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The second-order terms are:

$$
\begin{aligned}
f\left(x,y\right) \quad \approx \quad & \frac{1}{2!}\cdot\frac{d^2 f}{dx^2}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right)^2 \\
& + \frac{1}{2!}\cdot\frac{d^2 f}{dy^2}\left(x_{\max},y_{\max}\right)\cdot\left(y-y_{\max}\right)^2 \\
& + \frac{d^2 f}{dx\,dy}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right)\left(y-y_{\max}\right)
\end{aligned}
$$

The full third-order approximation is:

$$
\begin{aligned}
f\left(x,y\right) \quad \approx \quad & \frac{1}{1!}\cdot\frac{df}{dx}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right) \\
& + \frac{1}{1!}\cdot\frac{df}{dy}\left(x_{\max},y_{\max}\right)\cdot\left(y-y_{\max}\right) \\
\\
& + \frac{1}{2!}\cdot\frac{d^2 f}{dx^2}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right)^2 \\
& + \frac{1}{2!}\cdot\frac{d^2 f}{dy^2}\left(x_{\max},y_{\max}\right)\cdot\left(y-y_{\max}\right)^2 \\
& + \frac{2}{2!}\cdot\frac{d^2 f}{dx\,dy}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right)\left(y-y_{\max}\right) \\
\\
& + \frac{1}{3!}\cdot\frac{d^3 f}{dx^3}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right)^3 \\
& + \frac{3}{3!}\cdot\frac{d^3 f}{dx^2\,dy}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right)^2\left(y-y_{\max}\right) \\
& + \frac{3}{3!}\cdot\frac{d^3 f}{dx\,dy^2}\left(x_{\max},y_{\max}\right)\cdot\left(x-x_{\max}\right)\left(y-y_{\max}\right)^2 \\
& + \frac{1}{3!}\cdot\frac{d^3 f}{dy^3}\left(x_{\max},y_{\max}\right)\cdot\left(y-y_{\max}\right)^3
\end{aligned}
$$