Lec 19

Wave Function

$\psi(x,t) \Rightarrow$ 1D ; single particle

$\psi(x_1, y_1, z_1, x_2, y_2, z_2, \ldots, x_N, y_N, z_N, t)$

↳ N particle system

→ All physical observables can be expressed as operators.

Ex: linear momentum} $\hat{P}_x \equiv -i\hbar \dfrac{\partial}{\partial x}$

$i = \sqrt{-1}$

$\hbar = \dfrac{h}{2\pi}$ → Plancks' Constant 6.62×10^{-34} Js

Kinetic Energy $\hat{K} = \dfrac{\hat{P}\cdot\hat{P}}{2m} = \dfrac{-\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$

$\hat{P}_x\,\psi(x,t) = -i\hbar\dfrac{\partial}{\partial x}\psi(x,t)$

$= \alpha\,\psi(x,t)$

↳ scalar measure of linear momentum

- Schrodinger equation:

$\hat{H} = \hat{K} + \hat{U}$ – Hamiltonian

$\hat{H}\,\psi(x,t) = E\,\psi(x,t)$ → scalar (total energy)

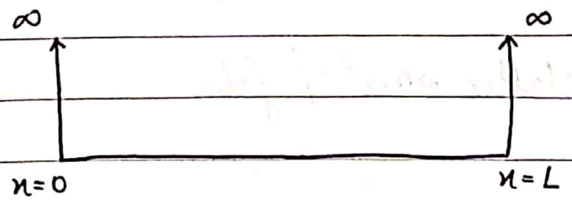
$-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\psi(x,t)}{\partial x^2} + \hat{U}(x)\,\psi(x,t) = E\,\psi(x,t)$

· time independent schrodinger

↓

At a given t, how ψ varies with x.

- Particle confined in a one-dimensional box.



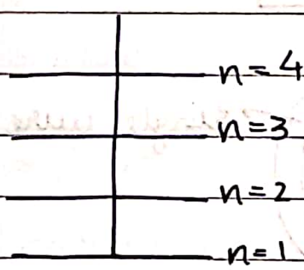
$U(x) = 0 \, , \; 0 < x < L$

$U(x) = \infty$ if $x = 0$ or $x = L$

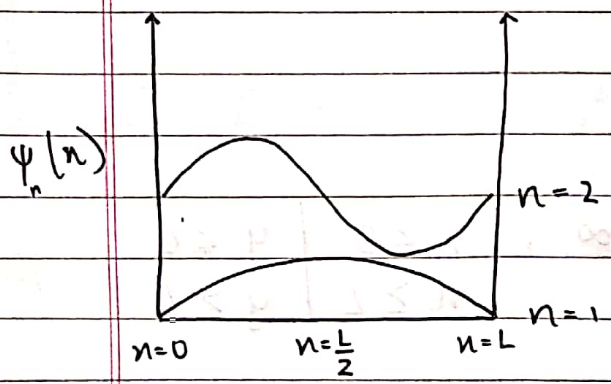
$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$$

$\psi_n^*(x) \, \psi(x) \Rightarrow$ Probability Density.

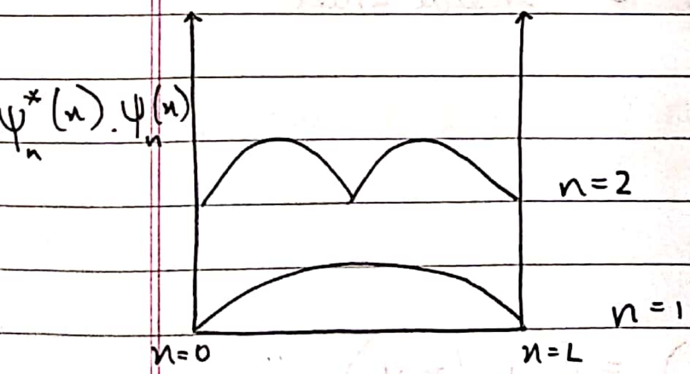
$$E_n = n^2 \frac{h^2}{8mL^2}$$



ground state.



ψ varies with x

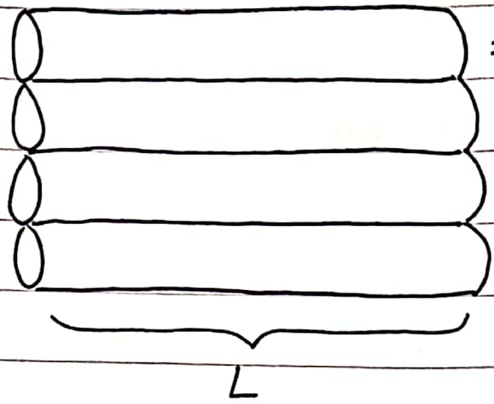


$$\langle x \rangle = \int_{x=0}^{x=L} x \, P(x) \, dx$$

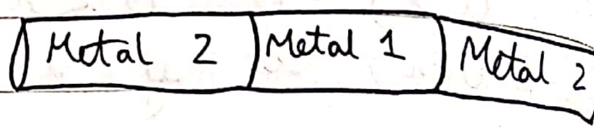
$$= \int_{x=0}^{x=L} x \, \psi^*(x) \, \psi(x) \, dx$$

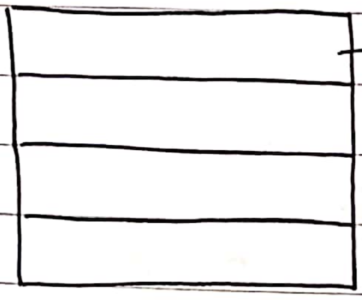
Scanned by CamScanner

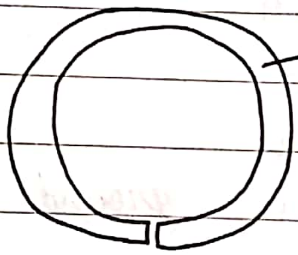
Extension of 1D Model

1) 

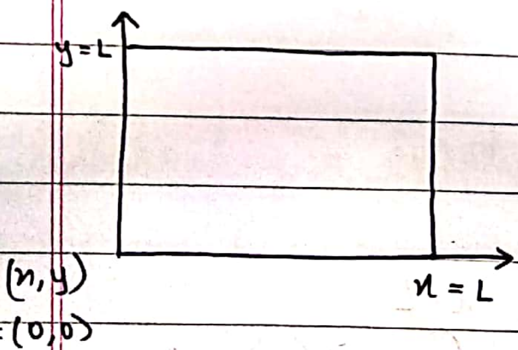
⇒ Metallic Sheet / film

4) 

2) 
→ film sheet

3) 
→ single wire (circular loop)

- Particle in a 2D box.



$U(x,y) = \infty \;;\; x \le 0 \;\; | \;\; y \le 0$
$\qquad\qquad\qquad\quad x \ge L \;\; | \;\; y \ge L$

$U(x,y) = 0$; inside sheet

Wave Function $\psi(x,y)$

- Inside the sheet : $U(x,y) = 0$

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\psi(x,y) = E\,\psi(x,y)$$

$$\psi(x,y) = X(x)\,Y(y)$$

$$\rightarrow Y(y)\cdot\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}X(x)\right)$$

$$+\,X(x)\cdot\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial y^2}Y(y)\right) = E.X(x).Y(y)$$

$\hookrightarrow E_1 + E_2$

divide by $X(x)\,Y(y)$ on both sides.

$$\frac{1}{X(x)}\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}X(x)\right) = E_1$$

$$\frac{1}{Y(y)}\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial y^2}Y(y)\right) = E_2$$

$$E_1 = \frac{n_1^2 h^2}{8mL^2} \; ; \quad E_2 = \frac{n_2^2 h^2}{8mL^2}$$

$$E = E_1 + E_2 = (n_1^2 + n_2^2)\frac{h^2}{8mL^2}$$

$$\psi_{n_1,n_2}(x,y) = \frac{2}{L}\sin\left(\frac{n_1\pi x}{L}\right)\sin\left(\frac{n_2\pi y}{L}\right)$$

Ground State : $n_1 = n_2 = 1$: $E_{1,1} = \dfrac{2h^2}{8mL^2}$

1st excited State : $n_1 = 1, n_2 = 2$; $n_1 = 2, n_2 = 1$: $E_{1,2} = E_{2,1} = \dfrac{5h^2}{8mL^2}$