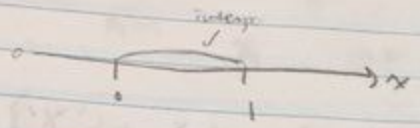
3/21 Stat 401 in class note

- $X \sim U(0,1)$



$V(X) = E[X^2] - E[X]^2 = 1/3 - (1/2)^2 = \boxed{1/12}$

$E[X] = \int_0^1 x \cdot 1 \, dx$

$= \int x \, dx$

$= \left(\frac{x^2}{2}\right)_0^1$

$= \frac{1}{2}$

$E[X^2] = \int_0^1 x^2 \cdot 1 \, dx = \frac{x^3}{3}\Big]_0^1 = 1/3$

Standard deviation $sd = \sqrt{\frac{1}{12}} = \frac{1}{2\sqrt{3}}$

- linearity does not hold variance

$E[X_1 + X_2] = E[X_1] + E[X_2]$

$E[aX_1 + b] = aE[X_1] + b$

$E[h_1(X) + h_2(X)] = E[h_1(X)] + E[h_2(X)]$

but, $Var(h_1(X) + h_2(X)) \neq Var(h_1(X)) + Var(h_2(X))$ (Var of linearity)

ex) $X \sim U(0,1)$

$h_1(X) = X \quad h_2(X) = X^2$

$\rightarrow Var(X + X^2) \neq Var(X) + Var(X^2) \Rightarrow \frac{31}{180} \neq \frac{61}{180}$

$= 1/12 \quad = 4/45$

$(E[X^2] = 1/4)$

$(E[X^4] = 1/5)$

$V(X^2) = E[X^4] - E[X^2]^2$

$1/5 - 1/9 = 4/45$

$V(X + X^2) = E[(X + X^2)^2] - E[X + X^2]^2$

$= E[X^2 + 2X^3 + X^4] - (E[X] + E[X^2])^2$

$= E[X^2] + 2E[X^3] + E[X^4] - (E[X] + E[X^2])^2$

$= 1/3 + 2 \cdot 1/4 + 1/5 - (1/2 + 1/3)^2$

$= \frac{61}{180} = LHS$

RHS $= \frac{1}{12} + \frac{4}{45} = \frac{31}{180}$