Example | 5% of all Men / 0.25% of all Women are Colorblind

→ If a community is 55% Women and 45% Men, if the person is colorblind, what is the probability that they are male.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

B ⇒ Colorblind
A ⇒ Male

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$
$$= \underset{\text{CB Men}}{(0.05)}\underset{\text{Men}}{(.45)} + \underset{\text{CB Not male / Women}}{(0.0025)}\underset{\text{Not male / Women}}{(0.55)}$$

$$P(A \cap B) = P(A)P(B|A)$$
$$= (.45)(0.05)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(0.05) \times (0.45)}{(0.05)(0.45) + (0.0025)(0.55)}$$

$$= \frac{20 \times 45}{(20 \times 45) + (55)}$$

$$\boxed{= 0.9424}$$

$$\underbrace{1 - P(A|B) = P(A^c|B)}$$

→ Probability that a person is colorblind and a women

Independence

- Events A and B are called <u>independent</u> if...

$$P(A \cap B) = P(A)P(B)$$

If A and B are not independent, they are called dependent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

and

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\cancel{P(A)}P(B)}{\cancel{PA}} = P(B)$$

Properties of Independents:

1. If A and B are independent, then so are A^c and B.

1. $P(A \cap B) = P(A)P(B)$
2. $P(A^c \cap B) \stackrel{?}{=} P(A^c)P(B)$
3. $P(A \cap B) + P(A^c \cap B) = P(B)$
4. $P(A^c \cap B) = P(B) - P(A \cap B)$
5. $P(A^c \cap B) = PB - P(A)P(B)$
6. $P(A^c \cap B) = P(B)[1 - P(A)] \rightarrow 1 - P(A) = P(A^c)$
7. $P(A^c \cap B) = P(B)P(A^c)$

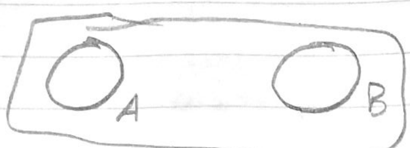
A^c and B are Independent

(2) $\underline{\phi}$ and $\underline{S}$ are independent of any other event

$$\underbrace{P(\phi \cap A)}_{\phi} = \underbrace{P(\phi)}_{\phi} P(A) \qquad \text{If } A \subseteq S$$

$$\underbrace{P(S \cap A)}_{P(A)} = \underbrace{P(S)\,P(A)}_{P(A)} \qquad \forall\, A \subseteq S$$

(3) Disjoint Events are not independent unless the prob of one of them is 0.



$(A \cap B) = \emptyset$

Disjoint of A, B not 0

$0 = P(A \cap B) \neq P(A)\,P(B) > 0$

if one has a prob of 0

$0 = P(A \cap B) \qquad 0 = P(A \cap B)$

$0 = 0 \times P(A) \qquad 0 = 0 \times P(B)$

$0 = 0 \times 0$

④ Events are independent if and only if...

Mutually Indep
$$\begin{cases} P(E_1 \cap E_2) = P(E_1)\,P(E_2) \\ P(E_1 \cap E_3) = P(E_1)\,P(E_3) \\ P(E_2 \cap E_3) = P(E_2)\,P(E_3) \end{cases}$$

$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cap P(E_2) \cap P(E_3)$$

Events E_1, E_2, E_3 are independent if and only if, for every subset,

$E_1, E_2, \ldots, E_k$ $\quad k \leq n$

$$P(E_1 \cap E_2 \ldots E_k) = P(E_1)\,P(E_2) \ldots P(E_k)$$

$$* \quad \boxed{\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots \binom{n}{n} = \boxed{2^n - n}}$$

$\frac{n!}{(n-1)!}$ Exclude $n=1$

How?

① ② ③ ④ .... ⓝ

Pull out without care to order.
No constraints on how many.
How many possible ways?