4.5-4 $h(y|x) = \frac{1}{\sigma_Y\sqrt{2\pi}\sqrt{1-\rho^2}}exp[-\frac{[y-\mu_y-\rho(\sigma_Y/\sigma_X)(x-\mu_X)]^2}{2\sigma_Y^2(1-\rho^2)}$ in this problem $\mu_X = 70, \quad \sigma_X^2 = 100, \quad \mu_Y = 80, \quad \sigma_Y^2 = 169, \quad \rho = \frac{5}{13}$

$$h(y|x) = \frac{1}{\sqrt{2\pi}12}exp[\frac{-1}{2\times 12^2}(y-81)^2]$$

Thus $(Y|X) \sim N(81, 12^2)$
Another Method

(a) $E(Y|X=72) = 80 + \frac{5}{13}(\frac{13}{10})(72-70) = 81$

(b) $Var(Y|X=72) = 169[1-(\frac{5}{13})^2] = 144$

(c) $P(Y \le 84|X=72) = P(\frac{Y-81}{12} \le \frac{84-81}{12}) = \Phi(0.25) = 0.5987$

5.1-2 proof: we set $x = \sqrt{y}$, then $\frac{dy}{dx} = \frac{1}{2\sqrt{y}}$ which $0 < x < \infty$ and $0 < y < \infty$. Thus

$$g(y) = \sqrt{y}e^{-(\sqrt{y})^2/2}|\frac{1}{2\sqrt{y}}| = \frac{1}{2}e^{-y/2}$$

5.1-6 proof: $Y = \frac{1}{1+e^{-X}}$, then we can compute $x = g(y) = ln(\frac{y}{1-y})$ then $g'(y) = \frac{1}{y(1-y)}$. Finally, he pdf of Y is

$$f_Y(y) = f_X(ln(\frac{y}{1-y}))|g'(y)| = \frac{exp(ln(\frac{1-y}{y}))}{[1+exp(ln(\frac{1-y}{y}))]^2}|g'(y)| = \frac{(1-y)/y}{1/y^2}|g'(y)| = 1, \quad 0 < y < 1$$

Thus $y \sim U(0,1)$

5.2-14 The joint pdf is

$$h(x,y) = \frac{x}{5^3}e^{-(x+y)/5}, \quad 0 < x < \infty, \quad 0 < y < \infty$$

where $z = \frac{x}{y}, \quad \omega = y$ i.e $x = z\omega, \quad y = \omega$. The Jacobian is

$$\mathbf{J}' = \begin{bmatrix} \omega & z \\ 0 & 1 \end{bmatrix} = \omega$$

. The joint pdf of Z and W is

$$f(z,\omega) = \frac{z\omega}{5^3}e^{-(z+1)\omega/5}\omega, \quad 0 < z < \infty, \quad 0 < \omega < \infty$$

The marginal pdf of Z is

$$\begin{aligned} f_Z(z) &= \int_0^\infty \frac{z\omega}{5^3}e^{-(z+1)\omega/5}d\omega \\ &= \frac{\Gamma[3]z}{5^3}(\frac{5}{z+1})^3\int_0^\infty \frac{\omega^2}{\Gamma[3](5/(z+1))^3}e^{-\omega/(5/(z+1))}d\omega \\ &= \frac{2z}{(z+1)^3}, \quad 0 < z < \infty \end{aligned}$$

5.3-20

$$\begin{aligned}\rho &= \frac{Cov(W,V)}{\sigma_W\sigma_V}\\ &= \frac{E(WV)-\mu_W\mu_V}{\sigma_W\sigma_V}\\ &= \frac{E(X^2)E(Y)-E(X)E(Y)E(X)}{\sigma_{XY}\sigma_X}\\ &= \frac{(\sigma_X^2+\mu_X^2)\mu_Y-\mu_X^2\mu_Y}{\sqrt{(\sigma_X^2+\mu_X^2)(\sigma_Y^2+\mu_Y^2)-\mu_X^2\mu_Y^2}\sigma_X}\\ &= \frac{\mu_Y\sigma_X}{\sqrt{\sigma_X^2\sigma_Y^2+\sigma_X^2\mu_Y^2+\sigma_Y^2\mu_X^2}}\end{aligned}$$

5.4-2

$$\begin{aligned}M_Y(t) &= E[e^{t(X_1+X_2)}] = E[e^{tX_1}]E[e^{tX_2}]\\ &= (q+pe^t)^{n_1}(q+pe^t)^{n_2}\\ &= (q+pe^t)^{n_1+n_2}\end{aligned}$$

Thus Y satisfies $b(n_1+n_2,p)$

5.4-6

(a)

$$\begin{aligned}E(e^{tY}) &= E(e^{t(\Sigma_{i=1}5X_i)})\\ &= E(\prod_{i=1}^{5}e^{tX_i})\\ &= \prod_{i=1}^{5}E(e^{tX_i})\\ &= [\frac{(1/3)e^t}{1-(2/3)e^t}]^5, \quad t<-ln(2/3)\end{aligned}$$

(b) So Y has a negative binomial distribution with $p=1/3$ and $r=5$.

5.4-8

$$\begin{aligned}E(e^{tY}) &= E(e^{t(\Sigma_{i=1}^{h}X_i)})\\ &= E(\prod_{i=1}^{h}e^{tX_i})\\ &= \prod_{i=1}^{h}E(e^{tX_i})\\ &= [\frac{1}{1-\theta t}]^h, \quad t<1/\theta\end{aligned}$$

The moment generating function for the gamma distribution with mean $h\theta$.

5.5-4 Set $Z=\frac{X-6.05}{\sqrt{\frac{0.0004}{9}}}$

(a) $P(X<6.0171)=P(Z<-1.645)=0.05$

(b) Let W equal the number of boxes that weigh less than 6.0171 pounds. Then W is $b(9,0.05)$ and $P(W\le 2)=0.9916$

(c)

$$\begin{aligned}P(\bar{X}\le 6.035) &= P(Z\le\frac{6.035-6.05}{0.02/3})\\ &= P(Z\le -2.25)=0.0122\end{aligned}$$

5.6-2 If $f(x) = \frac{3}{2}x^2, \quad -1 < x < 1$,

$$E(X) = \int_{-1}^{1} x\frac{3}{2}x^2 dx = 0$$

$$Var(X) = \int_{-1}^{1} \frac{3}{2}x^4 dx = [\frac{3}{10}x^5]|_{-1}^{1} = \frac{3}{5}$$

Thus Set $Z = \frac{X-0}{\sqrt{15(\frac{3}{5})}}$

$$P(-0.3 \leq Y \leq 1.5) \approx P(-0.1 \leq Z \leq 0.5) = 0.2313$$