7.1-16

$$P[a \leq \frac{(n-1)S^2}{\sigma^2} \leq b] = 1-\alpha$$
$$P[\frac{(n-1)S^2}{b} \leq \sigma^2 \leq \frac{(n-1)S^2}{a}] = 1-\alpha$$

Letting $a = \chi^2_{1-\alpha/2}(n-1)$ and $b = \chi^2_{\alpha/2}(n-1)$, a $100(1-\alpha)\%$ confidence interval σ^2 is

$$[\frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}(n-1)}]$$

Thus a 90% confidence interval for σ^2 is

$$[\frac{128.41}{21.03}, \frac{128.41}{5.226}] = [6.11, 24.57]$$

It follows that a 90% confidence interval for σ is

$$\sqrt{[6.11, 24.57]} = [2.47, 4.96]$$

.

7.1-17

$$P(-\frac{\sqrt{n}}{3} < \frac{\sqrt{n}(\bar{x}-\mu)}{3} < \frac{\sqrt{n}}{3}) = 0.9$$
$$P(-1.645 < \frac{\sqrt{n}(\bar{x}-\mu)}{3} < 1.645) = 0.9$$

$\frac{\sqrt{n}}{3} = 1.645$, thus $n \approx 25$

7.2-8

(a)

$$\bar{X} \sim N(\mu_x, \frac{\sigma_x^2}{n}) = N(\mu_x, \frac{d\sigma_y^2}{n})$$
$$\bar{Y} \sim N(\mu_y, \frac{\sigma_y^2}{n})$$
$$\bar{X}-\bar{Y} \sim N(\mu_x-\mu_y, \frac{d\sigma_y^2}{n} + \frac{\sigma_y^2}{n})$$
$$\frac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\sqrt{\frac{d\sigma_y^2}{n} + \frac{\sigma_y^2}{n}}} \sim N(0,1)$$

(b)

$$\frac{(n-1)S_x^2}{d\sigma_y^2} \sim \chi_2(n-1)$$
$$\frac{(m-1)S_y^2}{\sigma_y^2} \sim \chi_2(m-1)$$
$$\frac{(n-1)S_x^2}{d\sigma_y^2} + \frac{(m-1)S_y^2}{\sigma_y^2} \sim \chi_2(n+m-2)$$

(c) $\bar{X}$ and σ_X^2 are independent, and $\bar{Y}$ and σ_Y^2 are independent. Thus $\frac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\sqrt{\frac{d\sigma_y^2}{n}+\frac{\sigma_y^2}{m}}}$ and $\frac{(n-1)S_x^2}{d\sigma_y^2} + \frac{(m-1)S_y^2}{\sigma_y^2}$ are independent.

(d)

$$\frac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\sqrt{\frac{d\sigma_y^2}{n}+\frac{\sigma_y^2}{m}}} \Big/ \sqrt{\left[\frac{(n-1)S_x^2}{d\sigma_y^2}+\frac{(m-1)S_y^2}{\sigma_y^2}\right]/(n+m-2)} \sim t_{n+m-2}$$

Clearly, this ratio does not depend upon σ_Y^2; so

$$\bar{x}-\bar{y} \pm t_{\alpha/2}(n+m-2)\sqrt{\frac{(n-1)s_x^2/d+(m-1)s_y^2}{n+m-2}\left(\frac{d}{n}+\frac{1}{m}\right)}$$

provides a $100(1-\alpha)\%$ confidence interval for $\mu_X-\mu_Y$.

7.2-14

$$P\left(c \le \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \le d\right) = 1-\alpha$$
$$P\left(c\frac{S_X^2}{S_Y^2} \le \frac{\sigma_X^2}{\sigma_Y^2} \le d\frac{S_X^2}{S_Y^2}\right) = 1-\alpha$$

where $c = F_{1-\alpha/2}(8,12)$ and $d = F_{\alpha/2}(8,12)$, $12s_x^2 = 128.41$ and $8s_y^2 = 36.72$, a confidence interval for σ_x^2/σ_y^2 is

$$\left[\frac{10.7008}{5.67\times 4.59}, 4.50\frac{10.7008}{4.59}\right] = [0.41, 10.49]$$

For σ_x/σ_y, the 98% confidence interval is

$$\sqrt{[0.41, 10.49]} = [0.64, 3.24]$$

7.3-2

(a) $\hat{p} = \frac{142}{200} = 0.71$

(b) $[0.71 - 1.645\sqrt{\frac{0.71\times 0.29}{200}}, 0.71 + 1.645\sqrt{\frac{0.71\times 0.29}{200}}] = [0.657, 0.763]$

(c) $[0.655, 0.760]$

(d) $\tilde{p} = \frac{142+2}{200+4} = \frac{12}{17} = 0.7059$;

$$\left[\frac{12}{17} - 1.645\sqrt{\frac{(12/17)(5/17)}{204}}, \frac{12}{17} + 1.645\sqrt{\frac{(12/17)(5/17)}{204}}\right] = [0.653, 0.758]$$

(e) $[0, 71 - 1.282\sqrt{\frac{(0.71)(0.29)}{200}}, 1] = [0.669, 1]$

7.4-4 $n = \frac{(1.96)^2(34.9)}{(0.5)^2} = 537$, rounded up to the nearest integer.

7.4-14 For the difference of two proportions with equal sample sizes

$$\varepsilon = z_{\alpha/2}\sqrt{\frac{p_1^*(1-p_1^*)}{n}+\frac{p_2^*(1-p_2^*)}{n}}$$

For unknown p^*,

$$n = \frac{z_{\alpha/2}^2[0.25+0.25]}{\varepsilon^2} = \frac{z_{\alpha/2}^2}{2\varepsilon^2}$$

So $n = \frac{1.282^2}{2(0.05)^2} = 329$, rounded up.

7.5-1

(a) $P(Y_2 < \pi_{0.5} < Y_5) = \Sigma_{k=2}^{4}\binom{6}{k}(0.5)^k(0.5)^{6-k} = 0.7812$

(b) $P(Y_1 < \pi_{0.25} < Y_4) = \Sigma_{k=1}^{3}\binom{6}{k}(0.25)^k(0.75)^{6-k} = 0.7844$

(c) $P(Y_4 < \pi_{0.9} < Y_6) = \Sigma_{k=4}^{5}\binom{6}{k}(0.9)^k(0.1)^{6-k} = 0.4528$