Revision

- Inductive term cannot be a subterm of itself, but a coinductive term can be.

Bisimulation

- A quick flashback to term graphs and behaviours

$$\sigma$$

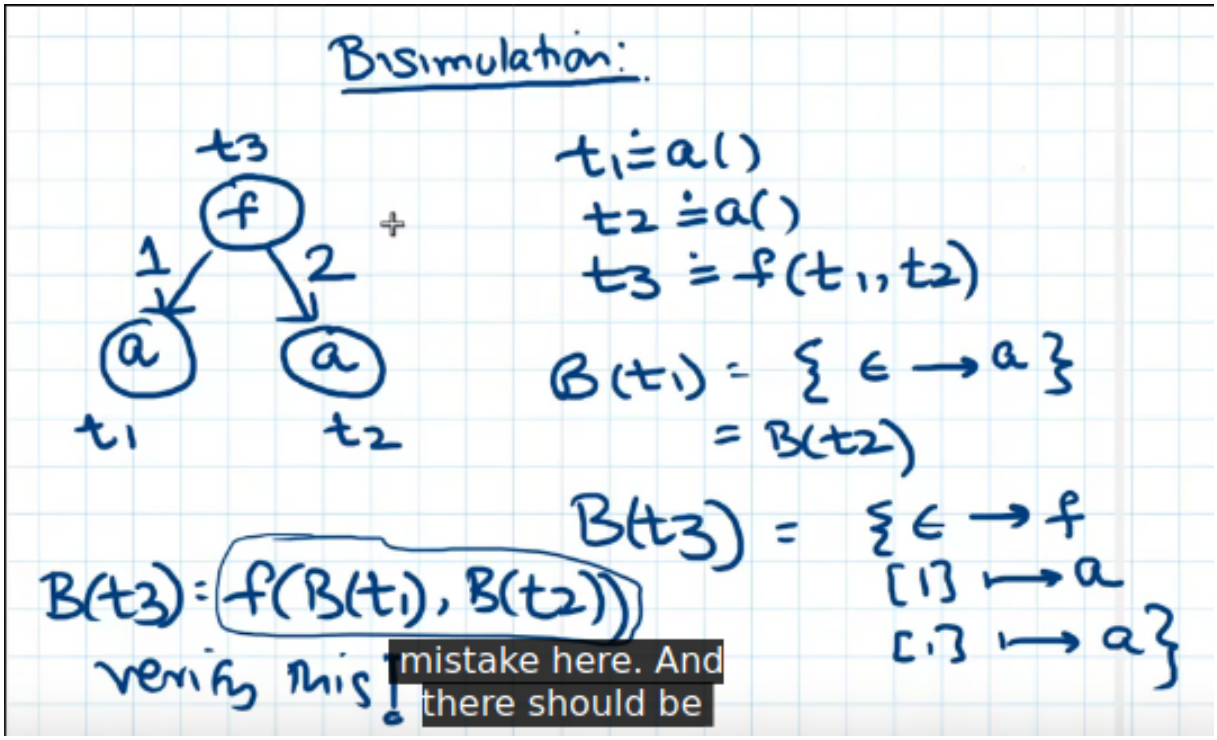


Figure 1: image

- A motivation for bisimulation: all the terms are intuitively the same

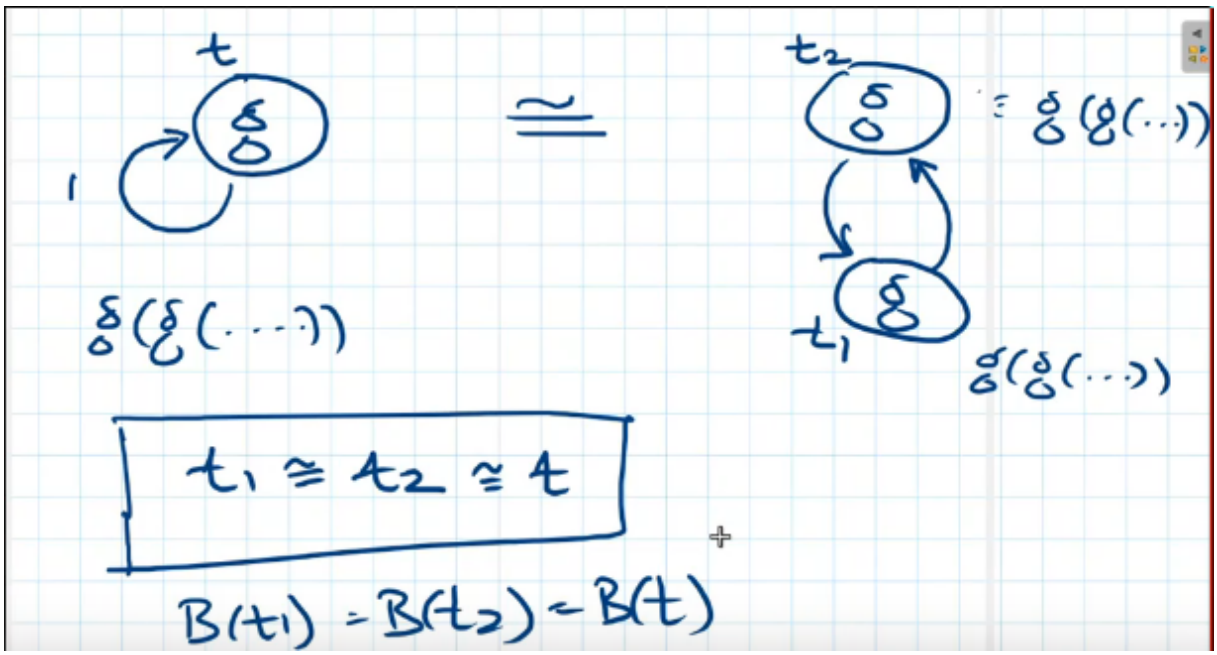


Figure 2: image

- **Bisimilarity** establishes equivalence of behavious by examining the structure of the term graph containing two vertices:
  - Equivalence of objects is modulo observation
  - In case of terms, observations are constructor symbols and we are allowed to look at the head of the term in order to identify the constructor
  - Equal terms should have equal heads + equality for corresponding pair of subterms –> *Inference rule down*, and *rule up*
  - Issues with circular reasoning, however
  - **Intuitive idea behind bisimulation**: just say that the 2 things are equal, and challenge anyone to show a mistake.
  - Unlike induction, here you construct an argument and then ask people to pick holes in it. Proof by construction, where it is entirely internally consistent.
- So, for bisimilarity `R`:
  - `hd(v)== hd(v')`
  - for each `i`,

    $$1 \leq i \leq \alpha(hd(v))$$

    , for all `i`: $subterm_i(v), subterm_i(v') \in R$
- Non-example

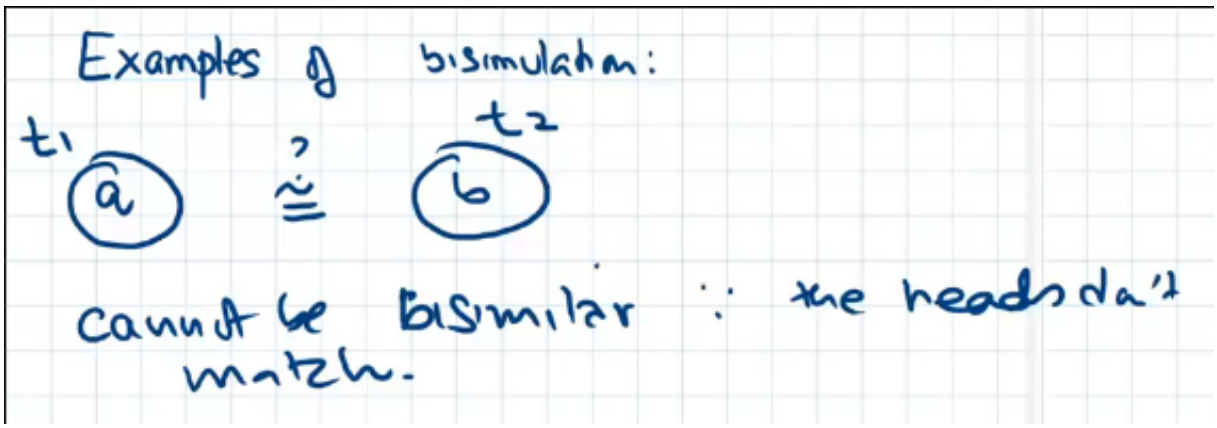


Figure 3: image

- Setup for a bisimilarity proof

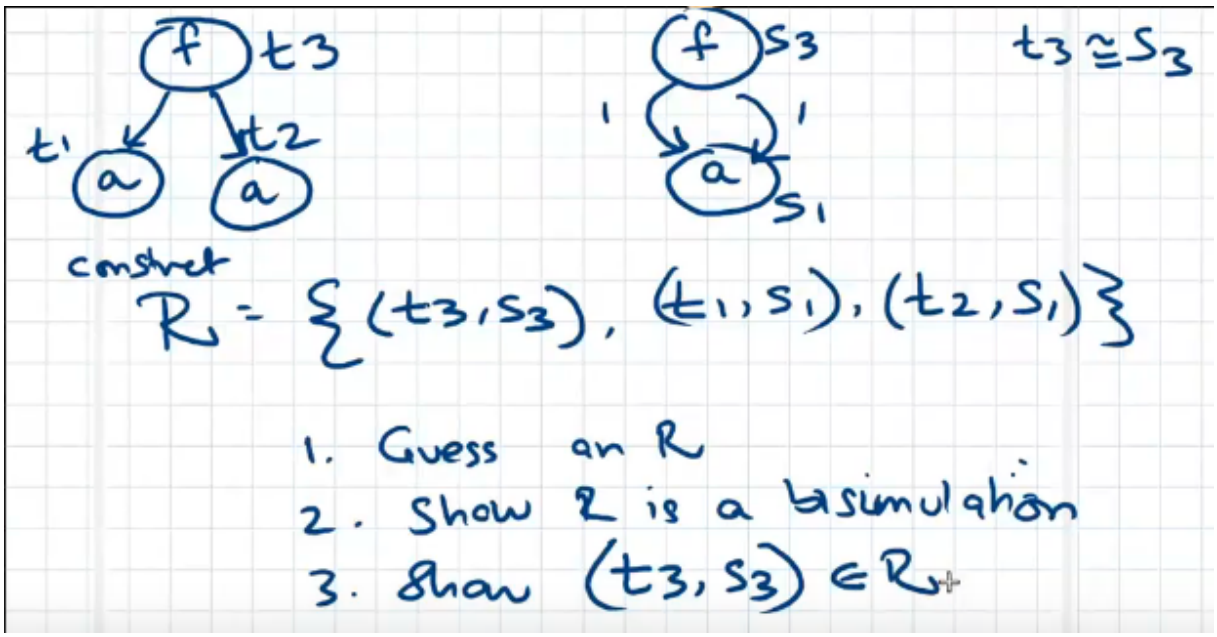


Figure 4: image

Demonstration of showing/proof:

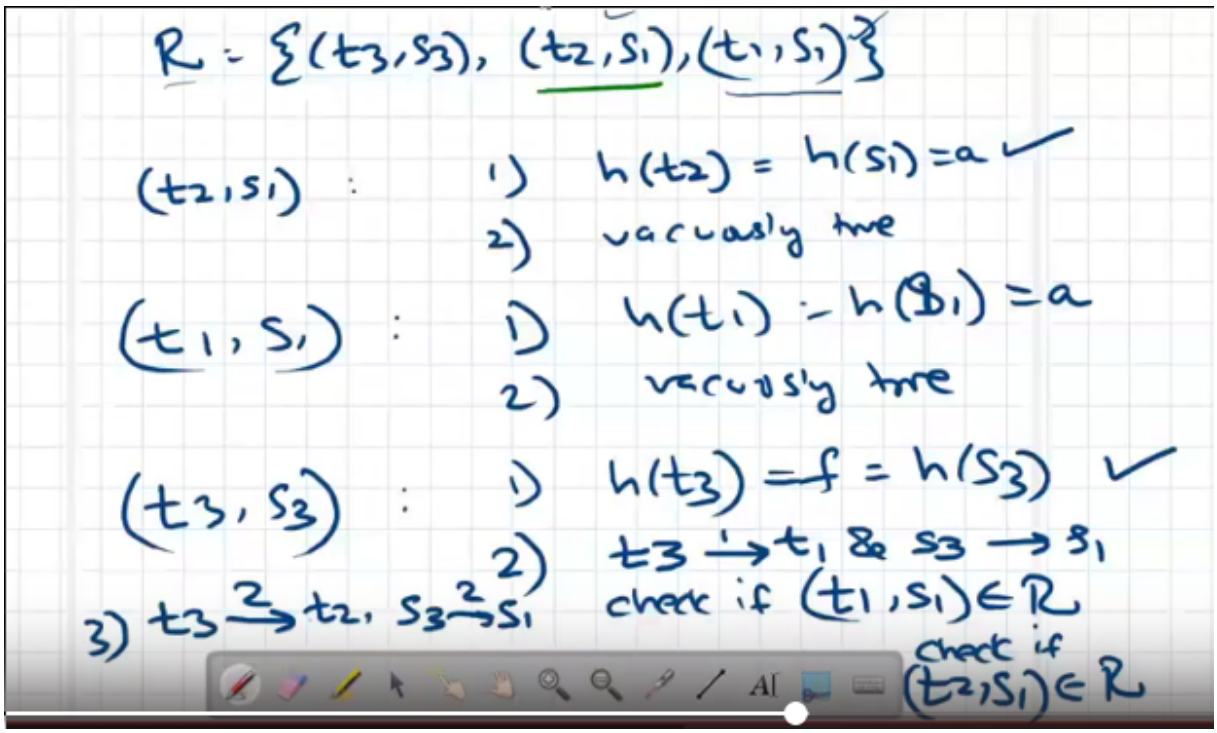


Figure 5: image

- Setup for a bisimulation break

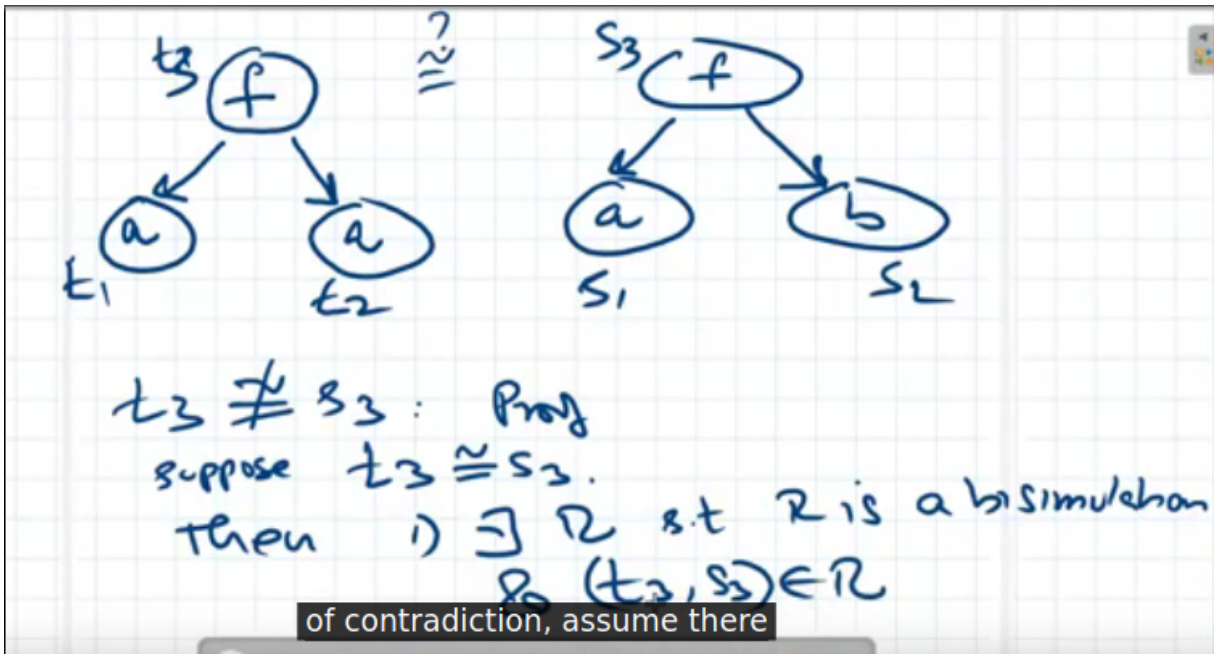
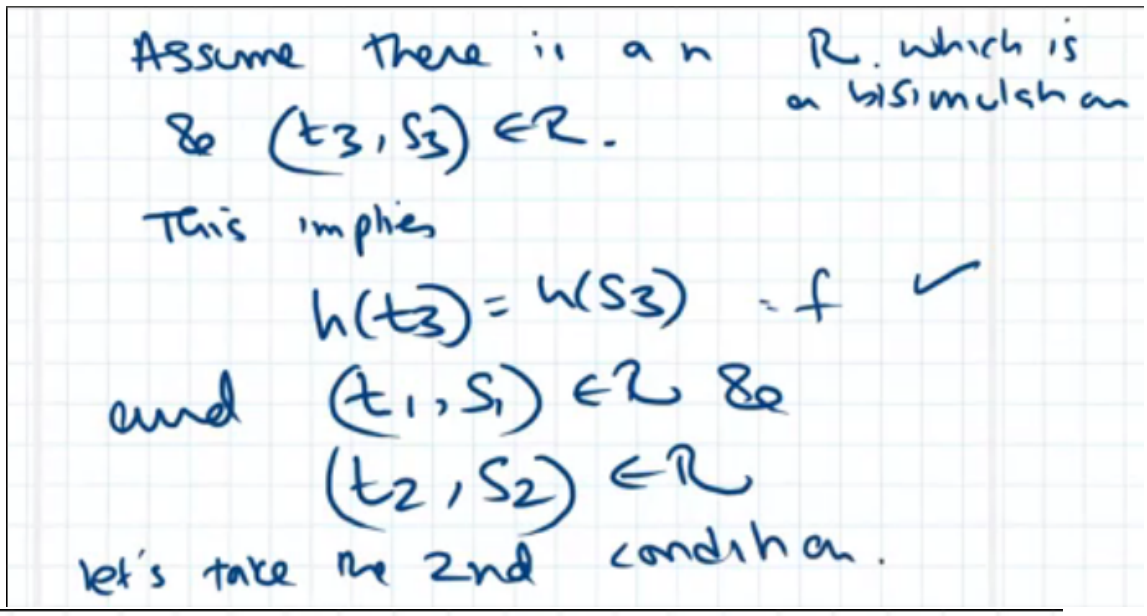
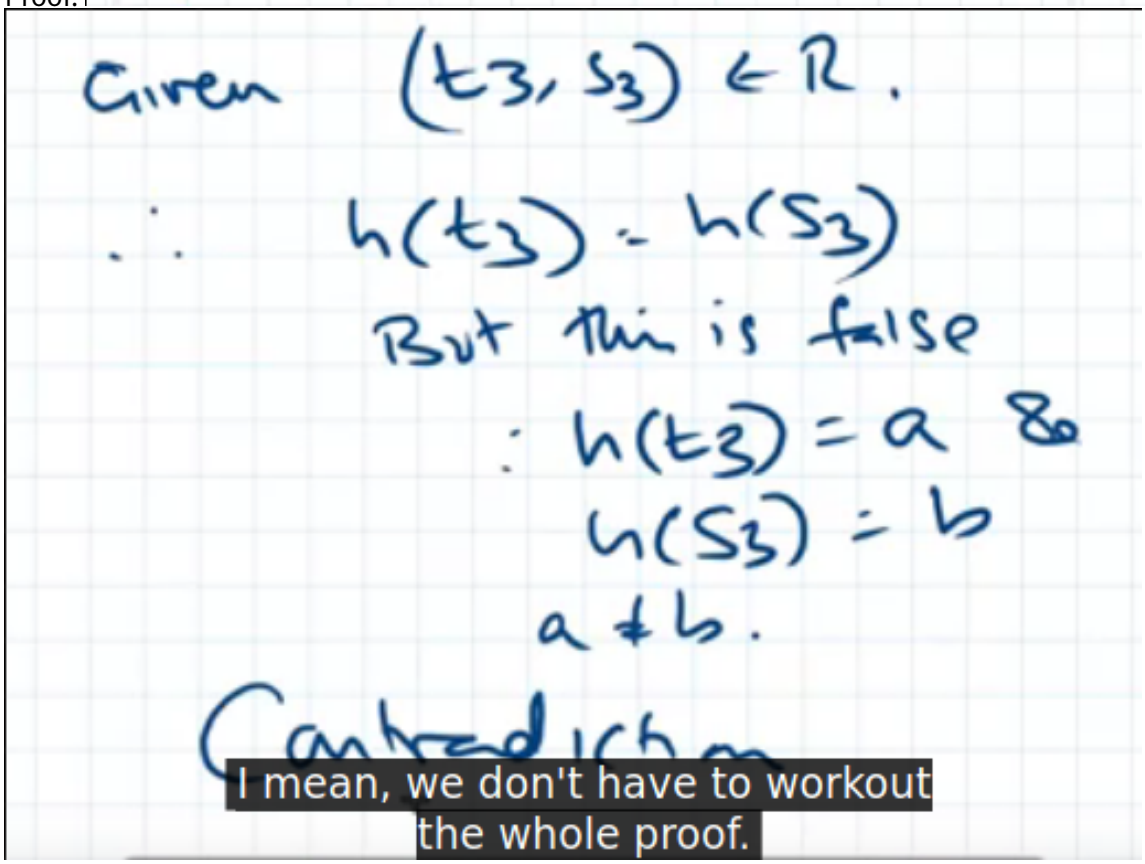


Figure 6: image

Proof:





Replacement for inductive terms

- Replace subterm `t'` at position `p`, with say `S`

Replacement for coinductive terms