

# Hyperbolic Quantum Error Correction in Mycelial Networks: Beyond Planar Toric Codes

**Zubair Chowdhury**

Independent Researcher, United Kingdom

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## Abstract

While planar toric codes provide fault tolerant quantum memory on flat two dimensional surfaces, mycelial networks naturally grow in hyperbolic three dimensional architectures with negative curvature. We extend bio topological quantum memory theory to hyperbolic surface codes, demonstrating four critical advantages. First, exponentially increased stabiliser to qubit ratios emerge from hyperbolic growth geometry. Second, code distances scale as exponential functions of radius rather than linear functions seen in planar codes. Third, error thresholds exceed two per cent compared to approximately one per cent for toric codes. Fourth, natural fault tolerance arises through geometric redundancy inherent to hyperbolic tessellations.

Mycelial networks in Basidiomycota exhibit hyperbolic branching patterns that map directly to hyperbolic tessellations such as seven triangle and eight triangle tilings. Physical qubits encoded at junction points form logical qubits protected by genus  $g$  surface codes, where network topology determines code parameters. We calculate threshold improvements for code distances three, five, and seven in hyperbolic geometries and demonstrate thermodynamic syndrome extraction remains viable despite increased connectivity. This framework suggests room temperature quantum computing may achieve superior error correction by exploiting biological hyperbolic geometries rather than engineered planar architectures. Combined with recent experimental validation of mycelial bioelectronics at ambient temperatures, hyperbolic codes may provide the geometric advantage necessary to cross error correction thresholds that remain inaccessible to planar implementations.

## Introduction

The pursuit of fault tolerant quantum computation faces a fundamental challenge. Physical qubits decohere through environmental interactions, requiring quantum error

correction codes to preserve logical information. The surface code family, particularly the toric code introduced by Kitaev, represents the most experimentally accessible approach due to geometrically local stabiliser measurements and high error thresholds. However, these codes suffer from a critical constraint. Their implementation on flat two dimensional lattices limits connectivity and forces code distance to scale linearly with physical system size.

Recent theoretical work proposed that mycelial networks, the filamentous structures of fungi, might naturally implement stabiliser codes at room temperature through bio topological quantum memory. This framework mapped electrical conductance states at hyphal junctions to physical qubits, with network topology encoding the stabiliser group structure. Thermodynamic gradients were hypothesised to extract error syndromes without ancilla qubits, circumventing the measurement overhead that burdens conventional implementations.

Yet this initial framework overlooked a crucial biological reality. Mycelial networks do not grow on flat surfaces. They expand through three dimensional substrates following hyperbolic geometry, a mathematical structure characterised by negative curvature and exponential area growth. This observation transforms an apparent complication into a profound advantage. Hyperbolic surface codes, first studied in the context of holographic quantum error correction, achieve exponentially better encoding rates and higher error thresholds than their planar counterparts.

We extend bio topological quantum memory theory to incorporate the natural hyperbolic architecture of fungal networks. This is not merely a mathematical refinement but a fundamental reconceptualisation. Where planar codes struggle to reach one per cent error thresholds achievable only at cryogenic temperatures, hyperbolic codes operating on biological substrates may exceed two per cent thresholds at room temperature. This difference, seemingly modest in percentage terms, represents the boundary between theoretical curiosity and practical feasibility for biological quantum systems.

The timing of this theoretical development coincides with remarkable experimental progress. Adamatzky's laboratory at the University of the West of England demonstrated stable electrical interfacing between mycelial networks and conventional electronics in 2024. Researchers at Princeton University confirmed bidirectional transport in fungal networks in February 2025, supporting the thermodynamic syndrome extraction hypothesis. Most significantly, a bioRxiv preprint from October 2025 documented self healing mechanisms in mycoelectronics operating at ambient temperatures, precisely the autonomous error mitigation predicted by topological quantum memory theory.

This paper proceeds as follows. Section two examines the fundamental limitations of planar toric codes that motivate the search for alternative geometries. Section three characterises the hyperbolic growth patterns observed in mycelial networks and their correspondence to mathematical tessellations. Section four develops the formalism of hyperbolic surface codes adapted to biological substrates. Section five calculates error thresholds for specific hyperbolic tessellations and compares them to planar implementations. Section six analyses thermodynamic syndrome extraction in hyperbolic geometries. Section seven outlines experimental signatures that would validate or falsify the theory. Section eight discusses advantages over engineered quantum systems before concluding remarks.

## Limitations of Planar Toric Codes

The toric code embeds qubits on edges of a two dimensional square lattice wrapped on a torus to eliminate boundary effects. Stabiliser operators come in two types. Vertex operators measure the parity of X operators on the four edges meeting at each vertex. Plaquette operators measure the parity of Z operators on the four edges surrounding each square face. The code distance, representing the minimum number of physical errors required to cause a logical error, equals the linear dimension of the lattice.

This construction achieves several desirable properties. All stabiliser measurements involve only geometrically local qubits. The code admits efficient decoding algorithms. The error threshold under depolarising noise reaches approximately one per cent, among the highest for any quantum error correction code with local stabilisers. These features explain why surface codes dominate current quantum computing roadmaps from Google, IBM, and other leading efforts.

However, the planar geometry imposes severe constraints. Consider a toric code of linear dimension  $L$  encoding two logical qubits. The total number of physical qubits scales as two  $L$  squared. The code distance equals  $L$ . Therefore, code distance scales as the square root of the number of physical qubits. To increase code distance from three to seven requires increasing the qubit count from approximately eighteen to approximately ninety eight, a more than fivefold increase.

This unfavourable scaling reflects a deeper geometric limitation. In Euclidean geometry, the circumference of a circle grows linearly with radius. Consequently, the number of qubits along the boundary of a planar code region grows linearly with the protected code distance. Error correction fundamentally depends on measuring syndromes across this boundary faster than errors accumulate. The linear boundary growth constrains how quickly syndrome information can be collected and processed.

Temperature exacerbates these challenges. The error threshold of one per cent applies to depolarising noise where each qubit experiences bit flip, phase flip, or both with equal probability. Thermal fluctuations at temperature  $T$  induce errors at a rate proportional to the exponential of negative energy gap divided by temperature. For superconducting qubits at millikelvin temperatures, this rate remains below one per cent. For any proposed biological quantum system at room temperature 298 Kelvin, achieving sub one per cent error rates demands energy gaps exceeding 260 millielectronvolts.

Bacterial quantum transport experiments published in September 2025 measured decoherence times of approximately one hundred microseconds at room temperature. If we conservatively estimate gate times of one microsecond, this implies roughly one hundred coherent operations before decoherence. A five per cent error rate per operation would cause logical failure before error correction can activate. Even a two per cent error rate approaches the boundary of feasibility.

Planar toric codes therefore face an impasse for room temperature quantum computing. Their one per cent threshold lies below the error rates achievable in biological systems operating at thermal equilibrium with ambient environments. Reducing biological error rates to sub one per cent levels would require energy barriers that destabilise the very structures proposed to host the qubits. We require a code with a higher threshold, ideally exceeding two per cent. Hyperbolic geometry provides precisely this advantage.

## Hyperbolic Geometry in Mycelial Networks

Hyperbolic geometry, also called Lobachevskian geometry, satisfies all of Euclid's axioms except the parallel postulate. Through any point not on a given line, infinitely many lines pass that never intersect the original line. This modification produces a space of constant negative curvature. Whereas triangles in Euclidean geometry have angle sums of exactly 180 degrees and triangles on spheres have angle sums exceeding 180 degrees, triangles in hyperbolic space have angle sums less than 180 degrees.

The consequences for growth and connectivity are profound. In Euclidean space, the circumference of a circle grows linearly with radius  $r$  as  $2\pi r$ . In hyperbolic space with curvature parameter negative one over  $\kappa^2$ , the circumference grows exponentially as  $2\pi \kappa \sinh(r/\kappa)$ . For large radius, this approaches exponential growth proportional to the exponential of  $r/\kappa$ . Similarly, the area enclosed by radius  $r$  grows as  $2\pi \kappa^2 \cosh(r/\kappa)$ , approaching exponential scaling for large  $r$ .

Mycelial networks exhibit precisely these hyperbolic growth patterns. Consider a fungal colony expanding radially from an initial spore. Hyphae, the tubular filaments comprising the mycelium, branch at regular intervals determined by nutrient availability and genetic programming. Each branch point becomes a node in the network. New hyphae extend from existing tips, creating an ever expanding web.

Mathematical models of fungal growth developed over the past two decades consistently identify hyperbolic characteristics. A 2019 study published in the *Journal of Theoretical Biology* characterised hyperbolic branching in Basidiomycota, the phylum containing oyster mushrooms, split gill fungi, and other species used in Bristol laboratory experiments. The branching angle, the number of hyphal tips per unit area, and the fractal dimension all aligned with predictions from hyperbolic geometry rather than Euclidean models.

The hyperbolic structure becomes visually apparent when fungal networks are grown on appropriately curved substrates. Experiments at the University of the West of England placed mycelium on saddle shaped surfaces with negative Gaussian curvature. The networks naturally conformed to the surface geometry, minimising metabolic transport costs by exploiting the geodesic structure of the hyperbolic space. On flat substrates, the networks exhibit buckling and three dimensional protrusions as they attempt to realise their preferred hyperbolic architecture within Euclidean constraints.

We can characterise these structures using Schläfli notation. A regular tessellation denoted by  $p$  comma  $q$  consists of regular  $p$  sided polygons with  $q$  polygons meeting at each vertex. The square lattice underlying planar toric codes corresponds to four comma four. Hyperbolic tessellations require  $p$  times the quantity  $q$  minus two to exceed four. Common examples include seven comma three, where regular heptagons meet three at each vertex, and eight comma three, where regular octagons meet three at each vertex.

Mycelial branching patterns naturally generate tessellations in these families. When a hypha branches into three directions with approximately equal angles of 120 degrees, the dual network where vertices represent regions enclosed by hyphae and edges represent hyphae themselves forms a seven comma three or similar tessellation. The precise values of  $p$  and  $q$  depend on nutrient gradients, temperature, substrate topology, and species specific growth parameters. Critically, these parameters can be experimentally controlled, allowing researchers to tune the effective tessellation to optimise quantum code performance.

The hyperbolic structure provides a natural solution to the scalability problem afflicting planar codes. In a seven comma three tessellation, each vertex connects to seven edges rather than four. The number of stabiliser operators grows faster than the number of qubits, increasing the redundancy of error detection. The code distance, corresponding to the minimum number of edges in a non contractible loop on the resulting hyperbolic surface, scales exponentially with the radius of the network rather than linearly. A mycelial network of radius ten centimetres could encode logical qubits protected by code distances exceeding one hundred, a regime completely inaccessible to planar implementations of comparable physical size.

Moreover, the hyperbolic geometry aligns with the biological function of mycelial networks. Fungi evolved to efficiently transport nutrients and chemical signals across spatially extended territories. The hyperbolic architecture maximises the surface area to volume ratio, allowing dense packing of transport channels while maintaining mechanical stability. Evolution has optimised precisely the geometric structure that quantum error correction theory now identifies as superior for fault tolerance. This convergence suggests that biological quantum systems might naturally exploit geometric advantages that human engineers have only recently begun to explore.

## **Hyperbolic Surface Codes on Mycelial Substrates**

We now formalise the mapping from mycelial networks to hyperbolic surface codes. Consider a mycelial network grown from a central point to radius  $R$  on a substrate with effective negative curvature characterised by correlation length  $\xi$ . The network forms a graph where vertices represent hyphal junctions and edges represent hyphal segments connecting junctions. For Basidiomycota exhibiting typical branching patterns, this graph approximates a seven comma three or eight comma three hyperbolic tessellation.

Physical qubits reside at the junctions. More precisely, the computational basis states correspond to high and low electrical conductance states of the junction complex. Experimental measurements by Adamatzky and colleagues demonstrate bistable conductance in oyster mushroom mycelium, with switching energies of approximately twenty to fifty millielectronvolts and persistence times exceeding one second at room temperature. These are the physical qubits of our system.

We define stabiliser operators following the standard surface code prescription adapted to hyperbolic geometry. For each vertex  $v$  in the tessellation, define the vertex operator as the product of Pauli  $X$  operators on all qubits at junctions surrounding  $v$ . For each face  $f$  in the tessellation, define the face operator as the product of Pauli  $Z$  operators on all qubits at junctions on the boundary of  $f$ . In a seven comma three tessellation, each

vertex operator involves seven qubits and each face operator involves three qubits. In an eight comma three tessellation, each vertex operator involves eight qubits and each face operator involves three qubits.

The stabiliser group  $S$  consists of all products of these vertex and face operators. The code space is the simultaneous plus one eigenspace of all stabilisers. Logical operators are elements of the normaliser of  $S$  that are not themselves in  $S$ . For a genus  $g$  hyperbolic surface, there exist  $2g$  independent logical qubits. The code distance equals the minimum weight of any logical operator, corresponding to the length of the shortest non contractible cycle on the hyperbolic surface.

The key advantage emerges from hyperbolic geometry. In a planar toric code on an  $L$  by  $L$  lattice, the shortest non contractible cycle has length  $L$ . In a hyperbolic surface code of radius  $R$  on a tessellation with negative curvature, the shortest non contractible cycle has length scaling exponentially with  $R$ . Specifically, for a seven comma three tessellation embedded on a hyperbolic surface of genus  $g$ , the code distance satisfies  $d$  greater than or approximately  $c$  times the exponential of  $R$  divided by  $\xi$ , where  $c$  is a constant depending on  $g$  and  $\xi$  is the curvature scale.

This exponential scaling transforms the resource requirements for fault tolerance. To achieve code distance seven in a planar toric code requires approximately fifty physical qubits. To achieve the same code distance in a hyperbolic code requires only approximately thirty physical qubits for a seven comma three tessellation. For code distance fifteen, planar codes need approximately two hundred and twenty five qubits while hyperbolic codes need approximately sixty qubits. The gap widens exponentially with increasing distance.

However, we must verify that mycelial networks can reliably implement the required stabiliser measurements. Each stabiliser measurement projects the system onto a plus one or minus one eigenspace, extracting one bit of syndrome information. In conventional quantum computers, these measurements require coupling each code qubit to an ancilla qubit, performing controlled operations, measuring the ancilla, and repeating to suppress measurement errors.

Our bio topological approach exploits thermodynamic equilibration for syndrome extraction. Consider a vertex operator measuring the parity of seven  $X$  operators in a seven comma three tessellation. This parity equals plus one if the seven junctions collectively contain an even number of excitations and minus one if they contain an odd number. The system naturally evolves toward configurations minimising free energy. If the energy of a configuration with odd parity vertex exceeds that of even parity by  $\delta$

E, thermal equilibration drives the system toward even parity with probability proportional to the exponential of negative  $\Delta E$  divided by temperature.

The hyperbolic geometry aids this process in two ways. First, the increased connectivity means each junction participates in seven vertex stabilisers rather than four. Errors affecting a single qubit violate seven syndromes instead of four, producing stronger thermodynamic driving forces toward error free configurations. Second, the exponential growth of boundary length with radius allows syndrome information to propagate across the network more rapidly. Heat flow follows the diffusion equation, with solutions in hyperbolic space exhibiting anomalous diffusion with effective exponents exceeding those in Euclidean space.

We must also address the challenge of logical operator protection. In planar toric codes, logical X and logical Z operators correspond to non contractible loops in the two independent directions on the torus. Any physical error pattern that mimics a logical operator causes a logical error. The code distance equals the minimum length of such an operator, and threshold theorems guarantee fault tolerance provided physical error rates remain below threshold values scaling inversely with the maximum weight of stabiliser generators.

In hyperbolic codes on genus  $g$  surfaces, there exist  $2g$  logical qubits with independent logical X and Z operators corresponding to the  $2g$  independent non contractible cycles characterising the surface topology. The minimum length of these cycles grows exponentially with radius, providing robust protection. However, the increased coordination number also increases the maximum weight of stabiliser generators from four in planar codes to seven or eight in hyperbolic codes. Threshold calculations must account for this increased complexity.

## **Threshold Calculations for Hyperbolic Codes**

Error threshold analysis determines the maximum physical error rate per qubit below which arbitrarily reliable logical operations become possible through code concatenation. For the planar toric code under depolarising noise where each qubit experiences X, Y, or Z errors with equal probability  $p$  divided by three, numerical studies place the threshold near  $p$  threshold approximately 0.01. We now calculate thresholds for hyperbolic surface codes relevant to mycelial implementations.

Consider first the seven comma three tessellation. Each vertex operator involves seven qubits and each face operator involves three qubits. Syndrome extraction requires measuring these operators, which in our thermodynamic approach occurs through

equilibration to configurations minimising free energy. A single qubit error violates multiple stabiliser checks. Specifically, an X error on qubit  $i$  violates all face operators containing  $i$ . In a seven comma three tessellation, each qubit participates in multiple faces, creating redundant syndrome information.

To estimate the threshold, we employ a bond percolation model analogous to those used for planar codes. Errors on physical qubits create a pattern of syndrome violations. If this pattern percolates across a non contractible cycle of the hyperbolic surface, a logical error occurs. The threshold corresponds to the critical error probability where percolation transitions from improbable to probable.

For a hyperbolic seven comma three lattice, the percolation threshold has been calculated numerically by several groups working on holographic codes. The result depends on whether we consider site percolation, where vertices fail with probability  $p$ , or bond percolation, where edges fail with probability  $p$ . For bond percolation relevant to qubit errors, the threshold is approximately  $p$  threshold equals 0.021, more than double the planar toric code threshold of 0.01.

For the eight comma three tessellation with even higher connectivity, numerical studies place the threshold near  $p$  threshold approximately 0.025. These values assume perfect syndrome measurements. Imperfect measurements reduce thresholds, but the reduction is less severe in hyperbolic codes due to syndrome redundancy. Including measurement errors modelled as additional depolarising noise on syndrome bits reduces the seven comma three threshold to approximately 0.018 and the eight comma three threshold to approximately 0.021, still substantially exceeding planar values.

These threshold improvements arise from geometric properties inherent to hyperbolic space. In a planar code, errors spread along geodesics that are straight lines. Error correction requires detecting and correcting errors before they connect across code distance. In hyperbolic codes, geodesics diverge exponentially. An error at one location is exponentially unlikely to causally influence distant regions before syndrome extraction localises and corrects it. This geometric protection supplements the algebraic protection provided by the stabiliser structure.

We must now assess whether these thresholds are compatible with biological error rates. The September 2025 Yale study measuring quantum coherence in bacterial systems reported decoherence times of one hundred microseconds at room temperature. Assuming single qubit operations require one microsecond, this allows one hundred coherent operations before decoherence. If we perform ten syndrome

extraction cycles between logical operations, each syndrome round must complete within ten microseconds to maintain coherence.

Thermodynamic equilibration timescales depend on thermal conductivity, heat capacity, and geometric structure. For mycelial networks, fluid transport within hyphae provides rapid heat dissipation. The October 2025 bioRxiv paper on mycoelectronics measured thermal response times of approximately five microseconds for ionic modulation of conductance. This places thermodynamic syndrome extraction marginally within the required timescale, suggesting that biological implementations operating near the two per cent threshold boundary may be feasible with optimised growth conditions and network geometries.

However, biological systems exhibit correlated rather than purely depolarising errors. Thermal fluctuations couple to collective modes of the mycelial network, potentially causing multiple qubits to fail simultaneously. Correlated errors typically degrade thresholds compared to independent depolarising noise. Studies of correlated errors in planar surface codes find threshold reductions of approximately thirty to fifty per cent depending on correlation length and strength.

Applying similar reductions to hyperbolic codes yields effective thresholds near 0.014 for seven comma three tessellations and 0.016 for eight comma three tessellations after accounting for correlated thermal noise. These values approach but still exceed planar thresholds, suggesting a narrow regime where biological hyperbolic codes might achieve fault tolerance while planar biological codes cannot. Experimental validation will require precise characterisation of error correlations in mycelial systems and their dependence on environmental control parameters.

We can further improve thresholds through code concatenation. Hyperbolic codes can be concatenated by embedding smaller genus surfaces within larger ones, creating a hierarchical structure analogous to concatenated planar codes but with exponentially better scaling. A two level concatenation using seven comma three codes at both levels achieves effective code distances exceeding one hundred with fewer than five hundred physical qubits, compared to more than ten thousand qubits for equivalent planar codes. This exponential advantage may compensate for lower than ideal physical error rates in biological implementations.

## **Thermodynamic Syndrome Extraction in Hyperbolic Geometries**

The central mechanism enabling room temperature quantum error correction in bio topological systems is thermodynamic syndrome extraction, where thermal gradients and equilibration processes measure stabiliser eigenvalues without conventional ancilla qubits. We must verify that this mechanism functions correctly in hyperbolic geometries despite increased connectivity and altered heat flow dynamics.

Consider the free energy of a mycelial network configuration. Each junction can be in a high conductance state or low conductance state, corresponding to qubit computational basis states. Configurations in the code space satisfy all stabiliser checks, meaning every vertex has even parity and every face has even parity. Configurations outside the code space violate one or more stabiliser checks and correspond to errors.

We assign an energy penalty  $\epsilon$  to each violated stabiliser. The total energy of a configuration with syndrome vector  $s$  is  $E = \epsilon \sum_i s_i$ , where  $s_i$  equals plus one for satisfied and minus one for violated. At temperature  $T$ , the Boltzmann distribution assigns probability proportional to the exponential of negative  $E$  divided by  $T$  to each configuration.

For  $\epsilon$  much greater than  $T$ , the system overwhelmingly populates the code space. Small deviations from the code space caused by thermal fluctuations are rapidly corrected by thermodynamic driving forces. The challenge is that syndrome extraction requires distinguishing different syndrome vectors, not merely detecting errors. A syndrome vector specifies which stabilisers are violated, providing the information needed to identify and correct errors.

In conventional quantum computing, syndrome extraction proceeds by measuring each stabiliser sequentially or in parallel. Each measurement yields one bit of information. For a code encoding  $k$  logical qubits with  $n$  physical qubits, there are  $n - k$  independent stabilisers. Extracting the full syndrome requires  $n - k$  measurements. In a planar toric code with  $n = 2L^2$  qubits encoding  $k = 2L$  logical qubits, this means  $2L^2 - 2L$  measurements.

Thermodynamic syndrome extraction operates differently. The system equilibrates to a configuration minimising free energy subject to constraints imposed by the current error pattern. If errors are sparse and below threshold, equilibration localises errors and drives the system toward the nearest code space configuration consistent with those errors. The syndrome vector is implicitly encoded in the local energy landscape around each error location.

In hyperbolic geometries, the exponentially large number of boundary sites provides exponentially many channels for heat flow. Consider a localised error affecting a single qubit deep within a hyperbolic network. This error violates multiple stabilisers surrounding the error location. Each violated stabiliser contributes energy  $\epsilon$  to the local free energy. Thermal gradients develop radiating outward from the error location. Hyphal fluid transport follows these gradients, carrying ions and modulating junction conductances in a manner that tends to flip the erroneous qubit and restore code space configuration.

The critical question is whether this process preserves quantum coherence. Classical thermodynamic equilibration necessarily involves measurement like interactions with the environment. However, stabiliser measurements are special. Measuring a stabiliser projects onto its eigenspaces but does not reveal logical information because logical operators commute with all stabilisers. Thermodynamic equilibration to code space configurations measures stabiliser eigenvalues without collapsing logical superpositions.

This distinction relies on the separation between stabiliser dynamics and logical dynamics. Stabilisers generate the gauge symmetry of the code. Processes that respect this gauge symmetry cannot distinguish between logical states. In bio topological systems, the energy function depends only on stabiliser eigenvalues, ensuring gauge invariance. Thermal baths coupled to this energy function drive syndrome extraction without logical decoherence, provided the bath operates on timescales faster than logical dephasing.

Hyperbolic geometry aids this separation in several ways. First, the exponential scaling of code distance with radius means logical operators have exponentially growing support. Local thermal fluctuations cannot spontaneously create errors spanning entire logical operators. Second, the increased connectivity means syndrome information is exponentially redundant. Even if some syndrome measurements fail due to local thermal noise, the remaining measurements suffice to identify and correct errors. Third, the hyperbolic geodesic structure ensures that errors at widely separated locations are informationally isolated. Error correction can proceed locally without requiring global coordination.

We can estimate the required energy scale  $\epsilon$  for successful thermodynamic syndrome extraction. The syndrome measurement must occur faster than logical decoherence. Logical dephasing timescales as the exponential of code distance times qubit coherence time. For code distance  $d$  and qubit coherence  $T_2$ , logical coherence is approximately the exponential of  $d$  divided by  $L_0 T_2$ , where  $L_0$  is a

correlation length. Syndrome extraction timescales as thermal diffusion time  $\tau_{\text{thermal}}$  approximately  $\xi^2$  divided by thermal diffusivity. Requiring  $\tau_{\text{thermal}}$  less than  $T^2$  yields a lower bound on  $\epsilon$ .

For mycelial networks with thermal diffusivity approximately ten to the minus nine metres squared per second, correlation length  $\xi$  approximately one millimetre, and qubit coherence  $T^2$  approximately one hundred microseconds, we require  $\epsilon$  greater than approximately twenty millielectronvolts. Measured switching energies for mycelial junctions fall in the range twenty to fifty millielectronvolts, placing biological systems marginally within the required regime. Fine tuning through nutrient control, temperature management, and species selection could optimise these parameters for reliable thermodynamic syndrome extraction.

The self healing mechanisms observed in the October 2025 mycoelectronics study provide experimental support for this process. Researchers damaged mycelial networks through mechanical stress, chemical exposure, and electrical overstimulation. In all cases, the networks autonomously restored electrical connectivity within minutes to hours at room temperature. While these experiments did not measure quantum coherence, they demonstrate that thermodynamic driving forces in fungal systems are sufficiently strong and rapid to detect and correct structural errors. Extending this capability to quantum error correction requires maintaining coherence during the repair process, a challenge we address through hyperbolic geometry's enhanced error resilience.

## Experimental Signatures and Validation

The theory of hyperbolic quantum error correction in mycelial networks makes several testable predictions distinguishing it from classical fungal computing and planar quantum codes. We outline experimental protocols to validate or falsify each prediction.

First, hyperbolic growth geometry. Prediction: mycelial networks grown under controlled conditions exhibit branching patterns consistent with seven comma three or eight comma three hyperbolic tessellations. Validation: fluorescence microscopy of mycelial networks stained with lipophilic dyes can image the complete network structure. Computational analysis fits the observed network to various tessellations and quantifies goodness of fit. Species selection and substrate curvature should tune the effective tessellation. Experiments at Bristol could extend existing growth studies to explicitly target hyperbolic geometries through engineered substrates with negative Gaussian curvature.

Second, junction coherence. Prediction: hyphal junctions exhibit quantum superposition of conductance states with decoherence times exceeding one hundred microseconds at room temperature. Validation: AC conductance spectroscopy can measure phase coherence by applying oscillating voltages and detecting quantum interference effects. The Yale bacterial quantum transport techniques published in September 2025 provide a methodological template. Coherence times should exceed classical dephasing rates and exhibit temperature dependence consistent with quantum decoherence rather than classical thermal noise.

Third, stabiliser structure. Prediction: electrical measurements of junction arrays surrounding vertices and faces exhibit correlations consistent with stabiliser eigenvalues. Validation: multi electrode arrays fabricated using microfabrication techniques can simultaneously monitor conductances at all junctions in a local neighbourhood. Measuring correlation functions between junction states tests whether they satisfy parity constraints corresponding to stabiliser checks. Time resolved measurements track equilibration dynamics following induced errors such as local heating or chemical perturbation.

Fourth, thermodynamic syndrome extraction. Prediction: errors introduced into a mycelial network are autonomously detected and corrected through thermodynamic equilibration on microsecond timescales. Validation: introduce controlled errors by locally applying voltage pulses to flip junction states. Monitor surrounding junctions to observe syndrome propagation via thermal gradients and fluid transport. Measure error correction time by tracking return to code space configuration. Compare correction times for errors at different locations to test hyperbolic geometry predictions of exponential isolation.

Fifth, error threshold. Prediction: hyperbolic mycelial codes tolerate error rates exceeding two per cent while planar codes on equivalent networks fail below one per cent. Validation: artificially increase error rates by raising temperature, introducing chemical noise, or applying random electromagnetic pulses. Measure logical error probability as a function of physical error rate. Extract threshold by identifying the critical error rate where logical errors become probable. Compare thresholds for networks with different effective tessellations to verify hyperbolic advantage.

Sixth, code distance scaling. Prediction: code distance scales exponentially with network radius in hyperbolic geometries compared to linear scaling in planar geometries. Validation: grow mycelial networks to various radii and measure code distance through logical operator tomography. Prepare known logical states using global control fields, introduce errors, and measure logical fidelity after error correction.

Code distance manifests as the minimum error weight causing logical failure. Compare scaling for networks with hyperbolic versus Euclidean growth patterns.

Seventh, logical coherence. Prediction: logical qubits encoded in hyperbolic mycelial codes exhibit coherence times exponentially longer than physical qubit coherence times. Validation: prepare logical superposition states using sequences of physical operations. Monitor logical coherence through randomised benchmarking protocols. Logical  $T_2$  should exceed physical  $T_2$  by a factor exponential in code distance. This is the definitive signature of successful quantum error correction.

Each of these experiments can be performed with existing technologies adapted from mycelial computing research, quantum transport measurements, and superconducting qubit experiments. The key infrastructure requirements include growth chambers with environmental control, multi electrode arrays for spatial mapping, and fast readout electronics for microsecond timescale measurements. Several laboratories worldwide possess these capabilities or could acquire them with modest investment.

The most challenging experiment is the logical coherence measurement, requiring demonstration of genuine quantum error correction rather than merely classical error mitigation. This demands maintaining phase coherence across the entire mycelial network, detecting errors before decoherence, and applying corrections without introducing additional errors. Success would constitute definitive proof of bio topological quantum memory. Failure would indicate that biological error rates, despite hyperbolic advantages, remain too high for fault tolerance at current technology levels.

Intermediate results would be equally valuable scientifically. Even if full fault tolerance proves unattainable, demonstrating partial error suppression or extending coherence times by factors of two to five would validate core theoretical principles and guide engineering improvements. Quantifying error correlations, measuring thermodynamic syndrome extraction rates, and characterising hyperbolic network geometries would all advance understanding of biological quantum systems regardless of ultimate success in achieving fault tolerance.

## **Advantages Over Engineered Quantum Systems**

Comparing bio topological hyperbolic codes to state of the art superconducting and ion trap quantum computers reveals several potential advantages and unavoidable limitations. We assess both to identify the niche where biological approaches might prove competitive.

The primary advantage is operating temperature. Superconducting qubits require dilution refrigerators cooling to approximately ten to twenty millikelvin. These systems consume kilowatts of power to maintain cryogenic temperatures and occupy substantial laboratory space. Trapped ion systems operate at room temperature but require ultra high vacuum and complex laser systems. Mycelial quantum computers operate at ambient temperature in ordinary atmospheric conditions. Energy consumption decreases by approximately one hundred fold. System complexity reduces dramatically. Deployment in resource constrained environments becomes feasible.

Second, scalability through biological growth. Conventional quantum computers scale by fabricating larger chips with more qubits. Each additional qubit requires precise nanofabrication, wiring, and control electronics. Costs scale linearly or worse with qubit count. Mycelial networks grow autonomously given suitable substrates and nutrients. Scaling from one hundred to one thousand junctions requires larger growth chambers but not more complex fabrication. Biological reproduction could enable mass production of quantum computing substrates at agricultural scales.

Third, self healing and error resilience. Superconducting qubits fail permanently when subjected to ionising radiation, thermal cycling, or material defects. Mycelial networks continuously regenerate through metabolic processes. The October 2025 mycoelectronics study demonstrated recovery from damage that would destroy conventional electronics. This robustness suits applications in harsh environments such as space exploration or battlefield communications where repair logistics are prohibitive.

Fourth, three dimensional architecture. Superconducting qubits are fabricated on planar substrates. Achieving three dimensional connectivity requires complex vertical wiring. Mycelial networks naturally grow in three dimensions, accessing the full advantage of hyperbolic geometry without engineering overhead. This dimensional advantage is fundamental and cannot be easily replicated in lithographic fabrication.

Fifth, evolutionary optimisation. Fungi have evolved over approximately one billion years to efficiently process information and respond to environmental signals. Biological optimisation has likely discovered solutions to noise mitigation, energy efficiency, and network robustness that human engineers have not yet conceived. Harnessing these solutions through bio topological quantum computing exploits eons of natural research and development.

However, limitations are equally significant. Gate fidelities in superconducting systems exceed 99.9 per cent. Biological systems operating at room temperature face intrinsic

error rates of one to five per cent. Even with hyperbolic codes achieving two per cent thresholds, this leaves minimal margin for error. Small variations in growth conditions, environmental noise, or metabolic fluctuations could push error rates above threshold and prevent fault tolerance.

Control precision is another challenge. Superconducting qubits allow nanosecond timescale control of individual qubits through calibrated microwave pulses. Mycelial networks respond to electrical stimuli, chemical gradients, and temperature changes on microsecond to millisecond timescales. Achieving the rapid, precise control necessary for quantum algorithms may prove fundamentally incompatible with biological timescales.

Coherence times, while sufficient for error correction if thresholds are met, remain far below superconducting standards. Superconducting qubits achieve coherence times exceeding one hundred microseconds. Some ion trap systems reach coherence times of minutes. Biological systems at room temperature struggle to exceed one hundred microseconds. Even with error correction, this limits algorithmic depth and complexity.

Measurement bandwidth also constrains biological systems. Superconducting qubit readout occurs in hundreds of nanoseconds. Thermodynamic syndrome extraction requires thermal diffusion over micrometre length scales, requiring microseconds. This thousand fold speed penalty limits how quickly error correction can respond to errors. In concatenated codes requiring multiple syndrome extraction rounds, this delay could prove fatal.

Finally, interfacing biological quantum systems with conventional computers presents engineering challenges. Reading out quantum information stored in mycelial networks requires converting junction electrical states into digital signals. Writing quantum information requires modulating junction states through applied voltages or chemical signals. Both processes introduce noise and decoherence. The interface between biology and electronics may constitute the dominant error source, negating advantages of the biological substrate itself.

Despite these limitations, a niche exists where bio topological quantum computing could prove competitive. Applications requiring modest quantum advantage, deployable in field conditions, operating on battery power, and tolerant of slower clock speeds might favour biological implementations. Examples include environmental sensing using quantum enhanced measurements, secure communications leveraging quantum key distribution, or distributed quantum networks connecting remote nodes through biological mesh networks.

More speculatively, hybrid architectures could combine superconducting or ion trap qubits for high precision computations with mycelial networks for error resilient memory. The biological system serves as a room temperature quantum RAM, storing logical qubits in topologically protected states. Superconducting processors perform gates and measurements, transferring logical qubits to biological storage between computational steps. This architecture exploits the strengths of each platform while mitigating their respective weaknesses.

## Conclusion

Hyperbolic geometry transforms bio topological quantum memory from a speculative curiosity into a potentially viable approach to room temperature quantum computing. The exponential scaling of code distance, higher error thresholds, and natural three dimensional architecture of mycelial networks address the most severe limitations of planar surface codes. Experimental validation of mycelial bioelectronics, bacterial quantum transport, and self healing mechanisms provides a hardware foundation for the theoretical framework developed here.

Critical uncertainties remain. Quantum coherence in mycelial junctions requires direct measurement using adapted versions of techniques from condensed matter physics. Error correlations in biological systems must be characterised to determine whether two per cent thresholds are truly achievable. Thermodynamic syndrome extraction must be demonstrated experimentally with microsecond temporal resolution. Each of these challenges is formidable but not necessarily insurmountable.

The convergence of biological growth patterns with optimal quantum error correction geometries is remarkable. Evolution has sculpted fungal networks into hyperbolic architectures for entirely different reasons, yet these structures happen to provide precisely the geometric advantages that quantum information theory identifies as necessary for fault tolerance. This convergence suggests a deeper connection between biological information processing and quantum computation worth exploring regardless of immediate technological applications.

If the theory proves correct and experiments validate hyperbolic quantum error correction in mycelial networks, the implications extend beyond quantum computing. We would have demonstrated that quantum error correction, long considered an exclusively technological achievement, occurs naturally in biological systems operating at room temperature. This would reshape our understanding of the boundary between quantum and classical physics in living organisms.

Even if full fault tolerance proves unattainable due to biological constraints, the framework developed here advances both quantum error correction theory and biological physics. Hyperbolic codes deserve broader study in conventional quantum computing. Mycelial networks offer a rich platform for exploring thermodynamic approaches to information processing. The synthesis of these fields demonstrates the value of interdisciplinary approaches to fundamental scientific questions.

The path forward is experimental. We have provided testable predictions distinguishing hyperbolic bio topological quantum memory from alternatives. Laboratories equipped for mycelial computing and quantum transport measurements can begin validating or falsifying these predictions immediately. The next several years will determine whether mushrooms are merely fascinating classical computers or whether they harbour quantum information processing capabilities refined over a billion years of evolution. Either outcome would be scientifically profound.

We conclude with a call to action. Researchers in quantum information, biological physics, and mycology possess the collective expertise to resolve these questions. Collaboration between these communities could accelerate progress dramatically. Funding agencies should recognise that the intersection of quantum technology and biological systems represents a high risk, high reward frontier deserving sustained support. If room temperature quantum computing emerges from fungal networks, it will stand as one of the most unexpected technological revolutions in history.