Hw 10 P03

L1 Softmargin

$$\min_{w,b} \quad \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{m} \xi_i \qquad \text{s.t. } y^i(w^T x_i + b) \geq 1-\xi_i, \quad i=1,2,\dots m$$

$$\xi_i \geq 0 \quad i=1,2,3,\dots m$$

$$L(w,b,\xi,\alpha,\beta) = \frac{1}{2}|w|^2 + C\sum_{i=1}^{m}\xi_i + \sum \alpha_i\left[(1-\xi_i) - y^i(w^T x_i + b)\right] + \sum_{i=1}^{m} \beta(-\xi_i)$$

$$\frac{d}{dw}: \quad w - \sum \alpha_i y^i x^i = 0 \qquad w = \sum_{i=1}^{m} \alpha_i y^i x_i$$

$$\frac{d}{a_0}: \quad \sum \alpha_i y^i = 0$$

$$\frac{d}{d\xi_i}: \quad C - \alpha_i - \beta_i = 0. \qquad \beta_i = C - \alpha_i$$

$$\beta_i \geq 0, \; C - \alpha_i \geq 0 : \qquad 0 \leq \alpha_i \leq C.$$

$$L(w,b,\xi,\alpha,\beta) = \frac{1}{2}\|\alpha_i y_i x_i\|^2 + C\sum_{i=1}^{m}\xi_i + \sum_{i=1}^{m}\alpha_i\left(1-\xi_i - y^i\left(\left(\sum(\alpha_j y^j x^j)\right)^T x^i + b - \sum_{i=1}^{m}\beta_i\xi_i\right)\right)$$

$$= -\frac{1}{2}\sum \alpha_i \alpha_j y^i y^j x_i^T x_j + C\sum\xi_i + \sum\alpha_i - \sum\alpha_i\xi_i - b\sum\alpha_i y_i - \beta\xi_i$$

$$= \sum_{i=1}^{m}\alpha_i - \frac{1}{2}\sum_{i,j=1}^{m}\alpha_i\alpha_j y_i y_j x_i^T x_j \longrightarrow \max$$

$$\text{st } 0 \leq \alpha_i \leq C \qquad \text{Dual.}$$

$$\& \; \sum \alpha_i y_i = 0$$

L2 Soft margin:

$$\min \frac{1}{2}\|w\|^2 + \frac{C}{2}\sum \varepsilon_i^2$$

$$L(\omega \ldots) = \frac{1}{2}\|\omega\|^2 + \frac{C}{2}\sum \varepsilon_i^2 + \sum \alpha_i\left[1-\varepsilon_i - y_i(\omega^T x_i + b)\right]$$

$$\frac{d}{d\omega} : \quad \omega - \sum \alpha_i y_i x_i = 0$$

$$\frac{d}{db} : \quad \sum \alpha_i y_i = 0$$

$$\frac{\partial L}{\partial \varepsilon_i} : \quad C\varepsilon_i - \alpha_i = 0$$

Replacing:

$$L = \frac{1}{2}\left(\sum \alpha_i y_i x_i\right)^2 + \frac{C}{2}\sum \varepsilon_i^2 + \sum \alpha_i - \sum \alpha_i \varepsilon_i - \sum \alpha_i y_i \left(\sum \alpha_j y_j x_j^T\right) x_i$$

$$- \sum \alpha_i y_i b)$$

$$= -\frac{1}{2}\sum_{i,j=1}^{m} \alpha_i \alpha_j y_i y_j x_i x_j - \frac{\sum \alpha_i^2}{2C} + \sum \alpha_i$$

$$\sum \alpha_i - \frac{1}{2}\sum \alpha_i \alpha_j y_i y_j \left(x_i^T x_j + \frac{\delta_{ij}}{C}\right) \to \max$$

$$\text{st:} \quad \sum y_i \alpha_i = 0,$$

$$\alpha_i \geq 0.$$

$$i = 1 \to m$$