ETC3530
Life Insurance Mathematics

Week 11
Reserving Aspects of Profit Testing

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Previously on ETC3530

- Life tables and survival models.
- Using life tables to evaluate EPV and variances of assurances and annuities.
- Calculation of premiums for contracts at time 0.
- Determine the gross premium reserve (prospectively or retrospectively) for policies in force at time t.
- Determine the mortality profit.
- Multiple sources of risks, multi-state models. Evaluating multiple decrement tables. Evaluating cash-flows dependent on multiple (more than 2) outcomes.
- Introduced combined life insurance and investment products: unit-linked and AWP contracts
- Introduced profit testing to evaluate profit and profit margin at $t = 0$.

Weeks 9 - 11: The business end ...

- Unit-linked (UL) and Accumulating-with-Profits (AWP) contracts. Policy benefits take the form of an accumulating fund of premiums.
- Profit testing. The process of projecting the income and outgo emerging from a policy, and discounting to the present. Used to set premium that will give a desired level of profitability.
- **Reserving aspects of profit testing**. Apply techniques for cash flow projections to setting reserves.

Today's goal

- Calculate reserves for unit-linked contracts with zeroisation.
- Examples of zeroisation.
- Extension to conventional products.

Outline

1. Calculating reserves for unit-linked contracts: Background and Overview.
2. Calculating reserves for unit-linked contracts: Process Derivation.
3. Examples of Reserve Calculations.
4. Calculating reserves for Conventional Products.
5. Impact of pricing and reserving bases.

Outline

1. **Calculating reserves for unit-linked contracts: Background and Overview.**
2. Calculating reserves for unit-linked contracts: Process Derivation.
3. Examples of Reserve Calculations.
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Calculating reserves for unit-linked contracts

Revision

- Unit linked contracts are administered through:
  - a *unit fund* representing the accumulation of the underlying investment benefit to the policyholder, and
  - a *non-unit fund* representing the insurer's income and out-go.
- Reserves:
  - The unit-fund requires a *unit reserve* which is equal to the unit fund value at any time. This does not require any amendment to cash-flow projections so far encountered.
  - The non-unit fund *may* require reserving. When required the technique of *zeroising negative cashflows* is used.

Calculating reserves for unit-linked contracts

Background

- A contract with a *single financing phase at the outset* is one where the profit signature has a single negative value over the initial period. E.g.

  $\{-20, 10, 8, 5, 5\}$

- The financing phase refers to the injection of funds at the start reflected by the negative first net cashflow in the profit signature. If there are no further negative values in the profit signature after the first value, then no further injection of funds (or reserving is required).

- Many unit-linked products produce profit signatures which have a single financing phase.

- Some unit-linked products with substantial outgo at later policy durations can have more than one financing phase, or more than one negative non-unit fund cashflow.

Calculating reserves for unit-linked contracts

Background

Where there are negative non-unit fund cashflows after the initial period:

- Later negative non-unit cashflows should be reduced to zero by establishing reserves in the non-unit fund at earlier durations.
- The reserves are funded by reducing earlier positive non-unit cashflows.
- Establish these reserves as late as possible during the term of the contract.

Calculating reserves for unit-linked contracts
Background

- Example: consider a unit-linked policy with net cash-flows: $\{-8, 15, -5, 8\}$

  The year 4 expected cashflow is positive; no reserves required at start of year 4 (or at $t = 3$).

  The year 3 expected cashflow is negative, so we need to set up a reserve at the start of year 3 to zeroise this.

  Ignoring interest and mortality, if we set up a reserve of 5 at the end of year 2 ($t = 3$), then net cash-flow in year 3 will be $5 + (-5) = 0$. The pattern of cash-flows becomes:

  $\{-8, 10, 0, 8\}$

Calculating reserves for unit-linked contracts

Background

Question: If expcted year 4 profit is 8, and can make up for the expcted loss of -5 in year 3, why bother with reserving?

- Positve cashflow in year 4 comes *after* the negative cashflow in year 3, it may be impossible to pay all claims and expenses at the required time.
- The fourth policy year may never happen (policy may lapse at the start of year 4). The insurer is potentially permanently short of money, and could become insolvent.

Outline

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Calculating reserves for unit-linked contracts

Process Derivation

Recall definitions:

$(PRO)_t$; $t = 1, 2, \ldots$; the total profit over time $t-1$ to t (element of the profit vector).

${}_tV$; the reserve held at time t.

i_s; rate of interest in the non-unit fund

$(ap)_x$; the 1 year survival probability of a life aged x

Now define:

$(NUCF)_t$; $t = 1, 2, \ldots$, the non-unit fund cash flows in year t from time $t-1$ to t. (see previous lecture). Note: this is first calculated without any reserves.

Calculating reserves for unit-linked contracts

Process Derivation

The equation of value at the end of policy year t, for cashflows in year t, per policy in force at time $t-1$, is:

$$(NUCF)_t + {}_{t-1}V(1+i_s) - {}_tV(ap)_{x+t-1} = (PRO)_t \qquad (1)$$

Interpretation: the profit vector for the period $t-1$ to t is made up of:

- $(NUCF)_t$; the non-unit cashflow at time t
- ${}_{t-1}V(1+i_s)$; the accumulated amount of the reserve set up at time $t-1$.
- $-{}_tV(ap)_{x+t-1\,t}$; minus the cost of setting up the reserve at time t for each policy that's remained in force over the policy year.

This is not new; recall lecture 10. Rewrite as:

$$(NUCF)_t + {}_{t-1}Vi_s + {}_{t-1}V - {}_tV(ap)_{x+t-1} = (PRO)_t$$

where;

- $(NUCF)_t$; the non-unit cashflow at time t, being the net income and expenses with interest accrued.
- ${}_{t-1}Vi_s$; the interest accrued on the reserve set up at time $t-1$.
- ${}_{t-1}V - {}_tV(ap)_{x+t-1}$; the amount from the total non-unit value at time t that must be transferred to reserves.

Calculating reserves for unit-linked contracts

Process Derivation

- The process of establishing reserves begins at the greatest duration t for which $(NUCF)_t$ is negative. Let this be duration $t = m$.
- Non-unit reserves will not be required at durations $t \geq m$ because during these policy years the product is expected to be self-financing. Hence we know that ${}_tV = 0$ for $t \geq m$.
- For policy year m, we can write:

$$(NUCF)_m + {}_{m-1}V(1 + i_s) - (ap)_{x+m-1} \times 0 = (PRO)_m$$

, where $(NUCF)_m < 0$, and we want to find ${}_{m-1}V$ so that $(PRO)_m = 0$. Thus;

$${}_{m-1}V = -\frac{(NUCF)_m}{(1+i_s)}$$

Calculating reserves for unit-linked contracts

Process Derivation

- This choice of ${}_{m-1}V$ will lead to $(PRO)_m = 0$.
- The required reserve ${}_{m-1}V$ for every policyholder alive at the start of the m-th year, must be taken out of the cash-flow for the previous year, $(NUCF)_{m-1}$.
- Taking the amount required to establish ${}_{m-1}V$ from $(NUCF)_{m-1}$ gives the adjusted cashflow:

$$(NUCF)^{adj}_{m-1} = (NUCF)_{m-1} - {}_{m-1}V \times (ap)_{x+m-2} \tag{2}$$

Calculating reserves for unit-linked contracts

Process Derivation

- If $(NUCF)^{adj}_{m-1} \geq 0$, then $(PRO)_{m-1} = (NUCF)^{adj}_{m-1}$.
- If $(NUCF)^{adj}_{m-1} < 0$, we repeat the process establishing non-unit reserves ${}_{m-2}V$ at policy duration $m-2$. We have:

$$(NUCF)^{adj}_{m-1} + {}_{m-2}V(1+i_s) = (PRO)_{m-1} \tag{3}$$

To see why (3) holds; set $t = m-1$ in (1) and use (2).

Choosing ${}_{m-2}V$ so that $(PRO)_{m-1} = 0$, gives

$${}_{m-2}V = -\frac{(NUCF)^{adj}_{m-1}}{1+i_s}$$

Calculating reserves for unit-linked contracts

Process Derivation

- The process is repeated until we reach a year $k < m$ where the adjusted cash-flow $(NUCF)_k^{adj} \geq 0$.
- To summarise; the zeroisation method:
  - Project cash-flows in non-unit fund assuming zero reserves.
  - Finds the last occurring negative cash-flow (other than the first period) in the projection.
  - Sets the reserve for the period where this negative cash-flow occurs such that the net-profit for period comes to 0.
  - Adjust the profit of the period immediately prior so that the required reserves are established for the next period.
  - Examine new profit vector; and repeat process if there is still a period (other than the first) with a negative cash-flow.

Outline

1. Calculating reserves for unit-linked contracts: Background and Overview.
2. Calculating reserves for unit-linked contracts: Process Derivation.
3. **Examples of Reserve Calculations.**
4. Calculating reserves for Conventional Products.
5. Impact of pricing and reserving bases.

Calculating reserves for unit-linked contracts

Examples

See document “Lecture Week 11 - Examples.pdf” for solution and explanations.

- Example 1: With an NUCF vector of $\{-60.20, -20.50, -17.00, 50.13, 85.75\}$; set reserve to zeroise negative cashflows ignoring mortality.
- Example 2: With an NUCF vector of $\{-10, -20, 5, -15, 40\}$; set reservs to zeroise negative cash flows with mortality accounted for.

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Calculating reserves for conventional products

- Zeroisation can also be used to determine reserves for a conventional (i.e. non unit-linked) policy.
- The process only takes as input a projection of cash flows to set reserves.
- These cash-flows need not be from the non-unit fund of a unit-linked policy; they can also be from a conventional contract.
- Recall: conventional endowment assurance example of Lecture 10.

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

- **A 5-year regular premium endowment assurance policy for a 55 year old male.**
- **Sum insured**: $\$10,000$ payable at the end of year of death.
- **Expenses**: Initial (year 1) expense is 50% annual premium, from year 2 on it is 5% of annual premium.
- **Premiums**: Paid annually in advance.
- **Surrenders**: An amount equal to a return of premiums paid, with no interest, paid at the end of year of withdrawal.
- **Interest rate**: 4% pa.

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

Decrement probabilities of mortality and surrender:

Age (x)	$(aq)^d_x$	$(aq)^s_x$	$(aq)_x$	$(ap)_x$
55	0.005	0.1	0.105	0.895
56	0.006	0.05	0.056	0.944
57	0.007	0.05	0.057	0.943
58	0.008	0.01	0.018	0.982
59	0.009	0	0.009	0.991

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

In lecture 10, we used resrves obtained from calculating the net premium reserves, suppose now we try to calculate the reserves requirements from the cashflows.

Removing reserves, the cash flow projection will look like:

Year (t)	1	2	3	4	5
Premium	2108.81	2108.81	2108.81	2108.81	2108.81
Expense	-1054.41	-105.44	-105.44	-105.44	-105.44
Interest	42.18	80.13	80.13	80.13	80.13
Expected Death Claims	-50	-60	-70	-80	-90
Expected Surrenders	-210.88	-210.88	-316.32	-84.35	0
Expected Maturities	0	0	0	0	-9910
Cashflow $(CF)_t$	835.70	1812.62	1697.18	1919.15	-7916.50

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

- With the reserves removed we can see clearly that there is a large negative cash flow in the final year.
- Conventional products with death or maturity benefits will tend to have more negative cashflows in later years.
- Reserves need to be established in earlier years with positive cashflows.
- The zeroisation technique applied to unit-linked policies can be applied here as well.

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

Apply the zeroisation process:

- Reserve required at start of Year 5:

$$_4V = \frac{7916.50}{1.04} = 7612.02$$

- Cost of setting up reserve at end of year 4:

$$-(ap)_{58} \times 7612.02 = -7475.00$$

- Adjusted cash-flow in Year 4:

$$(CF)_4^{adj} = 1919.15 - 7475.00 = -5555.85$$

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

Since the adjusted cash-flow in year 4 is negative; we need to set up a reserve at the start of year 4 to zeroise to net profit for that year.

- Reserve required at start of Year 4:

$$_3V = \frac{5555.85}{1.04} = 5342.16$$

- Cost of setting up reserve at end of year 3:

$$-(ap)_{57} \times 5342.16 = -5037.66$$

- Adjusted cash-flow in Year 3:

$$(CF)_3^{adj} = 1697.18 - 5037.66 = -3340.47$$

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

The adjusted cash-flow in year 3 is also negative; so we continue this process:

Year	**1**	**2**	**3**	**4**	**5**
Cashflow $(CF)_t$	835.70	1812.62	1697.18	1919.15	-7916.50
$(ap)_x$	0.895	0.944	0.943	0.982	0.991
Reserve at start of year	0.00	1172.60	3211.99	5342.16	7612.02
Interest on reserve	0.00	46.90	128.48	213.69	304.48
Cost of setting up reserve	-1049.47	-3032.12	-5037.66	-7475.00	0.00
Adjusted cash flow	-213.77	-1219.50	-3340.47	-5555.85	0.00
Profit EOY (PRO)t	-213.77	0.00	0.00	0.00	0.00

Calculating reserves for conventional products

Conventional Endowment Assurance from Lecture 11

Note that the values in the total profit row are given by:

$$(PRO)_t = (CF)_t + \{ \text{ Reserve at start of year } \} + \{ \text{ interest on reserve } \} + \{ \text{ cost of setting up reserve for next period } \}$$

Note also that this is exactly the same as the equation of value for the non-unit fund (1).

Calculating reserves for conventional products

- Zeroisation is a prudent way to set reserves; one directly evaluates the minimum required.
- The setting allows for premium levels, product design, and assumptions to be varied until desired profit margin is obtained.

Summary of zeroisation for non-unit funds

- Step 1: Starting from the last negative entry in the vector of non-unit cashflows (in year m say), calculate ${}_{m-1}V$ as:

$$ {}_{m-1}V = -\frac{(NUCF)_m}{1+i_s} $$

- Step 2: Calculate the adjusted expected non-unit cashflow in the previous year, as:

$$ (NUCF)^{adj}_{m-1} = (NUCF)_{m-1} - {}_{m-1}V \times (ap)_{x+m-2} $$

using the value of ${}_{m-1}V$ found from Step 1; and where (ap) is the dependent probability of staying in force over year $m-1$. Any surrenders as well as mortality needs to be taken into account in this probability.

Summary of zeroisation for non-unit funds

- Step 3: If the adjusted cashflow from Step 2 is negative, then find ${}_{m-2}V$ from:

  $${}_{m-2}V = -\frac{(NUCF)^{adj}_{m-1}}{1+i_s}$$

  using the adjusted cashflow value found from Step 2, and go to Step 4.

- Step 4: Carry on working backwards through the profit vector by repeated application of Steps (3) and (4) until either year 1 is reached, or a positive value for the adjusted cashflow is obtained.

  If year 1 is reached, the profit for year 1 is equal to the adjusted cashlow; i.e.

  $$(PRO)_1 = (NUCF)^{adj}_1 = (NUCF)1 - {}_0V \times (ap)_x$$

Summary of zeroisation for non-unit funds

- Step 4 (cont'd):

  If a positive adjusted cash-flow is obatined (in year k say, $k > 1$), the process srops for this run of negative entries. The profit for year k is equal to the adjusted cashflow for year k, and reserve at the start of year k, ${}_{k-1}V$ is equal to 0
- Step 5: Repeat steps 1 to 4 for any other run of negative entries.

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5. **Impact of pricing and reserving bases.**

Impact of pricing and reserving bases

From the perspective of the insurer:

- Apart from an initial cash injection, subsequent cash-flows need to be positive (non-negative). This ensures future claims are met.
- The insurer must manage the risk. This is done through either (or some combination of):
  - asset liability management (beyond course scope)
  - reserving; or ensuring enough capital is injected initially to meet claims in future years.

Impact of pricing and reserving bases

An insurer will want to:

- calculate reserves on a prudent bases (set assumptions in a conservative way, tend to overestimate reserves)
- calculate cash-flows on a realistic bases (best estimate of what the real experience may be).
- play around with premiums and product design until the desired profitability criterion is achieved with these "realistic experience but prudent reserves" cash-flows.

Impact of pricing and reserving bases

What are the different bases we have encountered in this course?

- **Pricing.** The rate of interest used for calculating premiums. What are some other assumptions in the pricing bases?
- **Risk discount rate.** The rate of interest used to discount profit cash-flows. Factors in cost of capital.
- **Experience.** The rate at which interest accrues on cash and reserves.

We can also calculate reserves required at a different (more prudent) rate to the above.

- **Reserving.** The rate at which reserve requirements in future years are discounted back to prior years.

Impact of pricing and reserving bases
Example

Consider the following simple example: a single premium 5-year pure endowment in a zero-mortality zero-expense world. The sum assured is $\$1,000$.

We have the following bases for the various rates of interest:

- Pricing: 6% pa
- Experience: 7% pa (on cash and reserves)
- Reserving: 5% pa
- Risk discount rate: 9% pa

Impact of pricing and reserving bases

Example

The premium amount is $1000v^5$ at 6%, so 747.26.

From the perspective of the product, the projected cash-flows are:

Year	1	2	3	4	5
Premium	747.26	0.00	0.00	0.00	0.00
Interest	52.31	0.00	0.00	0.00	0.00
Benefit outgo	0.00	0.00	0.00	0.00	-1000.00
Cashflow (before reserves)	799.57	0.00	0.00	0.00	-1000.00

Impact of pricing and reserving bases

Example

We need $\$1,000$ in reserve at the end of year 5.

Working backwards but this time using the reserving bases where interest rate is 5% pa (more prudent), we calculate:

For years $t = 1, 2, \ldots, 4$,

- note that 0 reserves are required at the start of year 1 and end of year 5.
- the reserve required at the end of the year, using ${}_tV = 1000v^{5-t}$ at 5% pa.
- calculate the interest on reserve in year $t + 1$, using ${}_tV \times i$, where $i = 7\%$ pa.
- the cost of increasing reserves ${}_tV - {}_{t-1}V$. Note there is 0 mortality. (this calc. also required for $t = 5$).
- the profit vector $(PRO)_t = \text{cash flow before reserves} + \text{interest on reserves} + \text{cost of increasing reserves}$

Impact of pricing and reserving bases

Example

From the perspective of the reserves, the projected cash-flows are:

Year	1	2	3	4	5
Start of year reserve	0.00	822.70	863.84	907.03	952.38
Interest earned on reserves	0.00	57.59	60.47	63.49	66.67
End of year reserve	822.70	863.84	907.03	952.38	0.00
Cost of increasing reserve	-822.70	-41.14	-43.19	-45.35	952.38
Cashflow (before reserves)	799.57	0.00	0.00	0.00	-1000.00
Profit Vector	-23.14	16.45	17.28	18.14	19.05

Note: the reserving basis (5% pa) being more prudent than the pricing basis (6% pa), leads to a negative cash-flow in the initial year.

Impact of pricing and reserving bases

Example

The final profit is calculated using the risk discount rate (9% pa) to discount profit cash-flows:

Year	1	2	3	4	5
Profit Signature	-23.14	16.45	17.28	18.14	19.05
Present value	-21.23	13.85	13.34	12.85	12.38
Total					31.19

See lecture workbook “Week11 examples.xlsx”. Vary the different bases on the sheet and observe the impact on the final profit.