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- 4 resistors, checking each of whether it is faulty [1] or functional [0]

E_i : ith resistor functional

$E_1 \rightsquigarrow (0, *, *, *)$

A_1: all resistors functional $= E_1 \cap E_2 \cap E_3 \cap E_4$: $(0,0,0,0)$

A_2: exactly one resistor faulty

$= \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

ex.) $S = \{1,2,3\}$

$A = \{1\}$

$A^c = A$ compliment $= \{2,3\}$

$\boxed{(1,0,0,0)}$ $E_1^c = \{(1, x_2, x_3, x_4) : x_2, x_3, x_4 \in \{0,1\}\}$

$E_1^c \cap E_2 \cap E_3 \cap E_4 = \{(1,0,0,0)\}$

$E_1 \cap E_2^c \cap E_3 \cap E_4 = \{(0,1,0,0)\}$

$E_1 \cap E_2 \cap E_3^c \cap E_4 = \{(0,0,1,0)\}$

$E_1 \cap E_2 \cap E_3 \cap E_4^c = \{(0,0,0,1)\}$

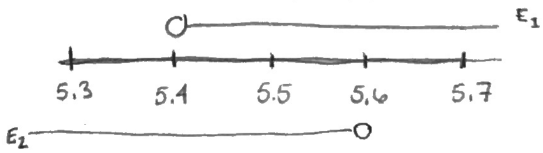
$F_1 \cup F_2 \cup F_3 \cup F_4 = A_2$

- Measure diameter of cylinder

$S = \{x : 5.3 \le x \le 5.7\}$

$E_1 = \{x : x > 5.4\}$

$E_2 = \{x : x < 5.6\}$



$E_1 \cup E_2 = S$ [all numbers between 5.3 + 5.7 - Union]

$E_1 \cap E_2 = \{x : 5.4 < x < 5.6\}$ [intersection]

$E_1 - E_2 = \{x : 5.7 \ge x \ge 5.6\}$

- General Definition of Probability Space / Model
  - The sample space S [set = collection of elements]
  - The collection $\mathcal{F}_1$ of set / events

    $S = \{1, 2, 3, 4, 5\}$

    $\mathcal{F}_1 = \{ \{1\}, \{2\}, \ldots, \{5\}, \Phi, \{1,2\}, \ldots, S \}$
  - A probability Measure P measuring uncertainties of members in $\mathcal{F}_1$ (subsets of S)

    $P(S) = 1$

    $P(\Phi) = 0$ * Φ = null set
- $(S, \mathcal{F}_1, P)$ → this we call probability space

Ex.) Throw a coin ⇒ $S = \{H, T\}$

$\mathcal{F} = \{\Phi, \underbrace{\{H\}}_{\text{set}}, \{T\}, S\}$

set, but H is only an element

$P(\Phi)$ = prob. of getting neither = $\underline{0}$

$P(\{H\})$ = prob. of getting H = $\underline{0.5}$

$P(\{T\})$ = prob. of getting T = $\underline{0.5}$

$P(\{H, T\})$ = Prob. of getting H or T = $\underline{1}$

- Axioms of Probability (something you assume to be true)

Must Be Satisfied

Axiom 1: $0 \leq P(E) \leq 1 \quad \forall E \in \mathcal{F}_1$ [$\forall$ = for all, E = set]

Axiom 2: $P(S) = 1$

Axiom 3: For any sequence of disjoint events/sets E_1, E_2

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

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Ex.) $A \cap B = \phi$, the A, B is disjoint

$$\underset{\overset{\shortparallel}{F_1}}{E_1}, \underset{\overset{\shortparallel}{F_2}}{E_2}, \underset{\overset{\shortparallel}{F_3}}{E_3}, \ldots, \underset{\overset{\shortparallel}{F_n}}{E_n}, \underset{\overset{\shortparallel}{F_{n+1}}}{\phi}, \underset{\overset{\shortparallel}{F_{n+2}}}{\phi}, \ldots \phi, \ldots$$

↑ infinite sequence of sets if $E_1, \ldots, E_n$ are mutually disjoint

then the infinite sequence sets is also disjoint

Has to be btw 0 & 1

$$P\left(\bigcup_{i=1}^{\infty} F_i\right) = \sum_{i=1}^{\infty} P(F_i)$$

$$P\left(\bigcup_{i=1}^{\infty} F_i\right) = \sum_{i=1}^{n} P(F_i)$$

Result 2 $$\left[P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{n} P(E_i) \right.$$

before this, we have to show $\underline{P(\phi)=0}$

$$0 \leq P\left(\bigcup_{i=0}^{\infty} F_i\right) \leq 1$$

$$= \sum_{i=1}^{\infty} P(F_i)$$

$$= \sum_{i=1}^{n} P(F_i) + \lim_{K\to\infty} (K-n)P(\phi) \leq 1$$

if not equal to zero, can't satisfy bounds

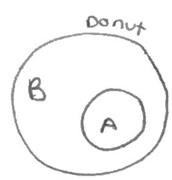
$\therefore \boxed{P(\phi)=0}$

Result 1

$A \subseteq B$ the $P(A) \leq P(B)$

B-A and A

$\Rightarrow (B-A) \cap A = \phi$ n=2



$$P((B-A) \cup A) = P(B-A) + P(A)$$

$$P(B) = P(A) + \boxed{P(B-A)} \quad 0 \leq \text{"} \quad \text{"} \leq 1$$

$$\Rightarrow P(B) \geq P(A)$$