8.6-1

(a)

$$\frac{L(80}{L(76)} = \frac{(128\pi)^{-n/2}exp[-\frac{1}{128}\Sigma(x_i-80)^2]}{(128\pi)^{-n/2}exp[-\frac{1}{128}\Sigma(x_i-76)^2]}$$
$$= exp[\frac{8}{128}\Sigma x_i - \frac{624n}{128}] \leq k$$
$$\bar{X} \leq \frac{16}{n}log(k) + 78 = c$$

The best critical region is $C = \{\mathbf{x}; \bar{X} \leq c\}$

(b)

$$P(\bar{X} \leq c; \mu = 80) = 0.05$$
$$P(\bar{X} > c; \mu = 76) = 0.05$$
$$\frac{c-80}{\sqrt{64/n}} = -1.645$$
$$\frac{c-76}{\sqrt{64/n}} = 1.645$$
$$c = 78$$
$$n = 43$$

8.6-3

(a)

$$\frac{L(3)}{L(5)} = (\frac{5}{3})^n exp[-\frac{2}{15}\Sigma x_i] \leq k$$
$$\Sigma x_i \geq -\frac{15}{2}[log(k) - nlog(\frac{5}{3})] = c$$

The best critical region is $C = \{\mathbf{x}; \Sigma x_i \geq c\}$

(b)

$$P(\Sigma x_i \geq c; \theta = 3) = 0.1$$
$$P(\frac{2}{3}\Sigma x_i < \frac{2}{3}c; \theta = 3) = 0.9$$
$$\frac{2}{3}c = \chi^2_{24}(0.1) = 33.2$$
$$c = 49.8$$

The best critical region is $C = \{\mathbf{x}; \Sigma x_i \geq 49.8\}$

(c) c only depends on Σx_i and $\theta = 3$, thus The best critical region holds i.e $C = \{\mathbf{x}; \Sigma x_i \geq 49.8\}$

(d) For any $\theta_1 > 3$,

$$\frac{L(3)}{L(\theta_1} = (\frac{\theta_1}{3})^n exp[(\frac{1}{\theta_1} - \frac{1}{3})\Sigma x_i] \leq k$$

Due to $\theta_1 > 3$

$$\Sigma x_i \geq -(\frac{1}{3} - \frac{1}{\theta_1})[log(k) - n(log(\theta_1) - log(3))] = c$$

where c is selected such that $P(\Sigma x_i \geq c; H_0 : \theta = 3) = \alpha$. Note that the same value of c can be used for each $\theta > 3$, but (of course) k does not remain the same.

8.6-5

(a) For any $\mu_1 < 50$,

$$\frac{L(50)}{L(\mu_1)} = exp\{-\frac{1}{72}[2(\mu_1 - 50)\Sigma x_i + n(50^2 - \mu_1^2)]\} \leq k$$

$$-\frac{1}{72}[2(\mu_1 - 50)\Sigma x_i + n(50^2 - \mu_1^2)] \leq log(k)$$

Due to $\mu_1 - 50 < 0$

$$\bar{X} \leq \frac{-72log(k) - n(50^2 - \mu_1)^2}{2n(\mu_1 - 50)} = c$$

The UMP critical region is $C_1 = \{\mathbf{x}; \bar{X} \leq c\}$

(b) Consider $H_0; \mu = 50; H_1; \mu > 50$. For any $\mu_2 > 50$, from similarly procedure.

$$-\frac{1}{72}[2(\mu_1 - 50)\Sigma x_i + n(50^2 - \mu_1^2)] \geq log(k)$$

Due to $\mu_1 - 50 > 0$

$$\bar{X} \leq \frac{-72log(k) - n(50^2 - \mu_1)^2}{2n(\mu_1 - 50)} = c$$

The UMP critical region is $C_2 = \{\mathbf{x}; \bar{X} \geq c\}$ Thus the two sided UMP critical region is $C_3 = \{\mathbf{x}; \bar{X} \geq c \& \bar{X} \leq c\}$ Thus $P(C_3) = 0$, the required uniformly most powerful test does not exist.

8.6-7

(a)

$$\frac{L(0.5)}{L(\mu_1} = (\frac{0.5}{\mu_1})^{\Sigma x_i} e^{10(\mu_1 - 0.5)} \leq k$$

$$\Sigma x_i \geq \frac{log(k) - 10(\mu_1 - 0.5)}{log(\frac{0.5}{\mu_1})} = c$$

where c is selected such that $P(\Sigma x_i \geq c; H_0 : 0.5) = \alpha$. So it is a UMP test.

(b) $\Sigma x_i \sim Poisson(\lambda)$, $\lambda = 10 \times \mu = 5$.

$$0.068 = 1 - P(\Sigma x_i < c;\ \mu = 0.5)$$

$c = 9$

(c)

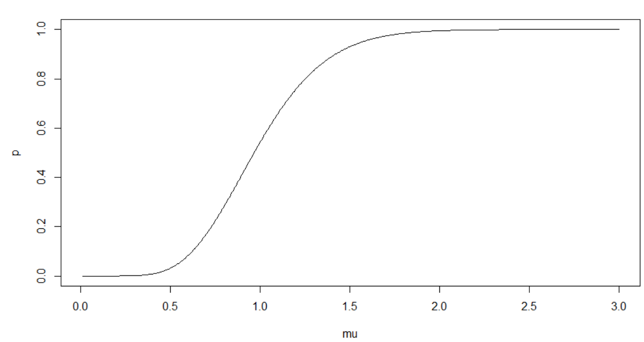

Figure 1: Sketch the power function of this test

8.6-9 For any $\theta_1 < 0.5$

$$\frac{L(0.5)}{L(\theta_1)} = \frac{(0.5)^{\Sigma x_i}}{\theta_1^{\Sigma x_i}(1-\theta_1)^{1-\Sigma x_i}} \leq k$$

$$\Sigma x_i \leq \frac{log(k) - 5log(1-\theta_1)}{log(0.5) - log(\theta_1) + log(1-\theta_1)} = c$$

where c is selected such that $P(\Sigma x_i \leq c; H_0 : 0.5) = \alpha$. So it is a UMP test.

$$K(\theta) = \Sigma_{k=0}^{1} \binom{5}{k} \theta^k (1-\theta)^{(5-k)}$$