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why the sample variance has division by $n-1$

$$(x_i - \bar{X})^2 = X_i^2 - 2\bar{X}X_i + \bar{X}^2$$

$$= x_i^2 - 2\frac{\left(\sum_{j=1}^{n} x_j\right)}{n} X_i + \frac{\left(\sum_{j=1}^{n} x_j\right)^2}{n^2}$$

$$= X_i^2 - 2\frac{\sum_{j=1}^{n} x_j X_i}{n} + \frac{\sum_{i=1}^{n} X_i^2 + 2\sum_{i>j} X_iX_j}{n^2}$$

$$= X_i^2 - 2\left(\frac{X_i^2 + \sum_{i\neq j} X_iX_j}{n}\right) + \frac{\sum_{i=1} X_i^2 + 2\sum_{i>j} X_iX_j}{n^2}$$

take it's expectation

$$E(X^2) - 2\left(\frac{E(X^2) + (n-1)E(X)^2}{n}\right)$$

$$+ \frac{nE(X^2) + n(n-1)E(X)^2}{^2}$$

$$+ \frac{n\,\mathbb{E}(\ldots)}{n^2}$$

so $$\mathbb{E}\left(\sum_{i=1}^{n} (X_i - \bar{X})^2\right) = n\,\mathbb{E}(X^2) - 2\mathbb{E}(X^2) - (n-1)\,\mathbb{E}(X)^2\, 2$$

$$+ \mathbb{E}(X^2) + (n-1)\,\mathbb{E}(X)^2$$

$$= (n-1)\,\mathbb{E}(X^2) - (n-1)\,\mathbb{E}(X)^2$$

So $$\mathbb{E}\left[\frac{1}{n-1}\sum_{i=1}^{n} (X_i - \bar{X})^2\right] = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

$$= \mathrm{Var}(X).$$