5.4-3

(a)

$$\begin{aligned} E(\exp\{t(X_1+X_2+X_3)\} &= E(\exp\{tX_1\})E(\exp\{tX_2\})E(\exp\{tX_3\}) \\ &= \exp\left[(e^t-1)(2+1+4)\right] \\ &= \exp\left[7(e^t-1)\right] \end{aligned}$$

(b) $Y \sim Poisson(7)$.

(c)

$$P(3 \le Y \le 9) = P(Y \le 9) - P(Y \le 2) = 0.80086$$

5.4-21 $X - Y \sim N(\mu, \sigma^2)$ where $\mu = 5 - 6 = -1$ and $\sigma^2 = 9 + 16 = 25$. Define $Z = \frac{X-Y+1}{5} \sim N(0,1)$.

$$P(X > y) = P(X - Y > 0) = P(Z > \frac{1}{5}) = 0.4207$$

6.4-5

(a)

$$\begin{aligned} logL(\theta) &= -2nlog(\theta) + \Sigma_{i=1}^n logx_i - \frac{1}{\theta}\Sigma_{i=1}^n x_i \\ \frac{dlogL(\theta)}{d\theta} &= \frac{-2n}{\theta} + \frac{\Sigma x_i}{\theta^2} = 0 \\ \hat{\theta} &= \frac{1}{2}\bar{X} \end{aligned}$$

(b)

$$\begin{aligned} logL(\theta) &= -nlog2 - 3nlog(\theta) + 2\Sigma_{i=1}^n logx_i - \frac{1}{\theta}\Sigma_{i=1}^n x_i \\ \frac{dlogL(\theta)}{d\theta} &= \frac{-3n}{\theta} + \frac{\Sigma x_i}{\theta^2} = 0 \\ \hat{\theta} &= \frac{1}{3}\bar{X} \end{aligned}$$

(c)

$$logL(\theta) = -nlog2 - \Sigma_{i=1}^n |x - \theta|$$

It is equivalent to maximize $\Sigma_{i=1}^n |x-\theta|$, from the hint $\hat{\theta} = x_5 = 1.7$ maximize the equation. Here we guess the median value of n sample point is the estimator we want. https://math.stackexchange.com/questions/1790622/show-that-the-mle-of-theta-is-the-median-of-a-sample You can see the detailed discussion in the website.

6.4-7

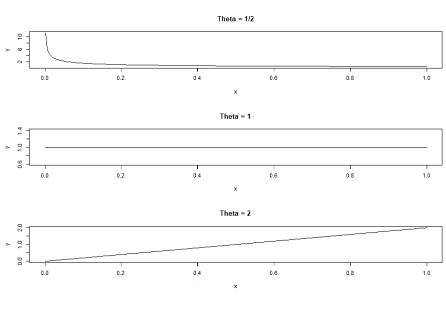

Figure 1: The pdf plot when $\theta = 0.5, 1, 2$

(a)

(b)

$$
\begin{aligned}
logL(\theta) &= nlog\theta + (\theta-1)log(\prod_{i=1}^{n} x_i) \\
\frac{dlogL(\theta)}{d\theta} &= \frac{n}{\theta} + log(\prod_{i=1}^{n} x_i) = 0 \\
\hat{\theta}_{MLE} &= \frac{-n}{log(\prod_{i=1}^{n} x_i)}
\end{aligned}
$$

(c) For moment-method, $E(X) = \frac{\theta}{\theta+1}$, thus $\hat{\theta}_{MM} = \frac{\bar{X}}{1-\bar{X}}$.

(a) $\hat{\theta}_{MLE} = 1.265$ and $\hat{\theta}_{MM} = 0.597$

(b) $\hat{\theta}_{MLE} = 5.089$ and $\hat{\theta}_{MM} = 2.4$

(c) $\hat{\theta}_{MLE} = 2.208$ and $\hat{\theta}_{MM} = 0.865$

6.4-9

(a)

$$
E(\bar{X}) = \frac{1}{n}\Sigma_{i=1}^{n} E(X_i) = \frac{1}{n}\Sigma_{i=1}^{n}\theta = \theta
$$

(b)

$$
\begin{aligned}
Var(\bar{X}) &= E(\bar{X}^2) - \theta^2 \\
&= \frac{1}{n^2} E\{(\Sigma_{i=1}^{n} X_i)^2\} - \theta^2 \\
&= \frac{1}{n^2}\{\Sigma_{i=1}^{n} E(X_i^2) + \Sigma_{i\neq j} E(X_i X_j)\} - \theta^2 \\
&= \frac{1}{n^2}\{2n\theta^2 + n(n-1)\theta^2\} - \theta^2 \\
&= \frac{\theta^2}{n}
\end{aligned}
$$

(c) Choose $\bar{X}$, and value is 3.48.

6.4-11

$$
\begin{aligned}
E(S^2) &= E\{\frac{1}{n-1}\Sigma_{i=1}^{n}(X_i - \bar{X})^2\} \\
&= \frac{1}{n-1} E\{\Sigma_{i=1}^{n}(X_i^2 - X_i\bar{X} + \bar{X}^2)\} \\
&= \frac{1}{n-1} E\{\Sigma_{i=1}^{n} X_i^2 - \bar{X}\Sigma_{i=1}^{n} X_i + n\bar{X}_2\} \\
&= \frac{1}{n-1} E\{\Sigma_{i=1}^{n} X_i^2 - n\bar{X}^2\} \\
&= \frac{1}{n-1}\{n(\mu^2+\sigma^2) - n\frac{1}{n^2}[n(\mu^2+\sigma^2) + n(n-1)\mu^2]\} \\
&= \frac{1}{n-1}(n-1)\sigma^2 \\
&= \sigma^2
\end{aligned}
$$

6.4-13

(a) $E(X) = \int_{\theta-1}^{\theta+1} \frac{x}{2} dx = \theta$, thus $\hat{\theta} = \bar{X}$

(b) $E(\bar{X}) = \frac{1}{n}\Sigma_{i=1}^{n} E(X_i) = \frac{1}{n} n\theta = \theta$. It is a unbiased estimator.

(c) 7.382.

(d) 7.485.

6.7-10 The joint pdf can be written as

$$[\prod_{i=1}^{n} x_i(1-x_i)]^{\theta-1}[\frac{\Gamma(2\theta)}{\{\Gamma(\theta)\}^2}]^n$$

So by definition

$$\prod_{i=1}^{n} X_i(1-X_i)$$

is a sufficient statistics.