15. (a) The probability is

$$\frac{48}{\binom{52}{5}} = 1.85 \times 10^{-5}.$$

(b) The probability is

$$\frac{\binom{4}{2}\binom{4}{2}44}{\binom{52}{5}} = 0.00061.$$

(c) The probability is

$$\frac{\binom{4}{3}\binom{12}{2}4^2}{\binom{52}{5}} = 0.0016.$$

16. The total number of possible assignments is $\binom{10}{2,2,2,2,2} = 113400$.

17. (a) There are $3^{15} = 14348907$ ways to classify the next 15 shingles in tow three grades.

(b) The number of ways to classify into three high, five medium and seven low grades is

$$\binom{15}{3,5,7} = 360360.$$

19.

$$\begin{aligned}
(a_1^2 + 2a_2 + a_3)^3 &= \binom{3}{0,0,3}(a_1^2)^0(2a_2)^0a_3^3 + \binom{3}{0,1,2}(a_1^2)^0(2a_2)^1a_3^2 \\
&+ \binom{3}{0,2,1}(a_1^2)^0(2a_2)^2a_3^1 + \binom{3}{0,3,0}(a_1^2)^0(2a_2)^3a_3^0 \\
&+ \binom{3}{1,0,2}(a_1^2)^1(2a_2)^0a_3^2 + \binom{3}{1,1,1}(a_1^2)^1(2a_2)^1a_3^1 \\
&+ \binom{3}{1,2,0}(a_1^2)^1(2a_2)^2a_3^0 + \binom{3}{2,0,1}(a_1^2)^2(2a_2)^0a_3^1 \\
&+ \binom{3}{2,1,0}(a_1^2)^2(2a_2)^1a_3^0 + \binom{3}{3,0,0}(a_1^2)^3(2a_2)^0a_3^0 \\
&= a_3^3 + 6a_2a_3^2 + 12a_2^2a_3 + 8a_2^3 + 3a_1^2a_3^2 + 12a_1^2a_2a_3 \\
&\qquad + 12a_1^2a_2^2 + 3a_1^4a_3 + 6a_1^4a_2 + a_1^6.
\end{aligned}$$

1. The probability can be calculated as

$$P(>3| >2) = \frac{P((>3)\cap(>2))}{P(>2)} = \frac{P(>3)}{P(>2)} = \frac{(1+3)^{-2}}{(1+2)^2} = 9/16.$$

3. (a) $P(A) = 0.132 + 0.068 = 0.2$.

(b) $P(A \cap B) = 0.132$, thus $P(B|A) = P(A \cap B)/P(A) = 0.132/0.2 = 0.66$.

(c) $P(X = 1) = 0.2$, $P(X = 2) = 0.3$, $P(X = 3) = 0.5$.