~~[illegible]~~ 4/9/18

- (X, Y) discrete r.v.
- Joint PMF $P(x,y)$
- Conditional PMF of Y gives $X=x$ is $P_{X|Y} = y(x) = \frac{P(x,y)}{P_Y(y)}$, $x \in S_x$

Connection w/ §2:

Take $A = \{X = x\}$, $B = \{Y = y\}$

$P(A \cap B) = (P(X=x), P(Y=y)) = P(x,y)$

$P(Y|x) = \frac{P(x,y)}{P_x(x)} = \frac{P(A \cap B)}{P(A)} = P(B|A)$

For fixed $x \in S_x$:

$P(Y|x) = {}_x(y)$ ← is this a valid univeriate point? why? Positivity ✓

$\sum_{y \in S_Y} P_{Y|X=x}(y) = \sum_{y \in S_Y} \frac{P(x,y)}{P_x(x)} = \frac{1}{P_Y(x)} \sum_{y \in Y} P(x,y) = \frac{P_x(x)}{P_x(x)} = 1$ ✓

Calculating conditional probability can be done as long as you have a joint PMF. From $p(x,y)$ you can get the $P_x(x)$ & $P_Y(y)$ w/ these three conditions, the probabilities can be written.

Table:

X \ Y	0	1	2	3
0	0.84	0.03	0.02	0.01
1	0.06	0.01	0.008	0.002
2	0.01	0.005	0.009	0.001

Find the conditional PMF of Y given $X = 0$

$P_x(0) = 0.9 \Rightarrow P_{Y|X=0}(0) = \frac{P(0,0)}{P_x(0)} = \frac{0.84}{0.9} = \frac{14}{15}$

$P_{Y|X=0}(1) = \frac{P(0,1)}{P_x(0)} = \frac{0.03}{0.9} = 1/30$

$$P_{Y|X=0}(2) = \frac{P(0,2)}{P_X(0)} = \frac{0.02}{0.9} = 1/45$$

$$P_{Y|X=0}(3) = \frac{P(0,3)}{P_X(0)} = \frac{0.01}{0.9} = 1/90$$

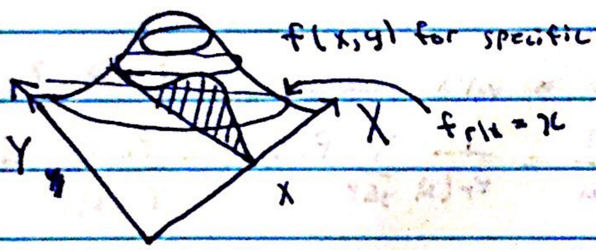
Conditional PDF (X,Y) is continuous random vector.

$P(X=x)=0 \quad \forall \; x \in \mathbb{R}^1$

So, $P(Y=y | X=x) = \dfrac{P(Y=y, X=x)}{P(X=x)}$ is not well defined.

So instead:

$$f_{Y|X=x}(y) = \frac{f(x,y)}{f_X(x)} \qquad f_{X|Y=y}(x) = \frac{f(x,y)}{f_Y(y)}$$



For specific x, $f_X(x)$ is area under curve.

* Same principle applies for specific Y, $f_Y(y)$ *

$$\int_{-\infty}^{+\infty} f_{Y|X=x}(y)\,dy = \int_{-\infty}^{\infty} \frac{f(x,y)}{f_X(x)}\,dy = \frac{1}{f_X(x)} \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \frac{1}{f_X(x)} f_X(x) = 1$$

For any $a < b$ conditional PDF provides way to evaluate $P(a < Y \leq b | X=x) = \int_a^b f_{Y|X=x}(y)\,dy$

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Connection to s^s 2:

- Multiplication Rule

$P(A \cap B) = P(B) P(A|B) = P(A) P(B|A)$

$f(x,y) = f_Y(y) f_{X|Y=y}(x) = f_X(x) f_{Y|X=x}(y)$

Law of Total Probability

$P(B) = \sum_{i=1}^{k} P(A_i) P(B|A_i)$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx = \int_{-\infty}^{\infty} f_X(x) f_{Y|X=x}(y)\, dx$

Ex. Joint PDF of (X,Y) $\quad f(x,y) = \begin{cases} \dfrac{e^{-x/y} e^{-y}}{y}, & x>0,\ y>0 \\ 0, & \text{otherwise} \end{cases}$

Find $f_{X|Y=y}(x)$

$F_Y(y) = \int_0^{\infty} \dfrac{e^{-x/y} e^{-y}}{y} dx = \dfrac{e^{-y}}{y} \int_0^{\infty} e^{-x/y} dx = \dfrac{e^{-y}}{y} \left(-y e^{-x/y}\right)\Big|_0^{\infty}$

$= \dfrac{e^{-y}}{y}(y) = e^{-y}$ for $y>0$ $\quad \left(y \leq 0,\ f_Y(y) = \int_0^{\infty} 0\, dy \right)$

$f_{X|Y=y}(x) = \dfrac{e^{-x/y} e^{-y}}{y e^{-y}} = \dfrac{e^{-x/y}}{y}, \ x>0$

↑ only exists for $y>0$

$P(X>1 \mid Y=3) \Rightarrow f_{X|Y=3}(x) = \dfrac{e^{-x/3}}{3} \Rightarrow \int_1^{\infty} \dfrac{e^{-x/3}}{3} dx$

$= \left[-e^{-x/3}\right]_1^{\infty} = e^{-1/3}$

Condition density (PMF) are valid to use them to obtain conditional expectation & variable.