Formal Methods

End Semester Examination(Spring 2015)

Please read the following instructions before answering questions.

1. Answer all questions.
2. If you feel any question is ambiguous, clearly state your assumptions and solve accordingly.
3. This is a **closed** book exam and use of calculators is **not** permitted.
4. You can **not** assume any theorem/lemma/proposition etc.. proved in class.

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1. For a fixed $k \in \mathbb{N}$, consider the language $L_k = \{w \mid k^{th}$ symbol from the end of w is $1\}$ over alphabet $\Sigma = \{0, 1\}$. Answer the following questions: [2+3]

   (a) Design a DFA for L_k.

   (b) Prove or Disprove : Any DFA recognizing L_k has at least 2^k states.

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2. Give a context free grammar for the following two languages: [2+2]

   (a) $L_1 = \{a^i b^j \mid 2i \leq j \leq 3i\}$ over alphabet $\Sigma = \{a, b\}$.

   (b) $L_2 = \{w \mid$ w contains more ones than zeros$\}$ over alphabet $\Sigma = \{0, 1\}$.

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3. Let L be the language of all palindrome strings over $\Sigma = \{0, 1\}$. Answer the following questions: [2+2+3]

   (a) Give a CFG for L.

   (b) Construct a PDA recognizing the language L.

   (c) Formally prove the correctness of your PDA construction.

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4. Prove that the following languages are not regular. [3+3]

   (a) $L = \{0^m 1^n \mid m \neq n \text{ and } m, n \geq 0\}$ over $\Sigma = \{0, 1\}$.

   (b) $L_p = \{a^k \mid k \text{ is prime}\}$ over $\Sigma = \{a\}$.

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5. Prove that the language $L = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$ over $\Sigma = \{0, 1\}$ is not context free. [4]
