

NO CLASS

03.03.2020

RSA

- "Textbook" RSA

- Some Attacks

- PKCS v1.5

- RSA signatures

Outline for today's class

and how the internet standards deal with it.

Textbook RSA

Gen: Choose 2 large distinct primes p, q and

$$N = pq$$

choose (e, d) st $ed \equiv 1 \pmod{(p-1)(q-1)}$

Public Key: $\langle N, e \rangle$

Private Key: $\langle d, p, q \rangle$

Given we know Primality testing (Miller Rabin, etc),

, [Algorithms course]
(Kannan, et al)
(Monsoon 2016)

so we do not prove that this is efficiently computable.

Enc: $m \in \{0, 1, 2, \dots, N-1\}$

$$C = m^e \pmod N$$

Efficient algs for both exist

padding technique

Dec: $c \in \{0, 1, 2, \dots, N-1\}$

$$m = c^d \pmod N$$

Now for correctness, by Euler's theorem

$$a^{\phi(N)} \equiv 1 \pmod N$$

Defn: $\phi(N)$ is the no. of nos. $< N$ that are co-prime to N

Euler's totient

euler's theorem

euler's totient

euler's totient
of $N=pq$

$$\text{Now, } (m^e)^d \pmod N = m^{ed} \pmod N$$

$$\text{Again, and now, } m^{\phi(N)} \pmod N = 1$$

$$\text{and } \phi(pq) = (p-1)(q-1)$$

$$\text{and we know } ed \equiv 1 \pmod N$$

$$\text{or } N \mid k(ed) + 1$$

Textbook RSA is not CPA-secure

Now going back to Textbook RSA,

- it is deterministic
- so, is ciphertext-only secure, not CPA secure

but in public key systems, notion of adversary does not have oracle access is meaningless. As it is public



so, CPA is easily mountable
 \Rightarrow minimum security required for PKC is CPA-secure

\Rightarrow there can be no PKC system that is deterministic, as

- deterministic \Rightarrow no CPA security

- PKC needs a min of CPA security.

Looking at other attacks

• If $e=3$ (or small)

and m is also small

eg: $m < \sqrt[3]{N}$

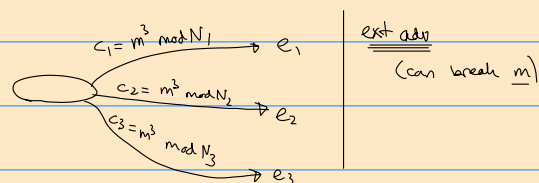
$$c = m^3 \bmod N$$

$$\Rightarrow c = m^3$$

$$\text{or } m = \sqrt[3]{c}$$

• Same e with multiple people

eg: $e=3$, N_1, N_2, N_3



consider: $\begin{cases} x = c_1 \bmod N_1 \\ x = c_2 \bmod N_2 \\ x = c_3 \bmod N_3 \end{cases}$ CRT says \exists unique x such that all 3 hold.

Unique $x \in [0, N_1 N_2 N_3 - 1]$ s.t. Chinese Remainder Theorem

Looking at the suggestions for modified RSA

"Recommended by current Internet": PKCS v.1.5

((\exists no proof that this rec is secure))

((but \exists other algo (nonstandard) that are provably secure))

Gen: • Same as regular RSA

• there are some restrictions on prime selection, but we can just say "any large primes should work"

we modify this to make it secure.

Enc:

$$\text{Ciphertext } C = [\text{padding}]^e \bmod N$$

$\log N$ bit no. / $(\log N)_8$ byte no. $\approx K$ -byte no.

Constructing the number

first byte : fixed (01000000, he says from memory)

second " : " all 0s

then, at least

8 more bytes

are filled with random (not truly random - none can be all 0s)

then, another set of 00000000

then, message m (at most $K-11$ bytes)

for decryption:

- ignore first byte
- " second "
- " byte after byte after byte until we hit another all-0 byte.
- then, print the rest as message.

General Idea of Working

Somehow, they decided

- first 16 MSBs are very weak
- ???

- at least 64 bits of random r recommended

↳ but not fixed. So message sent unknown, cannot build easy algo
⇒ getting to hard-core predicate (RSA vs LSB) difficulty.

$h(\text{RSA})$ is LSB

~ ~ this is modern internet.

Q: Can RSA be made provably secure?

A: Yes. RSA-OAEP (Optimal Asymmetric Encryption Padding)

RSA based schemes are hard to prove. For provable we use discrete log based schemes, like El Gamal.

RSA-based Digital Signatures

(will show deterministic → attacks → randomised)

Defining signatures

A \xrightarrow{m} B • auth data

• auth user

so

A $\xrightarrow{m, \sigma}$ B

s.t. • σ Alg known to everyone

• σ_{sign} uses private key of A

• \exists Verify, uses pubkey of A to verify signature done by A.

Req:

• Adv should not be able to produce new m, σ without sk.

Kannan Trivia

RSA-signature says,

$\text{sign}(d, m)$

$$m^d \bmod N = \sigma$$

Signature

$\text{verify}(m, e, \sigma)$

$$= \text{yes if } \sigma^e \bmod N = m$$

Sign yourself with sk
Verify by others with pk.

Breaking Vanilla (Textbook) RSA-sign

I. choosing m , then σ is hard.

But, do reverse:

1. Choose random signature σ
2. Compute $\sigma^e \bmod N$
3. Send $\langle \sigma^e \bmod N, \sigma \rangle \rightarrow$

$$\text{Verify}(\sigma^e \bmod N, \sigma, (N, e)) = \text{yes}.$$

II. Improved Attack

- suppose I have $\langle m_1, \sigma_1 \rangle$ sent by A.
- A also sent $\langle m_2, \sigma_2 \rangle$
- We produce third message that A never sent:

$$\langle m_1 m_2 \bmod N, \sigma_1 \sigma_2 \bmod N \rangle$$

now, CPA from this:

we want to sign M

- ask for σ for m_1
- ask for σ for m_2

$$\text{st } M = m_1 m_2$$

} Done

New Paradigm:

instead of signing directly

Hash and Sign Paradigm

signature $\sigma = [H(m)]^d \bmod N$

Yes if $\sigma^e \bmod N = H(m)$

Signing

Verifying

Why do old attacks not work?

Collision resistance

Proof: none for RSA

Rest of Course:

- a) A few more PKCs
- b) Proof of CPA security for El-Gamal PKC
- c) El-Gamal is not CCA-secure
- d) CCA-secure PKCs

This should be done by mid-march

then sir will "talk about what we actually want to do"
"freedom to work, not freedom from work"

06.03.2020

Digital signatures & Zero Knowledge Proofs

Q: How to Authenticate the user?

A: Ask user to prove they know the secret key.

Options: - Reveal the key ← pretty useless

5 minutes of
talking about this →

Problem: prove you know a secret without revealing it.

summary →

Q: is it only for special kinds of knowledge (like sk of PKC) that can be given ZKP for, or does ZKP exist for all kinds of knowledge.

More elaboration of problem

eg: • Suppose Alien says "I have winning strat. in chess"

• How would Alien prove this?

best: constructing a decision tree for every move

Useless: Exponential time to construct

Proof too long to comprehend.

We do not know if short
proof exists.

Aside: in complexity, compression problem? $PSPACE \neq P$?

Again,

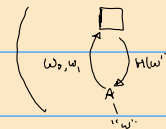
How to prove winning strat?

- Math proof impossible to show, if exists
- so, just play the game. Any loss/draw is proof of lie.
- with a few million games, the $P(\text{lie})$ is low

We will get back to this, hopefully.

eg: Alien needs to prove it can see more colours.

Proof: randomized blind testing



$$P(w=w_i) \leq \frac{1}{2^{100000}}$$

- any incorrect prediction
is proof of lie

- with a few million games,
the $P(\text{lie})$ is low

Q: where is crypto maths?

A: Instead of using random assumption (like alien cannot see through body) as a "shield"

use cryptography (like OWF exist) as shield.

This class:

- Showing all knowledge can be gamified such
- ZKP exists iff OWF exists
- ZKP \rightarrow Dsigns
- Dsigns do not need trapdoor OWF, as it is not PKC dependent.
- OWF \Leftrightarrow bit commitment

Concept of:

Bit Commitment

← same as blockchain

Binding \rightarrow \boxed{b}

Blinding $\rightarrow b$ is secret

eg: consider Use Case of Digital Auction.

- make a bid
- nobody else can see it
- you cannot modify it
- others can verify it when you reveal it after bidding.

Assume \exists one-way permutation $f()$, building Bit-Commitment scheme as:

Commit bit (b):

Pick a random s

Publish $\langle f(s), h(s) \oplus b \rangle$ eg: $\langle g^s \bmod p, \text{MSB}(s) \oplus b \rangle$

now, nobody knows anything about 's'

Reveal phase:

Give \underline{b} and \underline{s}

Now, everybody can check: if $f(s)$ is unchanged
and if so, $h(s) \oplus (h(s) \oplus b) = b$?

\therefore if DLP is a OWF, Bit-Commitment scheme exists

Something something ^{this is complex} NP_h problem

Opaque Locked box $\boxed{}$

Assume such boxes exist. (replace with bit-commitment)

Show it is in language, NP?

Looking at problem: Graph 3-Colouring ((is NP-complete, not proving))

given a graph $G=(V,E)$
is it 3-colourable or not?

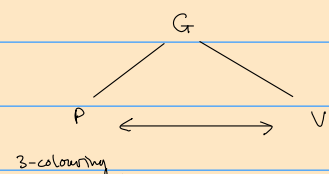
\exists exp algos

((eq. question: is G tri-partite))

pol algos: NP-hard, so can't say (certainly)

ZKP for graph 3-colouring:

(aka
Given a graph
you know the answer (it is 3-colourable)
" do not want to reveal solution
" " " prove to world you know solution)



ZKP has to satisfy:

- o Soundness condition
- o Completeness condition

((all "locked boxes" = bit commitment scheme))

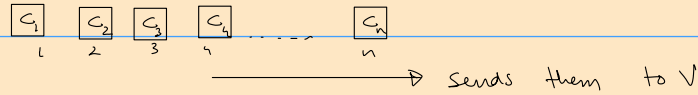
ZKP Algo:

Prover does:

- Prover knows 3 colours RGB

→ Prover permutes the three colours

→ " creates n locked boxes, each with the colour of the corresponding vector



Verifier (V) does:

→ V picks one edge (i, j) at random and asks to reveal C_i and C_j

→ If $C_i = C_j$ Reject

(convince)

If $C_i = C_j$ Accept if soundness, else repeat.

Verifying the proof

Completeness: if G is 3-colourable,

V always accepts

Soundness: if G is not 3-colourable,

V rejects with high p .

Seeing how high,

$|E|$ edges \Rightarrow rejection with $\Pr: \frac{1}{|E|}$ reject.

$(1 - \frac{1}{|E|})^k \leftarrow$ pr of acceptance with k trials.

At the end of this

V:- does not know any more about the solution

- is convinced P knows the solution

Now, we want to think of Dig. Sign based on DLP.

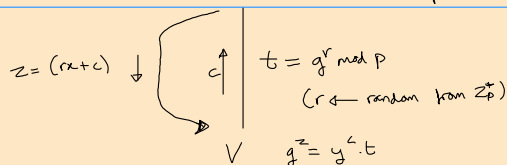
Considering interactive version

$$y = g^x \text{ mod } p$$

we will use ZKP for DL to get Dig. Sign for DL.

ZKP for DLP:

P \leftarrow wants to prove given $(y, g, p) \Rightarrow x$



1. P chooses r , sends $t = g^r \text{ mod } p$
2. V sends c (random from \mathbb{Z}_p^+)
3. P sends $z = (ct + r)$
4. V checks if $g^z = y^c \cdot t$

follows from,

DLP is one-way

This can act as signature scheme if interactive signatures allowed

Completeness:

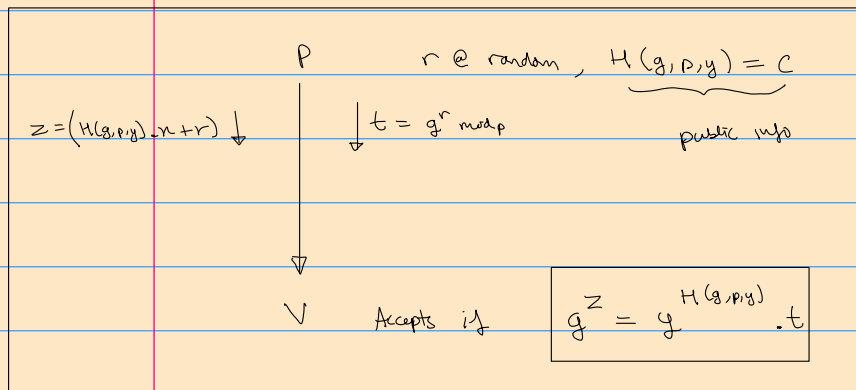
$$g^{ct+r} = (g^c)^t \cdot g^r = y^c \cdot t$$

Soundness:

if P does not know x , prob x', r' s.t.
 $P_r(Cx' + r' = ct + r) = \frac{1}{p}$

to get dig-sign (non interactive), we use **Random Oracle Model**:
 : if hash functions are random output, then oracle.???

If **assume hash functions give random outputs**,



← this is a signature scheme under this assumption
 (Schloss) signature

10.3.20

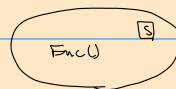
Secret Sharing

- the problem
- Shamir's solution
- A Generalisation
- Beautiful Relation to Computing (SOME OPEN PROBLEMS)

PROBLEM:

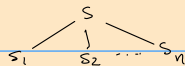
Secret key S

an adversary need not control system, just (access to) key S .



- if key is in a single location, there is no fault-tolerance
- to leverage adv. of dist. sys, fault-tolerance: key needs to be shared
- but then key is no longer secret

Need secret sharing:



$$0 < t < n$$

- Any $> (t-1)$ s_i 's can reconstruct S
- Any $< t$ s_i 's have no information about S

Shamir's Secret Sharing:

Define $s \in \mathbb{F}$ (for eg. $\mathbb{Z}_p, \text{prime } p$)

$$Q(x) = \sum_{j=0}^t r_j x^j$$

$$Q(0) = r_0 = S$$

$$r_j \xleftarrow{\text{rand}} \mathbb{F}$$

$$\alpha_1, \alpha_2, \dots, \alpha_n$$

we distinct public elements of \mathbb{F}

$$S_i = Q(\alpha_i)$$

General Access Structure

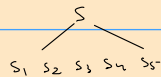
$$A \subseteq 2^{[n]}$$

- All subsets $B \in A$ can access S
- Others should have no info about S

$$B \in A \quad \text{if } |B| \geq t-1 \quad \text{else } B \notin A$$

$$A = \{B \mid |B| \geq t-1\}$$

eg: $n=5$



When 1,2 can reconstruct secret, every superset can automatically access it to

$$A = \{\{1,2\}, \{2,4,5,6\}\}$$

(monotone super-sonic property)

$\{1,2\}, \{2,4,5,6\}$ can access but not $\{2,3\}$.

I have entirely lost him here

take degree $t=3$ and 8 points on the polynomial.

Give 2 points to 1, 2 points to 2 and 1 each to rest of 3, 4, 5 and 6.

so the $A = \{\overset{\nearrow 4 \text{ points}}{\{1,2\}}, \overset{\nearrow 4 \text{ points}}{\{3,4,5,6\}}, \overset{\nearrow 4 \text{ points}}{\{1,3,4\}} \dots\}$

Will weights always work? Answer is NO in general but there are cases where YES but exponential number of weights required.

$A \rightarrow$ access structure

$$f: \{0,1\}^n \rightarrow \{0,1\}$$
$$A_f = \{B \mid f(B) = 1\}$$

0110...1 there is a canonical relation between 2 bit strings and subsets.

For any access structure, there will be a corresponding boolean functions. Boolean functions are not monotone. They will be monotone if they can be implemented using only AND and OR gates (monotone function theory).

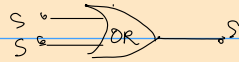
Conversion From AND-OR circuits for A to secret sharing schemes for A



$$A = \{\{1, 2\}\}$$

Give r to one and r-s to other

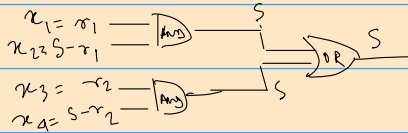
$$\bar{A} = \{\emptyset, \{1\}, \{2\}\}$$



$$A = \{\{1\}, \{2\}, \{1, 2\}\}$$

Give s to both.

$$\bar{A} = \{\emptyset\}$$

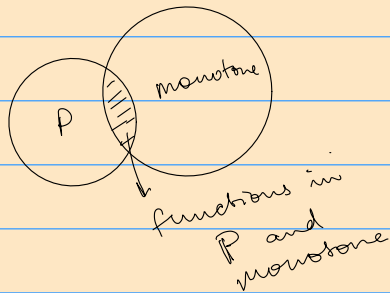


$$\text{Given } A \subseteq 2^{[1..n]}$$

Step 1: Model it as boolean function $f_A: \{0,1\}^n \rightarrow \{0,1\}$

Step 2: Design an AND-OR Boolean circuit for f_A

Step 3: Build the secret sharing scheme level-by-level.



mP \rightarrow poly sized AND-OR circuit

P \rightarrow poly sized AND-OR-NOT circuit

$$mP \stackrel{?}{=} P \cap \text{Monotone}$$

\neq Example: Perfect Matching.

what to do if $A \in P \cap \text{Monotone}$ but $A \notin mP$?

$\forall B \in A$, poly-time reconstruction algorithm

$\forall B \notin A$, $\forall \text{PPTM } D$ $P[D(\text{shares } B) = s] \leq \text{negl}(n)$.

$$|S| = \log |H| \text{ bits}$$

$$|S| = |S| \uparrow$$

$$n \log |H| \text{ bits.}$$

This can be broken

Secret S, $\text{Enc}_K(S) = C \rightarrow$ ^{Rabin's} Information Dispersal Algorithm ^{or} Reed-Solomon code $\geq t+1$
 \downarrow
 small \rightarrow Shamir's secret sharing
 any $t+1$ should be able to access.

$$\text{Complexity: } n|K| + \frac{n}{t+1} \log |H|$$

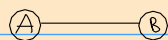
OPEN

Smallest computational secret sharing? $A \in P \cap \text{Monotone}$ but $A \notin mP$?
 NO known poly-sized soln EXISTS!

Oblivious Transfer (OT) and ^{General} Secure Two-Party Computation

13.3.2020

OT (Problem)



index i
Output = b_i

Array $B = [b_1, b_2, \dots, b_n]$

Shouldn't know i

Shouldn't know $b_i \neq i$

(How to query DB without DB revealing anything other than query without knowing query)

impossible to prove
but if PKC exists, provable

logically impossible (intuitively)

(can formalise)

Showing that solutions exist if PKCs exist (trapdoor OTs exist)

Secure General Two-Party Computation (General version)

$f(x, y)$



goal: compute some $f(x, y)$, but

A does not want to reveal x

B does not want to reveal y

((famous formulation: ——— millionaire problem))

Showing that OT is a special case of Secure Gen 2-party Comm

i

$B = [\quad]$

$f(i, B) = B[i]$

But it is a "special case", like

if OT solution is given to you, you can solve any SG2-PC

Showing:

$h(i, B) = B[i]$

$f(x, y)$ $f: D_A \times D_B \rightarrow D_C$

suppose



$x \in D_A$

$y \in D_B$

Solving as

1. WLOG, single bit answer ASSUMPTION:

2. WLOG, $D_A \rightarrow \{1, \dots, n\}$

3. now, B builds array as follows:

4. now, this is OT:

x

1st element, $[f(1, y),$

2nd " , $f(2, y),$

...

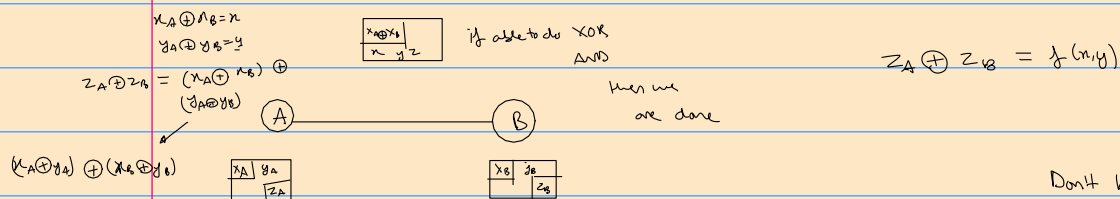
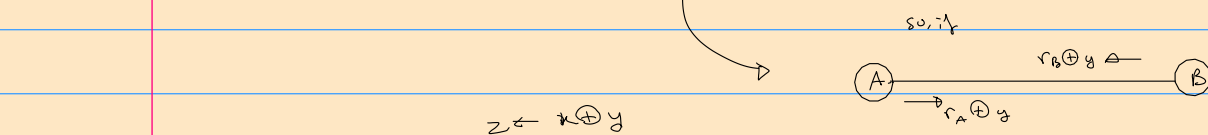
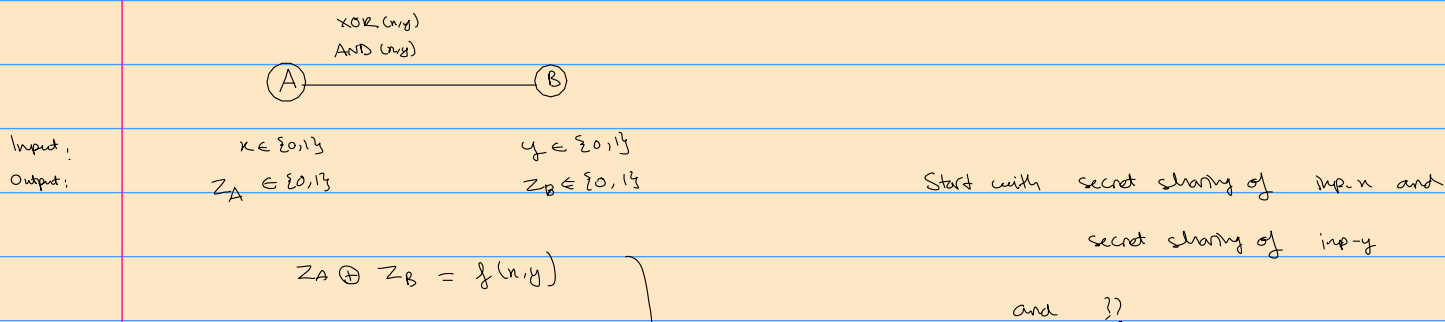
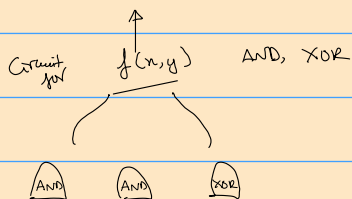
$f(n, y)]$

$h(x, B) = f(x, y)$

\Rightarrow if OT can be solved, SG2PC can be solved.

10 minute aside on why arrays are programs.

Now, similarly to last class, we are only interested in functions f that are efficiently computable (poly-efficient circuits)



Let us see protocols for XOR and AND

Can together simulate trusted third Party

Protocol for XOR

A: $z_A \leftarrow r_A \oplus y_A$
B: $z_B \leftarrow r_B \oplus y_B$

B defines $z_B \leftarrow_{\mathcal{R}} \{0,1\}$

B creates an array

b_{00}	b_{01}	b_{10}	b_{11}
----------	----------	----------	----------

x_A, y_A	
$z_B \oplus (x_B \wedge y_B)$	0, 0
$z_B \oplus (x_B \wedge \bar{y}_B)$	0, 1
$z_B \oplus (\bar{x}_B \wedge y_B)$	1, 0
$z_B \oplus (\bar{x}_B \wedge \bar{y}_B)$	1, 1

z_A

OT protocol

Step 1: A sends a random array $A = [r_1, r_2, \dots, \bar{x}_i, \dots, r_n]$ to B

$\text{enc}_B(r_i)$ (replaced by encrypted r_i)
 $= r_i^e \text{ mod } N$

Step 2: B decrypts the entire array A $D = [\text{dec}_B(r_1), \dots, x_i, \dots, \text{dec}_B(r_n)]$

Atk $D+B$, $DB = [\oplus_j \text{dec}(r_j) \oplus b_j]$

B sends DB to A

Step 3: A obtains $b_i = (DB[i] \oplus r_i)$

$F(n)$ f.w. [XOR of random subset of bits of x]