Stats April 13, 18 — Jose Bisono

(X, Y) discrete

$X \perp\!\!\!\perp Y \iff$

$P(x,y) = P_X(x) P_Y(y)$

$\forall (x,y) \in S_{x,y}$

X, Y Continuous

$X \perp\!\!\!\perp \iff$

$f(x,y) = f_X(x) f_Y(y) \quad \forall (x,y) \in \mathbb{R}^2$

$P(X \le x, Y \le x)$

$=$

$\sum_{t \le x} \sum_{s \le y} P_X{}' y' P_Y(s)$

$= \sum_{t \le x} P(x) + \sum_{s \le y} P(y)$

$= P(X \le x) P(Y \le y)$

$P(X \le x, Y \le y)$

$= P(X \le x) P(Y \le y)$

Smallest x & y

$P(X \le x, Y \le y_1)$

$= P(X \le x, Y \le y)$

$= P(x, y_1) = P_x(x_1) P_y(y_1) - \diamond$

X_2 second largest

$P(X \le x_2, Y \le y_1)$

$= P(X \le x_2) P(Y \le y_1)$

$P(X = x, x_2, Y = y_1)$

$= P(X = x_1, x_2) P(Y = y_1)$

$= [P(X = x_1) + P(X = x_2)] P(Y = y_1) - \star$

$P(X = x_2, Y = y_1)$

$= \star - \diamond$

$= P(X = x_2) P(Y = y_1)$

$F(x, y) =$

$P(X \le x, Y \le y)$

$= \int_{-\infty}^{x} \int_{-\infty}^{y} f_X(x) f_Y(y) \, dx \, dy$

$= \int_{-\infty}^{x} f_X(x) dx \int_{-\infty}^{y} f_Y(y) dy$

$F(x,y) = F_X(x) F_Y(y)$

$\frac{d}{dy} \frac{d}{dx} F(x,y) = f(x,y)$

$= F_X(x)\, F_Y(y)$

$$\frac{d}{dy}\frac{d}{dx} F_X(x) F_Y(y) = \frac{d}{dx} F_X(x) \frac{d}{dy} F_Y(y)$$

$$= f_X(x)\, f_Y(y)$$

Independence

tells you joint distribution can be obtained from marginals

under independence

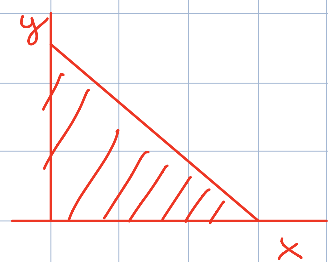
$$f_{X|Y} = \frac{f(x,y)}{f_Y(y)}$$

$$= f_X(x)$$

Conditional, is just the

$f(x,y)$

$$= \begin{cases} 24xy & 0 \le x \le 1,\ 0 \le y \le 1,\ x+y \le 1 \\ 0 & \text{otherwise} \end{cases}$$



$$f_X(x) = \int_0^{1-x} 24xy\, dy$$

$$= 24\left(\frac{xy^2}{2}\right)_0^{1-x} = 12x(1-x)^2$$

By symmetry

$f_Y(y) = 12$

$E[g(x)h(y)]$

$$\iint g(x)h(x)\, f_X(x) f_Y(y)\, dx\, dy$$

$$\int g(x) f_X(x)\, dx \int h(y) f_Y(y)\, dy$$

$$= Eg(x)\, Eh(Y)$$

$E(XY)$

$= E(X)E(Y)$

by taking h & g identity

$E(Y|X=x)$

is a constant for diff x

$$f_{Y|X=x} = \frac{f_Y(y) f_X(x)}{f(x)}$$

$= f_Y(y)$ same identity

generalization

$X \dots X_n$

independent

iff

$p(X_1, \dots, x_n)$

$= p_{X_1}(x_1) \dots p_{X_n}(x_n)$

$\forall (x_1, \dots, x) \in S X_1 \dots X_n$

$f(x_1, \dots, x_n)$

$= f_{X_1}(x_1) \dots f_{X_n}(x_n)$

$\forall (x_1, \dots, x_n) \in \mathbb{R}^n$

in addition if

$X_1, \dots, X_n$

has same

Distribution $F(p, f)$

then we say

$X_1, \dots, X_n$

are independently

and identically distributed

$X_1, \dots, X_n$

i.i.d. $\sim F(p, f)$

ex,

$X_1, \dots, X_{100}$

$\sim N(0,1)$

$h(x,y)$

$E(h(x,y))$

$$= \begin{cases} \sum_{(x,y)} h(x,y)\, p(x,y) \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) f(x,y)\, dx\, dy \end{cases}$$

$Var(h(x,y)) =$

$E(h^2(x,y))$

$- (E(h(x,y)))^2$

		1	2
x	1	.034	.134
	2	.066	.266
	3	.1	.400

$E(XY) = 1 \cdot .0034 + 2 \cdot .134$

$+ 2 * .066 + 4 * .266$

$+ 3 * .1 + 6 * .4$

$= \mu_{xy}$

$E(x^2y^2)$

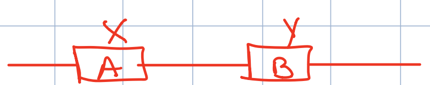
$= 1 * .034 + 4 \cdot .0134$

$+ 4 * .0666 + 16 * .02666$

$+ 9 * .011 + 36 * .4$

$= K$

$V(xy) = K - \mu_{xy}^2$



failure tue of A & B

are independent and $u(0,1)$

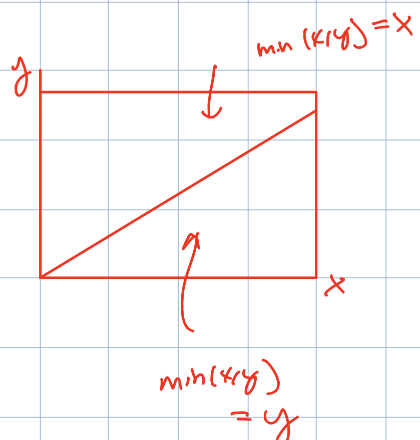
Find mean and variance of failure time of whole system

$E(\min(x,y))$

$V(\min(x,y))$

$x \sim (0,1)$ $\quad x \neq y$

$y \sim (0,1)$



$E(\min(x,y))$

$= \int_0^1 \int_0^y x \, dy$

$+ \int_0^1 \int_0^x y \, dx$