Wednesday, April 4

Let (X,Y) be a random vector such that both X & Y are discrete r.v.s.

The joint pmf of (X,Y) is defined to be

$$p(x,y) = P(X=x,\ Y=y)$$
$$(x,y) \in S_{X,Y}$$

(X,Y) can take $\{(1,2),(2,3),(3,1)\}$

$$\parallel$$

$S_{X,Y}$ (not all combinations of S_x + S_y)

If both $S_X = \{x_1, \dots, x_{N_1}\}$

$S_Y = \{y_1, \dots, y_{N_2}\}$ are finite, then the joint pmf of (X,Y) represented by a table would be:

	y_1	y_2	y_3	$\cdots$	y_{N_2}	$P_X(x)$
x_1	$p(x_1,y_1)$	$p(x_1,y_2)$	$p(x_1,y_3)$	$\cdots$	$p(x_1,y_{N_2})$	$p(x_1)$
x_2	$p(x_2,y_1)$	$p(x_2,y_2)$	$\cdots$			$p(x_2)$
$\vdots$						$\vdots$
x_{N_1}	$p(x_{N_1},y_1)$	$p(x_{N_1},y_2)$	$\cdots$		$p(x_{N_1},y_{N_2})$	$p(x_{N_1})$
$P_Y(y)$	$p(y_1)$	$p(y_2)$	$\cdots$		$p(y_{N_2})$	

pmf $p(x,y)$ of any random vector (X,Y) satisfies $p(x,y) \geq 0$ $(x,y) \in S_{X,Y}$

$$\sum_{(x,y)\in S_{X,Y}} p(x,y) = 1$$

For any $a<b$
$c<d$

$$P(a<X\leq b,\ c<Y\leq d) = \sum_{\substack{a<x\leq b \\ c<y\leq d}} p(x,y)$$

The joint pmf $p(x,y)$ contains more information than the marginal pmfs $p(x)$ + $p(y)$

$$p_X(x) = \sum_{y \in S_y} p(x,y), \quad x \in S_x \qquad p_Y(y) = \sum_{x \in S_x} p(x,y), \quad y \in S_y$$

	0	1	$p_Y(y)$
0	9/25	6/25	15/25 = 3/5
1	6/25	4/25	10/25 = 2/5
$p_X(x)$	15/25 = 3/5	10/25 = 2/5	

	0	1	$p_Y(y)$
0	3/10	3/10	6/10 = 3/5
1	3/10	1/10	4/10 = 2/5
$p_X(x)$	6/10 = 3/5	4/10 = 2/5	

Marginals $p_X(x)$ & $p_Y(y)$ are the same but the joint $p(x,y)$ aren't.

X \ Y	1	2
1	0.034	0.134
2	0.066	0.266
3	0.100	0.400

$p(1,2) + p(2,2) = 0.134 + 0.266$
$= 0.4$

$P(0.5 \le X \le 2.5)$

1 & 2 satisfy this

$P(0.5 < X \le 2.5, \; 1.5 < Y \le 2.5)$

1 & 2 satisfy this — 2 satisfies this

$P_X(1) + P_X(2) = P(1,1) + P(1,2) + P(2,1) + P(2,2)$
$= 0.4 + P(1,1) + P(2,1) = 0.5$

Find the marginal pmf of Y

$$P_Y(y) = \sum_{x \in S_x} P(x,y), \quad y \in S_y$$

$$P_Y(1) = \sum_{x=1,2,3} p(x,1) = P(1,1) + P(2,1) + p(3,1) = 0.2$$

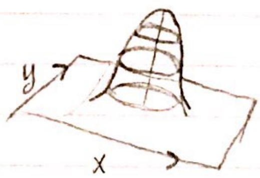
$$P_Y(2) = \sum_{x=1,2,3} p(x,2) = P(1,2) + P(2,2) + P(3,2) = 0.8$$

Let (X,Y) be a random vector such that both X & Y are continuous.
The joint pdf of (X,Y) is a non-negative bivariate function $f(x,y)$ s.t. for any region $A \subset \mathbb{R}^2$

$$P((X,Y) \in A) = \iint_A f(x,y)\,dx\,dy$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

bivariate pdf



$$P((X,Y) \in \mathbb{R}^2) = 1$$

Joint pdf $f(x,y)$ contains more information than marginal pdfs $f_X(x), f_Y(y)$.

you can obtain $f_X(x)$ & $f_Y(y)$ from $f(x,y)$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy, \ \forall x \in S_X \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx, \ \forall y \in S_Y$$

again, the joint pdf can't be determined by the marginal pdfs.

Example

$f_1(x,y) = x+y, \ 0<x,y<1 \qquad f_2(x,y) = (x+\frac{1}{2})(y+\frac{1}{2}), \ 0<x,y<1$

clearly $f_1(x,y) \neq f_2(x,y)$

$$\int_0^1\int_0^1 x+y\,dx\,dy = \left[\frac{y}{2} + \frac{y^2}{2}\right]$$