

Can deal with the microscopic & macroscopic world.

Macro

$$(P, T, V)$$

Pressure, temp, Vol

Thermodynamics.

Micro

$$\{\vec{r}\} = (\vec{r}_1, \vec{r}_2, \dots)$$

$$\{\vec{p}\} = (\vec{p}_1, \dots, \vec{p}_n)$$

$\uparrow \Leftrightarrow$ Av. kinetic energy

Extending concepts to

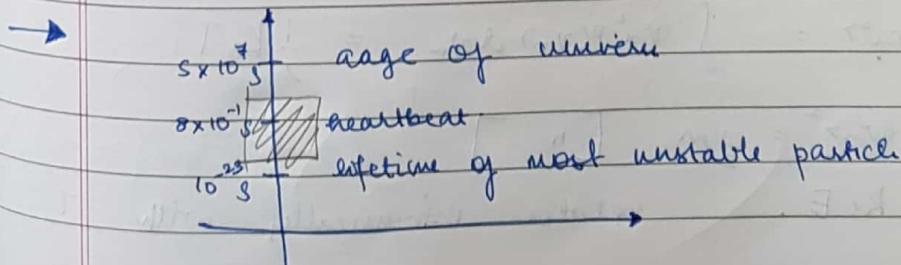
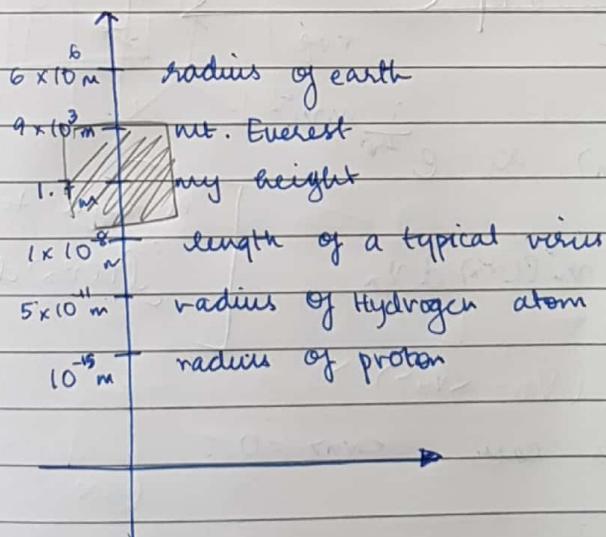
~ length scales 10^{-18} m to 10^{18} m.

~ Time scales 10^{-15} s to years

- Classical mechanics (large length scales)
- Quantum mechanics (smaller length scales)

■ Science is based on measurements & observations

→ A length scale:



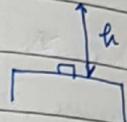
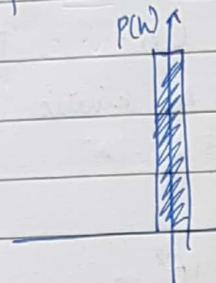
$$\langle \text{var} \rangle = \text{avg. of (var)}$$

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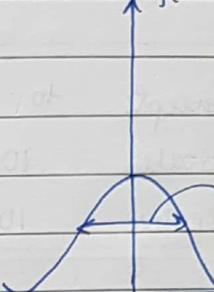
Measurements do not give you unique value

For eg: consider a corn bread on a hot plate

Expected curve:-

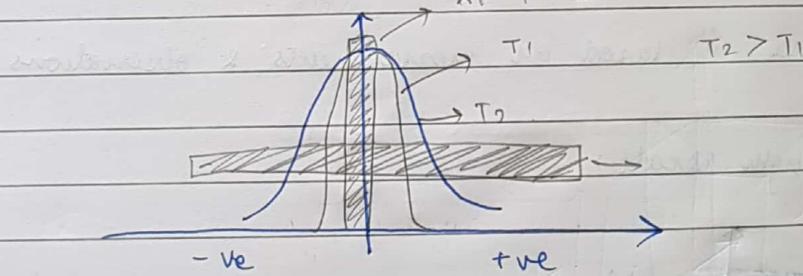


$$P(h)$$



→ Maxwell Boltzmann dist curve

At $T \rightarrow 0$,



$$\text{Now, } P(v_n) \propto e^{-\frac{mv_n^2}{2k_B T}}$$

Gives us we $\langle v_n^2 \rangle = \int_{-\infty}^{\infty} v_n^2 P(v_n) dv_n$
standard deviation

at high temperature
 $\int_{-\infty}^{\infty} P(v_n) dv_n \neq 0$

$$= \langle (v_n - \langle v_n \rangle)^2 \rangle$$

$$\text{In this case } \langle v_n \rangle = 0.$$

NOW,

$$\langle v_n^2 \rangle = \int_{-\infty}^{\infty} v_n^2 P(v_n) dv_n = \frac{1}{2} k_B T$$

$$\frac{1}{2} m \langle v_n^2 \rangle = \frac{1}{2} m ["]$$

K.E. relates intrinsically with T

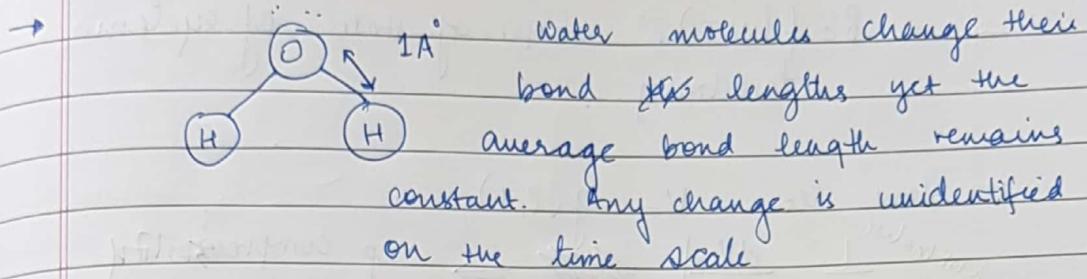
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ink! Mathematical model

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Proof that the H₂O molecules are vibrating: drop ink, it diffuses.

- Stability of Graphite & Diamond

- Graphite is more stable than diamond
- Diamond always tries to go back to the graphite form.
-
- Gibbs Free energy function is used to determine stability.

~ Cooling water rapidly will form water glass.
This is where kinetics comes into play!

To understand any system we could use 4 parameters

| Energy | Matter | Waves | Mathematics |
|--|---|---|------------------------------------|
| Bond Heat Thermodynamic Work Free energy | Statistical mechanics Solid state physics Matter - System is made of particles - define the micro state of the system | Waves Electromagnetic waves Particle waves Quantum Mech. | Diffusion eqn Random walk model |

Law:
PT, V, n
 $P = f(V, T)$

In the thermodynamic world we have 4 parameters

P, V, T, λ considered constant.

Each can be expressed as a function of the other law

$P = f(V, T) \rightarrow$ Eqn of state of system
+ arbitrary.

Now let $V = g(P, T)$.
Normalized
to get ratio $\frac{dV}{dP} \bigg|_T$ ~~1/V~~ \rightarrow rep compressibility
when $P \rightarrow P + dP$
 $V \rightarrow V + dV$.

• Matter:

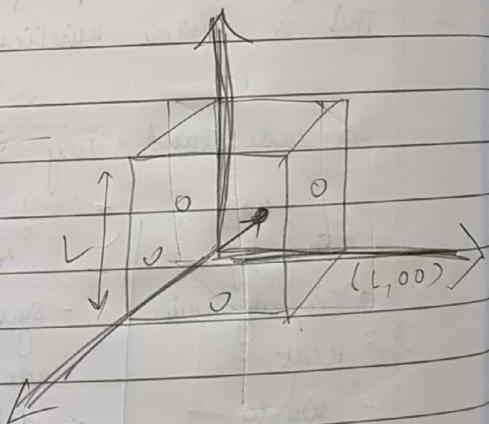
- System consists of N particles
- Positions & momenta of particles known
- Microscopic variables

Expt: N particles in the system

Let co-ordinates of particle

$$\vec{r} \rightarrow (x_i, y_i, z_i)$$

$$\text{also } \left. \begin{array}{l} 0 \leq x_i \leq L \\ 0 \leq y_i \leq L \\ 0 \leq z_i \leq L \end{array} \right\} + i$$



$$\text{Volume } V = L^3$$

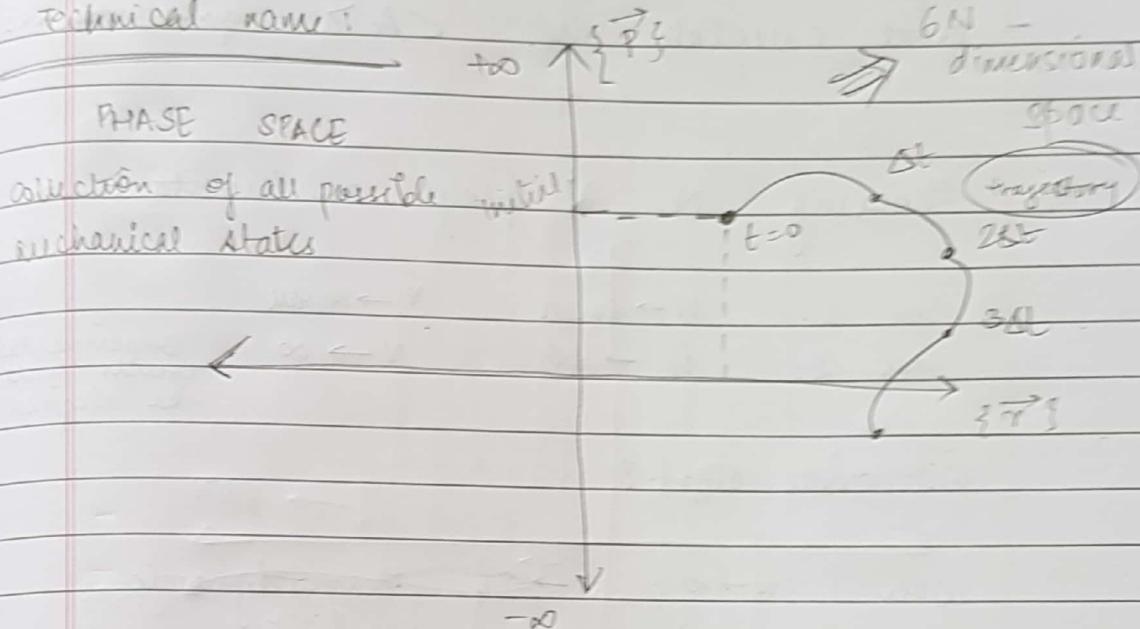
Notation: $\{\vec{r}\} = (\bar{x}_1, \bar{y}_1, \bar{z}_1, x_2, y_2, z_2, \dots, \bar{x}_N, \bar{y}_N, \bar{z}_N)$
($3N$ components)

$$\vec{r}_i = (x_i, y_i, z_i).$$

Similarly: $\{\vec{p}\} = (\bar{p}_{x1}, \bar{p}_{y1}, \bar{p}_{z1}, \dots, \bar{p}_{xN}, \bar{p}_{yN}, \bar{p}_{zN})$
($3N$ components)

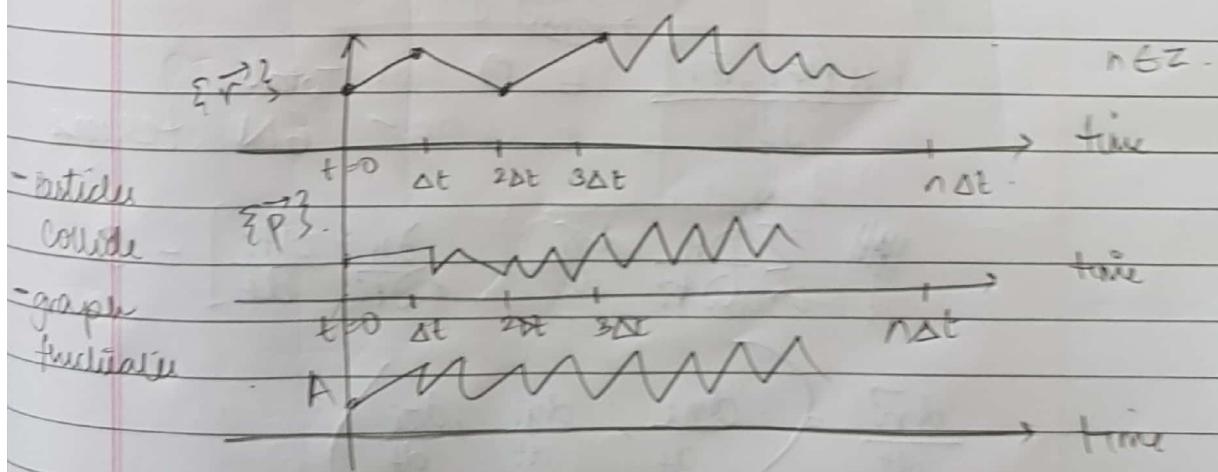
Now constructing a plane with $\{\vec{r}\}$ on x -axis & $\{\vec{p}\}$ on y -axis.

Technical name:



- A given pt. on this space rep. a mechanical state of the system.
- The system is going to evolve.
- When a particle moves, the system changes its state.
- All thermodynamic properties can be expressed in terms of the co-ordinates & momenta of particles.

arbitrary prop $\leftarrow A = A(\{\vec{r}\}, \{\vec{p}\})$



- A is a simple scalar function giving a single number
- Next calculate the $\langle A \rangle$ → comparable to experimental results
- choosing N, V plays an important role

Case 1: $N \rightarrow \text{small}$ $V \rightarrow \text{small}$

Case 2: $N \rightarrow \infty$ $V \rightarrow \infty$

Experimental results will be aligned.

Will results differ?

with $N \rightarrow \infty; V \rightarrow \infty \}$ thermodynamic limit
 $\frac{N}{V} = \text{finite}$

$$\rightarrow H(\{\vec{r}\}, \{\vec{p}\}) = U(\{\vec{r}\}) + k(\{\vec{p}\})$$

↑ scalar function ↑ potential energy func.
★ Hamiltonian function (Interaction b/w particles)
 (Total energy of system)

Consider an isolated system → Doesn't exchange energy with the surroundings

$$\frac{dH(\{\vec{r}\}, \{\vec{p}\})}{dt} = 0 \quad \text{Total } 6N \text{ terms}$$

$$\cancel{\frac{dH}{dt}} = \sum_{i=1}^N \left[\frac{\partial H}{\partial r_i} \cdot \frac{dr_i}{dt} + \frac{\partial H}{\partial p_i} \cdot \frac{dp_i}{dt} \right] = 0$$

$$\frac{d\vec{r}_i}{dt} = \left(\frac{dx_i}{dt}, \frac{dy_i}{dt}, \frac{dz_i}{dt} \right)$$

\vec{F}_i $\frac{\partial H}{\partial r_i}$ $\frac{\partial r_i}{\partial t}$

$$= - \cancel{\frac{\partial H}{\partial r_i}} \frac{d p_i}{d t}$$

force

Hamiltonian

eqn.

$$\frac{\partial H}{\partial p_i} = \frac{d r_i}{d t}$$

$$\frac{d r_i}{d t}$$

velocity

Recap:

→ microstate of a system consisting of N particles (\vec{r}, \vec{p})

$$\vec{r} = (x_1, y_1, z_1, x_2, y_2, z_2, \dots) \rightarrow S_N$$

$$\vec{p} = (p_{x1}, p_{y1}, p_{z1}, \dots, p_{xN}, p_{yN}, p_{zN})$$

in the hamiltonian scale:

$$\vec{F}_i = \frac{d \vec{p}_i}{d t} = - \cancel{\frac{\partial H}{\partial r_i}} = \boxed{- \frac{\partial U(\vec{r})}{\partial r_i}}$$

negative gradient of pot. energy

The direction of decrease in pot. energy is the direction of force

$$\text{Here } \vec{F}_i = F_{xi} \hat{i} + F_{yi} \hat{j} + F_{zi} \hat{k}$$

$$F_{xi} = \cancel{8} - \frac{\partial U}{\partial x_i}$$

$$F_{yi} = - \frac{\partial U}{\partial y_i}$$

$$F_{zi} = - \frac{\partial U}{\partial z_i}$$

for individual components.

Displace the i^{th} atom by an incremental amt ie:

$$x_i \rightarrow x_i + \delta x_i$$

$$y_i \rightarrow y_i + \delta y_i$$

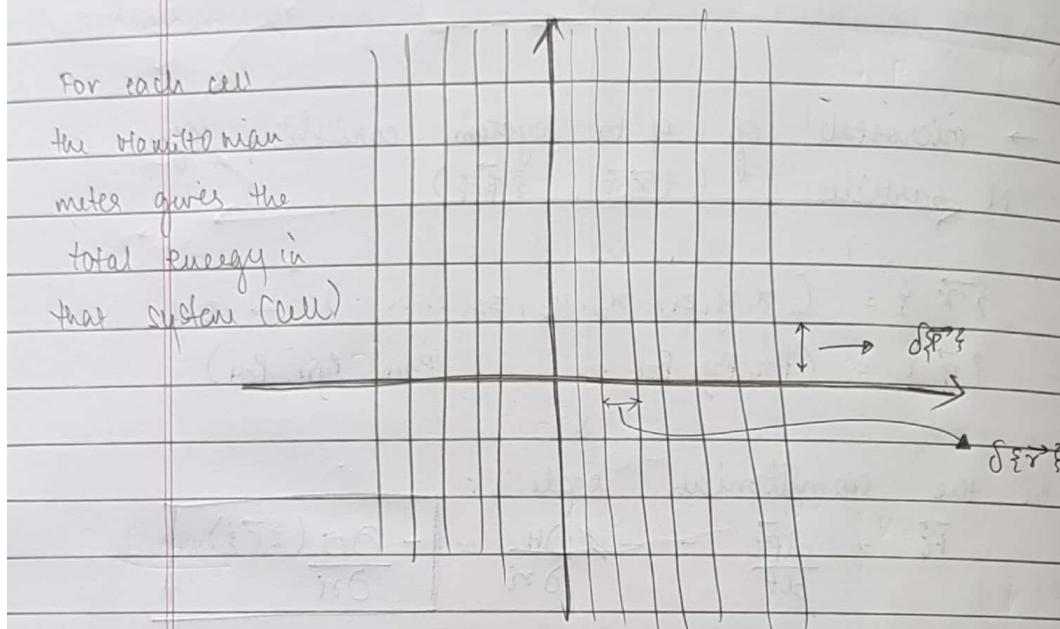
$$z_i \rightarrow z_i + \delta z_i$$

∴ change in pot energy is :-

$$U(x_i + \delta x_i, y_i + \delta y_i, z_i + \delta z_i) - U(x_i, y_i, z_i)$$

Aim : to convert the bN dimension space to $1D$

For each cell
the Hamiltonian
matrix gives the
total energy in
that system (cell)



$$\delta z_i \rightarrow 0 \quad \delta z_i \rightarrow 0 \text{ arbitrarily small}$$

ENERGY

dig.

E_{max} (highest energy)

$E_i \rightarrow \Omega(E_i)$ with
no. of cells
 E_i

$$E_{min} + 2\delta E$$

$$E_{min} + \delta E$$

E_{min} (lowest energy)

$$\delta E \rightarrow 0$$

Two overlapping particles could have ~~as~~ energy ^{classmate}

(E_i) No. of microstates accessible to the system with energy E_i

Boltzmann's claim :-

Entropy S of a system

$$S \propto \ln(\Omega(E_i))$$

microscopic

$$\xleftarrow{\text{thermodynamic quantity}} S = k_B \ln(\Omega(E_i))$$

Boltzmann const.

↓ statistical measure

→ This can be used to reason why ice has lower entropy than water. All atoms are frozen with 0 velocity & no change in momentum, all particles in ice are in one mechanical state. Then $S = k_B \ln(1) = 0$ (or tending to 0)

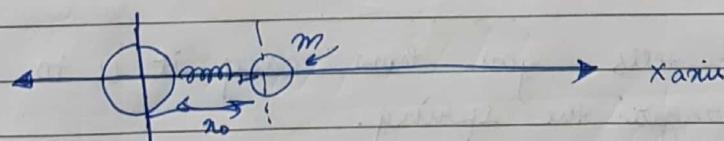
On heating, $\ln(E_i)$ increases. For an isolated system acceptable energy levels are where energy of the microstate is E_i

▲ Determine $\Omega(E_i)$ for:

① Model 1: One-dimensional harmonic oscillator

② Ideal Gas. $U(\vec{r}) = 0$

Model 1: Consider a heavy atom attached to a light O atom let a spring bond exist.



lighter atom will execute harmonic motion on being displaced.

Constructing phase space for model 1:

$$\Delta \text{Area} = Sx \times Sp.$$

dim ↑

angular
momentum ←

$$\Delta n \delta p = h$$

2D

Space

$\frac{\partial}{\partial p}$

$\frac{\partial}{\partial p}$

x

$$H(x, p) = \frac{p^2}{2m} + \frac{1}{2} K(x - x_0)^2 = E$$

Initial
Energy

Pot energy:

$U(x)$

K_1

K_2

$K_2 < K_1$

x_0

$x_0 + \Delta x$

x

$$U(x) = \frac{1}{2} K (x - x_0)^2$$

\downarrow
spring const.

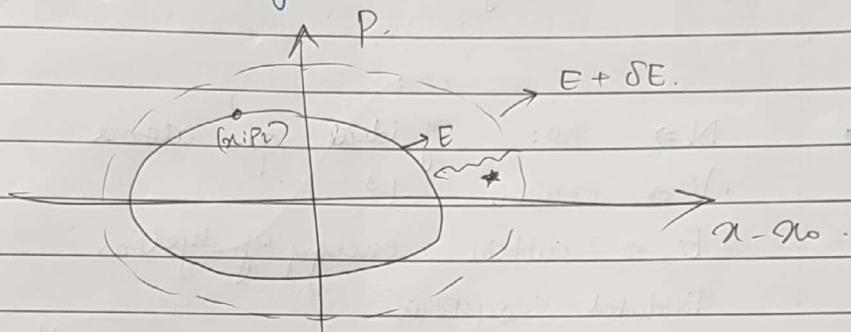
$$\left. \frac{d^2 U}{dx^2} \right|_x = k.$$

k tells you how easy it is to compress/elongate the spring.

Rearranging:-

$$\frac{(x - x_0)^2}{\left(\frac{\sqrt{2E}}{k}\right)^2} + \frac{P^2}{\left(\frac{\sqrt{2ME}}{k}\right)^2} = 1 \rightarrow \text{Ellipse}$$

The pts. on the circumference of this ellipse are all the accessible states. These are the states visited by the system.



$$\Omega(E) \propto \text{Circumference of Ellipse}$$

$$\Omega(E) \propto \pi \left(\frac{\sqrt{2E}}{k} + \frac{\sqrt{2ME}}{k} \right)$$

$$\text{i.e. } \Omega(E) \propto E^{1/2}$$

No. of microstates can increase with the energy of the system & consequently increase the entropy of the system.

* : relaxation time : how fast the system can come back to the ground state

→ For crystals at low temp:-

$$\begin{aligned} \Omega(E) &\rightarrow 1 \\ \Rightarrow S &\rightarrow 0 \end{aligned}$$

→ Ideal Gas (Model 1) :-

$$H(\vec{r}_1, \vec{p}_1) = U(\vec{r}_1) + K(\vec{p}_1)$$

if $U(\vec{r}_1) = 0$ atoms do not interact with each other \Rightarrow ideal gas
 Only energy that excites particles is thermal energy

- $N \Rightarrow$ no: of ideal gas atoms
- $V \Rightarrow$ volume L^3
- $E \Rightarrow$ initial energy of system
 Isolated system

It is a $6N$ dimension space.

Vol. of a $6N$ dimensional space

$$\delta\vec{r}_1 \delta\vec{p}_1$$

$=$

$$\delta x_1, \delta p_{x_1}, \delta y_1, \delta p_{y_1}, \delta z_1, \delta p_{z_1} \dots$$

$$\delta x_N, \delta p_{x_N}, \delta y_N, \delta p_{y_N}, \delta z_N, \delta p_{z_N}$$

$$= h^{3N} \quad h \rightarrow 0$$

h is arbitrary in classical mechanics

h is called Planck's const in quantum mechanics

Aim : calculate $S(E)$

But here more parameters exist

So calc: $S(N, V, E)$

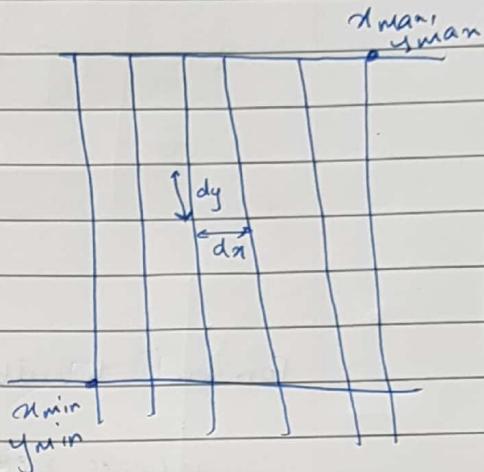
Ω : No: of accessible microstates.

System visits those states in which energy = E.
Check each cell.

Calc integral :-

$$\int_{x_{\min}}^{x_{\max}} \int_{y_{\min}}^{y_{\max}} dx dy .$$

visit all ~~for~~ cells in $6N$ dimensional phase



[Idea: Calc total area, \div by area of 1 cell \Rightarrow total no: of particles]

$$\iint \delta(H-E) \delta \vec{r} \delta \vec{p}$$

\downarrow \downarrow Hamiltonian $\int (H-E) = 1$ when $H=E$
 $3N$ $3N$ integrals integrals. \downarrow $= 0$ otherwise
 \downarrow direct delta function

$$\Omega(N, V, E) = \frac{1}{h^{3N}} \iint \delta(H-E) \delta \vec{r} \delta \vec{p} .$$

\downarrow
 counting cells that satisfy this condition

The Hamiltonian function:

$$H(\vec{r}, \vec{p}) = \frac{1}{m} \sum_{i=1}^N \vec{p}_i \cdot \vec{p}_i$$

\downarrow
 mass of an atom

$$0 \leq x_i, y_i, z_i \leq L .$$

$$\Omega(N, V, E) = \frac{1}{h^{3N}} \int_{\{\vec{r} = 0\}} d\vec{r} \int_{\{\vec{p} = 0\}} \delta(H-E) d\vec{p}$$

$$= \frac{1}{h^{3N}} \cancel{V^N} \int_{\{\vec{p} = 0\}} \delta(H-E) d\vec{p}$$

visiting only
the momentum
space of dim = $3N$

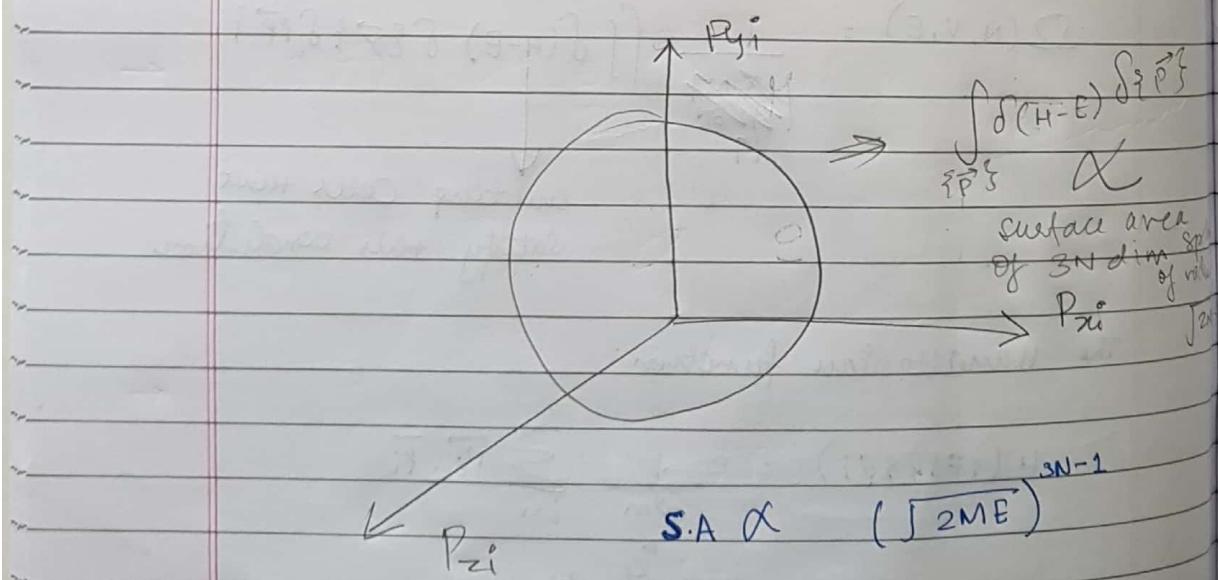
For the hamiltonian :-

$$H(\{\vec{r}\}, \{\vec{p}\}) = \frac{p_{x_1}^2}{2m} + \frac{p_{y_1}^2}{2m} + \frac{p_{z_1}^2}{2m} + \dots + \frac{p_{x_N}^2}{2m} + \frac{p_{y_N}^2}{2m} + \frac{p_{z_N}^2}{2m} = E$$

Also:-

$$p_{x_1}^2 + p_{y_1}^2 + p_{z_1}^2 + \dots + p_{x_N}^2 + p_{y_N}^2 + p_{z_N}^2 = (\sqrt{2mE})^2$$

This is the eqn of a $3N$ dimensional sphere of radius $\sqrt{2mE}$.



$$\Omega(N, V, E) \propto V^N E^{-2} h^{3N}$$

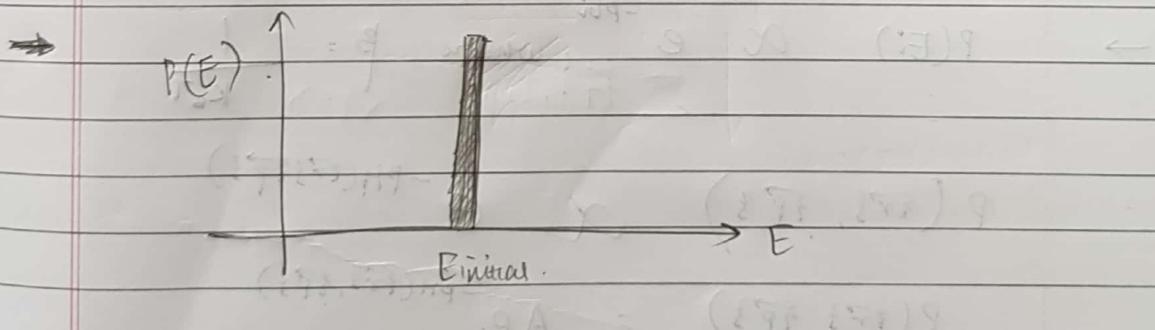
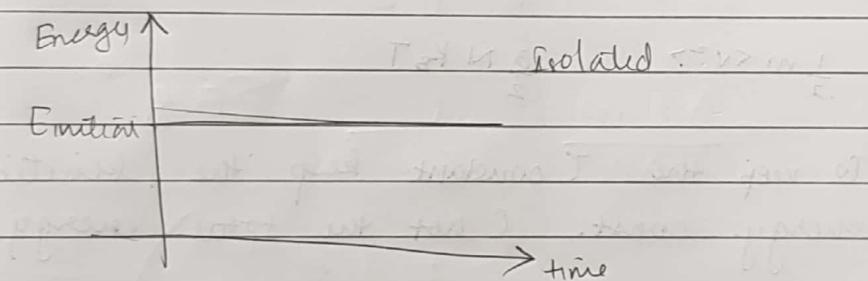
No: of accessible microstates increases with volume with Energy

Increasing no: of atoms increase no: of accessible microstates

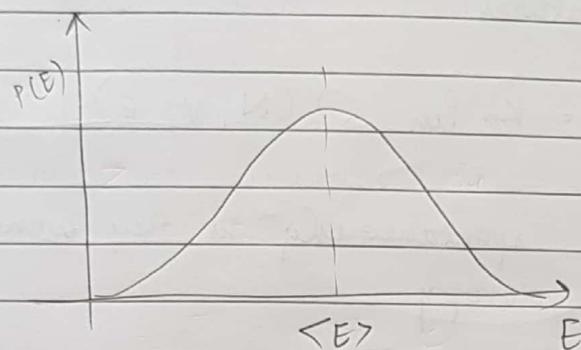
$$S = k_B \ln \Omega(N, V, E).$$

→ Gasses expand spontaneously as they want to increase their entropy

→ For an isolated system:-



For a closed system:-



$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} N k_B T$$

To keep the T constant keep the kinetic energy const. (not the total energy)

$$\rightarrow P(E_i) \propto e^{-\beta E_i}, \text{ where } \beta = \frac{1}{k_B T}$$

$$P(\vec{r}_1, \vec{p}_1) \propto e^{-\beta H(\vec{r}_1, \vec{p}_1)}$$

$$P(\vec{r}_1, \vec{p}_1) = A e^{-\beta H(\vec{r}_1, \vec{p}_1)}$$

$$\iint_{\vec{r}_1, \vec{p}_1} P(\vec{r}_1, \vec{p}_1) d\vec{r}_1 d\vec{p}_1 = 1$$

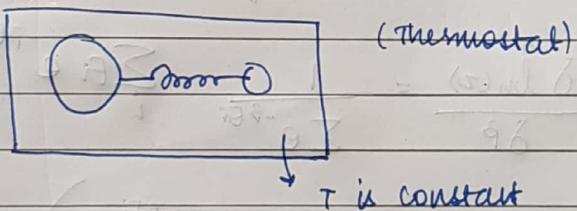
$$P(\{\vec{r}\}, \{\vec{p}\}) = e^{-\beta H(\{\vec{r}\}, \{\vec{p}\})}$$

$$Z = \frac{1}{h^{3N}} \int \int \left(e^{-\beta H(\{\vec{r}\}, \{\vec{p}\})} d\vec{r} d\vec{p} \right)$$

Z is called the partition function of the system.

Given Z we can calculate all thermodynamic properties of the system.

→ Model-1: One dimensional Harmonic Oscillator (closed)

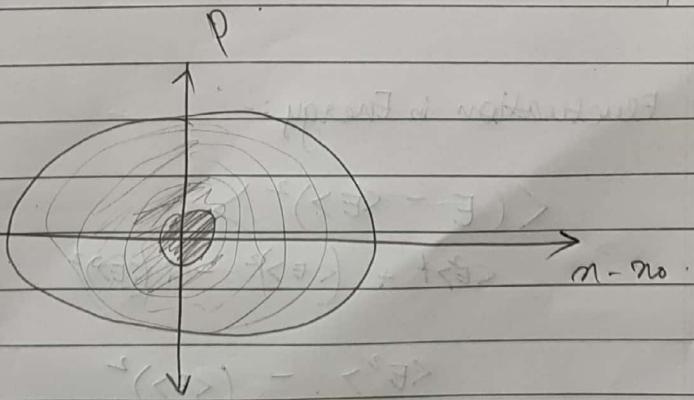


$$H(x, p) = \frac{1}{2} K(x - x_0)^2 + \frac{p^2}{2m}$$

$$P(x, p) \propto e^{-\frac{p^2}{2m} - \frac{K(x - x_0)^2}{2}}$$

Max prob when $x = x_0$ or $p = 0$

comb of 2 Gaussian func.



$$Z = \frac{1}{\beta h^{3N}} \iiint e^{-\beta E} d^3 p_1 d^3 p_2 d^3 p_3$$

Continuous :

$$Z = \sum_{i=1}^{\text{no. of microstates}} e^{-\beta E_i}$$

Discrete.

→ From Z we can calculate thermodynamic properties of the system :-

$$Z = \sum_i e^{-\beta E_i}$$

$$\ln Z = \ln \left(\sum_i e^{-\beta E_i} \right)$$

$$-\frac{\delta \ln(Z)}{\delta \beta} = \frac{1}{\sum_i e^{-\beta E_i}} \sum_i E_i e^{-\beta E_i}$$

$$= \sum_i E_i \left(\frac{e^{-\beta E_i}}{\sum_j e^{-\beta E_j}} \right) \quad \dots (1)$$

$$= \sum_i E_i P(E_i) = \langle E \rangle$$

↳ internal energy of the system

→ Fluctuation in Energy :-

$$\langle (E - \langle E \rangle)^2 \rangle$$

$$\langle E^2 \rangle + \langle E \rangle^2 - 2 \langle E \rangle^2$$

$$\langle E^2 \rangle - \langle E \rangle^2$$

Consider

$$\sum E_i e^{-\beta E_i} = -z \frac{\partial \ln z}{\partial \beta}$$

$$\frac{\partial}{\partial \beta} \left[\sum_i E_i e^{-\beta E_i} \right] = -\frac{\partial}{\partial \beta} \left(z \frac{\partial \ln z}{\partial \beta} \right)$$

$$-\sum_i E_i^2 e^{-\beta E_i} = -\left(\frac{\partial z}{\partial \beta}\right) \frac{\partial \ln z}{\partial \beta}$$

$$= -z \frac{\partial^2 \ln z}{\partial \beta^2}$$

$$\sum_i E_i^2 \left(\frac{e^{-\beta E_i}}{z} \right) = \left(\frac{1}{z} \frac{\partial z}{\partial \beta} \right) \left(\frac{\partial \ln z}{\partial \beta} \right) + \frac{\partial^2 \ln z}{\partial \beta^2}$$

$$\langle E^2 \rangle = \left(\frac{\partial \ln z}{\partial \beta} \right)^2 + \frac{\partial^2 \ln z}{\partial \beta^2}$$

$$\langle E^2 \rangle = (\langle E \rangle)^2 + \frac{\partial^2 \ln z}{\partial \beta^2}$$

$$\langle E^2 \rangle - (\langle E \rangle)^2 = \frac{\partial^2 \ln z}{\partial \beta^2}$$

$$= -\frac{\partial}{\partial \beta} \left(\frac{\partial \ln z}{\partial \beta} \right) = -\frac{\partial}{\partial \beta} \langle E \rangle$$

Now $\beta = \frac{1}{k_B T}$

$$\langle E^2 \rangle - (\langle E \rangle)^2 = \frac{\left(-\frac{\partial}{\partial T} \langle E \rangle \right)}{\left(\frac{\partial \beta}{\partial T} \right)}$$

$$\left\{ C_V = -\frac{\partial \langle E \rangle}{\partial T} \rightarrow \text{heat capacity} \right\}$$

$$\langle E^2 \rangle - (\langle E \rangle)^2 = \frac{C_V}{(\frac{\partial P}{\partial T})}$$

Energy fluctuation is related to the heat capacity of the system.

$$\left(\frac{\partial P}{\partial T}\right)$$

$$\frac{\partial \ln P}{\partial T} = \frac{C_V}{k_B T}$$

- Energy fluctuation is related to the heat capacity of the system.
- C_V is always positive as $C_V = \frac{\partial \langle E \rangle}{\partial T}$ and $\langle (E - \langle E \rangle)^2 \rangle$ is positive
- Fluctuation in energy can be calculated at constant temperature.

▲ For an isolated system:-

isolated

- entropy $S = k_B \ln \Omega$
- $S = -k_B \sum_i p_i \ln p_i \rightarrow$ Expectation
Prob of finding the system in i^{th} microstate
- $\underline{-k_B \langle \ln p_i \rangle}$

If the system was isolated

$$p_i = \frac{1}{\Omega} \quad (\text{The prob of equal a priori probabilities})$$

all states equally probable

Closed systems \rightarrow energy exchange only
 Open systems \rightarrow both energy & matter

classmate

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For a closed system :- $P_i = e^{-\frac{E_i}{k_B T}} / Z$

$$S = -k_B \sum_i \left(\frac{e^{-\frac{E_i}{k_B T}}}{Z} \right) (-\beta E_i - \ln Z)$$

Sum over microstates = $\beta k_B \sum_i E_i P_i + k_B \ln Z \sum_i P_i$

$$= \frac{1}{k_B T} \sum_i E_i P_i + k_B \ln Z \sum_i P_i \underbrace{\quad}_{=1}$$

$$S = \frac{1}{T} \langle E \rangle + k_B \ln Z$$

$$-k_B T \ln Z = \langle E \rangle - TS$$

RHS \rightarrow Helmholtz free energy $A = U - TS$.

$\therefore A = -k_B T \ln Z$ \rightarrow In terms of the partition func.

Keeping all parameters - no: of moles, V, T const
 & ~~then~~ ^{only} Helmholtz free energy will be minimized

$$S = -k_B \sum_i P_i \ln P_i \quad \text{If } P_i = 1 \text{ then } S = 0$$

Whenever the P.D is delocalized ~~the~~ ie: prob is distributed to various states entropy increases
 You don't know where the system exactly is at a given pt. of time.

MICROSCOPIC DEFINITION OF PRESSURE:

$$P_i = - \frac{\partial E_i}{\partial V} = - \left[\frac{(E_i(v + \Delta v)) - E(v)}{\Delta v} \right]_{\Delta v \rightarrow 0}$$

↑
pressure of
the microstate i

expand the vol. of the system & measure the energy of the same microstate. This change is the microscopic defn of pressure.

$$\langle P_i \rangle = \sum_i P_i P_i = \frac{1}{Z} \frac{\partial \ln Z}{\partial V}$$

For an ideal gas (closed)

$N \Rightarrow$ no: of ideal gas atoms.

$V \Rightarrow$ volume.

$T \Rightarrow$ temperature

$$H(\{\vec{r}\}, \{\vec{p}\}) = \frac{1}{2m} \sum_{j=1}^N \vec{p}_j \cdot \vec{p}_j$$

↑ Pot energy
↓ Total KE

To calc Z:-

$$Z = \frac{1}{h^{3N}} \int \int e^{-\frac{p_i \vec{r}_i \cdot \vec{p}_i}{2m}} d\vec{r}_i d\vec{p}_i$$

$$= \frac{1}{h^{3N}} \int_{\vec{r}_1} \int_{\vec{p}_1} d\vec{r}_1 d\vec{p}_1 \int_{\vec{r}_2} \int_{\vec{p}_2} \cdots \int_{\vec{r}_N} \int_{\vec{p}_N} d\vec{r}_N d\vec{p}_N$$

\uparrow 3N terms

$$Z = \frac{1}{h^{3N}} V^N \left(\int_{n=-\infty}^{\infty} e^{-\frac{\beta n^2}{2}} \right)^{3N}$$

$$Z = \frac{1}{h^{3N}} V^N \left(\sqrt{\frac{2m\pi}{\beta}} \right)^{3N}$$

$$= \frac{1}{h^{3N}} V^N \left(2m\pi k_B T \right)^{3N/2}$$

$$\langle P \rangle = \frac{1}{\beta} \frac{\partial \ln Z}{\partial V}$$

$$\langle P \rangle = \frac{1}{\beta} \frac{N}{V}$$

$$\boxed{\langle P \rangle V = N k_B T} \rightarrow \text{Ideal Gas eqn of state}$$

→ Feynman's trick to solve ① :-

$$I = \int_{n=-\infty}^{\infty} e^{-\alpha n^2} dn.$$

$$I = \int_{y=-\infty}^{\infty} e^{-\alpha y^2} dy$$

$$I^2 = \iint_{n=-\infty}^{\infty} e^{-\alpha(n^2+y^2)} dndy$$

$$n^2 + y^2 = r^2 \\ dndy = r dr d\theta$$

$$n = r \cos \theta \\ dn = r \cos \theta dr \\ d\theta = -r \sin \theta dr$$

Transforming
to polar coords

~~Method~~

$$\begin{aligned} I &= \int_0^\infty \int_0^{2\pi} e^{-\alpha r^2} r dr d\theta \\ &= 2\pi \int_0^\infty e^{-\alpha r^2} d\left(\frac{r}{\alpha}\right)^2 \\ &= \pi \int_0^\infty e^{-\alpha u} du. \end{aligned}$$