H_0 : p \leq 0.2 \quad \text{vs.} \quad H_a : p > 0.2.$$

(b) The standardized test statistic is

$$Z_{H_0} = \frac{9/42 - 0.2}{\sqrt{0.2 \times 0.8/42}} = 0.23.$$

The RR is $Z_{H_0} > z_\alpha = z_{0.01} = 2.33$. Since $0.23 < 2.33$, we should not reject H_0. The p-value is calculated as 0.41 by using R command *1-pnorm(0.23)*. Thus, there is not enough evidence that the marketing would be profitable.

1. (a) When a null hypothesis is rejected, there is risk of committing Type I error.

   (b) When a null hypothesis is not rejected, there is risk of committing Type II error

2. (a) True

   (b) False

   (c) False

3. (a) To calculate Type I error, we have

$$
\begin{aligned}
P(\text{Type I Error}) &= P(H_0 \text{ is rejected when it is true}) \\
&= P(X \geq 8 | p = 0.25, n = 20) = \sum_{k=8}^{20} \binom{20}{k} 0.25^k 0.75^{20-k},
\end{aligned}
$$

   because under this situation, the random variable X has a binomial distribution with $n = 20$ and $p = 0.25$. This probability can be calculated using R command *1-pbinom(7,20,0.25)*, which gives us 0.1018 as the probability of Type I error.

   (b) We first calculate the probability of Type II error as

$$
\begin{aligned}
P(\text{Type II Error when } p = 0.3) &= P(H_0 \text{ is not rejected when } p = 0.3) \\
&= P(X < 8 | p = 0.3, n = 20) = \sum_{k=0}^{7} \binom{20}{k} 0.3^k 0.7^{20-k},
\end{aligned}
$$

   because under this situation, the random variable X has a binomial distribution with $n = 20$ and $p = 0.3$. This probability can be calculated using R command *pbinom(7,20,0.3)*, which gives us 0.7723 as the probability of Type II error. Finally, the power is 1-0.7723 = 0.2277.

   (c) When $n = 50$ and rejection region is $X \geq 17$, the probability of Type I error can be found by the command *1-pbinom(16,50,0.25)*, which gives us 0.0983. The power at $p = 0.3$ can be found by the command *1-pbinom(16,20,0.3)*, which gives us 0.316. We found that as the sample size increases, we have a smaller probability of Type I error and more power.

4. (a) The R command *1-pwr.t.test(36, (2-1)/4.1, 0.05, power=NULL, "one.sample", "greater")$power* returns 0.583 as the probability of Type II error when the true concentration is 2 ppm.

   (b) The R command *pwr.t.test(n=NULL, (2-1)/4.1, 0.05, 0.99, "one.sample", alternative="greater")* returns a sample size of 266.46, which is rounded up to 267.

5. In this problem, the hypotheses tells us that $\mu_0 = 8.5$. We require that the probability of delivering a batch of acidity 8.65 should not exceed 0.05, thus, $\mu_a = 8.65$ and The type II error is 0.05, therefore the power is 0.95. We also know that the standard deviation from a preliminary study is 0.4, therefore, $S_{pr} = 0.4$. Combining this information, we use the commands *library(pwr); pwr.t.test(n=NULL, (8.65-8.5)/0.4, 0.05, 0.95, "one.sample", alternative="greater")*, which returns a sample size of 78.33 and is rounded up to 79.

6. (a) In this test, the testing statistic is

$$Z_{H_0} = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

and because of $H_a : p > 0.2$, the rejection region is $Z_{H_0} > z_\alpha$. Thus, the probability of Type II error at $p_a = 0.25$ is

$$\begin{aligned}\beta(0.25) = P(\text{Type II error}|p = 0.25) &= P\left(\frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} < z_\alpha \middle| p = 0.25\right) \\ &= P\left(\hat{p} < p_0 + z_\alpha\sqrt{p_0(1-p_0)/n} \middle| p = 0.25\right) \\ &= \Phi\left(\frac{p_0 + z_\alpha\sqrt{p_0(1-p_0)/n} - p}{\sqrt{p(1-p)/n}}\right).\end{aligned}$$

For the calculation, we use the command
*pnorm((0.2+qnorm(1-0.01)*sqrt(0.2*0.8/42)-0.25)/sqrt(0.25*0.75/42))*
and it gives 0.9193 as the probability of Type II error.

(b) To achieve power of 0.3 at $p_a = 0.25$ while keeping the level of significance at 0.01, we should use the commands *library(pwr); h=2*asin(sqrt(0.25))-2*asin(sqrt(0.2)); pwr.p.test(h, n=NULL, 0.01, 0.3, alternative="greater")*. The code returns a sample size of 225.85 and is rounded up to 226.

7. (a) In this test, the testing statistic is

$$Z_{H_0} = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

and because of $H_a : p < 0.05$, the rejection region is $Z_{H_0} < -z_\alpha$. Thus, the probability of Type II error at $p_a = 0.04$ is

$$
\begin{aligned}
\beta(0.04) &= P(\text{Type II error}|p = 0.04) = P\left(\frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} > -z_\alpha \middle| p = 0.04 \right) \\
&= P\left(\hat{p} > p_0 - z_\alpha \sqrt{p_0(1-p_0)/n} \middle| p = 0.04 \right) \\
&= 1 - \Phi\left(\frac{p_0 - z_\alpha \sqrt{p_0(1-p_0)/n} - p}{\sqrt{p(1-p)/n}} \right).
\end{aligned}
$$

For the calculation, we use the command
*1-pnorm((0.05-qnorm(1-0.01)*sqrt(0.05*0.95/500)-0.04)/sqrt(0.04*0.96/500))*
and it gives 0.926 as the probability of Type II error.

(b) To achieve power of 0.5 at $p_a = 0.04$ while keeping the level of significance at 0.01, we should use the following commands *library(pwr); h=2*asin(sqrt(0.04))-2*asin(sqrt(0.05)); pwr.p.test(h,n=NULL, 0.01, 0.5, alternative="less")*. The code returns a sample size of 2318.77 and is rounded up to 2319.