Stat 401

Normal distribution (Gaussin distribution)

Gauss was person who first presented the Normal distribution

defining parameter

$\mu \in R^1$ location parameter

$\sigma > 0$ dispersion parameter

Family of normal distributions

$Z \sim N(0,1)$

location-scale transformation

$X = \mu + \sigma Z$

$E[X] = E[\mu + \sigma Z]$

$= E[\mu] + E[\sigma Z]$

$= \mu + \sigma \underbrace{E[Z]}_{=0}$

$= \mu$

$V[X] = V[X+$

$V[\mu + \sigma Z]$

$= V(\sigma Z)$

$= V(\sigma Z) = \sigma^2 \underbrace{V[Z]}_{=1}$

$= \sigma^2$

$X \sim N(\mu, \sigma^2)$

$f_{\mu,\sigma} = (x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{1}{2}x^2\right\}$

change of variables

$X = \mu + \sigma Z$

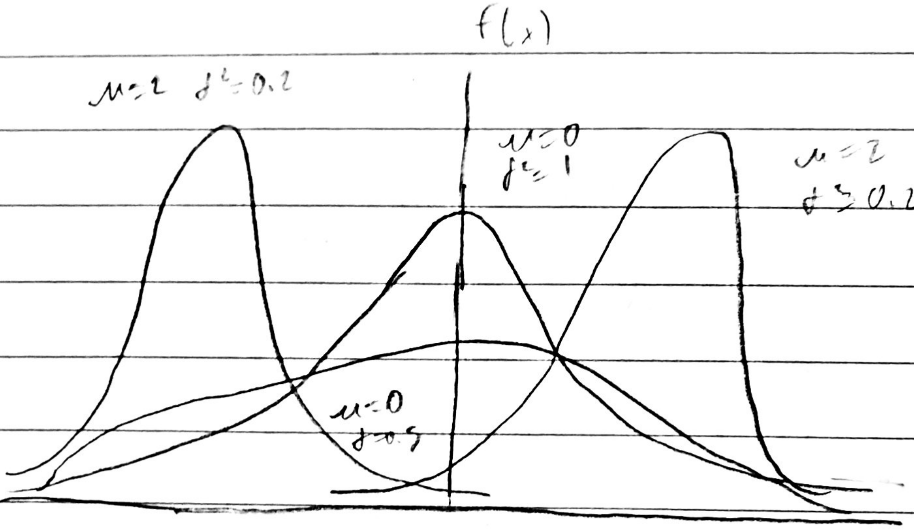
$\frac{X-\mu}{\sigma} = Z$

$\frac{1}{2\pi} \exp\left\{-\frac{1}{2}x^2\right\}$

$\frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma}(\sigma-\mu)^2\right\} \frac{d\sigma}{\sigma}$

$\frac{dx}{dz} = \sigma \qquad dx = \sigma\, dz$

$dz = \frac{dx}{\sigma}$



if $x \sim N(\mu, \sigma^2)$
true

$$Z = \frac{x-\mu}{\sigma}$$

$$\sim N(0,1)$$

$$E[Z] = E\left[\frac{X-\mu}{\sigma}\right]$$

$$= \frac{1}{\sigma} E[X-\mu] = \frac{1}{\sigma}\left[E[X]-\mu\right]$$

$$= \frac{1}{\sigma}[\mu-\mu] = 0$$

$$V[Z] = V\left[\frac{X-\mu}{\sigma}\right]$$

$$= \frac{1}{\sigma} = V[X-\mu]$$

$$= \frac{1}{\sigma^2} V[X] = \frac{1}{\sigma^2}\sigma^2 = 1$$

$$X_\alpha = \mu + \sigma Z_\alpha$$

$$P(X \leq X_\alpha) = \alpha$$

$$P(Z \leq Z_\alpha) = \alpha$$

$X_\alpha + Z_\alpha$ denote 100^{th} percentile of X and Z

$$P(X \leq \mu + \sigma Z_\alpha)$$

$$P\left(\frac{X-\mu}{\sigma} < Z_\alpha\right)$$

$$= P(Z \leq Z_\alpha) = \alpha$$

checked ✓

for any $a \in \mathbb{R}^1$
and $b \in \mathbb{R}^1$

$a + bX \sim N(a + b\mu)$

$E[a+bX] = E[a] + bE[X]$

$= a + b\mu$

$V[a+bX]$

$= V[bX]$

$= b^2 V[X] = b^2\sigma^2$

$x + c$

x

$sd[bX] = b\, sd[X]$

Variance is square of sd

$Z = \dfrac{X - \mu}{\sigma}$ standardization

it implies that you only need the percentiles for the standard normal

$a + bx$ becomes normal again is X is normal

The Normal family is closed under linear transformation

$X \sim N(1.25, 0.46^2)$

$X_{0.75}$

$P(X < X_{0.75})$

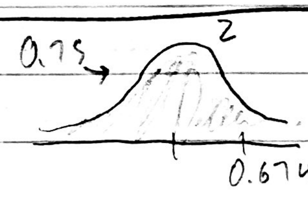
$P\left(\dfrac{X - 1.25}{0.46} \leq \dfrac{X_{0.75} - 1.25}{0.46}\right) = 0.75$

$P\left(Z \leq \dfrac{X_{0.75} - 1.25}{0.46}\right) = 0.75$

$P(Z < Z_\alpha) = 0.75$

$\dfrac{X_{0.75} + 0.75}{0.46} = Z_\alpha$

$Z_{\frac{1}{4}} = 0.674$



$X_{0.75} = 0.674 \cdot 0.46$
$+ 1.25$
$= 1.56$

$P(1 \leq X \leq 1.75$

$= P(X \leq 1.75)$

$- P(X \leq 1)$

$\left(\underset{s}{1} X \leq 1.75 \right)$

$P\left(\frac{X-1.25}{0.46} \leq \frac{1.75-1.25}{0.46} \right)$

$= P\left(\frac{X-1.25}{0.46} \leq \frac{1-1.25}{0.46} \right)$

$\searrow Z$

$= P\left(Z \leq \frac{0.5}{0.46}\right) - P\left(Z \leq \frac{-0.25}{0.46}\right)$

$P(Z \leq 1.086)$

$= P(Z \leq -0.543)$

$= 0.861 - 0.293$

$= 0.568$

$P(X > 2)$

$P(X \leq 2) + P(X > 2) = 1$

$P(X > 2) = 1 - P(X \leq 2)$

$P(X > 2) = P\left(\frac{X-1.25}{0.46} \leq \frac{2-1.25}{0.46} \right)$

$= P\left(Z \leq \frac{0.75}{0.46}\right)$

$= 0.948$

$1 - 0.948 = 0.052 = P(X > 2)$