Asset Pricing Theory

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1.1 Holding Period Return

The holding period return measures the total return earned on an investment over a specific period. It consists of two components:

1. **Income:** Dividends or interest received.
2. **Capital Gain/Loss:** The change in the price of the asset.

The general formula for Holding Period Return (HPR) is:

$$R = \frac{P_1 - P_0 + D_1}{P_0}$$

Where:

- P_0: Initial Price (Purchase Price)
- P_1: Price at the end of the period
- D_1: Dividend paid during the period

Example: You buy a stock for \$50 (P_0). After one year, the price is \$55 (P_1) and it paid a dividend of \$1 (D_1).

$$R = \frac{55 - 50 + 1}{50} = \frac{6}{50} = 0.12 \text{ or } 12\%$$

1.2 Expected Return and Risk

In reality, future prices (P_1) and dividends (D_1) are unknown. Therefore, the return is a **random variable** characterized by possible outcomes and their probabilities.

- **Expected Return ($E[R]$):** The probability-weighted average of all possible returns. It is used as the discount rate for present value calculations.
- **Risk:** The uncertainty of future returns (both good and bad outcomes). It is typically measured by the **Variance** (σ^2) or **Standard Deviation** (σ).

1.3 Comparison: Risk-Free vs. Risky Investment

Consider an investor with \$100 and two investment choices:

1.3.1 Investment 1: Risk-Free

Guaranteed payoff of \$105 after 1 year.

- **Return:** $\frac{105-100}{100} = 5\%$

- **Risk:** Since there is no uncertainty, Risk (σ) = 0.

1.3.2 Investment 2: Risky

Payoff depends on probabilities:

- 60% probability of \$150 (Return: $\frac{150-100}{100} = 50\%$)
- 40% probability of \$80 (Return: $\frac{80-100}{100} = -20\%$)

Calculating Expected Return:

$$E[R] = (0.6 \times 0.50) + (0.4 \times -0.20) = 0.30 - 0.08 = 0.22 \text{ or } 22\%$$

Calculating Risk (Variance and Standard Deviation): Variance is the weighted average of squared deviations from the expected return.

$$\sigma^2 = \sum P_i(R_i - E[R])^2$$

$$\sigma^2 = 0.6(0.50 - 0.22)^2 + 0.4(-0.20 - 0.22)^2$$

$$\sigma^2 = 0.6(0.28)^2 + 0.4(-0.42)^2 = 0.6(0.0784) + 0.4(0.1764)$$

$$\sigma^2 = 0.04704 + 0.07056 = 0.1176$$

$$\text{Standard Deviation } (\sigma) = \sqrt{0.1176} \approx 34.29\%$$

1.3.3 Conclusion

Investment 2 offers a higher expected return (22% vs 5%) but comes with significantly higher risk (34.29% vs 0%). Whether the additional 17% return is sufficient compensation for the risk depends on the investor's **utility** (attitude toward risk) and **asset pricing models**.