401 Midterm 2

TIME ALLOCATION WILL BE CRUCIAL AS YOU WILL LIKELY NOT HAVE ENOUGH TIME TO FINISH. FIND EASY PROBLEMS, MAKE SURE YOU SOLVE THEM.

1. (20 pts) Answer T (true) or F (false) to the following statements.

- It is always the case that $P(A|B) = P(B|A)$.
- You can talk of conditional probability $P(A|B)$ for any set B.
- $P(A|B) = 1$ if B is a subset of A.
- $c + X$ and X has the same variance for any c.
- If A is independent of B and B is independent of C, A is independent of C.
- If two events are disjoint, they cannot be independent.
- An event A can be independent of itself.
- The pmf is only defined for random variables that take a finite set of possible values.
- Even if you change the value of the pdf on a countably infinite number of points, you still get the same distribution.
- Uniform distributions are closed under linear transformation. That is, if we have U, a uniform from a to b, $cU + d$ is also going to be a uniform.

2. (20 pts) For each of the following problems, choose the most suitable option.

- If you take a uniform from 0 to 1, and map it with a negative of a logarithm, you get back a,
  (a) geometric distribution
  (b) exponential distribution
  (c) normal distribution
  (d) binomial distribution
- If a continuous distribution takes values on a region from 0 to 0.5, there must be some x between 0 and 0.5 such that,
  (a) $f(x) \geq 2$
  (b) $f(x) = 0$
  (c) $f(x) < 2$
  (d) $f(x) < 0$
- Generate points within a sphere with radius 1 uniformly. Record the distance of the point from the origin. The range of this random variable is,
  (a) $\{x, y, z : x^2 + y^2 + z^2 \leq 1\}$

(b) $[-1, 1]$

(c) $[0, 1]$

(d) $\{x, y : x^2 + y^2 \leq 1\}$

- In the above setting, probability of getting a point with radius smaller than $0 < k < 1$ is,

  (a) k over 1

  (b) area of disc with radius k over area of disc with radius 1

  (c) volume of sphere with radius k over volume of sphere with radius 1

  (d) none of the above

- You throw two die. Record the larger of the two as the outcome of this random variable. What is its expected value?

  (a) $-\frac{5}{2}$

  (b) $\frac{20}{3}$

  (c) $\frac{160}{35}$

  (d) $\frac{161}{36}$

3. (10 pts) Explain how to reproduce a fair die from repeated flips of a fair coin. That is you take a coin, repeatedly flip them, and assign numbers 1 to 6 to the outcomes so that the probability of getting each number is the same. There is no restriction on how many times you flip the coin. You may be able to assign a number with just 3 flips or it might be that it takes much more flips like 1000.

4. (10 pts) Explain how to reproduce a fair random generator of numbers from 1 to 10 with repeated throws of a fair die. That is you take a die, repeatedly throw them, and assign numbers 1 to 10 to the outcomes so that probability of getting each number is the same. There is no restriction on how many times you throw the die. You may be able to assign a number with just 3 throws or it might be that it takes much more throws like 1000.

5. (10 pts) What is the expected number of coin flips needed to reproduce a dice throw outcome in problem 3?

6. (10 pts) What is the expected number of dice throws needed to reproduce a random number from 1 to 10 in problem 4?

7. (5 pts) Think of the first eight prime numbers. A random variable takes these numbers with probability proportional to its size. Calculate its pmf.

8. (5 pts) Calculate the variance of the above random variable.

9. (5 pts) The pdf of a random variable supported from 1 to ∞ is $k\frac{1}{x^4}$. Where k is some constant. Find k.

10. (5 pts) Find the variance of the random variable in the above problem.