2018年4月23日 12:31

two results from the normal equations.

$$\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 = Q$$

$$\frac{\partial Q}{\partial \beta_0} = -2\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\frac{\partial Q}{\partial \beta_1} = -2\sum_{i=1}^{n} x_i (y_i - \beta_0 - \beta_1 x_i) = 0$$

take away the -2 factor.

$$\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\sum_{i=1}^{n} x_i (y_i - \beta_0 - \beta_1 x_i) = 0$$

for the second equation,

$$y_i - \beta_0 - \beta_1 x_i = \varepsilon_i$$

So we have

$$\sum_{i=1}^{n} x_i \varepsilon_i = 0.$$

this means that the errors and the predictors are uncorrelated.

So we need to have uncorrelatedness of X and ε to use L.S. So in fact independence of X and ε is more than necessary. Just X uncorrelated with ε is enough.

Now if you look at the first equation.

$$\sum y_i - n\beta_0 - \beta_1 \sum X_i = 0$$

so

$$\frac{\sum y_i}{n} - \beta_0 - \beta_1 \frac{\sum X_i}{n} = 0$$

$$\bar{Y} - \beta_0 - \beta_1 \bar{X} = 0.$$

$$\therefore \beta_0 = \bar{Y} - \beta_1 \bar{X}.$$

$$\underline{SSE} = \frac{1}{n-2} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

$$\hat{\sigma}^2 = \frac{SSE}{n-2} = \frac{1}{n-2}\sum_{i=1}(Y_i - \hat{Y}_i)$$

the integer $n-2$ is called the degree of freedom.

n : number of observations

2 : number of parameters (β_0, β_1)

Under the Analysis of Variance framework, $\frac{SSE}{n-2}$ is also known as the Error Mean Square (MSE) which is of great importance in testing statistical hypothesis.