7.5-2

(a) $(y_3 = 5.4, y_{10} = 6.0)$ is a 96.14% confidence interval for the median, m.

(b) $(y_1 = 4.8, y_7 = 5.8)$;

$$P(Y_1 < \pi_{0.3} < Y_7) = \Sigma_{k=1}^{6} \binom{12}{k} (0.3)^k (0.7)^{12-k}$$
$$= 0.9614 = 0.0138 = 0.9476;$$

Using Table II with $n = 12$ and $p = 0.30$.

7.5-12

(a) $P(Y_7 < \pi_{0.70}) = \Sigma_{k=7}^{8} \binom{8}{k} (0.7)^k (0.3)^{8-k} = 0.2553$.

(b) $P(Y_5 < \pi_{0.70} < Y_8) = \Sigma_{k=5}^{7} \binom{8}{k} (0.7)^k (0.3)^{8-k} = 0.7483$.

8.1-4

(a)

$$|t| = \frac{|x - 7.5|}{s/\sqrt{10}} \geq t_{0.025}(9) = 2.262$$

(b)

$$|t| = \frac{|7.55 - 7.5|}{0.1027/\sqrt{10}} = 1.54 < 2.262$$

So do not reject H_0.

(c) A 95% confidence interval for μ is

$$[7.55 - 2.262(\frac{0.1027}{\sqrt{10}}), 7.55 + 2.262(\frac{0.1027}{\sqrt{10}})] = [7.48, 7.62]$$

Hence, $\mu = 7.50$ is contained in this interval. We could have obtained the same conclusion from our answer to part (b).

8.1-14

(a) $\frac{(23-1)s^2}{100} = \frac{715.44}{100} = 7.1544 < 10.98 = \chi^2_{0.975}(22)$, so she would reject H_0.

(b) Reject H_0, if $\frac{(23-1)s^2}{100} = \frac{715.44}{100} = 7.1544 < 10.98 = \chi^2_{0.025}(22)$. or if $\frac{(23-1)s^2}{100} > \chi^2_{0.025}(22) = 36.78$ So again she would reject H_0

8.1-15

(a)

$$df = 19 - 1 = 18$$
$$\chi^2_{0.05} = \frac{n-1}{\sigma_0^2} s^2 = 28.87$$
$$s^2 = 28.87 \times 30/18 = 48.12$$

$C: \quad s^2 \in (48.12, \infty)$

(b) $\chi_2 = \frac{n-1}{\sigma^2} s^2 = \frac{18}{80} \times 48.12 = 10.83$ $\beta = P(\chi^2_{18} < 10.83) = 0.0984$

8.2-2

(a)

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{15s_x^2 + 12s_y^2}{27}\left(\frac{1}{16} + \frac{1}{13}\right)}} \geq t_{0.01}(27) = 2.473$$

(b)

$$t = \frac{415.16 - 347.40}{\sqrt{\frac{15(1356.75)+12(692.21)}{27}\left(\frac{1}{16} + \frac{1}{13}\right)}} = 5.570 \geq t_{0.01}(27) = 2.473$$

Reject H_0

(c) For small n and m we can use the following approximation

$$W = \frac{415.16 - 347.40}{\sqrt{\frac{1356.75}{16} + \frac{692.21}{13}}} = 5.767$$

$$c = \frac{1356.75/16}{1356.75/16 + 692.21/13} = 0.614$$

$$\frac{1}{r} = \frac{0.614^2}{15} + \frac{0.386^2}{12} = 0.0375$$

$$r = 26.63 \approx 26.$$

The critical region is therefore $t \geq t_{0.01}(26) = 2.479$. Since $W = 5.767 > 2.479$, we again reject H_0.

8.2-15

1. If H_0 is true,

$$F = \frac{S_x^2/\sigma_x^2}{S_y^2/\sigma_y^2} = \frac{S_x^2}{S_y^2} \sim F_{n-1,m-1}$$

.

2. $F = \frac{1.410}{0.4399} = 3.2053$. And the critical value $F_{31-1,31-1}(0.01) = 2.39 < 3.2503$, so reject H_0

3. 2.39 is the critical value of $F_{30,30}$ with $\alpha = 0.01$.

8.2-17 Since the variance is known i.e $\sigma_x = 20, \quad \sigma_y = 15$

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - \theta}{400/n + 225/n} = \frac{\bar{X}_1 - \bar{X}_2 - \theta}{25/\sqrt{n}}$$

The power function is $K(\theta) = 1 - \phi(\frac{\bar{X}_1 - \bar{X}_2 - \theta}{25/\sqrt{n}})$ where $\phi(.)$ is the cdf of standard normal distribution.

$$K(0) = 1 - \phi(\sqrt{n}\frac{c-0}{25}) = 0.05$$

$$K(10) = 1 - \phi(\sqrt{n}\frac{c-10}{25}) = 0.90$$

$$\sqrt{n}\frac{c-0}{25} = 1.645$$

$$\sqrt{n}\frac{c-10}{25} = -1.282$$

$$c = 5.62$$

$$n = 53.55 \approx 54$$