$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Calculating eigenvalues: $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 4 & 5-\lambda & 6 \\ 7 & 8 & 9-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[45 - 14\lambda + \lambda^2 - 48\right] - 2(36 - 4\lambda - 42) + 3(32 - 35 + 7\lambda) = 0$$

$$\lambda^2 - \cancel{14\lambda} - \cancel{3} - \lambda^3 + 14\lambda^2 + 3\lambda + 8\lambda + \cancel{12} + 21\lambda - \cancel{9} = 0$$

$$-\lambda^3 + 15\lambda^2 + 18\lambda = 0$$

$$-\lambda(\lambda^2 - 15\lambda - 18) = 0$$

$\Rightarrow \lambda = 0$

$$\lambda = \frac{15 \pm \sqrt{225 + 72}}{2} = \frac{15 \pm \sqrt{297}}{2}$$

$\lambda = 0$

16.1168

-1.1168

Now

for $\lambda = 0$:

$AX = 0$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -12 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \leftarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x - z = 0$

$y + 2z = 0$

$x = z$

$y = -2z = -2x.$

vector: $(1, -2, 1)$

$\lambda = 16.1168$ $\quad (A - \lambda I) v = 0$

$$\begin{bmatrix} -15.1168 & 2 & 3 \\ 4 & -11.1168 & 6 \\ 7 & 8 & -7.1168 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Likewise, vector: $(0.2833, 0.6417, 1)$

$\lambda = -1.1168$

vector: $(-1.2833, -0.1417, 1)$

1. Dimensions of matrix A: $p \times q$

2. Yes.

<u>Conditions:</u>

1. Dimensionality reduction preserves ~~at~~ ~~least two dimens~~ the common dimensions in vectors p and q.

2. The points in the original vector had 0-valued destructive dimension values.

3. Eg:

$$(1, 2, 0) \,|\, (2, 4, 0) \rightarrow (1, 2)\ (2, 4)$$

using A to reduce originals by eliminating the z axis.

3. a) $q=2$, $p=2$.

Original: $(1, 2)$; $(3, 4)$

$A = I_2$

New: $(1, 2)$; $(3, 4)$

b). $q=2$, $p=1$

Original: $(3, 5)$; $(1, 5)$

New: (3), (1)

A: $[1 \quad 0]$

c). $q=4$ $p=2$

Original: $(1, 4, 7, 3)$; $(7, 9, 7, 6)$

New: $(1, 3)$; $(7, 6)$

A: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

PSET 3, 3.3

3-3

$L: w_1x_1 + w_2x_2 + w_3x_3 = 0$

~~1. depends on con.~~
~~$D_i = \{ x_i = [x_1^i, x_2^i]^T \}$~~

1. $A^i = [x_1^i - \mu_{1}, x_2^i - \mu_2][x_1^i - \mu_1, x_2^i - \mu_2]^T$

for a line, say $y = mx + c$

$x = \{ x_1^i - \mu_1, \forall i \}$

$y = \{ x_2^i - \mu_2, \forall i \}$

$A^i = [x \;\; y][x \;\; y]^T$

$= \cancel{[x \;\; mx+c][x \;\; mx+c]^T}$

$= [x^2 + y^2]$

$= [x^2 + m^2x^2 + c^2 + 2mxc]$

nonzero eigenvalue: 1

2. $A = \frac{1}{N} \sum_{i=1}^{N} [x_i - \mu][x_i - \mu]^T$
covariance matrix.

nonzero eigenvalues: 1

7. Line perpendicular to L, passing through μ:

$$\frac{y-\mu_2}{x-\mu_1} = \frac{\omega_2}{\omega_1} \quad \text{or} \quad \frac{x_2-\mu_2}{x_1-\mu_1} = \frac{\omega_2}{\omega_1}$$

$D_j = \{x_j = [x_1^j, x_2^j]^T\}$

Since $\mu = [\mu_1, \mu_2]^T$

$$B^i = [x_1^i - \mu_1, x_2^i - \mu_2][x_1^i - \mu_1, x_2^i - \mu_2]^T$$

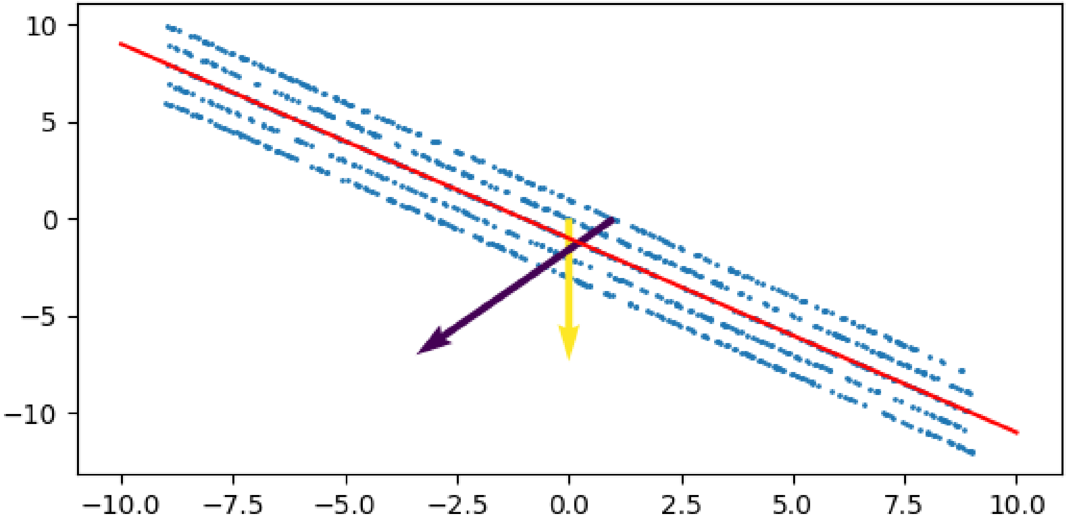
= 1x1 matrix.

1 non zero eigenvalue.

Likewise for
$$B = \frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)(x_i-\mu)^T$$

1 non zero eigenvalue.

8. Reasoning: Since points have uniform distribution just around line, 2 eigenvalues exist. Second smaller than first due to shape's proximity to 1D line. Of the 2 vectors(eigen), one will be almost along the line and the other perpendicular.


10
5
0
−5
−10
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0