

6. (a) $P(\text{no defect} \cap A) = P(\text{no defect}|A)P(A) = 0.99 \times 0.3 = 0.297$.
 (b) $P(\text{no defect} \cap B) = P(\text{no defect}|B)P(B) = 0.97 \times 0.3 = 0.291$, and $P(\text{no defect} \cap C) = P(\text{no defect}|C)P(C) = 0.92 \times 0.3 = 0.276$.
 (c) $P(\text{no defect}) = P(\text{no defect} \cap A) + P(\text{no defect} \cap B) + P(\text{no defect} \cap C) = 0.297 + 0.291 + 0.276 = 0.864$.
 (d) $P(C|\text{no defect}) = P(\text{no defect} \cap C)/P(\text{no defect}) = 0.276/0.864 = 0.3194$.

9. Let A be the event that the plant is alive and let W be the event that the roommate waters it. Then, from the given information, $P(W) = 0.85$ and $P(W^c) = 0.15$; $P(A|W) = 0.9$ and $P(A|W^c) = 0.2$.
 (a) $P(A) = P(A|W)P(W) + P(A|W^c)P(W^c) = 0.9 \times 0.85 + 0.2 \times 0.15 = 0.795$.
 (b) $P(W|A) = P(A|W)P(W)/P(A) = 0.9 \times 0.85/0.795 = 0.962$.

11. Let L_1, L_2, L_3, L_4 be the event that the radar traps are operated at the 4 locations, then $P(L_1) = 0.4, P(L_2) = 0.3, P(L_3) = 0.2, P(L_4) = 0.3$. Let S be the person speeding to work, then $P(S|L_1) = 0.2, P(S|L_2) = 0.1, P(S|L_3) = 0.5, P(S|L_4) = 0.2$.
 (a) $P(S) = P(S|L_1)P(L_1) + P(S|L_2)P(L_2) + P(S|L_3)P(L_3) + P(S|L_4)P(L_4) = 0.2 \times 0.4 + 0.1 \times 0.3 + 0.5 \times 0.2 + 0.2 \times 0.3 = 0.27$.
 (b) $P(L_2|S) = P(S|L_2)P(L_2)/P(S) = 0.1 \times 0.3/0.27 = 0.11$.

12. Let D be the event that the aircraft will be discovered, and E be the event that it has an emergency locator. From the problem, $P(D) = 0.7$ and $P(D^c) = 0.3$; $P(E|D) = 0.6$ and $P(E|D^c) = 0.1$.
 (a) $P(E \cap D^c) = P(E|D^c)P(D^c) = 0.1 \times 0.3 = 0.03$.
 (b) $P(E) = P(E|D^c)P(D^c) + P(E|D)P(D) = 0.1 \times 0.3 + 0.6 \times 0.7 = 0.45$.
 (c) $P(D^c|E) = P(E \cap D^c)/P(E) = 0.03/0.45 = 0.067$.

1. From the given information $P(E_2) = 2/10$ and $P(E_2|E_1) = 2/9$, thus $P(E_2) \neq P(E_2|E_1)$. Consequently, E_1 and E_2 are not independent.

2. We can calculate from the table that $P(X = 1) = 0.132 + 0.068 = 0.2$ and $P(Y = 1) = 0.132 + 0.24 + 0.33 = 0.702$, thus $P(X = 1)P(Y = 1) = 0.2 \times 0.702 = 0.1404 \neq 0.132 = P(X = 1, Y = 1)$. Thus, the events $[X = 1]$ and $[Y = 1]$ are not independent.

4. A total of 8 fuses being inspected means that the first 7 are not defective and the 8th is defective, thus the probability is calculated as $0.99^7 \times 0.01 = 0.0093$.
6. Yes. By the given information, $P(T|M) = P(T)$, we see that T and M are independent. Thus, T and $F = M^c$ are also independent; that is, $P(T|F) = P(T)$.
9. Since E_1, E_2, E_3 are independent, we have

$$\begin{aligned}
 P(E_1 \cap (E_2 \cup E_3)) &= P((E_1 \cap E_2) \cup (E_1 \cap E_3)) = P(E_1 \cap E_2) + P(E_1 \cap E_3) \\
 &\quad - P(E_1 \cap E_2 \cap E_3) \\
 &= P(E_1)P(E_2) + P(E_1)P(E_3) - P(E_1)P(E_2)P(E_3) \\
 &= P(E_1)[P(E_2) + P(E_3) - P(E_2 \cap E_3)] = P(E_1)P(E_2 \cup E_3),
 \end{aligned}$$

which proves the independence between E_1 and $E_2 \cup E_3$.

10. Let E_1, E_2, E_3, E_4 be the events that components 1, 2, 3, 4 function, respectively, then

$$\begin{aligned}
 P(\text{system functions}) &= P((E_1 \cap E_2) \cup (E_3 \cap E_4)) = P(E_1 \cap E_2) + P(E_3 \cap E_4) \\
 &\quad - P(E_1 \cap E_2 \cap E_3 \cap E_4) \\
 &= P(E_1)P(E_2) + P(E_3)P(E_4) - P(E_1)P(E_2)P(E_3)P(E_4) \\
 &= 2 \times 0.9^2 - 0.9^4 = 0.9639.
 \end{aligned}$$