HW6 P3

a) Prior probabilities:

$$P(\omega_1) = P(\omega_2) = \frac{1}{2}$$

b) classes:

$\omega_1 = (0,0), (0,1), (2,0), (3,2), (3,3), (2,2), (2,0)$

$\omega_2 = (7,7), (8,6), (9,7), (8,10), (7,10), (8,9), (7,11)$

Means:

ω_1: $(1.7143, 1.1428)$

ω_2: $(7.7143, 8.5714)$

Covariance matrices

ω_1: $\begin{bmatrix} 1.5714 & 0.8809 \\ 0.8809 & 1.4762 \end{bmatrix}$

ω_2: $\begin{bmatrix} 0.5714 & -0.6428 \\ -0.6428 & 3.6190 \end{bmatrix}$

c) At the decision boundary, a point is equally likely to be either ~~on the~~ classified as ω_1 or ω_2. Let $p(x,y)$ be a such a point.

$$\frac{e^{-\frac{1}{2}\left(\frac{p-\mu_1}{\sigma_1}\right)^2}}{\sigma_1\sqrt{2\pi}} P(\omega_1) = \frac{e^{-\frac{1}{2}\left(\frac{p-\mu_2}{\sigma_2}\right)^2}}{\sigma_2\sqrt{2\pi}} P(\omega_2)$$

$P(\omega_1) = P(\omega_2) = \frac{1}{2}$.

$$\frac{e^{-\frac{1}{2}(p-\mu_1)^T \Sigma_1^{-1}(p-\mu_1)}}{|\Sigma_1|^{\frac{1}{2}}} = \frac{e^{-\frac{1}{2}(p-\mu_2)^T \Sigma_2^{-1}(p-\mu_2)}}{|\Sigma_2|^{\frac{1}{2}}}$$

$$-\frac{1}{2}(p-\mu_1)^T \Sigma_1^{-1}(p-\mu_1) - \frac{1}{2}\ln|\Sigma_1| = -\frac{1}{2}(p-\mu_2)^T \Sigma_2^{-1}(p-\mu_2) - \frac{1}{2}\ln|\Sigma_2|$$

$$\ln|\Sigma_1| + (p-\mu_1)^T \Sigma_1^{-1}(p-\mu_1) = \ln|\Sigma_2| + (p-\mu_2)^T \Sigma_2^{-1}(p-\mu_2)$$

$$\ln|\Sigma_2| - \ln|\Sigma_1| = 0.3552$$

$$\begin{bmatrix} x-1.7143 & y-1.1429 \end{bmatrix} \begin{bmatrix} x-1.7143 \\ y-1.1428 \end{bmatrix} 0.7206 - \begin{bmatrix} x-7.7143 & y-8.5714 \end{bmatrix} \begin{bmatrix} x-7.7143 \\ y-8.5714 \end{bmatrix} 0.5052 = 0.3552$$

$$\left[(x^2 - 3.4286x + 2.9388) + (y^2 - 2.2856y + 1.3059)\right]0.7206$$
$$- \left[(x^2 - 15.4286x + 54.5104) + (y^2 - 17.1428y + 73.4689)\right]0.5052$$
$$= 0.3552.$$

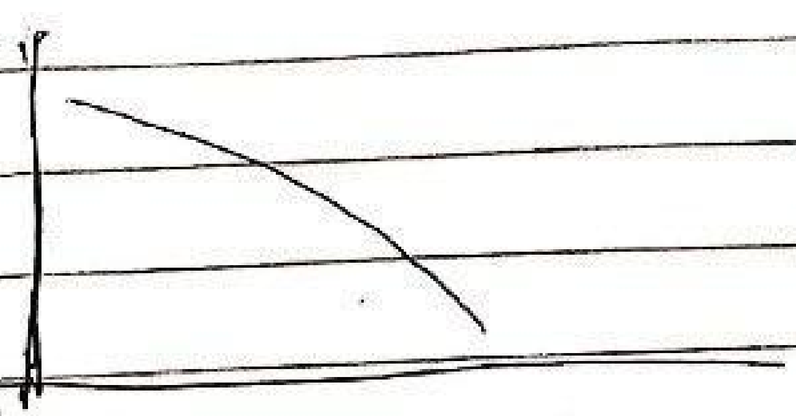
$$0.2154x^2 + 0.2154y^2 + 5.3239x + 7.0135y = 64.4776$$

$$(0.4641)^2 y + 7.0155y + 57.0935 = 121.5711 - 0.2154x^2 - 5.3239x$$

$$(0.4641y + 7.5560)^2 = \sqrt{121.5711 - 5.3239x - 0.2154x^2}$$

$$0.4641y + 7.5560 = \sqrt{121.5711 - 5.3239x - 0.2154x^2}$$

$$y = \frac{\sqrt{121.5711 - 5.3239x - 0.2154x^2} - 7.5560}{0.4641}$$



shape of rough boundary

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In [27]: import matplotlib.pyplot as plt

         xs = np.arange(10)
         ys = np.array([((math.sqrt(121.5711 - 5.3239*x - 0.2154 * x**2) - 7.5560)/0.4641) for x


         plt.scatter(w1[:,0], w1[:,1])
         plt.scatter(w2[:,0], w2[:,1])
         plt.plot(ys)

         plt.show()
```