A.1 $0 \leq P(E) \leq 1 \quad \forall E \in \mathcal{F}$ where $\mathcal{F}$ is collection of events (subset of sample space)

A.2 $P(S) = 1$

A.3 for disjoint events $E_1, E_2, \ldots$

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

1. $P(\phi) = 0$

for finite collection of disjoint events/sets

$E_1, \ldots E_2, \ldots E_m$

2. $P(E_1 \cup E_2 \cup \ldots E_m) = P(E_i) + \ldots + P(E_m)$

$$P\left(\bigcup_{i=1}^{m} E_i\right) = \sum_{i=1}^{m} P(E_i)$$

3. if $A \subseteq B$ then $P(A) \leq P(B)$

$P(A^c) = 1 - P(A)$

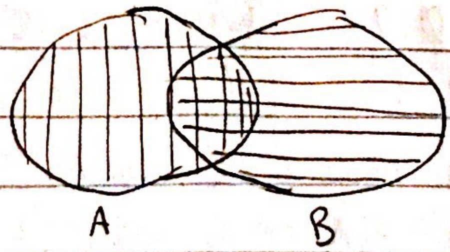
$m = 2$

A^c and A are disjoint, $A^c \cap A = \phi$

$P(\underbrace{A^c \cup A}_{S}) = P(A^c) + P(A) \qquad 1 = P(A^c) + P(A)$

4. $P(A^c) = 1 - P(A)$ Inclusion – Exclusion Formula

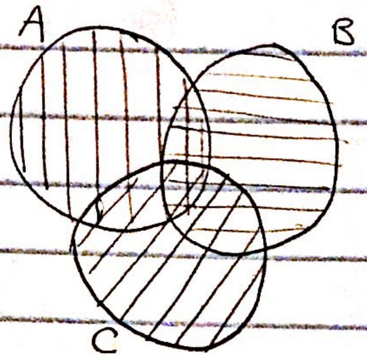
$P(A \cup B)$ in terms of $P(A)$, $P(B)$, $P(A \cap B)$



A |||| $\quad$ $A \cap B$ ####

B ≡

$P(A)+P(B)-P(A\cap B)=P(A\cup B)$



A |||| $\quad$ $A\cap B$ ⊞

B ☰ $\quad$ $A\cap C$

C //// $\quad$ $B\cap C$

$A\cap B\cap C$

$P(A\cup B\cup C)$
$= P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)$
$\quad - 3P(A\cap B\cap C)$
$+ P(A\cap B\cap C)$

Example

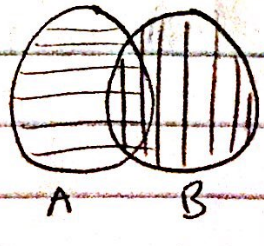
60% of families own a dog in a town, 70% own a cat, and 50% own both. What is the probability that a randomly selected family has

a) dog but no cat $\rightarrow A$

b) cat but no dog $\rightarrow B$

c) at least one of the two kinds of pets $\rightarrow A\cap B$

$P(A)=0.6 \qquad P(B)=0.7 \qquad P(A\cap B)=0.5$



a) $P(A\cap B^c) = P(A-B) = 0.6-0.5=\boxed{0.1}$

b) $P(B\cap A^c)=P(B-A)=0.7-0.5=\boxed{0.2}$

c) $P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.6+0.7-0.5$
$=\boxed{0.8}$

Classical Probablity Models

probability $(S, \mathcal{F}, P)$

The size of S is finite, say N

$S = \{x_1, \dots, x_N\}$

$\mathcal{F}$ is the collection of subsets of S

$P(\{x_1\})$

$= \dots P(\{x_N\})$

$= \frac{1}{N}$

Dice throw

$S = \{1,2,3,4,5,6\}$

$\mathcal{F}$ has $2^6 = 64$ elements (sets)

$\mathcal{F}_1 = \{\phi, \{1\}, \{2\}, \dots S\}$

$P(\{\phi\}) = P(\{2\}) = P\{(6)\} = 1/6$

$E_1 = \{\text{outcome is odd}\}$

$= \{1,3,5\} = P(E_1) = 1/2$

$E_2 = \{\text{outcome greater than } 2\} \rightarrow P(E_2) = \frac{2}{3}$

$E_3 = E_2 \cap E_1$