1/29

$\forall E \in \mathcal{F}$

under the classical probability model, we have

$$P(E) = \frac{N(E)}{N}$$

figure out S

determine N

figure out E

determine $N(E)$

there are also counting techniques

ex. find probability that 5 cards randomly selected from a deck of 52 cards will form a full house. (one pair, one triple)

Fundamental Principle of Counting

if a task can be completed in K stages, and stage ~~i~~ i has n_i outcomes, regardless of the outcomes of the previous stages, then the task has $n_1, n_2, \ldots, n_K$ outcomes

How many ways to pick 5 cards?

52 for first card

51 for second card

50 for third card

49 for fourth card

48 for fifth card

$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$

How many ways to get full house?

$K = 2$

first stage: pair

$13(4 \cdot 3)$

second stage: triple

$K = 3 \quad n = 4$

$12(4 \cdot 3 \cdot 2)$

Permutations

number of permutations of K units selected from n

* the units must be distinct
* order matters

$$P_{K,n} = \frac{n!}{(n-K)!}$$

choose $K = 2$ from $n = 3$ units $\{A, B, C\}$

AB and BA are distinct permutations

$$P_{2,3} = \frac{3!}{1!} = 6$$

$K = 5 \quad n = 52$

$$P_{5,52} = \frac{52!}{47!} =$$

$K = 2 \quad n = 4$

$$P_{2,4} = \frac{4!}{2!} = 12$$

$$\frac{13\left(\frac{4!}{2!}\right) \cdot 12\left(\frac{4!}{1!}\right)}{\frac{52!}{47!}}$$

Combinations of k from n

$$\binom{n}{k} = \frac{P_{k,n}}{P_{k,k}} = \frac{n!}{(n-k)!\,k!}$$

$$\frac{k!}{(k-k)!} = \frac{k!}{0!} = k! \qquad 0! \overset{*}{=} 1$$

* order doesnt matter

$$\binom{3}{2} = \frac{3!}{1!\,2!} = 3$$

without considering order!

$$N = \binom{52}{5} = \frac{52!}{47!\,5!}$$

$$\binom{13}{1}\binom{4}{2} \cdot \binom{12}{1}\binom{4}{3}$$

$$= \frac{13!}{\cancel{12!}}\ \frac{4!}{2!\,2!}\ \frac{\cancel{12!}}{11!}\ \frac{4!}{3!}$$

divide this by $\dfrac{52!}{47!\,5!}$