$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Calculating eigenvalues: $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 4 & 5-\lambda & 6 \\ 7 & 8 & 9-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[45 - 14\lambda + \lambda^2 - 48] - 2(36 - 4\lambda - 42) + 3(32 - 35 + 7\lambda) = 0$$

$$\lambda^2 - 14\lambda - 3 - \lambda^3 + 14\lambda^2 + 3\lambda + 3\lambda + 12 + 21\lambda - 9 = 0$$

$$-\lambda^3 + 15\lambda^2 + 18\lambda = 0$$

$$-\lambda(\lambda^2 - 15\lambda - 18) = 0$$

$$\Rightarrow \lambda = 0$$

$$\lambda = \frac{15 \pm \sqrt{225 + 72}}{2} = \frac{15 \pm \sqrt{297}}{2}$$

$$\lambda = 0$$
$$16.1168$$
$$-1.1168$$

Now

for $\lambda = 0$:

$$AX = 0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -12 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right)$$

b

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \leftarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x - z = 0$$

$$y + 2z = 0$$

$$x = z$$

$$y = -2z = -2x.$$

vector: $(1, -2, 1)$

$\lambda = 16.1168$ $\quad (A - \lambda 2)\, v = 0$

$$\begin{bmatrix} -15.1168 & 2 & 3 \\ 4 & -4.1168 & 6 \\ 7 & 8 & -7.1168 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvector, vector: $(0.2833, 0.6417, 1)$

$\lambda = -1.1168$

vector: $(-1.2833, -0.1417, 1)$