$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Calculating eigenvalues: $(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 4 & 5-\lambda & 6 \\ 7 & 8 & 9-\lambda \end{vmatrix} = 0$$

$(1-\lambda)[45 - 14\lambda + \lambda^2 - 48] - 2(36 - 4\lambda - 42) + 3(32 - 35 + 7\lambda) = 0$

$\lambda^2 - 14\lambda - 3 - \lambda^3 + 14\lambda^2 + 3\lambda + 8\lambda + 12 + 21\lambda - 9 = 0$

$-\lambda^3 + 15\lambda^2 + 18\lambda = 0$

$-\lambda(\lambda^2 - 15\lambda - 18) = 0$

$\Rightarrow \lambda = 0$

$$\lambda = \frac{15 \pm \sqrt{225 + 672}}{2} = \frac{15 \pm \sqrt{297}}{2}$$

$\lambda = 0$
$\quad 16.1168$
$\quad -1.1168$

Now

for $\lambda = 0$:

$$AX = 0 \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 2 & 8 & 9 & 0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -12 & 0 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \leftarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$x - z = 0$$

$$y + 2z = 0$$

$$x = z$$

$$y = -2z = -2x.$$

vector: $(1, -2, 1)$

$$\lambda = 16 \cdot 1168 \qquad (A - \lambda 2) \, v = 0$$

$$\begin{bmatrix} -15 \cdot 1168 & 2 & 3 \\ 4 & -14 \cdot 1168 & 6 \\ 7 & 8 & -7 \cdot 1168 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Likewise, vector: $(0 \cdot 2833, 0 \cdot 6417, 1)$

$$\lambda = -1 \cdot 1168$$

vector: $(-1 \cdot 2833, -0 \cdot 1417, 1)$

1. Dimensions of matrix A: $p \times q$

2. Yes.

Conditions:

1. Dimensionality reduction preserves at least ~~two dimens~~ the common dimensions in vectors p and q.

2. The points in the original vector had 0-valued destructive dimension values.

3. Eg:

$$(1, 2, 0) | (2, 4, 0) \rightarrow (1, 2) (2, 4)$$

using A to reduce originally eliminating the Z axis.

3. a) $q=2$, $p=2$.

Original: $(1, 2)$; $(3,4)$

$A = I_2$

New: $(1,2)$; $(3,4)$

b). $q=2$, $p=1$

Original: $(3,5)$; $(1;5)$

New: (3), (1)

A: $\begin{bmatrix} 1 & 0 \end{bmatrix}$

c) $q=4$ $p=2$

Original: $(1, 4, 7, 3)$; $(7, 4, 7, 6)$

New: $(1,3)$; $(7,6)$

A: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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c: $w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$

$D_c = \{ x_i^c = [x_1^c, x_2^c] \}$

1. $A^c = [x_1^c - \mu_1, x_2^c - \mu_2][x_1^c - \mu_1, x_2^c - \mu_2]^T$

For the, say $y = w^T x + c$

$$x = \{ x_1^i - \mu_1, \forall i \}$$
$$y = \{ x_2^i - \mu_2, \forall i \}$$

$A^c = [x \ y][x \ y]^T$

$s = [x \ y]^T [x \ y]^T$

$= [x^2 + y^2]$

$= [x^2, x y, x y, c^2, 2 x y]$

non-zero eigenvalue: 1

2. $A = \frac{1}{2} \sum_{i=1}^{N} [x_i - \mu][x_i - \mu]^T$ interclass matrix

non-zero eigenvalues: 1

2. Line perpendicular to L_1 passing through μ:

$$\frac{y - \mu_2}{x - \mu_1} = \frac{\omega_2}{\omega_1} \qquad \text{or} \qquad \frac{x_2 - \mu_2}{x_1 - \mu_1} = \frac{\omega_2}{\omega_1}$$

$$D_2 = \{x_2 = [x_1^i, \, \emptyset \, x_2^i]^T\}$$

Since $\mu = [\mu_1, \mu_2]^T$

$$B' = [x_1^i - \mu_1, \, x_2^i - \mu_2][x_1^i - \mu_1, \, x_2^i - \mu_2]^T$$

$$= 1 \times 1 \text{ matrix.}$$

1 non zero eigenvalue.

Likewise for

$$B = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)(x_i - \mu)^T$$

1 non zero eigenvalue.

3. Reasoning: Since points have uniform distribution just around line, 2 eigenvalues exist. Second smaller than first due to shape's proximity to 1D line. Of the 2 vectors (eigen), one will be almost along the line and the other perpendicular.