PSET 3, 3.3

3-3

i: $w_1x_1 + w_2x_2 + w_3x_3 = 0$

~~1. depends on calc~~

~~$D_i = \{ x_i = [x_1^i, x_2^i]^T \}$~~

1. $A^i = [x_1^i - \mu_1, x_2^i - \mu_2][x_1^i - \mu_1, x_2^i - \mu_2]^T$

for a line, say $y = mx + c$

$x = \{ x_1^i - \mu_1, \forall i \}$

$y = \{ x_2^i - \mu_2, \forall i \}$

$A^i = [x \quad y][x \quad y]^T$

$= \cancel{[x \quad mx+c][x \quad mx+c]^T}$

$= [x^2 + y^2]$

$= [x^2 + m^2x^2 + c^2 + 2mxc]$

nonzero eigenvalues: 1

2. $A = \frac{1}{N} \sum_{i=1}^{N} [x_i - \mu][x_i - \mu]^T$

covariance matrix.

nonzero eigenvalues: 1

7. Line perpendicular to L, passing through μ:

$$\frac{y-\mu_2}{x-\mu_1} = \frac{\omega_2}{\omega_1} \quad \text{or} \quad \frac{x_2-\mu_2}{x_1-\mu_1} = \frac{\omega_2}{\omega_1}$$

$D_j = \{ x_j = [x_1^j, x_2^j]^T \}$

Since $\mu = [\mu_1, \mu_2]^T$

$$B^i = [x_1^i - \mu_1, x_2^i - \mu_2][x_1^i - \mu_1, x_2^i - \mu_2]^T$$
$= 1 \times 1$ matrix.

1 non zero eigenvalue.

Likewise for
$$B = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)(x_i - \mu)^T$$

1 non zero eigenvalue.

8. Reasoning: Since points have uniform distribution just around line, 2 eigenvalues exist. Second smaller than first due to shape's proximity to 1D line. Of the 2 vectors (eigen), one will be almost along the line and the other perpendicular.