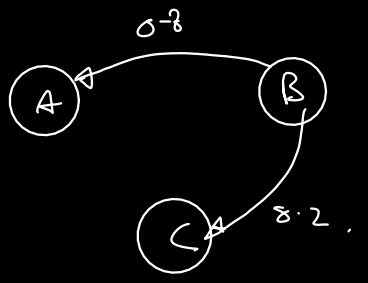
# POS Tagging

— Most frequent class baseline.

Markov Assumption : $P(q_i = a \mid q_1 \cdots q_{i-1}) = P(q_i = a \mid q_{i-1})$

$P(A|B) = 0.8$

$P(C|B) = 0.2$




| A | $a_1$ | $a_2$ | - - - | $a_N$ |
|---|---|---|---|---|
| $a_1$ | $a_{11}$ | $a_{12}$ | | $a_{1N}$ |
| $a_2$ | $a_{21}$ | $a_{22}$ | | |
| ⋮ | | | | |
| $a_N$ | $a_{N1}$ | | | $a_{NN}$ |

$Q: q_1 q_2 \cdots q_n$ $\quad A = a_{11} a_{12} \cdots a_{nn}$

$\Pi = \pi_1 \pi_2 \cdots \pi_n$

## HMM Model

Tags: "hidden". Words observed.

$t : N$ $\quad w : O$ → word tags

$Q: q_1 q_2 \cdots q_N$

$A: a_{11} a_{12} \cdots a_{NN}$

$O: o_1 o_2 \cdots o_T$ → sequence of observations (words)

$B: b_i(o_t)$

$\pi = \pi_1 \pi_2 \cdots \pi_N$

| A | $a_1$ | $a_2$ | - - - | $a_N$ |
|---|---|---|---|---|
| $a_1$ | $a_{11}$ | $a_{12}$ | | $a_{1N}$ |
| $a_2$ | $a_{21}$ | $a_{22}$ | | |
| ⋮ | | | | |
| $a_N$ | $a_{N1}$ | | | $a_{NN}$ |

$b_i(o_t) : P(o = o_t \mid q = q_i)$

each $o_t$: $N$ 'b's

$T$ 'o's

$P(q_i \mid q_1 \cdots q_{i-1}) = P(q_i \mid q_{i-1})$

$P(o_i \mid q_1 \cdots q_T, o_1 \cdots o_T) = P(o_i \mid q_i)$

# Basic HMM problems

## 1. Likelihood of a sequence

given observation sequence $O = \{o_1, o_2 \dots o_n\}$

efficiently estimate $P(O|\lambda)$

$O = \{o_1, o_2, \dots o_T\}$

$Q = \{q_1, q_2, \dots q_T\}$

Complexity:

$O(2TN^T)$

Now,

$$P(O|Q,\lambda) = \prod_{i=1}^{T} P(o_i|q_i,\lambda)$$

$$= b_{q_1}(o_1) \dots b_{q_T}(o_T)$$

$$P(Q|\lambda) = \pi_{q_1} a_{q_1 q_2} \dots a_{q_{T-1} q_T}$$

$$P(O|\lambda) = \sum \pi_{q_1} a_{q_1 q_2} \dots a_{q_{T-1} q_T} b_{q_1}(o_1) \dots b_{q_T}(o_T)$$

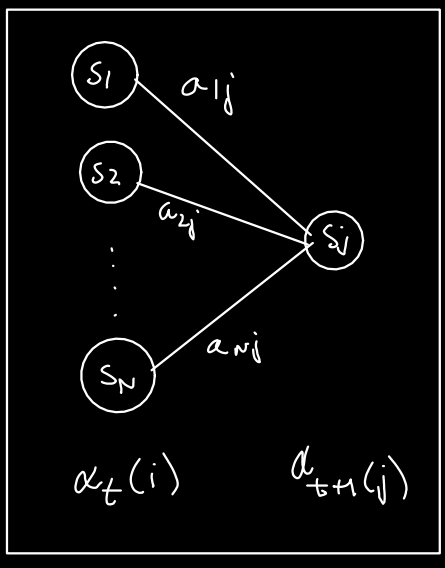
$[P(A,B) = P(A|B) \cdot P(B)]$

## Forward Procedure :

DEF:

$$\alpha_t(i) = P(o_1 \dots o_t, q_t = S_i | \lambda)$$

given partial obs. seq, — Prob. that — State at position $t$ is $S_i$




STEP:

1. Init

$$\alpha_1(i) = \pi_i b_i(o_1)$$

2. Induction

$$\alpha_{t+1}(j) = \left[ \sum_{i=1}^{N} \alpha_t(i) \cdot a_{ij} \right] b_j(o_{t+1})$$

3. Termination

$$P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i)$$

# Backward Procedure

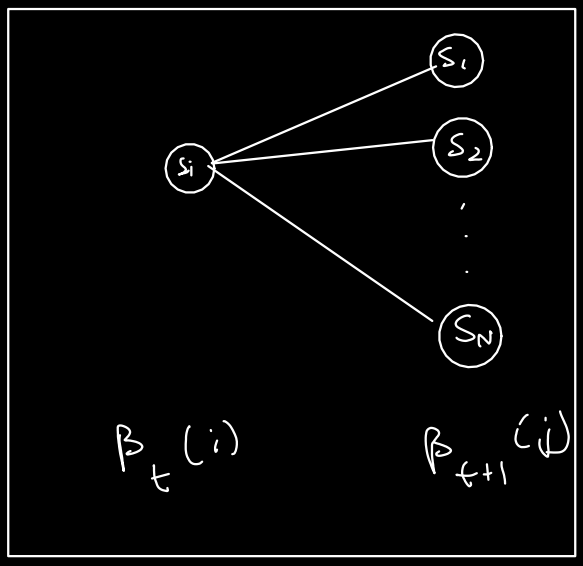
$$\beta_t(i) = P(o_{t+1} \dots o_T \mid q_t = S_i, \lambda)$$

1. Init

$$\beta_T(i) = 1$$

2. Induction

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)$$




$O(N^2 T)$

2. Given observation sequence $O = \{o_1, \dots o_T\}$

Get best $Q = \{q_1 \dots q_T\}$

Solution:

Best state individually likely at a position i

Best state given all the previously observed states and observations

- Viterbi Algorithm

## Viterbi Algorithm

DEF

$$\delta_t(i) = \max_{q_1 \dots q_{t-1}} P(q_1 q_2 \dots q_t = i, o_1 o_2 \dots o_t \mid \lambda)$$

$$\rightarrow \delta_{t+1}(j) = \left[ \max_i \delta_t(i) a_{ij} \right] b_j(o_{t+1})$$

Init

$$\delta_1(i) = \pi_i \, b_i(o_1)$$

$$\psi(i) = 0$$

Recursion

$$\delta_t(j) = \max_{1 \le i \le N} \left[ \delta_{t-1}(i) \, a_{ij} \right] b_j(o_t)$$

$$\psi_t(j) = \operatorname*{argmax}_{1 \le i \le N} \left[ \delta_{t-1}(i) \, a_{ij} \right]$$

Termination

$$p^* = \max_{1 \le i \le N} \left( \delta_T(i) \right)$$

$$q_T^* = \operatorname*{argmax}_{1 \le i \le N} \left( \delta_T(i) \right)$$

Backtracking

$$q_t^* = \psi_{t+1}\left( q_{t+1}^* \right)$$

# Tokenisation, LM, Smoothing

## Tokenization

Challenges:
- hyphenation
- number
- dates
- abbreviation
- punctuation
- URLs
- sentencification

## Ngrams

$$P(w \mid s(w)) = \frac{c(w, s(w)\ldots)}{c(s(w)\ldots)}$$

## Language Model

A module that computes $P(W)$ or $P(w_n \mid w_1, w_2, \ldots w_{n-1})$

## Perplexity and Entropy

Entropy over RV: X

$$H(X) = -\sum_{x=1}^{n} p(x) \log p(x)$$

Perplexity:

$$PP(w) = 2^{H(w)}$$

Perplexity = $2^{\text{entropy}} = 2^{\text{avg. \# of bits}}$ (of our model $q$)

$= 2^{-\frac{1}{N}\sum_{i=1}^{N} p(x_i) \cdot \log_2 q(x_i)}$ (tot # of words; $x$ can be any RV. In NLP, it's each word; $p(x_i)$: real dist.; $q(x_i)$: our prediction!)

$= e^{-\frac{1}{N}\sum_{i=1}^{N} p(x_i) \cdot \ln q(x_i)}$

We assume all words have the same frequency.

$= e^{-\frac{1}{N}\sum_{i=1}^{N} \frac{1}{N} \cdot \ln q(x_i)}$

$= q(x_1)^{-\frac{1}{N}} \cdot q(x_2)^{-\frac{1}{N}} \cdots q(x_N)^{-\frac{1}{N}}$

$= \prod_{i=1}^{N} q(x_i)^{-\frac{1}{N}} = \sqrt[N]{\frac{1}{q(x_1)\, q(x_2) \cdots q(x_n)}}$

## Smoothing

Laplace: Add 1 to all counts.

Linear Interpolation:

$$\hat{P}(w_n \mid w_{n-1} w_{n-2}) = \lambda_1 P(w_n \mid w_{n-1} w_{n-2}) + \lambda_2 P(w_n \mid w_{n-1}) + \lambda_3 P(w_n)$$

$$\sum_i \lambda_i = 1$$

Weights from a held-out corpus.

Katz

$$P_{katz}\left(w_n \mid w_{n-N+1}^{n-1}\right)=\begin{cases}P^*\left(w_n \mid w_{n-N+1}^{n-1}\right) & \text{if } C\left(w_{n-N+1}^{n}\right)>0\\ \alpha\left(w_{n-N+1}^{n-1}\right)P_{katz}\left(w_n \mid w_{n-N+2}^{n-1}\right) & \text{otherwise}\end{cases}$$

The $w_i$, $w_{i-1}$, $w_{i-2}$ for clarity are referred as a sequence $x$, $y$, $z$.
Katz method incorporates discounting as integral part of the algorithm.

$$P_{katz}(z \mid x,y)=\begin{cases}P^*(z \mid x,y) & \text{if } C(x,y,z)>0\\ \alpha(x,y)P_{katz}(z \mid y) & \text{else if } C(x,y)>0\\ P^*(z) & \text{otherwise}\end{cases}$$

$$P_{katz}(z \mid y)=\begin{cases}P^*(z \mid y) & \text{if } C(y,z)>0\\ \alpha(y)P_{katz}(z) & \text{otherwise}\end{cases}$$

$$P^*\left(w_n \mid w_{n-N+1}^{n-1}\right)=\frac{c^*\left(w_{n-N+1}^{n}\right)}{c\left(w_{n-N+1}^{n-1}\right)}$$

Kneser – Ney

- Re-estimated counts $c^*$ for greater than 1 counts could be estimated pretty well by just subtracting 0.75 from the MLE count $c$.
- Absolute discounting method formalizes this intuition by subtracting a fixed (absolute) discount d from each count.
  - The rational is that we have good estimates already for the high counts, and a small discount d won't affect them much.
  - The affected are only the smaller counts for which we do not necessarily trust the estimate anyhow.
- The equation for absolute discounting applied to bigrams (assuming a proper coefficient $\alpha$ on the backoff to make everything sum to one) is:

$$P_{absolute}(w_i \mid w_{i-1})=\begin{cases}\frac{c(w_{i-1}w_i)-D}{c(w_{i-1})}, & \text{if } c(w_{i-1}w_i)>0\\ \alpha(w_i)P_{absolute}(w_i) & \text{otherwize}\end{cases}$$

- In practice distinct discount values d for the 0 and 1 counts are computed.
- Kneser-Ney discounting augments absolute discounting with a more sophisticated way to handle the backoff distribution. Consider the job of predicting the next word in the sentence, assuming we are backing off to a unigram model:
  - I can't see without my reading XXXXXX.
- The word "*glasses*" seem much more likely to follow than the word "*Francisco*".
  - But "*Francisco*" is in fact more common, and thus a unigram model will prefer it to "*glasses*".

1. Thus we would like to capture that although "*Francisco*" is frequent, it is only frequent after the word "*San*".
2. The word "*glasses*" has a much wider distribution.

Thus the idea is instead of backing off to the unigram MLE count (the number of times the word ***w*** has been seen), we want to use a completely different backoff distribution!

- We want a heuristic that more accurately estimates the number of times we might expect to see word ***w*** in a new unseen context.
- The Kneser-Ney intuition is to base our estimate on the number of ***different contexts word w has appeared in***.
- Words that have appeared in more contexts are more likely to appear in some new context as well.
- New backoff probability can be expressed as the "***continuation probability***" presented in following expression:

- Continuation Probability:

$$P_{continuation}(w_i) = \frac{|\{w_{i-1} : c(w_{i-1}w_i) > 0\}|}{\sum_{w_i} |\{w_{i-1} : c(w_{i-1}w_i) > 0\}|}$$

- Kneser-Ney backoff is formalized as follows assuming proper coefficient $\alpha$ on the backoff to make everything sum to one:

$$P_{KN}(w_i \mid w_{i-1}) = \begin{cases} \frac{c(w_{i-1}w_i) - D}{c(w_{i-1})}, & \text{if } c(w_{i-1}w_i) > 0 \\ \alpha(w_i) \frac{|\{w_{i-1} : c(w_{i-1}w_i) > 0\}|}{\sum_{w_i} |\{w_{i-1} : c(w_{i-1}w_i) > 0\}|} & \text{otherwize} \end{cases}$$

- Kneser-Ney **backoff** algorithm was shown to be less superior to its **interpolated** version. **Interpolated Kneser-Ney** discounting can be computed with an equation like the following (omitting the computation of $\beta$):

$$P_{KN}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}w_i) - D}{c(w_{i-1})} + \beta(w_i) \frac{|\{w_{i-1} : c(w_{i-1}w_i) > 0\}|}{\sum |\{w_{i-1} : c(w_{i-1}w_i) > 0\}|}$$

- Practical note – it turns out that any interpolation model cab be represented as a backoff model, and hence stored in ARPA backoff format. The interpolation is done when the model is built, thus the 'bigram' probability stored in the backoff format is really 'bigram already interpolated with unigram'.

WITTEN BELL

# Witten-Bell Smoothing

HYDERABAD

- Let's consider bi-grams
  - $T(w_{i-1})$ is the number of different words (types) that occur to the right of $w_{i-1}$
  - $N(w_{i-1})$ is the number of all word occurrences (tokens) to the right of $w_{i-1}$
  - $Z(w_{i-1})$ is the number of bigrams in the current data set starting with $w_{i-1}$ that do not occur in the training data

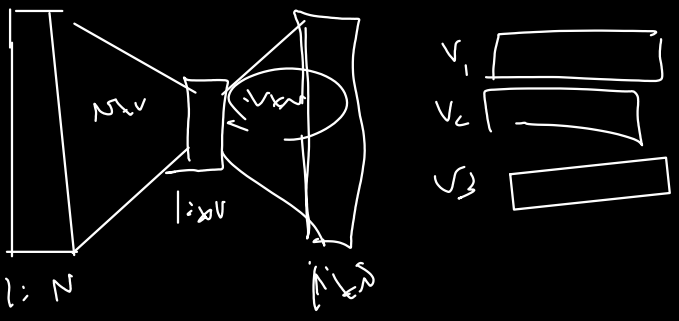
- If $c(w_{i-1},w_i) = 0$

- If $c(w_{i-1},w_i) > 0$

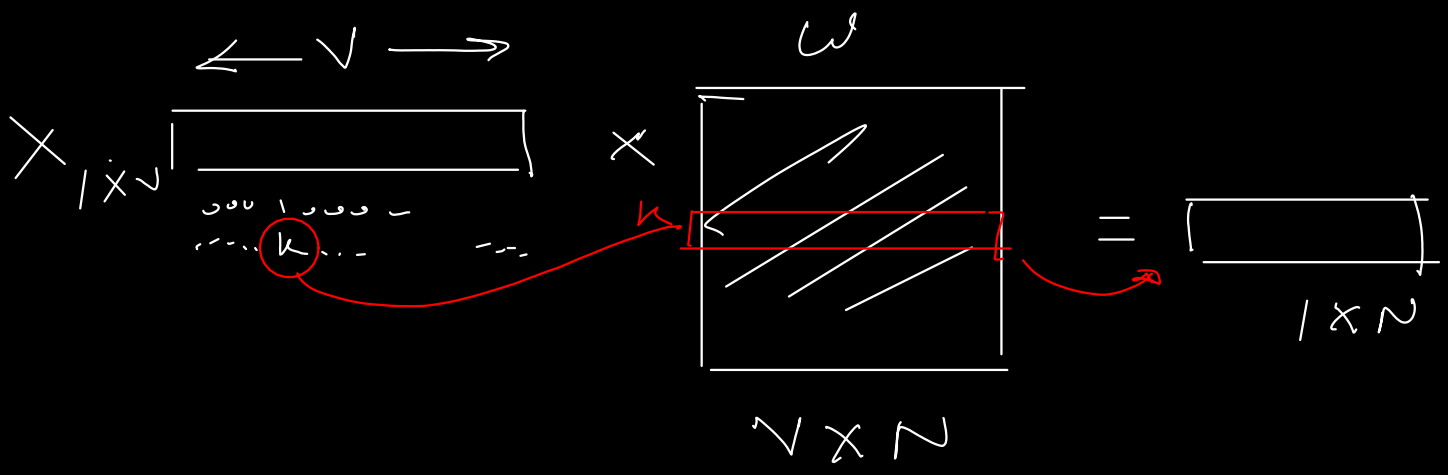
$$P^{WB}(w_i \mid w_{i-1}) = \frac{T(w_{i-1})}{Z(w_{i-1})(N + T(w_{i-1}))}$$

$$P^{WB}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}w_i)}{N(w_{i-1}) + T(w_{i-1})}$$

# Word2Vec

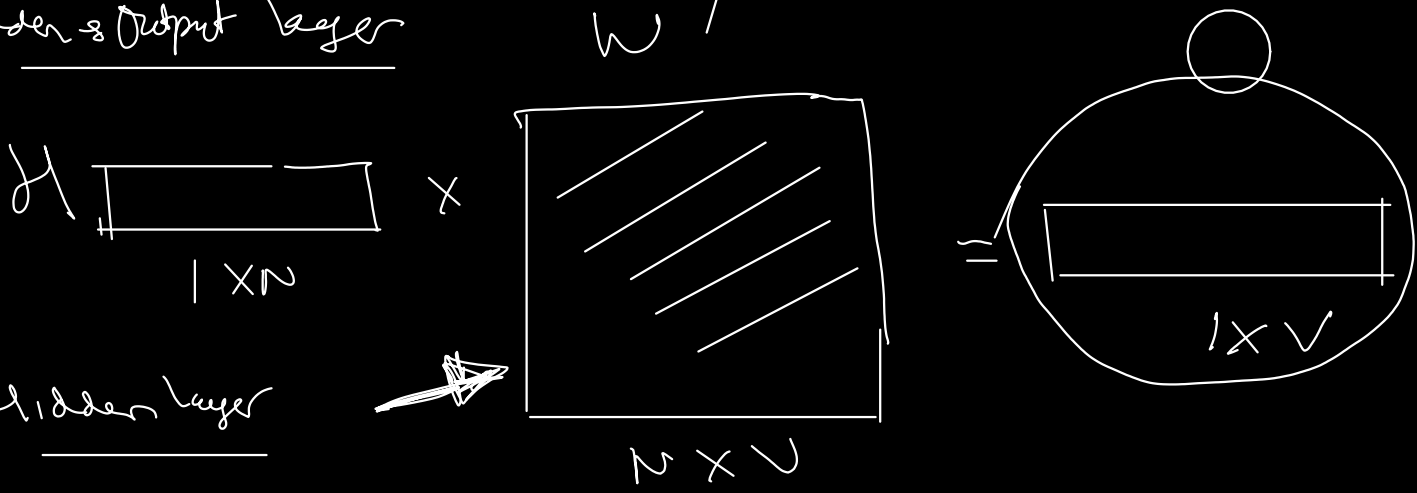


## Input→hidden layer

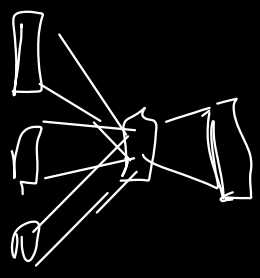


$$x_{1\times V} \times W_{V\times N} = h_{1\times N}$$

## Hidden→Output layer



## Hidden layer



$h = xW$

$h = \frac{1}{C}(x_1 + \dots + x_C)W$

https://arxiv.org/pdf/1411.2738.pdf

https://ronxin.github.io/wevi/

# GloVe

- uses global count statistics.
- uses co-occurence matrix

We have P ratios,

As you can see, words that are related to the nature of "ice" and "steam" ("solid" and "gas" respectively) occur far more often with their corresponding words that the non-corresponding word. In contrast, words like "water" and "fashion" which are not particularly related to either have a probability ratio near 1. Note that the

$$\frac{P(k|ice)}{P(k|steam)}$$

k = solid : 8.9

k = gas : $8.5 \times 10^{-2}$

k = fashion, water $\sim = 1$

Building co-occurence can be done during vocab building, so does not affect complexity.

# Karakas

k1: karta

k2: karma

k3: karan (instrument)

k4: recipient

k6: source

k7: → 
- t → time
- p → place

## Thematic Roles

- Agent
  - sentient (ate → Hari)
- Instrument
  - (opened → key)
- Cause
  - (killed → epidemic)
- Experiencer
- Recipient
- Path
- Location
- Measure
- Theme

# Shift-Reduce Parsing

The boy plucked the flower .

| det | n | v | det | n |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |

Rules :

$R_1$ $S \rightarrow NP\ VP$

$R_2$ $NP \rightarrow det\ n$

$R_3$ $PP \rightarrow prep\ NP$

$R_4a$ $VP \rightarrow v$

$R_4b$ $VP \rightarrow v\ NP$

$R_4c$ $VP \rightarrow v\ NP\ NP$

$R_4d$ $VP \rightarrow VP\ PP$

| Sl. No | stack | Inp rem. | Action | Rules rem. |
|---|---|---|---|---|
| 1 | – | 1 | | nil. |
| 1' | det | 2 | shift | R1 |
| 1'' | det n | 3 | Shift | R3 |
| 2 | NP | 3 | reduce | R1 |
| 2' | NP v | 4 | Shift | R1 |
| 3 | NP VP | 4 | reduced | R4b. |
| 4 | S | 4 | reduce | R2 |
| 4' | S det | 5 | shift | R1 |
| 4'' | S det n | null | shift | R1 |

Sl. No

| | | | |
|---|---|---|---|
| 5 | S NP | null | reduce R3 |

Error ∴ Backtrack

| | | | |
|---|---|---|---|
| 3' | NP VP det | 5 | shift R1 |
| 3'' | NP VP det n | null | shift R1 |
| 4 | NP VP NP | null | reduce R3 |

Error ∴ Backtrack

| | | | |
|---|---|---|---|
| 2'' | NP v det | 5 | shift R1 |
| 2''' | NP v det n | null | shift R1 |
| 3 | NP v NP | null | reduce R3 |
| 4 | NP VP | null | reduce R4c |
| 5 | S | null | reduce R2 |

Accept